ΛΟΓΙΣΤΙΚΗΛΟΓΙΑ,

OR

ARITHMETICK

Surveighed and Reviewed:

In Four BOOKS.

VIZ.

1 *Book* { 1 *Part* Integers. 2 *Part* Fractions.

2 *Book* { 1 *Part* Geodæticals. 2 *Part* Figurals.

3 *Book* { 1 *Part* Decimals. 2 *Part* Astronomicals. 3 *Part* Logarithmes. 4 *Part* Cossicks. 5 *Part* Surds. 6 *Part* Species.

4 *Book* { 1 *Part* Ratios. 2 *Part* Proportions disjunct. 3 *Part* Proportions continued. 4 *Part* Æquations.

R. Potts
Trin. Coll. Cantab.

Wherein the Nature of Numbers absolutely abstract, generally and specially contract, with their Simple and Comparative Elements, are plainly declared, and fully handled.

Every Part furnished with such necessary Rules, Cases, Theoremes, Questions, Observations, and Varieties of Operation, as principally to them belong, and for the most part Illustrated by sundry Tables, Diagrams, and very many Examples: Together with divers Etymologies, Symboles, Characters, and Abbreviations for Artificial Termes, Words, Names, and Denominations, and all digested into so succinct, and orderly a Method, and delivered in so familiar a Style, as may befit Mean Capacities, and if practically applied, become more than ordinarily Useful both in Mechanical and Mathematical Arts and Sciences.

By SAMUEL JEAKE Senior.

Non partis Studiis agimur, &c.

LONDON,
Printed by *J. R.* and *J. D.* for *Walter Kettilby* at the *Bishops-Head* in St. *Paul's Church-Yard*, and *Richard Mount* on *Tower-Hill*, near the *Postern*, M DC XCVI.

5/16

To the Honourable Sir *ROBERT SOUTHWELL*, Knt. PRESIDENT OF THE ROYAL SOCIETY.

SIR,

ALthough it was the Author's Pleasure to prefix my Name to the Ensuing Treatise; which I am therefore necessitated to insert: lest I should be accused of altering that Work, the Impression whereof was recommended to me; and whereto I am obliged without any respect to Profit. Yet as I thereby received a Right to dispose of the Dedication: So I could not otherwise acquiesce, than in the Resignation of my Interest therein, to some greater Name;

eſteeming my own too minute to ſtand before it. And becauſe I am not under any particular Obligation to whom I ſhould preſent it: I have adventured to ſhroud it under your Honourable Patronage. Preſuming that where no Perſonal Advantages are expected; no imputation of Flattery can be charged: But that the Motives were alone that Merit and Generoſity which muſt always be venerated by

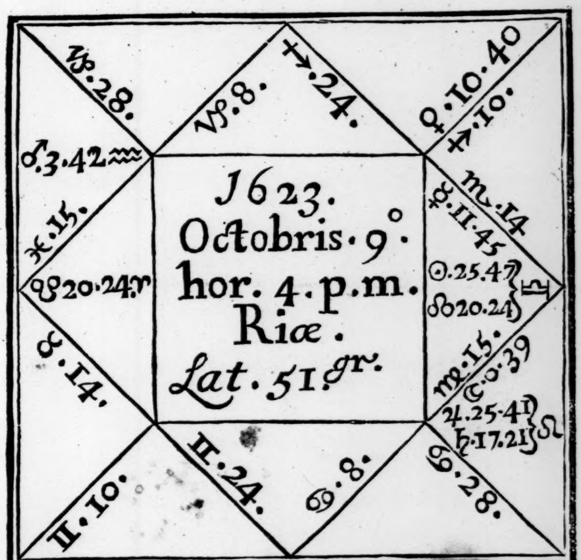
SIR,

Your moſt

Humble Servant,

S. JEAKE.

Defectum Iconismi Placuit supplere Themate Natalitio Authoris.



NO Planets here by Exaltation proud:
None by a Rest Supine, in House bestow'd.
But congruous Heav'n at this Birth dispos'd,
T'inspirit a clear Soul in Flesh enclos'd.
The mildest Dodecatemorie springs
In beauteous Orient: the encircling Rings
Of her Cœrulean Lord's Quaternion,
By Starry Regulus in Triumph shone.
That bright Superior's Domination fixt
In Heav'n's Culmen. Gen'rous Aspects mixt:
His Fiery Partil Trine to actuate
The Active House to a more Active Fate.
Nor was it vain: the happy Site of this
Æthereal Ruler of the Genesis,

A Judg-

A Judgment firmly form'd; whose Adjutant
Mnemonick pow'r, did by Cœlestial grant
Of Saturn's seminated Beams ensue,
In Platique Synod, with Proportion due.
As when the skilful Artist to compose
His mighty Theriaque; Weighs the Critick Dose
Of Theban Opium; which with Virtue full
Quickens that Brain, it's least Excess would dull.
The Wit's Dictator from the brighter Scale
Suits his harmonious Trine, whose Rays may fall
On th' Eastern Point: Whilst the Hesperian Face
Resplendent Venus doth the Ninth House grace.

S. J. Authoris Filius.

THE

THE
Author's Epistle.

TO
His Well-Beloved Son,
SAMUEL JEAKE.

My Son,

PEradventure the Reader (if ever the ensuing Piece be made Publick) may expect here, more than will be found; and yet find more than he expects: But sure I am, Thou wilt not find here all I could wish thee, nor yet all I intend thee. What thou findest, was mine before thine, and though thine is mine notwithstanding. The Gift thereof may inrich thee, but cannot impoverish me. And the surest way to make it thine is, to make it others too by Publication. By which though others, it will be nevertheless secured to thee, as being then uncapable of perishing in Private Papers. And because thy Right to inherit what is mine is indubitable, and thy Duty to defend what I leave thee, (though but a small Patrimony:) I have sought no other Patron, nor (seeing *Vino Vendibili non opus est hederâ*) do I want any, or shroud it under thy Patronage, thereby to gain the more respect or honour to my self or it, (the great cause of Dedications) if Art will not Patronize it, I am content to bear the accruing blame, whatever it be.

But seeing how good soever the Wine of the Book be (to make it the more Vendible) it is now grown Customary to hang out the Ivy-Bush of an Epistle as an Apology for the Author, or his Work, or both: Or lest it should be thought so useless or unprofitable,

that nothing can be ſaid to commend it or its Publication: I ſhall add a few things, and but a few on that Accompt.

Among the many Authors of Arithmetick that have come to my hands, I have ever obſerved that each pleaſeth himſelf with his own Method, as I my ſelf have done with mine. But this I muſt needs ſay, that I never met with any Single Piece but left me diſſatisfied in ſome or other parts of Arithmetick. Some handling only the Operations in *Whole Numbers* and *Extraction of Roots*, Others *Whole Numbers* and *Fractions*, Some all theſe with ſome *Rules in Proportions*, Others together with them have taught *Decimals*; Some have dealt only with *Logarithmes*, Others with *Coſsicks* and *Algebraical Notes*, *&c.* So as none I have yet ſeen gives a Compleat Accompt of ſome neceſſaries thereto. And beſides, the Accompt given by ſeveral, is ſo diſordered and imperfect, as the Art hath been but a little beholding to them for preſenting her to the World in ſo rural a Dreſs. Wherefore if the labour of a Complete Collection of the Cream of other Authors may be acceptable to any; or the Foundation of a Method large enough to bear all the parts of the Building whereon may be faſtned, and from whence may be drawn, the Reſolution of any Queſtion concerned in Arithmetick; this Piece may as well as others, that want both, be welcome to the Preſs, and crowd in for a place among the multitude of Books now Printed, wherein I hope 'twill neither ſhame the Author, nor be aſhamed of the Title of *Logiſticelogia*, ſeeing all the Concerns and Appurtenances of Arithmetick are therein diſcourſed of, and largely Surveyed and Reviewed.

Perhaps ſome may think, it is but to light a Candle to the Sun, ſince ſo many already have wrote on the Subject; as if *Nihil dictum quod non dictum prius.* To which I may plead with the Lawyers, *Non modo & forma*, and put the Iſſue on the Countrey to try.

True it is, moſt new Models are but the Light that ſometime ſhined in anothers Lamp, with an addition of freſh Oyl out of a new Veſſel, *Et facile eſt inventis addere.* But he that is ſenſible of the charge of buying, and trouble of turning over many Books to learn ſome one thing, will I doubt not excuſe my further plea herein, and plead for me; eſpecially if he knew that I ſpeak not without Experience, of no little time and trouble to glean ſo many Fields for one Griſt, having pickt up the knowledge of *Integers*, *Fractions*, *Figurals*, *Coſſicks*, and *Surdes* principally from *Record*, *Decimals* from *Johnſon*, *Aſtronomicals* from *Blundevile*, *Logarithmes* from *Briggs*, *Species* and *Æquations* from *Oughtred*, with a conference of many others. It follows therefore that each may have his due, what is here may be accompted anothers, yet is it all my own, and ſome things therein ſo far my own, as will be found in none extant that I know of. And becauſe this

may

may prove Beneficial to ſome, unleſs ſuch prove remiſs to themſelves in the peruſal; it may ſerve for a ſecond incouragement to the Publication.

Beſides ſome Pieces are wrote in *Latine*, and of thoſe in *Engliſh*, ſome too ſhort in their Rules, or too long or dark in their Examples, or in the Reaſon and Ground thereof, and worſe than that have Examples inſtead of Rules, or Rules which will not hold generally, nor anſwer to ſeveral Caſes, ſo as ſetting aſide all the Errors of the Preſs, (a great Miſchief to a Learner of theſe things by Book, where the Sence is not guided by the Antecedent or Subſequent Matter) all the Learner reapes after the Expence of his Time, Coſt, and Travel, expecting with the Mower to fill his Lap; is but a handful of the Graſs of the Houſe-top, to wit, the Reſolution of ſome few Queſtions. And therefore the ſpreading this Table with ſuch Varieties of Rules for almoſt all Caſes, and fitting Examples to them (and not the Rules to the Examples) and over and above the Explanation of both in very many places, may I ſuppoſe paſs for a further full and ſufficient plea for it's *Imprimatur*.

As to the Work it ſelf, the occaſion of it's firſt penning was to help an Imperfect Memory, not once then thinking it ſhould ever have ſeen the Sun. Moſt of whoſe rough drawn, and unpoliſht Papers have layn by me above Twenty Years, in which time there have not wanted the often preſſures of Friends for a Tranſcription, which to them yet on this ſide the Grave will I know be grateful. If others undeſervedly ſlight it, it will but give occaſion for the deſerved ſlighting of their own Opinions, and not at all hurt me or it. *Alij quidem neſciunt, neque quidem curant ſcire, & quia neſciunt nolunt ſcire. Nihil enim deſiderabile eſt dum ignotum, nec amatum, niſi cognitum.*

Neverthleſs to take away occaſion from ſuch as ſeek occaſion, (it being too common for ſome to ſeek in Books for advantages againſt their Authors;) and to obviate ſeeming Objections, as alſo that all cauſeleſs Scruples and Calumnies intended to blurr both Author and Work may be wiped off (if poſſible) and utterly to raſe the foundation thereof, and abſterge ſuch rubbiſh, I ſhall add,

1. Wherein I diſſent from others in the Method, placing Continued Proportions after disjunct, and both after Figural Numbers, it is ſufficient there is a neceſſity for it, becauſe without the knowledge of Figural Numbers, and Extraction of their Roots, *Progreſſion* and ſeveral of the Dependants thereof cannot be Learned, nor Doubled nor Tripled *Diſjunct Proportions* wrought, as the new-born *Scioliſt* will eaſily ſee: And whether it be not prepoſterous to teach the more hard and Sublime parts of any Science, before the Introductory; let any *Tyro* judge.

2*dly*, *Fractions*

2*ly.* *Fractions* upon the ſame account of neceſſity are here made to precede them all; for otherwiſe it can never be underſtood, how to Square or Cube, *&c.* a *Fraction*, nor how to value the Remain upon any Diviſion in the *Golden Rule*, and other *Proportional Operations*; if it be not known, what a *Fraction* is, and how to work therewith.

3*dly.* For the ſame reaſon, *Æquations* are ſet after *Surdes*, and *Surdes* before *Species*, ſeeing as in *Species* there happen *Surdes*; ſo both *Surdes* and *Species* ariſe in *Æquations*. But if no ſuch neceſſity were for theſe diſagreements; yet the ſymmetry and ſuitable agreement of the ſeveral parts with the whole in the Method purſued, and their concatenate concurrence or dependance one with, or upon another, with the reaſons here and there in places apt for the purpoſe rendred, may put a ſtop to any further diſpute concerning the ſame.

4*ly.* Neither may the placing of *Reduction in Geodæticals* before *Addition*, &c. be any Stone of Stumbling, conſidering *Geodæticals* are now acknowledged to be *Contract Numbers*, as limited to *Denominations*. And in *Contract Numbers* it is no ſtrange thing to ſee *Reduction* an Ortive part of *Numeration* put before the Original. And how helpful *Geodætical Reduction* is to other the Prime parts of *Geodætical Numeration*, nothing can ſo well diſcover as the Survey it ſelf, where it will eaſily be perceived without that Order, eſpecially *Multiplication* and *Diviſion* of thoſe Numbers if placed before, muſt have been imperfect and diſorderly.

5*ly.* If ſome of the Characters in *Coſſicks* and *Surdes* anciently uſed are changed for others: I think it Apology enough to ſay, they are arbitrary, and if for expedition (the reaſon of their alteration) other diſtinct Notes may be found, the Practitioner is at liberty to change all the reſt.

6*ly.* Varieties of working a Queſtion, if objected as troubleſome, are added as a benefit, like multiplicity of words in a Language for the ſame thing; if one be not hit upon, the other may; and where the one is dark, the other helps to Illuſtrate, and each ſerve to prove the other.

7*ly.* Many of the Examples may be thought needleſs; but it may be remembred, that *Profunda luſtrare abſque exemplis arduum*; and oftentimes one Example is not enough to ſhew the ſufficiency of a Rule, wherefore ſeveral Examples are added, where the Rule is dark, or the Work difficult, and ſeveral of them explained, leſt I ſhould ſeem to walk in the Clouds, accompting it for a *Maxime*, That nothing is worth the Writing, which cannot be underſtood when wrote. And ſurely plurality of Examples of ſo general Approbation and Practice with others, cannot come under diſlike here, and if it be a fault, muſt needs be one pardonable.

8*ly.* When

8*ly*. When Termes are used promiscuously, as *Mixt Numbers*, for *Numbers* and *Fractions*, and sometimes for Numbers made up of Digits and Articles, or if *Denominator* and *Denomination* be used either for other, or *Ratio* and *Proportion*, or such like; Yet will the place where, and Matter about which they are so used, easily discover to the Observant Reader, which Term is properly intended without further help of an Expositor. And thus in the Common Elements of *Addition*, *Substraction*, *Multiplication* and *Division*, if any of them be mentioned when *Integers* are in hand, it shall be taken for *Addition*, *Substraction*, *Multiplication*, or *Division of Integers*: But when *Fractions* or other *Numbers* are in hand, then those *Elements* shall accordingly be understood to be used, and wrought after the manner of *Fractions*, &c. respectively, without annexing to every repetition words at length to direct it.

9*ly*. Where ever any Term or Phrase hath escaped Exposition, and may seem discrepant from the more common Road of acceptation in other Writings, yet will not this widen the difference, if such Termes or Phrases be proper to *Arithmetick*: For all things here are to be taken in congruity thereto. Wherefore a Perfect Number shall not be Chymically, but Arithmetically so; for with the *Chymists* 10 is a Perfect Number. *Chymical Collections*, *p*. 92. but seeing the Aliquot parts of 10 will make but 8, Arithmeticians count it Imperfect and Defective.

10*ly*. It's more than probable, That in the First Chapter of *Geodæticals* some of the Divisions of Foreign or Domestick Denominations may have lost, or in time may lose much of their Propriety to the present State of Affairs, or new Accompts of such things as there declared, they having been adapted to the Elder Laws, Customs, and Usages of Kingdoms and Countries, yet mutable and alterable as Reasons of State or other Contingencies in every Kingdom, or Countrey shall or may enforce. Wherefore (this Exception, if not provided against by sufficient Caution and warning thereof given in the Chapter it self) let allowances be made as occasion shall require, either by taking them in the *Præterperfect-Tense*, or making all not certainly known, but Suppositions, which nevertheless will not prejudice the truth of the Conclusions where such Suppositions are but Conditionals, since it is not necessary in the Resolution of a Question, that the Suppositions be true, but that the Conclusion be true according to such a Conditional Supposition. For if 120 be counted for an Hundred, then 2 Hundred must be 240, but it will not follow that the Hundred must alwaies be reckoned at that rate.

11*ly*. No Man may stumble at the use of the Word Infinite or endless, it being intended only beyond the Power or Skill of Man to count or cast up, not Infinite in a proper or abstract Sence, for so nothing is

Infinite but God, and it is impoſſible there ſhould be two Infinites.

And laſtly, if beſides all this, the Lawyers will favourably overlook the unintentional miſquoting or repreſenting any Statute or Law and Conſtruction thereof, (if any ſuch be,) And Merchants, Goldſmiths, and other Artificers will bear with the Termes in Queſtions of their particular Concerns, as poſſibly not ſo proper to their Profeſſions as others beſt known to themſelves, And Grammarians add their uſual Grains of Allowance (due on the Score of humane frailty) for miſplacing of Letters, miſſpelling of Syllables, miſpointing of Sentences, Omiſſion of Points, Parentheſes, *&c.* There cannot be much left to need excuſe; but if there be, it muſt now ſtand or fall to it's own Maſter.

Whilſt we all live in the Atmoſphære, no doubt but in a clear day ſome Motes may be ſeen; and conſidering the Imperfections of the Penman, diſcompoſures and diſadvantages under which, moſt was wrote or tranſcribed (enough to have diſtracted a more accurate Accomptant and abler Pen:) 'tis well, if there be no more faults, than the unprejudiced Reader in common Charity can or ought to remit or paſs by. And inaſmuch as none that may be found are wilful or intentional, I will not be ſo uncharitable as to think I ſhall want their Charity or candid Conſtruction thereof.

I never thought *Humanum eſt errare*, was, or ever will be a warrant or plea ſufficient to commit or excuſe the commiſſion of Wilfull Tranſgreſſions; but for involuntary Errors, Miſtakes, and Aberrations, it hath always been accompted ſufficient to cover them when committed.

To pleaſe or diſpleaſe, is an inconſiderable and unprofitable end in Writing, and they that aim at the former commonly miſs. It was ſaid of old, *Ne Jupiter quidem omnibus placet.* I write neither to praiſe or diſpraiſe, flatter or beſpatter any. Some who being dead yet ſpeak by their Writings I have here and there as worthy of Commendation valued, and am yet of Opinion this Nation ſtands much obliged among others to *Record* of old, and *Oughtred* of late, for their labours in this Art; the former for his plainneſs and clearneſs in thoſe things he hath handled, and the latter for his piercing apprehenſion into the more lofty and myſterious parts of the Mathematicks: But none dead or living have I any where wrongfully charged, or mentioned without due reverence to their Perſons and reſpect to their works. So far as I might not contract a guilt of leading or miſleading others into a dark unpleaſant and perplexed Path where one more plain was nearer hand. And many of their Examples (whether corruptly wrought or printed I cannot ſay) I have purpoſely choſen to correct the Errors found therein, that if this

Impreſſion eſcape the *Errata* will need no further amendment.

However whether their Lines or theſe be faulty, it is not ſo much material, if no blemiſh thereby be imputed to the Art it ſelf. *Arithmetick* is a Noble and high born Science, uſeful and profitable in ſeveral things both divine and humane, lends to many, but borrows of few, and hath midwifed into the World divers excellent Atchievements, which without her aid and aſſiſtance would have been Still-born, and ſlept an Everlaſting Sleep. And becauſe a fit Encomium thereof deſerves a moſt clear and unclouded capacity I ſhall deſiſt. Nor ſhall I undertake to exhibit a Narrative of the riſe, growth, production or propagation of *Arithmetick*, nor yet to catalogize ſo much as the *Engliſh* Authors thereof; all I have further to ſay is

Whoever would profit hereby, muſt begin at the beginning, and go through every part gradually, but (if like moſt Scholars a little will content) the Two Firſt Books, and the Second Part of the Fourth, will furniſh the Learner for Merchants Affairs Trade and ordinary Commerce in a good meaſure; the other is written *Non cuivis*, but for thoſe who ſhall not think their labour loſt in ſuch Speculations, to whom though *Studiorum radices amaræ*, yet *fructus erunt ſapidi.*

Thus (my Son) as in ſome Projections of *Geometry* and *Aſtronomy* my thoughts have out-done me, ſo in this of *Arithmetick* I have out-done my thoughts in the length both of the Epiſtle and Book; to ſhorten which I have forborne to exemplifie the Prints of the ſeveral Coines mentioned in the Firſt Chapter of *Geodæticals*, and to purſue my intentions as well about the Addition and Subſtraction of *Surdes*, eſpecially of Univerſals, as the Diviſion of *Surdes* and *Species*, wherein ſomethings might not have been unprofitably added, as likewiſe in *Progreſſion* of both kinds by Inſtances in *Fractions* and *Decimals*: Nevertheleſs what is written may be ſufficient to the ingenuous, and therefore I ſhall ſay no more but conclude, The Lord bleſs thee and it, and make both inſtrumental to his glory, which is and will be the hearty and earneſt Prayer of

Rye, March 25th.
1674.

Thy Loving Father

SA. JEAKE.

Books principally consulted with in the ensuing **Treatise**, besides the Sacred Scriptures, divers Histories, Lexicons, Dictionaries, *&c.*

Authors.		Titles.	Impressions.	
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Richard	*Balam.*	Algebra.	*Lond.*	1653.
Isaac	*Barrow.*	*Euclide's* Elements.	*Lond.*	1660.
Thomas	*Blundevile.*	Exercises.		
Henry	*Brigges.*	Logarithmica Arithmetica.	*Lond.*	1624.
		Logarithmical Arithmetick.	*Lond.*	1631.
Michael	*Dalton.*	The Countrey Justice.	*Lond.*	1643.
Michael	*Dary.*	*Dary's* Diary.	*Lond.*	1650.
John	*Dee.*	Mathematical Preface, *&c.*	*Lond.*	1651.
Richard	*Delamain.*	Grammelogia.	*Lond.*	1630.
Thomas	*Digges.*	Pantometria.	*Lond.*	1591.
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William	*Eldred.*	The Gunners Glass.	*Lond.*	1646.
Henry Van	*Etten.*	Mathematical Recreations.	*Lond.*	1633.
Giles	*Fletcher.*	History of *Russia.*	*Lond.*	1657.
Thomas	*Fuller.*	Pisgah Sight of *Palestine.*	*Lond.*	1650.
Edward	*Grimstone.*	Estates *&c.* of the World.	*Lond.*	1615.
Edmund	*Gunter.*	Works, *&c.* of the *Sector*, *&c.*	*Lond.*	1653.
Peter	*Heylin.*	Cosmography.	*Lond.*	1657.
Nicolas	*Hunt.*	Handmaid to Arithmetick.	*Lond.*	1633.
John	*Johnson.*	*Johnson's* Arithmetick.	*Lond.*	1657.
Gerard	*Malines.*	Lex Mercatoria.	*Lond.*	1636.
Jonas	*Moore.*	*Moore's* Arithmetick.	*Lond.*	1650.
John	*Nepair.*	Rabdologia.	*Lugd.*	1626.
William	*Oughtred.*	Clavis Mathematicæ limata.	*Oxon.*	1652.
		Circles of Proportion.	*Oxford*	1660.
Seth	*Partridge.*	Rabdologia.	*Lond.*	1648.
Fardinando	*Pulton.*	Collection of Statutes.	*Lond.*	1640.
Peter	*Ramus.*	Way to Geometry.	*Lond.*	1636.
William	*Rastal.*	Collection of Statutes.	*Lond.*	1572.
Robert	*Record.*	Ground of Arts.	*Lond.*	1636.
		Whetstone of Wit.	*Lond.*	1557.
John	*Speidel.*	Treatise of Sphærical Triangles.	*Lond.*	1627.
Christian	*Urstitius.*	Elementa Arithmeticæ.	*Basil*	1602.
William	*Webster.*	The Principles of Arithmetick.	*Lond.*	1634.
Vincent	*Wing.*	Harmonicon Cœleste.	*Lond.*	1651.
		Astronomia Instaurata.	*Lond.*	1656.
Richard	*Wit.*	Arithmetical Questions.	*Lond.*	1613.
Edward	*Wright.*	Description, *&c.* of Logarithmes	*Lond.*	1618.
		The Pathway to knowledge.	*Lond.*	1596.

&c.

The

The Contents of the Chapters.

The First Book.

The Second Book.

The Third Book.

The Contents of the Chapters.

A

Chap. 15.

The Fourth Book.

An Appendix.

ARITHMETICK.

The First BOOK,

CONCERNING

Numbers absolutely abstract;

In Two PARTS.

WHEREIN

INTEGERS } are { Examined,
FRACTIONS } are { Explained,

AND THEIR

SIMPLE ELEMENTS.

CHAP. I.

A Brief Introduction to the Parts of Arithmetick *hereafter handled.*

THE Noble Science of *Arithmetick* is the very Foundation of the *Mathematicks*, whose *Roots* are *Number* and *Magnitude*. The Original of the one an *Unit*, of the other a *Point*; but the *Products* of either are Infinite. The *Mathematical Science* specifically appropriate to *Number* is *Arithmetick*; to *Magnitude*, *Geometry*. From these as Two Principal Springs Mathematical considered simply or *per se*, and mixt or *inter se* are derived many other Excellent Arts and Mysteries; useful not only in Natural, Substantial, Accidental, Visible, and Invisible things, but also in Supernatural and Divine, as the Learned *John Dee* affirmeth in his *Mathematical Table* inserted in *John-Henry Alsted* his *Encyclopœdia*, *lib.* 2. *p.* 70. which I have thought fit here to Translate.

Arithmetick *what the ground of.* *The Original an* Unit.

Dr. Dee *his Mathematical Table.*

Sciences and Arts Mathematical are

- Principal
  - *Arithmetick.*
    - *Simple,* Which only treateth of *Numbers*, and demonstrates their Properties. Here Unity is Indivisible.
    - *Mixt,* Which by the help of *Principal Geometry* demonstrateth certain Arithmetical Conclusions.
  - *Geometry.*
    - *Simple,* Which treateth only of *Magnitudes*, and demonstrateth all the Properties and Passions of them. Here a Point is Indivisible.
    - *Mixt,* Which by the help of *Principal Arithmetick* demonstrateth certain Geometrical things. as *Euclides Elements.*
  - The use of these Sciences is either in things
    - Supernatural, Eternal and Divine by Application *Ascending.*
    - Mathematical without any other Application.
    - Natural, Substantial, and Accidental Visible, and Invisible, by Application *Descending.*
    - The like Uses (only inferiour in one degree) are in the Mathematical Derivative Arts
- or Derivative some of which
  - Retain the Names of the Principal, as
    - Common Arithmetick, which considereth
      - Arithmetick of Whole Numbers and their Fractions.
      - Arithmetick of Proportion.
      - Circular Arithmetick.
      - Arithmetick of Rooted Numbers and their Fractions.
      - Arithmetick of Cossick Numbers and their Fractions.
    - Common Geometrry, which teacheth to measure a thing.
      - Not distant hence
        - *Mecometria.*
        - *Embadometria.*
        - *Stereometria.*
      - Distant hence
        - *Apomecometria.*
        - *Hyposometria.*
        - *Platometria.*
        - from these arise these Arts, *viz.*
          - *Geodesia.*
          - *Geographia.*
          - *Chorographia.*
          - *Hydrographia.*
          - *Stratarithmetria*
  - Have peculiar Names, As
    - *Perspectiva.*
    - *Astronomia.*
    - *Musica.*
    - *Cosmographia.*
    - *Astrologia.*
    - *Statica.*
    - *Anthropographia.*
    - *Trochilica.*
    - *Helicosophia.*
    - *Pneumatithmia.*
    - *Menedria.*
    - *Hypodeiodia.*
    - *Hydragogia.*
    - *Horometria.*
    - *Zographia.*
    - *Architectura.*
    - *Navigatio.*
    - *Thaumaturgica.*
    - *Archimagisterica.*

Number *helpful to search out the Creature Vertues, Natures,* &c.

Number *of Universal Use.*

And the same Doctor *Dee* in his Mathematical Preface to the first Six Books of *Euclides* Elements further assures us, That *Number* seems to be so Immaterial and Pure, that thereby we may wind our selves into the deep Search and View of all Creatures distinct Vertues, Natures, Properties, Forms, *&c.* The Universal Use of *Number* is Witnessed to also, by the Noble *Picus*, Earl of *Mirandula*, who had set up in *Rome* Nine Hundred Conclusions in all kinds of *Sciences* openly to be disputed of; and in the Eleventh Conclusion saith, By *Numbers*, a way is had to the Searching out, and Understanding of every thing able to be known.

Without Numbers, *no Knowledge of the Mathematicks.*

Boëtius and others are not far behind them in Commending *Number*; but had it none of their *Encomiums*, yet certain it is, That whosoever would fit himself for the *Mathematicks*, unless he begin with the Science of *Numbers*, will quickly find himself in a Labyrinth, from which he can never escape, nor deliver himself from many Inextricable Doubts, without the Assistance thereof.

Threefold Consideration of Number.

Great *Mathematicians* and *Philosophers* have considered *Number*: First, In Respect of the Creator, Simple, Pure, and Immixt. Secondly, In Reference to Spiritual and Angelical Minds, Including the Soul of Man: And, Thirdly, In relation to every Creature, and their compleat Constitution: And in the First and Second respect, term it *Number* numbring; but in the Third *Number*, numbred. In the sence of the Two former I shall not intermeddle to treat of *Numbers*, referring it to a more Able and Divine Pen. But in the latter respect, and that in the lowest and most gross consideration of *Numbers*, *Viz.* In Conference and Coincidence to Visible, Material, and Corporeal things, I have shadowed out this Science of *Numbers* in the Ensuing Treatise, under the Name of *Arithmetick Surveyed and Reviewed*: Containing Four Books. The First

Here, under the Third Consideration to be Viewed in Four Books.

1. Treating of *General Arithmetick*, Examining the *Simple Elements* of *Abstract Numbers*, in Two Parts, *viz. Integers* in the First Part, and *Fractions* in the other Part; although this latter after a Sort may be called Contract, in respect to their *Denominators.*
2. The Second and Third Books handle *Special Arithmetick*, Explaining the *Simple Elements*

ments of *Contract Numbers*, more generally as of *Geodeticals* and *Figurals*, in the Second Book, in Two Parts: Those more Specially Contract, are dealt with in the Third 3.
Book, in Six Parts; *viz. Decimals*, *Astronomicals*, *Logarithmes*, *Cossicks*, *Surdes*, and
Species, in each Part distinctly. In the Fourth Book is taught the Whole Doctrine of 4.
Proportions, *Disjunct*, *Continual*, and *Æquated*. And in one or other of these, (especially as to the Practick Part of *Arithmetick*) is included what Others otherwise have divided and delivered concerning *Numbers*, under that Name of *Arithmetick*.

Arithmetick derives its *English* Name from the Latine *Arithmetica*; and this again from the *Greek* 'Αριθμὸς, signifying as *Keckerman* saith, *Phys. Lib.* 1. both to Number and Measure; and so may truely include *Arithmetick* mixt with *Geometry*, as before hinted; but with us is generally taken for the Art of Numbring Restrictively, intending only its Essential Consistence in *Numbers*, and but Collateral Converse with Magnitudes: In which Notion I am contented likewise to take it, reserving not only a Liberty to understand it in a higher Note, when Occasion serves; but also to accept it under the Vulgar Terms of *Casting Accompt* and *Cyphering*, till they that so Abase this high born *Science* shall see it worthy of a more Excellent Name than those.

Arithmetick, *whence the Name.*

Is the Science of Numbring generally taken.

Cyphering *too vulgar a Name.*

The Antient Hebrews *and* Greeks *Numbred by their Letters; as Ensueth in these Tables.*

Hebrew Account.

Numeral Let. of the Hebrews

Sec. Masorethas. ך	ר	Two Hundred.	ל	Thirty.	יא	Eleven.	א	One.
ם	ש	Three Hundred.	מ	Forty.	יב	Twelve.	ב	Two.
ן	ת	Four Hundred.	נ	Fifty.	יג	Thirteeen.	ג	Three.
ף	תק	Five Hundred.	ס	Sixty.	יד	Foutteen.	ד	Four.
ץ	תר	Six Hundred.	ע	Seventy.	טו	Fifteen.	ה	Five.
	תש	Seven Hundred.	פ	Eighty.	יו	Sixteen.	ו	Six.
	תת	Eight Hundred.	צ	Ninety.	יז	Seventeen.	ז	Seven.
	תתק	Nine Hundred.	ק	One Hundred.	יח	Eighteen.	ח	Eight.
	א̈	One Thousand.			יט	Nineteen.	ט	Nine.
	&c.				כ	Twenty.	י	Ten.

Greek Account.

Numeral Let. of the Greeks.

I α'	One.	ΔΠII. ιζ'.	Seventeen.	𐅅 H. χ'.	Six Hundred.	MMMM. μ.	Forty Thousand.
II. β'.	Two.	ΔΠIII ιη'.	Eighteen.	𐅅 HH. ψ'.	Seven Hundred.	IMI. ,ν.	Fifty Thousand.
III γ'.	Three.	ΔΠIIII. ιθ'.	Nineteen.	𐅅 HHH. ω'.	Eight Hundred.	IMIM. ,ξ.	Sixty Thousand.
IIII δ'.	Four.	ΔΔ. κ'.	Twenty.	𐅅 HHHH. ϡ'.	Nine Hundred.	IMIMM. ,ο.	Seventy Thousand
Π. ε'.	Five.	ΔΔΔ. λ'.	Thirty.	X. ,α.	One Thousand.	IMIMMM. ,π.	Eighty Thousand
ΠI. ϛ'	Six.	ΔΔΔΔ. μ'.	Forty.	XX. ,β.	Two Thousand.	IMIMMMM. ,ϛ.	Ninety Thousand.
ΠII ζ'	Seven.	𐅄. ν'.	Fifty.	XXX. ,γ.	Three Thousand.	,ρ.	One Hundred Thousand.
ΠIII. η'	Eight.	𐅄 Δ. ξ'.	Sixty.	XXXX ,δ.	Four Thousand.	,σ.	Two Hundred Thousand.
ΠIIII. θ'.	Nine.	𐅄 ΔΔ. ο'.	Seventy.	𐅆. ,ε.	Five Thousand.	,τ.	Three Hundred Thousand.
Δ. ι'.	Ten.	𐅄 ΔΔΔ. π'.	Eighty.	𐅆X. ,ϛ.	Six Thousand.	,υ	Four Hundred Thousand.
ΔI. ια'	Eleven.	𐅄 ΔΔΔΔ ϛ.	Ninety.	𐅆XX. ,ζ.	Seven Thousand.	,φ.	Five Hundred Thousand.
ΔII. ιβ'.	Twelve.	H. ρ'.	One Hundred.	𐅆XXX. ,η.	Eight Thousand.	,χ.	Six Hundred Thousand.
ΔIII. ιγ'	Thirteen.	HH. σ'.	Two Hundred.	𐅆XXXX. ,θ.	Nine Thousand.	,ψ.	Seven Hundred Thousand.
ΔIIII. ιδ'.	Fourteen.	HHH. τ'.	Three Hundred.	M ,ι.	Ten Thousand.	,ω.	Eight Hundred Thousand.
ΔΠ. ιε'	Fifteen.	HHHH. υ'.	Four Hundred.	MM ,κ.	Twenty Thousand.	,ϡ.	Nine Hundred Thousand.
ΔΠI. ιϛ'.	Sixteen.	𐅅. φ'.	Five Hundred.	MMM ,λ.	Thirty Thousand.		&c.

The Latines *made use only of Seven of their Letters*: Viz. C. D. I. L. M. V. X.

Latine Account.

Numeral Let. of the Latines.

I.	One.	XII.	Twelve.	L.	Fifty.	DCC.	Seven Hundred.
II.	Two.	XIII.	Thirteen.	LX.	Sixty.	DCCC.	Eight Hundred.
III.	Three.	XIIII. XIV.	Fourteen.	LXX.	Seventy.	DCCCC.	Nine Hundred.
IIII. IV.	Four.	XV.	Fifteen.	LXXX.	Eighty.	M. CIↃ. ∞.	One Thousand.
V.	Five.	XVI.	Sixteen.	XC.	Ninety.	V̄. V∞. IↃↃ.	Five Thousand.
VI.	Six.	XVII.	Seventeen.	C.	One Hundred.	X̄. CCIↃↃ. ↃMↃ.	Ten Thousand.
VII.	Seven.	XXII. XIIX.	Eighteen.	CC. ↅ.	Two Hundred.	L̄. L∞. IↃↃↃ.	Fifty Thousand.
VIII.	Eight.	XIX.	Nineteen.	CCC.	Three Hundred.	C̄. CM. C∞. CCCIↃↃↃ.	One hundred Thousand.
VIIII. IX.	Nine.	XX.	Twenty.	CCCC.	Four Hundred.	D̄. IC. D∞. IↃↃↃↃ.	Five hundred Thousand.
X.	Ten.	XXX.	Thirty.	D. IↃ.	Five Hundred.	CCCCIↃↃↃↃ.	One Million.
XI.	Eleven.	XL.	Forty.	DC.	Six Hundred.	&c.	

The

Numeral Let. of the English.

The *English* following the *Latine* made up their Antient Accompts by the same Seven Letters: Accompting I. one. V. five. X. ten. L. fifty. C. a hundred. D. five hundred. and M. a Thousand, *&c.*

Arithmetick *best performed by Figures.*

The Art of *Numbers* being best performed by the Pen with the *Arabick* Notes, or Characters, commonly called Figures. I shall altogether wave the Practise thereof by Counters, and Letters, and all other Instrumental Arithmetick (save only a little touch of *Nepairs* Bones) and remit those desirous to learn such kind of Operations to the several Treatises of the respective Authours who have dealt therewith.

CHAP. II.

Of the Nature of Numbers.

What Arithmetick *is.* — *The Subject thereof.* — *Number taken restrictively.* — *How an* Unit *is a Number.* — *Number taken largely.* — *Parts of an* Unite *how Numbers.*

IN pursuance of what hath been said, and hereafter followeth *Arithmetick* may be defined, The *Science* of *Numbring* well. In *Arithmetick* two things are principally to be considered; The Subject, in which the Nature of *Numbers* is declared: And the Affection of the Subject, wherein the Elements of *Numbers* are exercised. The Subject of *Arithmetick* is *Number*, all the Precepts and Doctrines of the whole Art, having a particular Relation thereunto. *Number* is first taken restrictively, and so *Number* is considered only according to it's multitude or capacity of Units; hence *Euclide, lib.* 7. *def.* 2. will have *Number* to be a multitude of Units. An Unite is indeed the Original of *Number*, but being in the Prædicament of Quantity, and *in potentia*, may well pass for a *Number*. And *Euclide* himself in the 18, 19, and 20. Definitions of the same Book, cannot be excused from accompting an Unite for a *Number*. Secondly, *Number* is taken more largely for the Quantity according to which any thing is Numbred, or Accompted; So according to an Unite any singular thing is said to be One; as one Man, one House, and such like; and two things, as Stars, Ships, Men, *&c.* are said to be two, according to that *Number*: And in this Metaphysical Sense, not only the Unit it self, but the parts of an Unite, as one half of a Day, two third parts of a Pound, *&c.* are reckoned for *Numbers*.

Numbers therefore largely taken, may be better discern'd in their proper Natures, as they are capable of being Divided: A brief *Synopsis* whereof followeth.

A Synopsis *of the Nature of Numbers.*

- *Nature of Numbers.*
  - Homogeneal
    - Proper
      - Essential
        - Whole
          - Unity or Multitude.
          - Digit, Article, or Mixt.
          - Prime, or Compound.
          - Even, or Odd.
          - Perfect, or Imperfect.
        - Broken
          - Part.
          - Parts
            - Commensurable.
            - Incommensurable.
      - Accidental
        - Abstract
          - More Absolute. *Integers.*
          - Lesser Absolute *Fractions.*
        - Contract
          - More General.
            - *Geodæticals.*
            - *Figurals.*
          - More Special.
            - *Decimals.*
            - *Astronomicals.*
            - *Logarithmes.*
            - *Cossicks.*
            - *Surdes.*
            - *Species.*
    - Common
      - *Ratios.*
      - *Proportions.*
        - *Discontinual.*
        - *Continual.*
      - *Æquations.*
  - Heterogeneal
    - Primary. *Integers,* and *Fractions.*
    - Secondary. *Abstract,* and *Contract.*

For

For the Explanation whereof obſerve *Numbers* according to their Nature have ſome peculiar properties to themſelves, and ſomething common with others; and are of two ſorts, *Homogeneal*, and *Heterogeneal*; *Homogeneal* are of the ſame Nature or Kind, whether Proper, or Common. Eſſential, or Accidental. Whole, or Broken, Abſtract, or Contract: *Numbers Heterogeneal* are mixt *Numbers* of Whole and Broken, Abſtract and Contract.

Numbers Homogeneal *are proper and common.* *Proper* Eſſential, *or* Accidental.

Whole Numbers are called *Integers* from the *Latine*, *Integrum*, and *Broken Numbers*, *Fractions*, from the *Latine*, *Fractio*.

Integers *and* Fractions, *whence the Names.*

A proper *Whole Number* according to his Eſſence, paſſeth under a five-fold Conſideration, Firſt, Whither it be Unite or Multitude. Secondly, Digit, Article, or Mixt, Thirdly, Prime, or Compound. Fourthly, Even, or Odd: And Fifthly, Perfect, or Imperfect.

Integers *conſidered five wayes* Eſſentially.

An Unite is the beginning of *Multitude*, and foundation of *Number*; never more than one. *Multitude* is the Collection of Unites, as one, one, one, are equal to three, being Collected together, *&c.*

1. *One or Many.*

Secondly, *Integers* are again divided into Digits, Articles, and Mixt Numbers. A Digit is any *Integer* under Ten, as One, Two, Three, Four, Five, Six, Seven, Eight, Nine; Digits are ſometimes called *Monades*. An Article is Ten, with all Whole Numbers that may be juſtly divided into Ten parts, as Twenty, Thirty, Forty, *&c.* Theſe are called Round Numbers, and ſometimes *Decades*. A Mixt Number is compound of an Article, and a Digit, as Eleven, Twelve, *&c.* Twenty-one, Twenty-two, *&c.*

2. *Digits, Articles, and mixt.* *Digits called Monades.* *Articles called* Decades.

Thirdly, A Number, Prime, called alſo Simple, and ſometime Uncompound, is a Number made only by *Addition*, or Collection of Units, and not by *Multiplication*, ſo an Unit only can meaſure it; as Two, Three, Five, *&c.* Compound Numbers are ſuch as are made by *Multiplication* of two Numbers together, and not by *Addition*, though they may ſeem to be made of both: Yet becauſe they may be meaſured only by the Numbers of which they are compounded, they are not to be accounted Prime, as Six made by *Multiplication* of Two and Three, is Compound, becauſe by either of them it may be meaſured, or divided; and though it may be made by *Addition* of Five and One, yet ſhall it not be Prime; for that neither Five nor One, can equally meaſure Six.

3. *Prime or Compound.*

Fourthly, *Numbers* are again conſidered as they are, Even or Odd; Even may be divided into Two Equal Parts, Odd cannot; Even *Numbers* are diſtinguiſhed into Three Sorts. Firſt, Even *Numbers* Evenly, theſe continually may be parted into halves, till you come to an Unite, as Sixteen, into Eight, Four, Two, One, *&c.* Secondly, Even *Numbers* Oddly, theſe may be parted into equal halves, but the halves will be odd *Numbers*, as Ten into Five, and Five, *&c.* Thirdly, Even *Numbers*, Evenly and Oddly, as thoſe which may a while be parted into even halves; but before you come to one, the halves will be odd *Numbers*, as Twelve into Six, Three, *&c.*

4. *Even or Odd. Even of Three ſorts.* 1. 2. 3.

Fifthly, *Integers* are differenced as they are Perfect or Imperfect. Perfect *Numbers* are ſuch whoſe aliquot or even parts joyned together will exactly return the whole *Number*, as Six, Twenty-Eight, *&c.* for of Six the half is Three, the Third part Two, and the Sixth part One, which added together, make Six; and it hath no more aliquot parts in whole *Numbers*, for the fourth part, is one and an half, and the fifth part, one and a fifth. So Twenty-Eight, whoſe parts being Fourteen, Seven, Four, Two, and One, exactly return Twenty-Eight, which therefore appears to be a perfect *Number*. Perfect *Numbers* are almoſt as rare as perfect Men; for between One, and One Million of Million there are but Ten; and the Twentieth perfect *Number* exceeds the value of Hundred Thouſand Million of Million of Million. *Mathemat. Recreations. p.* 92. Imperfect *Numbers* on the Contrary, are thoſe whoſe even parts added together, will not return the Primary *Number*, whoſe parts they be: And theſe are either *Abundant*, or *Defective*. *Abundant* called alſo ſuperfluous, whoſe parts added together make more than the whole *Number*; as Twelve, whoſe parts being one, two, three, four, and ſix, together make ſixteen; So the parts of Twenty make Twenty two, *&c.* *Defective* are ſuch, whoſe parts added together make leſs than the *Integer*; as Eight, whoſe parts being one, two, and four, make but ſeven; likewiſe the parts of Sixteen make but Fifteen, and of Forty Five make but Thirty Three.

5. *Perfect or Imperfect.* *Perfect Numbers very rare.* *Imperfect of two ſorts.* *Abundant.* *Defective.*

Fractions or *Broken Numbers* ariſe from the Diviſion of the Unite into parts, and therefore are properly parts of *Numbers*, becauſe every proper *Fraction* is leſs than an Unite; as when a Unite ſignifieth any Denominate Quantity, in reſpect to that Quantity, it may be divided into leſſer parts than that one Quantity; So one Pound parted into Four Equal parts, then ſhall one half, or three Quarters thereof, be leſs than one whole Pound, and theſe, and ſuch parts be called *Fractions* or *Broken Numbers*. Theſe are conſidered as they contain, either one part of the whole, or more parts; as one half; then

Fractions, *whence they ariſe.* *Doubly conſidered Eſſentially.*

then shall the whole be divided but into two parts, but three quarters denote the Unite to be parted into four parts, and the Fraction to contain three of them: These parts are either Commensurable, or Incommensurable.

The latter sort either Commensurable.

Commensurable, when the two Termes of the *Fraction* have any common part, that will equally divide them both; as Three being a part of Twelve and Fifteen will divide them both equally: therefore Twelve Fifteenths being the Termes of a Fraction, are parts Commensurable, and may be measured by Three, *&c.*

or,

Incommensurable.

Incommensurable on the contrary have no such parts for a common Divisor, as Eighteen Twenty Fifths, for Twenty Five can be equally divided by no Number but Five, and Five cannot divide Eighteen.

Homogeneal Numbers considered. *Accidentally as Abstract and Contract.*

The Nature of *Numbers Homogeneal* are further discerned in some properties accidental to them singly, or amongst themselves, and that is to be Abstract, or Contract: That called numbring, or formal; this numbred, or material.

Abstract more or less Absolute. More Integers.

Abstract are such as have no Denomination annexed to them; These are more or less Absolute: The more Absolute are *Integers*, as One, Two, Three, or any other whole Number, without any denomination, or sirname, relation, or comparison, *&c.* thereto belonging, but are free to value Men, Money, Years, or any other quantity. Of *Integers* and their Affections, *Vide plus lib.* 1. *par.* 1.

Less Fractions.

The less Absolute are *Fractions*, as one quarter, one half, two thirds, *&c.* though Absolute without respecting Denominations, and therefore left free to be parts of Weight, Measure, Time, or any other quantity, or thing whatsoever; yet must be considered in respect of the one number to the other. So that *Fractions* are not so Absolute as *Integers*, but in a sort, as I said before, may be accompted for a sort of Denominate Numbers, relation being had between the Numerator, and Denominator. Of these *Vide plus. lib.* 1. *par.* 2.

The same Numbers Abstract, *are again to be consider'd, as they may be* Contract.

Contract.

Contract, called also *Concrete Numbers*, proceed from Abstract, and are such as are refringed by the Annexion of some or other Denomination, as Two Groats, Ten Shillings, Two Thirds of a Pound, *&c.* where the Numbers are restrained from the liberty of valuing any thing else, but Groats, Shillings, *&c.* according to the annexed Denomination. Some *Contract Numbers* are contracted more generally, as *Geodæticals* and *Figurals*: Others more specially, in respect to the several sorts of Denominations, by which they are contracted.

Generally.

Geodæticals.

Numbers Geodætical, which are considered according to those Vulgar Names or Denominations, by which Money, Weights, Measures, *&c.* are generally known, or particularly divided by the Laws and Customs of several Nations; these spring from Denominate *Fractions*, and by omitting the Denominators, when the parts are commonly known, and annexing names Artificial or Inartificial, making some Mark or Sign to distinguish them, they pass into the Catalogue of vulgar Denominate Integral *Numbers*; As because the twentieth part of a pound *sterling* is well known by the name of a Shilling, we use to say Three, Four or Five Shillings, *&c.* and not Three Twentieths, Four Twentieths, and Five Twentieths of a Pound. The like is done also in the Twelve parts of a Shilling; as for One Twelfth, Two Twelfths, Three Twelfths, *&c.* we say One penny, Two pence, Three pence, *&c.* The same also may be understood of the parts of Weights, Measures, Time, Motion, *&c.* Of these more particularly may be seen, in the 1st *Part* of the 2d *Book.*

These pass for vulgar denominate Numbers.

Figurals.

The second sort of more Generally *Contract Numbers* are Figural, so called because they do or may represent some Figure Geometrical; in relation to which they are ever considered, and thereupon by some are reckoned absolutely among Denominate *Numbers*. Figural *Numbers* are either *Lineary*, *Superficial*, or *Solid*.

These of 3. sorts.

1. *Lineary.*

Lineary, which have relation to length only, and are considered Universally or Restrictively. Universally, and so though most aptly it be referred to such a Number as will make no other form duely, yet may it also be applyed to any Number Abstract. Restrictively, and so *Numbers Lineary* are taken for the sides of all Æquilateral Figures, or Forms, and Metaphorically these are called *Roots*.

2. *Superficial.*

Superficial, Called also *Flat Numbers*, are considered in the Formes they make by *Progression* or *Multiplication*, whereof there be as many varieties as there be like Figures in *Geometry*; as *Numbers Triangular*, *Quadrangular*, *Quinquangular*, *&c.* *Numbers Circular*, *Diametral*, *Oblong*, *&c.* all which have length, and breadth, but no depth.

Solids

2. Solids.

Solids, or *Sound Numbers*; otherwise termed Bodily, or Cubical, as by the first Multiplication they take length, and breadth like *Flat Numbers*, so by the next Multiplication they take depth, or thickness; which thickness or solidity is increased according to the Number of Multiplications, and accordingly from thence do they take their Names; as Cubes, Squared Squares, Sursolids, *&c.* *ad Infinitum.* These take up the 2^d *Part* of the 2^d *Book.*

Numbers Specially Contract, of six sorts.

1. Decimals.

Again *Numbers* Specially Contract are considerable, as first *Decimals*, which arise from the Abbreviation of *Fractions* before mentioned, or rather from the Conversion of one kind of *Fraction* into another; For let the *Integer* be what it will, it shall be broken but into Ten parts, and one of these Ten parts, into other Ten, and so infinitely decreasing by Ten only, whereas in other *Fractions* the Denominators might be any other Number; and because the Denominators here are still known to be One with Cyphers more by One place than the Figures of the Numerator, the Denominator is alway omitted as needless; and the Numerator only set down. As is further declared, *Book* 3^d *Part* 1^st.

2. Astronomical.

Secondly, *Numbers Astronomical*, which are conversant about *Astronomy*, in Time, and Motion, have their Denominators Sixty certain; and so also omitted and ordered very like Decimals. As appeareth *Book* 3^d *Part* 2^d.

3. Logarithmes.

Thirdly, *Numbers Logarithmical*, or *Rational*, which have the same foundation with *Decimal* and *Astronomical Arithmetick*, and like them have their Denominators with the Numbers they come of also omitted. These are of such singular use, as they will for ever Renown the Honourable Lord *Nepair* who Invented them, and may be further viewed in *Book* 3^d. *Part* 3^d

4. Cossicks.

Fourthly, *Numbers Cossical*, which proceed from Figural Numbers Abstract, and may be accompted Rational-Contract-Compound-Figural-Numbers, As Two *Roots*, Three *Squares*, Three *Cubes*, and such like. *Cossicks* are Simple, or Compound, and of either sort, Whole, and Broken, as is demonstrated in *Book* 3^d. *Part* 4^th.

5. Surdes.

Fifthly, *Numbers Surde*, or *Irrational*, which are such Numbers set for *Roots*, as cannot be expressed by any other Number Absolute, arising from Lines, or Figures Inequilateral, whose measure is a whole Number and a Fraction, and in a sort are to Figurals like Fractions to Integers. These are also Simple, and Compound; and take up the fifth Part of the 3^d *Book.*

6. Species.

Sixthly, Because the Characters used in many Denominations are Arbitrary, and marked with Letters, also in Geometrical Figures for brevity and distinction sake one line is noted with one Letter, and another with another; hence the Alphabetical Notes are called *Species*, and are ordered in Arithmetical form in the 6^th Part of the 3^d *Book*; And are duely inserted among the rest of Contract Numbers, because every Magnitude, or Number, *&c.* is at pleasure denominate A. B. C. *&c.* or by any such other Letter, or Mark.

Homogeneal Numbers considered in Common.

Related two wayes.

Ratio.

Homogeneal Numbers, whether Abstract, or Contract, are considerable in their Common Nature that is Relative; as when Five is compared to Ten, it is but one half; so Eighteen is tripple to Six; here the Numbers being compared together, are fitly termed Relative. This Relation is taken differently, as when two Numbers only; or more than two are compared together: That called *Ratio*; this *Proportion*, or *Analogy.*

This two-fold Equality and Inequality.

Inequal of two sorts.

Greater is Simple, or Compound.

Simple 2. fold.

Compound 3. fold.

Lesser is 2. fold.

Ratio hath a property common with *Fractions*, to be Commensurable, or Incommensurable; and peculiar to it self is bisected into Equality, or Inequality. *Ratio* of Equality, is when two Equal Numbers are compared together, as Two to Two, *&c.* *Ratio* of Inequality, when one Number is compared to another different from him, *viz.* Greater, or Lesser. Greater, as Six to Two. Lesser, as Two to Six. The Greater is of two kinds; Prime, or Simple, Conjunct, or Compound. The Simple are again divided into two sorts. *First*, When the Greater Number containeth the Lesser, once and a part more, whether one half, one third, *&c.* as Six to Five, *&c.* *Secondly*, When the Greater containeth the Lesser with some parts of him, as Five to Three; which containeth the Lesser once, and two Thirds more, *&c.* *Ratio* of the Greater Inequality Compound, are distinguished into three sorts. *First*, When the Greater Number containeth the Lesser divers times, whether twice, four times, or such like, as Four to Two, Nine to Three, *&c.* *Secondly*, When the Greater containeth the Lesser many times, and a part of him besides, as Five to Two, Sixteen to Five, *&c.* *Thirdly*, When the Greater containeth the lesser many times, and also many parts thereof, as Eight to Three, Eleven to Three, *&c.* *Ratio* of the Lesser Inequality, like *Fractions*, either contain a part of the Number, as one Third, one Fourth, *&c.* So Two to Six, Two to Eight, *&c.* Or else many parts, as Three Quarters, Two Thirds, *&c.* So Four to Six is Two Thirds, and Nine to Twelve, Three Quarters, *&c.* These are further treated of in the 1^st *Part* of the 4^th *Book.*

Proportion,

Proportion 2. fold.

Proportion or *Analogy* when more Numbers than two are compared together, then there may be a conference of the former several *Ratios* in their several termes, and this is different either in *Discontinual*, or *Continual Proportion.*

Discontinual, Direct, and Indirect.

Discontinual, Is when in four termes the first, and second, third, and fourth termes are compared together, but not the second and third; so Five to Fifteen, is as Six to Eighteen: Where the second term Fifteen, and the third term Six are not compared together. *Discontinual Proportion* is again divided into *Direct* and *Indirect Proportion*; and either of these double. *Indirect*, if the Greater requireth a Lesser, or the Lesser, a Greater to be found. *Direct*, when sometimes the Lesser requireth the Lesser, and sometime the Greater requireth the Greater to be found out.

Continual Arithmetical, Geometrical.

Continual Proportion, is when three or more Numbers bear like Difference, or Proportion in their Progression. This is double, *First*, When between every two Numbers the Difference, or Excess is Equal, as between Three, Six, Nine, Twelve, *&c.* the Difference is Three. *Secondly*, When the *Ratio* is equally alike, as Four, Eight, Sixteen, *&c.* the *Ratio* is Two. These are dealt with in the 2^d^ and 3^d^ *Parts* of the 4^th^ *Book.*

Æquations are Compound Proportions or Numbers in the Ratio of Equality.

Again *Homogeneal Numbers* may hold community in Special Denominations, being considered in the *Ratio*, or *Proportion* of *Equality*; and such are called *Æquations*, and by some *Algebra*; So Numbers Æquational are Numbers equal one to the other, though the Denominations of such Numbers are different, as one Square shall be Equal to Three Roots, if the *Root* be Three; wherefore *Æquations* may be rightly placed amongst Numbers Denominate, the Operations of *Algebra* annexing, and of necessity requiring *Cossical*, *Speciosal*, or other Denominations unto Absolute and Undenominate Numbers, without which they must still remain uncouth and indeterminable. The Mysteries of these are unvailed in the 4^th^ *Part* of the 4^th^ *Book.*

Numbers Heterogeneal.

Enough in this place hath been said of *Homogeneal Numbers*, the other sort are *Heterogeneal*, called also Mixt Numbers, so that Mixt Numbers admit of a double acceptation as well in difference to Digits and Articles (as before noted) as in opposition to *Homogeneal Numbers* which are immixt, or of one sort.

Integers and Fractions.

Abstract and Contract.

Briefly, to conclude, Numbers are *Heterogeneal*, or of divers kinds when they are mixed primarily, or secundarily. The First are made up of *Integers* and *Fractions*, as One and a half, Two, and Three quarters, *&c.* The other are Mixt of Abstract, and Contract Numbers, as Three Squares, and Two Thirds, Thirteen Roots, and Four Integers, *&c.* The latter sort of them may be observed in the Three Last *Books*, Generally. The Former particularly *Book* 1^st^ *Part* 2^d^, and Occasionally throughout the whole Volumne.

CHAP.

CHAP. III.

Of the Elements of Numbers.

Affections of Numbers further to be seen in their Elements.

WHAT hath not been seen of the Affections of *Numbers* in the former Chapter of their *Nature*, may be further discovered in their *Elements*: A Type whereof followeth.

A Type of the Elements of Numbers.

- Elements of Numbers.
  - Simple
    - Notation
      - Determinate.
      - Customary.
      - Arbitrary.
    - Numeration
      - Original
        - Expression
          - by Word
          - by Writing
        - Composition
          - Prime, in Addition.
          - Compound, in Multiplication.
        - Dissolution
          - Prime in Substraction.
          - Compound, in Division.
      - Ortive
        - Reduction
          - Multiplication.
          - Division
        - Figuration
          - Production
            - Multiplication.
            - Addition.
          - Extraction
            - Substraction.
            - Division.
  - Comparative
    - Description
    - Computation
      - of Ratios.
      - of Proportions.
      - of Æquations.

Simple Elements, 2. fold:

In this Table the *Elements* of *Numbers* appear double, *viz.* Simple, and Comparative. Simple consisteth either in Notation, or Numeration.

1. Notation and this.

Notation, which in some Authours passeth for Numeration teacheth the Notes, Marks, and Characters whereby Numbers, Quantities, Denominations, *&c.* are described, and accompted as valuable as expressed in words at length. And these are either Certain, and Determinate; or Uncertain and Arbitrary; or else Customary, nevertheless variable, being partly certain, and partly uncertain.

Certain in the use of Figures.

Certain Notation I call the most excellent invention of expressing all Cardinal Numbers by those Ten Characters; *viz.* 1. 2. 3. 4. 5. 6. 7. 8. 9. 0. Generally called Figures, the Tenth is called properly in Latine, *Circulus*, or *Ciphra*, in English a Circle, or Cypher, and vulgarly Nought, and of it self signifieth nothing, but being joyned to the right hand of the other, helpeth to increase their value by accident; the other Nine are called signifying Figures, as 1. doth signifie One, 2. Two. 3. Three. 4. Four. 5. Five. 6. Six. 7. Seven. 8. Eight. 9. Nine. All Ordinal Numbers are expressed by the same Figures, only at top inclining to the right hand, is placed one, or two of the final Letters of the Word, as the First, 1st. Second, 2d. Fourth, 4th. *&c.*

Arbitrary in several Marks and Characters.

Notation Uncertain, and Arbitrary is either in form, or in quantity. In Form, and so in Cossical, and Surde Denominations some Authors use to write them in one form, and others in another; as some write thus, *Z.* for *Zenzike*, others. *Q.* and sometime, q. for Quadrate: Some thus, ℨ. and others thus, ℨ. at pleasure. Likewise in Surds this $\sqrt{c}$. standeth for a Cube Root in one, for a Squared Square Root in another. Characters both in Cossicks, and Surdes used in this Treatise may be seen, *Book* 3d. *Part* 4th. and 5th. And so from each Authour is to be expected an Explanation of their own Marks, alwayes minding that every man in his private practise useth those he hath most accustomed, or best fancieth, and may change them for others sooner made. Secondly, some Notes are uncertain in their Quantities, as in Species; for though any Quantity, or Number, whole, or broken, for the time present (or during the work of a question) may be noted by any Letter of the Alphabet, *Greek*, or *Roman*, small or Capital; as Α. Β. Γ. Δ. *&c.* α. β. γ. δ. *&c.* A. B. C. D. *&c.* a. b. c. d. *&c.* yet when the Question is ended, and another in working, the same Letters shall signifie other Magnitudes, or Numbers greater or lesser, at the will of the Arithmetician, as is evident by the practise of the 6th *Part* of the 3d *Book.*

Customary in several Symboles and Characters of common use.

Customary Notation is certain in respect of the General use, yet uncertain in respect of the end for which used; for if any form of better representation, may be more commodiously made, they may be altered for others. And such are all those Symbols, or Characters used for Vulgar Denominations with many that are frequented, to avoid prolixity, and the tedious, and often rescription of some Words, and termes of Art: Some of the most certain, and chiefly in use the following Table will easily demonstrate.

D Vulgar

Vulgar Denominations marked, Words abbreviated, and Termes of Art Characterised.

		Latine.	*English.*		Characters.
Geodeticks.	Money	Libræ	Pounds, Liures, Gilders		l. ŧ. £.
		Solidi	Shillings; Solx, Stivers		s. ſ. ß.
		Denarij	Pence, Deniers		d.
		Oboli	Half-pence		ob: ob.
		Quadrantes	Farthings		qd. q.
		Coronati	Crownes		Δ. ϖ.
	Weight	Centum	Hundreds		c.
		Quadrantes	Quarters		qr. q.
		Libræ	Pounds		℔. lb.
		Unciæ	Ounces		℥.
		Pondo-denarij	Penny-weights		p. pd. pw. ts
		Drachmæ	Drams		ʒ.
		Scrupuli	Scruples		℈.
		Graines	Graines		Gr. gr.
		Caractæ	Karacts		K. Kr.
		Per Centum	By the Hundred		p°.
		Continens	Containing		ton.
		Superpondium	Tare		ts.
		Abſque Superpondio	Without Tare		Netto.
Astronomicks.	Time and Motion	Anni	Years		Ai.
		Anno Domini	In the Year of Our Lord		An.° D.
		Anno Chriſti	In the Year of Chriſt		An.° C.
		Dies	Daies		D. d.
		Horæ	Hours		H. h.
		Signa	Signs		S.
		Gradus	Degrees		G. g. d. deg.
		Minuta, Secunda, *&c.*	Minutes, Seconds, *&c.*		ı. ıı. &c.
		Ante Meridiem	Before Noon		a.m.
		Poſt Meridiem	After Noon		p.m.
	Sexagenæ	Primæ, Secundæ, Tertiæ, *&c.*	Primes, Seconds, Thirds, *&c.*		\ \\ \\\ *vide plus lib. 2. par.*
	Sexageſimæ	Primæ, Secundæ, Tertiæ, *&c.*	Primes, Seconds, Thirds, *&c.*		/ // ///
	Signes	Aries	Ram	Head and Face	♈.
		Taurus	Bull	Neck and Throat	♉.
		Gemini	Twinns	Arms and Shoulders	♊.
		Cancer	Crab	Breaſt and Stomack	♋.
		Leo	Lion	Heart and Back	♌.
		Virgo	Virgin	Bowels and Belly	♍.
		Libra	Ballance	Reins and Loins	♎.
		Scorpio	Scorpion	Secrets	♏.
		Sagitarius	Archer	Thighs	♐.
		Capricornus	Goat	Knees	♑.
		Aquarius	Waterman	Leggs	♒.
		Piſces	Fiſhes	Feet	♓
	Planets and Mettals.	Saturnus Plumbum	Saturn	Lead	♄.
		Jupiter Stannum	Jupiter	Tinn	♃.
		Mars Ferrum	Mars	Iron	♂.
		Sol Aurum	Sun	Gold	☉.
		Venus Ars Cuprum	Venus	Braſs, Copper	♀.
		Mercurius Argentum vivum	Mercury	Quick-Silver	☿.
		Luna Argentum	Moon	Silver	☽.
	Nodes	Caput Draconis	Dragons head		☊.
		Cauda Draconis	Dragons tail		☋.
		Pars Fortunæ	Part of Fortune		⊕.
		Antiſcia	Antiſcions		Ant.
		Contrantiſcia	Contrantiſcions		CA.
	A	Conjunctio	Conjunction		☌.
		Vigintilis	Vigintil		Vg.
		B Quindecilis	Quindecil		Qd.

Aſpects

A	B Semisextilis	Semisextil	SS.⚺.
	Semiquintilis	Semiquintil, or Decil	Dc.⊥.
	Semiquartilis	Semiquartil	Sq.∠.
	Sextilis	Sextil	⚹.
	Quintilis	Quintil	Q.♡.
	Quartilis	Quartil	□.
Aspects	Sesquiquintilis	Sesquiquintil or Tredecil	Td.⚼.
	Trigonus	Trine	△.
	Sesquiquartilis	Sesquiquartil	SSQ.⚼
	Biquintilis	Byquintil	Bq.±.
	Quincunx	Quincunx	Vc.
	Oppositio	Opposition	☍.
Houses	Horoscopus, Ascendens	The Angle of the East, or 1^{st} house	Hor.Ascen.
	Medium Cœli	Midheaven or 10^{th} house	M.C.Med.C.
	Angulus Occidentis	West Angle, or 7^{th} house	Ang: Occ.
	Sinum Cœli	The Angle of the Earth, or 4^{th} house	IC.
Cardinal Points	Septentrio	North	N.
	Oriens	East	E.
	Meridies	South	S.
	Occidens	West	W.
Affections	Longitudo	Longitude, or Length	Long.
	Latitudo	Latitude, or Breadth	Lat.
	Altitudo	Altitude, or Height	Alt.
	Recta Ascentio	Right Ascention	R.Asc.
	Recta Descentio	Right Descention	R.Desc.
	Obliqua Ascentio	Oblique Ascention	Ob.Asc.
	Obliqua Descentio	Oblique Descention	Ob.Desc.
Logarithmeticks. Canon Artificial	Logarithmus	Logarithme	Log.
	Sinus	Sine	s.
	Sinus Complementum	Cosine	cos.
	Tangens	Tangent	t.
	Tangentis Complementum	Cotangent	cot.
	Secans	Secant	sec.
	Secantis Complementum	Cosecant	cosec.
Cossicks.	Radix	Root	ʒ.
Surdes.	Radix	Root	√.
	Radix Universalis	Universal Root	√.
Species	Radix, sive Latus	Root, or Side	l.
	Quadratus	Square	q.
	Cubus	Cube	c.
	Diameter	Diameter	D.
	Diametri Quadratus	Square of the Diameter	Dq.
	Diametri Cubus	Cube of the Diameter	Dc.
	Radius	Semidiameter, or Radius	R. Rad.
	Radij Quadratus	Square of the Radius	Rq.
	Radij Cubus	Cube of the Radius	Rc.
	Peripheria	Circumference, or Periphery	P.
	Peripheriæ Quadratus	Square of the Periphery	Pq.
	Radix Binomii	Binomial Root	√b.
	Radix Residui	Residual Root	√r.
	Radix Suppositia	Supposititious Root	A.
	Duo Numeri	Two Numbers	AE.
	Major	Greater	A.
	Minor	Lesser	E.
	Summa	Sum	Z.
	Differentia	Difference	X.
	Rectangulum	Rectangle, or Product	Æ.AE.P.
	Summæ Quadratus	Square of the Sum	Zq.
	Differentiæ Quadratus	Square of the Difference	Xq.
	Summæ Cubus	Cube of the Sum	Zc.
	Differentiæ Cubus	Cube of the Difference	Xc.
a	b		

Summa

Vocabula Artis.

Summa Quadratorum	Sum of the Squares	Zq.
Summa Cuborum	Sum of the Cubes	cZ.
Differentia Quadratorum	Difference of the Squares	Xq.
Differentia Cuborum	Difference of the Cubes	cX.
Tres continuè proportionales	Three Numb. contin. proportional	A, M, E.
Quatuor continuè proportionales	Four Numb. continually proportional	A, M, N. E
	Arithmetical Progression.	
Primus Terminus minimus	The First Term, or least	α.
Ultimus maximus	The Last Term, or greatest	ω.
Numerus Terminorum	The Number of Termes	T.
Differentia communis	The Difference, or Excess common	X.
Summa omnium Terminorum	The Sum of all the Termes	Z.
	Geometrical Progression.	
Terminus primus	The first Terme	α.
Terminus secundus	The second Terme	β.
Terminus tertius	The third Terme	$\frac{\beta q}{\alpha}$.
Terminus quartus	The fourth Terme	$\frac{\beta c}{\alpha q}$.
&c.	&c.	

Signal Symboles for Terms of Art.

+ Commonly called St. *Georg's* Cross, is the sign of *Addition*, and being set before a Number, or quantity, signifieth *More*, *More by*, *and*, or any such word that may shew the same Number, or Quantity to be affirmative.

− A *Right Line* is the sign of *Substraction*, and set before a number or quantity, imports *less*, *less by*, *lacking*, *wanting*, or any such word that may declare the same number or quantity to be negative.

X Commonly called St. *Andrews's* Cross, is the sign of *Multiplication*, and set between two Numb. or Quant. implies multiplyed *by*, *in*, *into*, *times*, or any such word, that may denote the numb. or quant. multiplyed one into the other.

) The *Lunular Increscent* is the sign of *Division*, and set between two numbers or quantities, signifyes dividing, and declares the right hand number or quantity of the Two, to be divided by the left.

(The *Lunular Decrescent* is the sign of the *Quotient* of any *Division*, and the Number there standing demonstrates how often the *Divisor* is contained in the *Dividend*.

···· *Four Points* in a Right Line, is used sometime in *Reduction of Fractions*, to separate the old Numerators from the new, and the old Denominators from their least termes.

—— A *Right Line* between two numbers, or quantities, one standing above, and the other beneath, commonly denotes a *Fraction*, and that the upper number, or quantity is to be divided by the neather. And sometimes is used to separate one Number from another.

⌊— The *Rectangle* is the *Seperatrix* between *Integers*, and *Decimals*, of which and other Distinctions thereof, *Vide plus, lib.* 3. *par.* 1. & 2. Sometime also is instead of the *Decrescent Lunular* in *Division*.

, The *Comma*, is the Distinction also between *Integers* and *Decimals*; and between the *Logarithme* and his *Characteristique*.

: The *Colon*, Including *Numbers* or *Quantities*, is a Note of the *Universal Root* of such Number, or Quantity.

. The *Period*, is often used for Distinction-sake, but in disjunct Proportions, between two Numbers or Quantities, understands the word, *To*.

⋮ *Three Pricks* or *Points*, in the middle of four numbers, or quantities, are sometimes used in Disjunct Arithmetical Proportions, for the words, *is as*

:: *Four Points* thus, in the middle of four numbers or quantities is the sign of *Analogy*, and frequently used in Disjunct Geometrical Proportions for the words, *is as*; or *so is*.

═ *Parallels*, are the sign of *Æquality*, and implies the numbers or quantities of the one side thereof, are equal to the numbers or quantities of the other side thereof.

⊏ Greater. **⊏.** Next Greater. **⊐.** Lesser. **⊐.** Next Lesser.

⊏ Not Greater. **⊐.** Not Lesser. **⊑.** Equal or Less. **⊒.** Equal or Greater.

·· Between

- Between two Numbers, or Quantities, the one above, and the other below, is the sign of a *Ratio.*
- Greater Ratio. Lesser Ratio. Continual Proportionals.
- Commensurable. Incommensurable
- Commensurable in Power. Incommensurable in Power.
- Rational. Irrational. Medial.
- A Line cut according to extreme, and mean Proportion.
- The greater Portion thereof. The lesser Portion thereof.
- A mean Proportional. Divided by.
- The Difference of the two Magnitudes.

Common Termes.	
	By.
	Book
	Creditor.
	Debitor.
	Leafe.
	Number.
	Paid.
	Received.
	&c.

Other words not having any proper Symbole are abbreviated when occasion requires by two, or three of the first Letters of the word, as *Duc.* for *Ducats. Mon.* for *Moneths*, &c. D. and *Q.* besides what as before they stand for Notes of, may sometime peradventure be found for *Data*, and *Quæsita*, which termes import the One such *Numbers* or *Magnitudes*, *&c.* as are given for the Resolution of a Question, the other the *Numbers* or *Magnitudes* sought.

Nothing need be added for demonstration of the Table, but that some Difference may be observed between the Characters used in Printed Books, and those practised by the Pen, according as the written, and Printed Alphabets differ.

Where denominations have two or more marks any one of them will serve, as may be further observed in the parts of the Treatise to which they belong.

The next part of the *Simple Elements* of *Numbers* is *Numeration*, which is both *Original* and *Ortive*. *Original* is of double use; for it serveth either rightly to Express, and Accompt the Value of Numbers by their Notes, Symboles, Characters, Places, *&c.* or else to find out, and procure Numbers valuable Greater, or Lesser. This last is sometime called *Algorithme*, though *Algorithme* more properly is *Cossical Arithmetick*. *2. Simple Element of Numbers is Numeration, and this either Original, which is of 3 sorts.*

1. The first kind teacheth the Order, which *Arithmeticians* do observe, in the usual expressing, and valuing of *Numbers*, *Quantities*, *&c.* either by Word or Writing; and with several Authours passeth for the only Numeration: This for *Integers* is to be sought in the next Chapter, and for *Fractions*, and all the other sorts of Numbers, in the first Chapter of every Part of the other Books.

2. The second sort of *Original Numeration*, sheweth the Method of Increasing Numbers. The Increasing of Numbers is called Composition, or the Genesis of Numbers; This is Simple in *Addition*, and Compound in *Multiplication*.

3. The third sort of *Numeration Original*, is the Diminishing of Numbers, or their Dissolution, which likewise is Simple in *Substraction*, and Compound in *Division*, and both these may be called the *Analysis* of *Numbers*. So that *Addition* and *Substraction*, are both the *Prime*, or *Simple* parts of *Numeration*; and therefore shall in order precede *Multiplication*, and *Division*, which are the Conjunct parts thereof: The several Chapters whereof are to be seen for each sort of Numbers distinctly, in the Parts where they are handled.

Or, Ortive, which is 2 fold.

Numeration Ortive, ariseth from the former Species of *Numeration Original*, and consisteth in two things, *Reduction*, and *Figuration*. *Reduction* is useful in *Fractions*, and *Contract Numbers*, and performed by *Multiplication*, or *Division*: As will sufficiently be evident in the second part of this Book, and elsewhere in Contract Numbers. *Reduction.*

Figuration, serveth for Figural Numbers, to produce them, or extract their *Roots*. *Production*, or their *Genesis*, is effected by *Multiplication*, and *Addition*. The Extraction of their *Roots*, or *Analysis* is made up principally of *Substraction*, and *Division*, occasionally using *Multiplication* and *Addition*; which being further set forth in the second Part of the Book may here be spared. *Figuration.*

The *Comparative Elements* of *Numbers*, belong to the Work of *Proportions*; The Description, and Computation whereof, both *Arithmetical*, or *Numeral*, and *Geometrical*, or *Mensural*, and of each kind *Disjunct*, and *Continued*, together with *Æquations*, are reserved for the Fourth Book, and therefore omitted here. *Comparative Elements to be seen in the 4th. Book.*

CHAP. IV.

Of INTEGERS.

Integers firſt proceeded with.

THE *Nature*, and *Elements* of *Numbers* ſeen in general, it is now requiſite to deſcend into particulars, and to proceed in Order according to the foregoing Method. The firſt ſort of Numbers that preſent themſelves, are proper, and homogeneal Numbers, called *Integers*, Conſidered abſtractively.

All Numbers expreſſed by 9. Figures and the Cypher.

The Notes, or Characters, frequented both in *Integers*, and *Fractions* abſolute, or of common Denomination, and generally in all Arithmetical Operations, are the forementioned Figures, 1. 2. 3. 4. 5. 6. 7. 8. 9. 0. the which thereby that Arithmeticians may expreſs all Numbers, they have ordered them into certain places, and periods, proceeding alwayes from the right hand towards the left in a Decimal progreſſion, ſo as the aforeſaid Figures may expreſs greater Numbers, or leſſer as neceſſity requireth.

Place of a Figure, what.

A Place is the Seat or Room that a Figure ſtandeth in, and ſo many Figures and Ciphers as there are in one Number or Sum, ſo many places hath that whole Number.

No certain Number of Places.

Of theſe Places there is no certain Number, but that is called the firſt place that is next to the right hand, and reckoning in order towards the left hand, the next is the ſecond, then the third, and ſo *ad infinitum*, as a. b. c. d. thus ſtanding, d. thus ſtandeth in the firſt place, c. in the ſecond, b in the third, and a. in the fourth, *&c.*

Every Place hath ſome Denomination.

Every place hath a certain denomination properly belonging to it. Whereby a figure according to its ſtanding comes to be valued many times more than the Figure ſtanding ſingle would import. For 1. 2. 3. *&c.* ſhall not only ſignifie ſo many entire Unites, or Ones, according to their formes, but may alſo ſignifie 1. 2. 3. *&c.* Tens Hundreds, Thouſands, *&c.* according to the place the figure occupieth, for by ſo much as any figure inclineth towards the left hand, by ſo much is the value thereof increaſed.

Unites Place

The firſt place, hath the denomination of Units, and doth ſignifie that every figure ſtanding there, betokeneth his own ſimple value according to his form, as the figure 1. to ſignifie but One, the figure 2. but Two, and ſo of the reſt.

Tens

The ſecond place, toward the left hand hath the denomination of Tens, and every Figure here ſtanding ſhall betoken his own certain value Ten Times; as 1. if it ſtands in the ſecond place ſhall be one Ten. 2. Twice Ten, or Twenty. 3. Three times Ten, or Thirty, *&c.*

Hundreds.

The third place, to the left hand hath the denomination of Hundreds, and ſo every Figure there ſtanding ſhall betoken his own value a hundred times, as 6. ſtanding there denotes ſix hundred, *&c.*

Thouſands, &c.

In the fourth place, every figure ſtanding, ſignifieth his value a Thouſand times, In the fifth place, Ten thouſand times. In the ſixth place, one hundred thouſand times. In the ſeventh place, one Thouſand Thouſand times, or one Million (which is called by ſome the firſt great Thouſand.) In the Eight place, Ten Millions In the Ninth place, one hundred Millions. In the Tenth place, one Thouſand Millions, (called by ſome the ſecond great Thouſand.) In the Eleventh place, Ten Thouſand Millions. In the Twelfth place, One Hundred Thouſand Millions. In the Thirteenth place One Million of Millions, (or the third great Thouſand.) So infinitely Names may be given to every place, each ſucceeding place exceeding the former Ten times; though in ordinary practiſe we ſeldom need thirteen places, yet if any liſt to exceed, it is but doubling the Millions to begin as at the Eight place, for the fourteenth place is Ten Million of Millions, The Fifteenth like the Ninth is Hundred Million of Millions, and thus proceeding till the Nineteenth place, where tripling the Millions, go on as before to the Twenty fifth place, and then quadruple the Millions, and ſo as before *ad Infinitum*.

Others inſtead of doubling the Millions in the thirteenth place, call it Billion; the nineteenth place Trillion, inſtead of Millions of Millions of Millions; the twenty fifth place Quadrillion, and the Thirty firſt place Quintillian, *&c.*

Denomination of the Places reſpect, only Quantity.

Note here, That a Number though thus denominate according to his place, yet is accompted Abſtract, without another denomination, this denomination only reſpecting the quantity, number, or multitude of the thing or things propounded, and that denomination which truely decyphers a Number Contract, reſpects the quality, or nature of the thing numbred; yet notwithſtanding it bears ſome ſimilitude thereto, for as the

denomination of quantity is the last recited in the verbal expression of any Number, as to say Two Hundred, where Two is the Numerator, or Number, and Hundred the denomination of quantity which is last repeated. So is the denomination of quality repeated after the quantity; As in saying Two Hundred Pounds, after the Sum of Two Hundred the quantity, is last of all named Pounds the denomination of quality; likewise Thirty Men, here Men is the Contract Denominator, and Thirty the Numerator, or Valuer, &c.

How different to the denomination of Quality.

These places are distinguished into Degrees, and Periods. Degrees are three; Once, Ten times, a Hundred times. A Period is a comprehension of Degrees, and is Simple, or Compound. Simple is made up of one Ternary of Degrees, containing three places, as 123. Compound is double, as 12345. Or treble, as 12345678, or fourfold, &c. A further view whereof may be had in the following Table, which may be set in several formes.

Places are distinguished into Periods.

Numeration Table.

A.	z. y. x.	w. u. t.	s r q	p. o. n.	m. l. k.	i. h. g.	f. e. d.	c. b. a
5.	6. 7. 8.	9. 0. 1.	2. 3. 4.	5. 6. 7.	8. 9. 0.	1. 2. 3.	4. 5. 6.	7. 8. 9
M	M M M	M M M	M M M	M M M	M M M	M M M	M M M	C X I
M	M M M	M M M	M M M	M M M	M M M	M M M	C X	
M	M M M	M M M	M M M	M M M	M M M	C X		
M	M M M	M M M	M M M	M M M	C X			
M	M M M	M M M	M M M	C X				
M	M M M	M M M	C X					
M	M M M	C X						
M	C X							
&c. ad Infinitum	The eight Period of Thousands of Millions of Millions of Millions.	The seventh Period of Millions of Millions of Millions.	The Sixth Period of Thousands of Millions of Millions.	The Fifth Period of Millions of Millions.	The Fourth Period of Thousands of Millions.	The Third Period of Millions.	The Second Period of Thousands.	The First Period of Units.

Here the order of the places is noted by the *Roman Letters* at top. The Value or denomination by the Numeral Capital Letters. The *Periodical Division* shews the thousandth place of the Number.

These things observed it will not be difficult to express the true content of any Number Integral by word or writing: For the former consider the figures and the places they stand in, and call their values such as the figure is augmented by his standing; wherefore if I see 98. standing, and would express the right value thereof, I know the first figure is but Eight, for it beareth that form, and standing in the Units place, and therefore can signifie himself but once; but the 9. must be Nine times Ten, because it standeth in the second place, and so is the value, or content of that quantity Ninety Eight, as if it should stand thus 90. 8. and be expressed Nine times Ten and Eight. And this 398. Three hundred ninety eight, as if it stood thus 300. 90. 8. and after this manner are all quantities valued that are beneath a Thousand. If the quantity exceed, for the better remembring the periods, and so thereby the more readily to express great Sums, or quantities, mark the periodical Division of the Table on the given Number with a prick above or below the figure standing in that place, beginning at the right hand set one prick over the first Figure, another over the fourth, &c. as to numerate or express the value of this Sum 3479841234. The pricks are placed thus $\dot{3}479\dot{8}41\dot{2}3\dot{4}$. then beginning at the left hand read as it were every pricked parcel apart, with the denomination of the place to it: So at last you shall have the whole value; as first One Thousand, Two hundred thirty four; Then, Nine Million, eight hundred forty one thousand, two hundred thirty four; Lastly, Three Thousand four hundred seventy nine million, eight hundred forty one thousand, two hundred thirty four. And so much is the value, or quantity of that Number; the like is to be done with others.

To express a Number by word.

To express in Writing, any propounded Integer, remember all numbers from one to ten, are expressed by a digit in the Units place; All numbers above ten, and under a hundred with two Figures, or one Figure, and one Cipher, to wit Tens in the place

To express a Number by writing.

place of tens, and Units or Cyphers in the place of Units, according to the Numbers to be expressed; likewise all Numbers under a thousand, and above a hundred are expressed by three Figures, or two Figures and one Cipher, &c. Then observe the places as before, and begin at the right hand, put down the quantity of Unites propounded, and so proceed unto the left. If the given Number have any rooms void, they are alwayes to be supplyed with Cyphers, as to write down one hundred the places of Units and tens being void are filled up with Cyphers, and one is set in the hundreds place thus 100. So one thousand thus 1000. and nine thousand and seventy million ten thousand one hundred and one, thus 9070010101. where Cyphers occupy several places instead of Figures, because neither Tens, Thousands, Hundred Thousands, Millions, nor Hundred Millions were given in the Number.

Void places to be supplyed with Cyphers.

CHAP. V.

Addition of Integers.

HAving seen the due ordering and placing of the Notes for the Expressing of Numbers by them, it remaineth to declare the other parts of Numeration, and first the Prime Part of

Addition, *what it is.*

Addition is that part of *Arithmetick* whereby divers Numbers are collected, and added together into one total Sum.

Addition of *Integers*, hath respect to *Collocation*, *Operation*, and *Probation*.

Addends, what. Total, what.

Collocation, is the due placing the Numbers given to be added, called the *Addends*, and distinguishing them from the Number found out by *Addition*, called the Sum Total, or Aggregate.

How to place the Addends.

Place the *Addends* in rank and file one directly under another, beginning at the right hand; it matters not whether the greatest or least be uppermost, so that Numbers of one quantity, may stand under Numbers of the same quantity, that is to say, Units under Units, Tens under Tens, &c. then draw a right line under them, to distinguish them from the Total.

Operation, hath *Induction*; and *Consummation*, or *Perfection*.

Induction *what it is, called* Simple Addition.

Induction, teacheth the Sum that any two digits added together make, called sometime *Simple Addition*; and had need be ready in Memory with the Accomptant; This is declared in the following Table.

Addition *Table.*

0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

The use of the Table.

By this Table the sum of any two digits is easily had, for entring with one at the head, and the other at the left hand side, the Common Angle is the Sum, as 3. and 7. make 10. and so much is found over against 7. in the side, and 3. in the head, or against seven in the head and three in the side, if the Table be quadrangular as this is; but for that

that the lower part of the Table beneath the black scale is sufficient, it may be set in that Triangular form, for 2. and 1. and 1. and 2. are all alike, *&c.*

By the help of these Digits added, the Sum of the Decades or Articles are also known, because their signifying figures are but digits, as 20. and 20. make 40. because 2. and 2. are 4 the Cyphers being only reserved to keep place as before, but increase not the Sum, for 1. and 0. is but 1.

Perfect Operation called Compound Addition how it is wrought.

Perfect Operation, sometime called *Compound Addition* finisheth the Addition of Mixt Numbers thus; begin at the right hand, and take all the figures or digits in the right hand file, as they stand one over another, and putting them together consider what the result or total thereof is, and if it be a digit write it under the right line directly in the same file, and so do in every of the files; but if any file being cast up amount to an Article write the Cypher under the file, and reserve in mind the figure of the Article to be added in the next place, and when the Sum of the next file is found add that reserved Article, to wit, for every ten reserved, one; as two for 20. three for 30. *&c.* and this result or total subscribe accordingly; and if the sum or total of any file with or without such reserved Article, if any be, amount to a mixt number, then set down the digit of that mixt Number, and reserve the Article thereof, as before.

Example 1.

Example. There are two Numbers propounded, whereof the one was 234. and the other was 342. What is the Sum of both? The numbers set as at A. in the right hand file are found 2. and 4. which put together make 6. to be set under the file because a digit, as is represented by the work standing at B. Then going forward to the left hand in the next file are 4. and 3. which make 7. to be subscribed as at C. Lastly in the third place are 3. and 2. which make 5. and the whole work stands as at D. where the Total appears to be 576.

Rank

A	B	C	D	
234	234	234	234	Addends.
342	342	342	342	
	6	76	576	Total.

Example 2.

Second Example. Suppose Two Numbers whereof the Sum is desired be 40235. and 34973. then are they set as before, and in the right hand file 3. and 5 make 8. to be there subscribed. Then 7. and 3. make 10. an Article, the Cypher therefore is set down under the line in the second file, and the Article reserved. Then 1. carried in mind, and 9. is 10, and 2. is 12. which being a mixt number, two the digit thereof is subscribed, and the Article reserved as before. Again 1. reserved and 4. is 5. to be set down under the 4. not regarding the 0. because the number is not augmented thereby in Addition. Lastly adding 3. and 4. together they make 7. which is set in the last place, and the work standeth as at K. where the total 75208. is the number desired. The several other Paragraphs stand as followeth.

E	F	G	H	I	K
40235	40235	40235	40235	40235	40235
34973	34973	34973	34973	34973	34973
	8	08	208	5208	75208

Proof of Addition. Common.

Probation, is the *Examen* or *Demonstration* whereby the Operation may be proved true. The vulgar proof is by casting all the Nines that can be had both out of the Aggregate, and Numbers added, which done will leave like figures if the work be right. To do this begin with the Total, or the Addends, at the left or right hand, not regarding the places of the Numbers, but as though they were all Units add them together, and as the Numbers increase above 9. reject 9. and go forward with the rest, and what remains when all the Numbers are gone over, set down at the one side or end of a right line; Then do so with the other part of the Addition, and this remain place at the other side or end of the same line: As to instance in the first Example 4. and 2. make 6 in the right hand file, then 6. and 4. make 10. coming to the next file, from which 9. rejected there resteth 1. that 1. and 3. make 4. which 4. and 3. make 7. and 2. is 9. which cast away there remaineth 0. to be set on the line as at M. Then in the

Total 6. and 7. is 13. from which 9 cast there remaineth 4. which 4. and 5. are 9. to be again cast away, and there resteth 0. also to be set under the line as at N.

$$L\ \begin{array}{r}234\\342\\ \hline 576\end{array}\qquad M\ \underline{0}\qquad N\ \frac{0}{0}\ \text{or thus}\ 0-0$$

This Proof by 9. uncertain.

Although if the work be right, it will never vary from leaving equal remains, yet by reason some digit or other may happen through inadvertency to be misplaced, this kind of proof is uncertain, for it is evident, that if the places of 7. and 6. in the Total should be unhappily changed, or of 5. and 7. yet would the remains by rejection of nines be as before, though the Sum much altered in the value. For both 567. and 576. also 756. and 765. likewise 657. and 675. after the nines are cast away leave 0. yet is their value greatly different: Which thing may fall out in every other Number.

Best Proof by Substraction.

Therefore the best Proof is by *Substraction*, which may be seen in the next Chapter.

CHAP. VI.

Substraction of Integers.

Subduction or Substraction, what it is.

THE first part of the Genesis of Numbers unfolded, The prime part of their *Analysis* followeth, which is *Substraction*, called also *Subduction*.

Substraction, is that numerative part of *Arithmetick*, which teacheth how to deduct one Number from another, and to shew what remaineth.

Substraction of *Integers*, like *Addition*, respecteth *Collocation*, *Operation*, and *Probation*.

How to place the Numbers.

Subtrahend what.

Remainder what and how called.

Collocation, placeth in the uppermost Rank the greater number, (or number from which *Substraction* is to be made,) for in *Integers* no greater number can be taken from a lesser; and underneath the same (with or without an interjacent line) the number to be substracted or deducted called the *Subtrahend*, so orderly that every figure may stand under his like, as Units under Units, Tens under Tens, &c. Then with an interjacent right line seperate these two given Numbers from the Number found out by *Substraction*, which is called the Remainder, Remain, Rest, Difference, or Excess.

Operation hath *Induction* and *Perfection*.

Induction called Simple Substraction.

Induction, sometime called *Simple Substraction* sheweth the Difference between any two digits substracted one from the other, and is requisite to be remembred by the Practitioner, according to the following Table.

Substraction *Table.*

0	1	2	3	4	5	6	7	8	9
1	0	1	2	3	4	5	6	7	8
2	1	0	1	2	3	4	5	6	7
3	2	1	0	1	2	3	4	5	6
4	3	2	1	0	1	2	3	4	5
5	4	3	2	1	0	1	2	3	4
6	5	4	3	2	1	0	1	2	3
7	6	5	4	3	2	1	0	1	2
8	7	6	5	4	3	2	1	0	1
9	8	7	6	5	4	3	2	1	0

This

This Table would be sufficient, if only the upper part above the black Scale be used. The difference between two Digits is found thus; enter with the lesser or substracting digit at the left side, and the other at the head, and in the Common Angle is the difference found, as 2 from 9 leave 7. which stands just against 2 in the side, and 9 in the head; the like also will be found against 2 in the head, and 9 in the left side of the Table, if the Table be quadrangular as this is. *The Use of the Table.*

By help of the difference of these *Monades* the difference of the Articles also are had, as 30 from 40 leaves 10, because 3 from 4 leaves 1. the Cypher, only keeping their place as before.

Consummate Substraction sometime called *Compound* perfecteth the work with mixt Numbers in this manner. It beginneth at the right hand, and withdraweth or abateth the lower Numbers or Figures, particularly one after another, out of the upper standing over them, and subscribeth the Number remaining, if any be, but if nothing remain, 0. except it happen to be the last file, and then the Cypher need not be set down. *Compound Substraction how wrought.*

Example. If 241. were to be taken from 343. and it were desired to know the Remain. The numbers being placed as at A. then 1. out of 3. leave 2. to be subscribed under the Line in the right hand file, as at B. then 4. from 4. there resteth 0. as at C. and 2. from 3. there remaineth 1. and the compleat work stands as at D. So the whole difference between the two given Numbers is found to be 102. *Example without any borrowing.*

A (Rank / File.)	B	C	D	
343	343	343	343	Greater Number.
241	241	241	241	Subtrahend.
	2	02	102	Remain.

If it happen that the neather figure be greater than the upper, so that *Substraction* cannot be made, then in imagination borrow Ten, and adding it to the upper Figure make *Substraction* from both, and for that borrowed ten account one back in the next file, either by counting the next figure to be substracted one more than it is, or the next Figure to be substracted from one less than it is.

Example. Suppose the difference between 30971. and 12381. be required. The numbers are set as at E. and 1. from 1. taken, 0. is left, as at F. but 8. from 7. cannot be taken without borrowing 10. which put to 7. makes 17. from which 8. taken leaves 9. to be subscribed in the second file, as at G. for which 10. reckoning the next 3. for 4. or the 9. but 8. and taking the lesser from the greater, that is either 4. from 9. or 3. from 8. the remain is 5. subscribed as at H. then 2. from 0. cannot be substracted, wherefore 2. from 10. as before leaves 8. as at I. and lastly 1. borrowed, and 1. are 2. abated from 3. leaves 1. as at K. so the whole excess is found to be 18590. *Example with borrowing.*

E	F	G	H	I	K
30971	30971	30971	30971	30971	30971
12381	12381	12381	12381	12381	12381
	0	90	590	8590	18590

When one Number is to be substracted from many, or many from one, first add all the Plural Numbers into one, and therewith proceed to *Substraction* as before.

Examples of both. In the first 3496001 taken from 3424501. 1042601. and 200000. whose total is 4667102. and the Remainder 1171101. as at A. In the second 5423401. 800100. and 10081. are subducted from 6245002. and the Remain is 11420. as at M. *Example where the data are many.*

L

3424501 1042601 200000	Numbers from which *Substraction* is made.
4667102	Total of the Addends.
3496001	Subtrahend
1171101	Remain.

M

6245002	Greater Number.
5423401 800100 10081	Subtrahends.
6233582	Total of the Subtrah.
11420	Remain.

Probation

Proof of Substraction. Common.

Probation serveth to demonstrate the Truth or Error of the Work, and is vulgarly performed by casting away nines from the Number from which *Substraction* is made, and keeping the remain thereof equal to the remain left after rejection of nines from the Subtrahend, and Remain; as in the first Example above in this Chapter. 9. cast from 343. leaves 1. So also from 241. and 102. to be set thus $\frac{1}{1}$, or thus 1—1.

This Proof by 9. uncertain.

But for the Reason shewed before in Addition this way of Proof is uncertain. As the best proof therefore of *Addition* is by *Substraction*; so the best proof of *Substraction* is by *Addition*.

Proof of Addition by Substraction 2. waies.

Addition may be proved by *Substraction* two wayes. First by beginning at the left hand, and deducting in order from the particular places of the total, the Sum of the particular Files added, for if the work be right, 0. will remain at last.

1. As in the former instance where the total of 5423401. 800100. and 10081. was found to be 6233582. here 5. taken from 6. in the total leaves 1. then 8. and 4. in the next File make 12. abated from the 1. left, and 2. in the total, which are 12, leave 0. then 2. and 1. make 3. taken from 3. there rest 0. and so in all the rest; as at N.

2 Secondly, By cutting off from the Addends any Rank, and adding the Residue into one total; then substracting this total from the first total, the remain will be the numbers first cut off; if the work be right.

As in the former instance, if 5423401. be cut off, and the total of 800100. and 10081. which is 810181. as at O. abated from 6233582. the remain will be 5423401. as at P.

N	O	P	
5423401	5423401	6233582	First Total.
800100	800100	810181	Second Total.
10081	10081	5423401	Numbers cut off.
6233582	810181		
1000000			

Proof of Substraction by Addition.

Substraction is proved by *Addition*. For if the Remainder, and Subtrahend be added together, the Number from which *Substraction* is made will be returned.

As in the last instance, if 5423401. be added to 810181. the Total will be 6233582. as at Q.

	6233582	Greater Number.	
Q	810181	Subtrahend	added.
	5423401	Residue	
	6233582	Total and Proof.	

Both bottomed on 2. Theorems.

The Proof of *Addition*, and *Substraction* is bottomed on the two fundamental *Theorems* following, which give life mutually to *Addition*, and *Substraction*.

1. In *Addition* the Aggregate is equal to all the Addends, and *contra*, as $3+2=5$. and $5=2+3$.

2. In *Substraction* the Number to be subducted, and the difference are together equal to the Number from which *Substraction* is made, and *contra*, because $3+2=5$. therefore $5-2=3$. and $5-3=2$.

Consectaries from thence.

From hence spring these two Consectaries.

1. To know the one Addend, if the Total, and other Addend be given, substract the given Addend from the Total.

2. To know the Subtrahend, if the Number from which *Substraction* is made, and the Remain be given, substract the Remain from the Number from which *Substraction* is made.

Examples in Questions of Addition *and* Substraction.

Questions in Addition.

1. What Number is that to which if 52. be added it makes 397?

Answer. Substract 52. from 397. and there remaineth 345. the Number sought.

2. What Numbers are those to which if there be added severally 32. 42. and 52. their Totals will be alike, and the Sum of each will be 130?

Answer. Substract the given Numbers severally from 130. and the remains are the Numbers quesited, which are 98. 88. 78.

3. What

3. What Number is that from which if 40. be taken, the remain will be 375? *Questions in Substraction.*
Answer. Add 50. to 375. and the total is 415. the desired Number.
4. What Numbers are they from which if 14. 15. or 16. be deducted, yet the remains will be equally alike 226? 1. 2.
Answer. Add severally to 226. the same Numbers, and the Totals will be 240. 241. 242. the Numbers required.

	1	2			3	4.		
	397	130.	130.	130.	375	226.	226.	226.
Operation	52	32.	42.	52.	40	14.	15.	16.
	345	98.	88.	78.	415	240.	241.	242.

CHAP. VII.

Multiplication of Integers.

THE Prime, and Uncompound parts of *Numeration*, *Addition*, and *Substraction* preceeding, next succeed the Compound, which are *Multiplication* and *Division*. *Multiplication what it is.*

Multiplication is the Compound Genesis of Numbers, and a Conjunct part of *Numeration*, whereby one Number is led into, or increased by another; for the Number given to be multiplyed is so often added to it self as there be Units in the multiplying number, wherefore the third number, or number produced thereby, shall so often contain the first number, as there be Units in the second.

Multiplication serveth instead of many *Additions*, with more speed to augment a lesser number than *Addition* can. For *Multiplication* effecteth at once, what *Addition* could but do at many times. *Serveth instead of many Additions.*

Multiplication observeth *Collocation*, proceedeth to *Operation*, and concludeth with *Probation*.

Collocation, respecteth the *Nomenclator* or Artificial termes, and duely placeth every number thus. First set down the greater of the two given numbers for the number to be increased, or multiplyed, which is called the *Multiplicand*; and under him beginning at the right hand (observing due place, and denomination of Units, Tens, *&c.*) place the lesser given number for the multiplying number, called the *Multiplier*, yet as in *Addition* it is not material which standeth uppermost, for either may be exchanged for other, but it is most orderly to place the greater number at top; then draw a right line under them to seperate them from the number found out by their *Multiplication*, which is called the *Multiplee*, and ordinarily the *Product* because produced by the other two, sometime the *Offcome*, as coming off them, sometime the *Factus*, or *Fact*, being made by them, in reference to which the two given Numbers, *Viz.* the *Multiplicand* and *Multiplier* are called *Factores*, or *Factors*, and sometime also the *Product* is called the *Rectangle* or *Plain*, which implies that one of the propounded numbers is taken for the length and the other for the breadth of a *Rectangle*, or plain Figure geometrical, in respect to which the given numbers are called sides, and so shall the *Product* be understood for the *Content* or *Area* of that *Rectangle* figure. *How to place the Numbers. Multiplicand what. Multiplier what. Product what, and how called. Factors and Rectangle what.*

Operation hath *Induction*, and *Perfection*; That sometime called *Simple*, and this *Compound Multiplication*.

Induction helpeth to know the product of any two digits multiplyed into themselves; without which knowledge great Numbers cannot be multiplyed; which is commonly thus done at the two ends of a Cross; on the left side place the two digits to be multiplyed, the one over the other; then substract each digit from 10. and set the several remains respectively collateral to the digit from whence it came; again substract either of these differences, it matters not which from the other digit, cross-wise, and this remain place under the digits: Lastly, Multiply the two first differences, and the number amounting place under them to the right hand of the number before set down, and this shall be the product of the two digits. As to know how much 7 times 8 is, they are placed as at A. then 7 abated from 10 rest 3, and 8 from 10 rest 2, *Induction called Simple Multiplication. Used by Record Example.*

as at B. Lastly 2 from 7, or 3 from 8 the remain is 5, and 2 times or twice 3 is 6, as at C. So is 7 times 8 found to be 56.

A $^7_8$ **X** B $^7_8$ **X** $^3_2$ C Sub. *Dat.* $^7_8$ **X** *Diff.* $^3_2$ Mult. 56

Used by Alsted.

This way is used both by *Alsted*, our Countrey-man *Record*, and Common School-masters, yet *Alsted* seems not well-pleased with it, calls it the Sluggards Rule, and prefers another before it, which is to break one of the digits into several parts, and multiply those small parts by the other digit, and add all these little Products together, as if in the former Example, 8 should be disected into 2. 2. 2. 2. and each of them multiplyed by 7 make 14. which set down 4 times, and added make 56 as before.

Example.

The Rule is undoubtedly true this Authour grounds this Operation on, (which is, If two Numbers one be cut into several parts, the product of the two Numbers shall be equal to the product of the one by the several parts of the other) but the purpose for which both this, and the former are brought, is not without exception; for to produce Rules to teach any thing, which require the knowledge of the thing pretended to be taught by the Rule, is not proper, and surely in *Logick* such reasoning would be accompted begging the Question, as to bring a thing doubtful to prove a thing in dispute: Yet such is the Mishap in these Rules which instead of teaching to know the Sum of any two digits, in order to learn to multiply; set the ignorant to multiply to find them out.

Best way of Induction.

The better way for the Learner, is to set down one digit so many times as there be Units in the other digit, and add these into one total, as to set down 8 seven times, and the added total will be 56, as before.

But to avoid the prolixity of this, the best way to have *ad Unguem*, the value of any two digits multiplyed together, is to learn by heart the Table commonly called *Multiplication Table*, where they are all expressed thus.

Multiplication Table.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Table how far inlarged by Pythagoras. *Use of the Table.*

This Table *Pythagoras* had enlarged to 12, others to 10, but 9 being sufficient, it is contracted here, and because 8 times 7, and 7 times 8, or any such like, is all one, some convert it into a Triangular form, using only the part above the black Scale.

The Table is thus to be read, 2 times, or twice 2 is 4, twice 3 is 6, twice 4 is 8, *&c.* So 3 times 2 is 6, and 3 times 3 is 9, and 3 times 4 is 12. and so of the rest. So that to find the Product of any two digits multiplyed together, is to find one of the digits in the left side, and the other in the head of the Table, and in the Angle common to both, is the product required: As to know how many 5 times 6 is, against 5 in the side, and 6 in the head, (or alternately contrary, 5 in the head, and 6 in the side) is found 30, and so much is 5 times 6, or 6 times 5.

The Compound Multiplication how wrought commonly.

Compleat Operation, proceedeth first, to multiply by Digits, and Articles, and afterward by mixt Numbers, and in either, the common, and select way of proceeding is observable. The

By Digits. The Common way to multiply by digits, is after the numbers are placed as before, to begin at the right hand, and multiply every figure of the *Multiplicand* by the multiplying digit, and what doth amount thereof place under the same if it be a digit, if an Article subscribe the Cypher, and reserve the figure of the Article (as in *Addition* was taught) to be added unto the Sum of the next multiplyed figure; and if it be a mixt number, set down the digit, and reserve the Article as before.

Example. *Example.* 5291 is to be multiplyed by 2, after the Numbers set, as at D. say 2 times or twice 1 is 2, which is subscribed, as at E. then twice 9 is 18, whereof 8 is subscribed, as at F. and 1. the Article borne in mind, again twice 2 is 4, and 1 carried is 5, to be set in the next place, as at G. and lastly twice 5 is 10. set as at H. So is the product of 5291 doubled or multiplyed by 2, the Sum of 10582.

D	E	F	G	H	
5291	5291	5291	5291	5291	Multiplicand.
2	2	2	2	2	Multiplier.
	2	82	582	10582	Product.

By Articles. In the same manner proceed to multiply by Articles, increasing the *Multiplicand* by the signifying figure of the Article, the Cyphers whereof are best placed as at the 10. 11. and 12. Sections of this Chapter, but with inartificial Artists are set down as in the following instance of multiplying 5291 by 20, as at I.

Example.

	5291	Multiplicand.
I	20	Multiplier.
	0000	
	10582	
	105820	Product.

By mixt numbers. The Common way to multiply by mixt Numbers is, after all the figures of the *Multiplicand* be gone through as before by the first multiplying figure of the *Multiplier* then take the second figure of the *Multiplier*, and proceed therewith as before, and so with the third, fourth, fifth, *&c.* multiplying figures, increase each figure of the one given number by each figure of the other, till all be gone over, observing to set the product of the first multiplyed digit of each, directly under the multiplying figure; and the rest gotten thereby orderly in a straight line to the left hand thereof; then under all these lines of production sometime called *Multiplees*, which will ever be as many, as there be figures or places in the *Multiplier*, draw another right line, and add all the particular products into one total product, and this shall be the desired number.

Example. *Example.* To increase 402 by 349, the numbers placed as at K. after the *Multiplicand* is gone over with 9 the first multiplying digit, the work stands as at L. then begin with 4, and say 4 times 2 is 8, which subscribe under 4, and 4 times 0 is 0, which set down to keep place, also 4 times 4 is 16, which makes the work as at M, and so proceeding do the like with the multiplying 3. Lastly, Adding those three lines of production together the total product of 402 multiplyed by 349 is found as at N. to be 140298.

K	L	M	N	
402	402	402	402	Multiplicand.
349	349	349	349	Multiplier.
	3618	3618	3618	Multiplees added.
		1608	1608	
			1206	
			140298	Total Product.

Select wayes of Multiplication. This Select way of *Multiplication* consisteth either in certain brief or compendious Rules, or other choice Methods of proceeding, whereby the work of the Common way

way is abbreviated or meliorated, and these are both comprehended in these 18 following Sections.

1. If an Unite be in the Multiplier. Because an Unite neither multiplieth nor divideth, wheresoever 1 is found in the *Multiplier*, it is but only to subscribe the *Multiplicand* in its due place among the *Multiplees*, and add them as before; as to multiply 342 by 12, the product is 4104. Set as at O or P.

Example.

	342 Multiplicand.	P 342	342
		684	12 Common way.
O	684 Double.	4104	684
	342		342
	4104 Product.		4104

2. If 2 be the Multiplier. To multiply by 2 called *Duplication*, is nothing else but to double every figure of the *Multiplicand*, and subscribe the number amounting, if the double be not above a digit, but if an Article or mixt number subscribe the Cypher, or remaining figure, and for the Article account the next figure to the left hand one more than the double; as

Example. 4372 duplicated or multiplyed by 2, for 2 is subscribed 4, for 7 the next amounting to an Article 4, and go 1. wherefore 3 the next is made 7, that is 1 more than the double, and the double of 4 is 8. Thus,

4372 Multiplicand.	4372
	2 Common way.
8744 Product.	8744

3. If 3. be the Multiplier. Example. *Triplication*, or to multiply by 3, is to add the given number to the double of the same, as to multiply 4372 by 3 produce 13116, for 4 the double of 2 added to 2 make 6, and 14 the double of 7 added to 7 make 21, of which 1 subscribed, and 2 carried away, which 2, 3 and 6 the double of 3 is 11, whereof 1 set down in the next place, and the other 1 reserved, which at last added to 4 and 8 makes 13, and for more ease, some first put down the double: Thus,

4372 Multiplicand.	4372
8744 Double.	3 Common way.
13116 Product.	13116

4. If 4. be the Multiplier. Example. *Reduplication*, *Quadruplication*, or *Multiplication* by 4 is to double the *Duplication*, as to multiply 15 by 4 is to double 30 the double of 15, and the 60 amounting is the Product; so in the former Example 17488 is the double of 8744, which was the double of 4372, therefore 17488 is the Product of 4372 multiplyed by 4.

8744 Multiplicand doubled.	4372
	4 Common way.
17488 Product.	17488

5. Multiplication by 5. Example. To multiply by 5, called *Quintuplication*, adjoyn a Cypher to the right hand of the *Multiplicand*, and then take the half thereof as to multiply 468 by 5, the Product is 2340.

468.0 Cypher adjoyned.	468
	5 Common way.
2340 Product.	2340

6. By 6. *Sextuplication*, or to multiply by 6, adjoyn a Cypher to the given number as before, take half thereof, beginning at the right hand, and to the half add the figure

Example. standing next before; as to multiply 468 by 6, the Cypher adjoyned makes it 4680, then

then the half of 0 is 0, but 8 next before is 8, so the half of 8 is 4, and 6 next make 10, also the half of 6 is 3, to which the next 4 and the 1 reserved of 10 make 8 for the next place, and at last the half of 4 is 2. So the Product is found to be 2808, or the half may be set down and added.

468.0 Cypher adjoyned.	468	468
	2340	6 Common way.
2808 Product		
	2808	2808

Septuplication, or to multiply by 7, add half each figure to the double of the figure next before, a Cypher being first adjoyned, as to multiply 468 by 7, the half of 0 is 0. but the double of 8 is 16, of which 6 is subscribed, and 1 reserved, then the half of 8 is 4, being added to 1 before reserved, and the double of 6 make together 17, whereof 7 is set down, and 1 reserved, which with the half of 6 and double of 4 make 12, the reserved 1 of which with the half of 4 make 3 to be set at last, and so the Product is 3276 — 7. *By* 7. *Example.*

468.0 Cypher adjoyned.	468
	7 Common way.
3276 Product.	
	3276

Octuplication, or to multiply by 8. substract the double of the given number from the same increased by a Cypher adjoyned to the right hand thereof, as to multiply 468 by 8. the double is 936, which taken from 468.0 leaves 3744 the Product desired. — 8. *By* 8. *Example.*

468.0 Cypher adjoyned.	468
	8 Common way.
936 Double substracted.	
	3744
3744 Product.	

Noncuplication, or to multiply by 9, adjoyn a Cypher to the Right hand of the given Number, as before, and from thence substract the same *Multiplicand*, as to multiply 468 by 9, the Product will be — 9. *By* 9. *Example.*

468 0 Cypher adjoyned.	468
	9 Common way.
468 Multiplicand sustracted.	
	4212
4212 Product.	

To multiply by 10, 100, 1000, *&c.* adjoyn so many Cyphers to the Right hand of the *Multiplicand*, as there be Cyphers in the *Multiplier*, as to multiply 468 by 100 two Cyphers being adjoyned, the *Product* is 46800. — 10. *By* 10. *&c.* *Example.*

468 Multiplicand.	468
100 Multiplier.	100 Common way.
46800 Product.	000
	000
	468
	46800

To multiply by 20. 30. 40. *&c.* 200. 300.400. *&c.* multiply the given number by the signifying Figure of the *Multiplier*, and to the right hand of the *Product* place so many Cyphers as shall be in the *Multiplier*, as to multiply 468 by 200, the Product is 93600. for the 468 doubled or multiplyed by 2, is 936, to which the 2 Cyphers in 200 are adjoyned. — 11. *By* 20. 30. &c. *Example.*

468 Multiplicand.	468
200 Multiplier.	200 Common way.
93600 Product.	000
	000
	936
	93600

12. When Cyphers are at the right hand.

Example.

To multiply when Cyphers are at the right hand of the *Multiplier* or both the given Numbers, first work with the signifying figures, and to the right hand of the Product place so many Cyphers as the *Multiplier* and *Multiplicand* had at the right hand places. As to multiply 3400 by 40, first 34 multiplyed by 4 produce 136, to which 3 Cyphers adjoyned make the true Product 136000.

3400	Multiplicand.	3400	
40	Multiplier.	40	Common way
136000	Product.	0000	
		13600	
		136000	

13. When Cyphers come between.

Example.

To multiply when one Cypher, or more, fall between the signifying figures of the *Multiplier*; instead of making a line of Cyphers, place only one Cypher to keep place and then proceed with the next figure of the *Multiplier*. As to multiply 1432 by 204, the *Product* will be 292128.

1432		1432	
204		204	Common way.
5728		5728	
28640		0000	
		2864	
292128	Product.	292128	

14. To multiply by nines.

Example.

To square any number of Nines, that is to multiply them into themselves, as 4 nines by 4 nines, &c. to the right hand of the number of nines to be multiplyed place so many Cyphers, as there be nines, then add 1 to the first Cypher, and substract 1 from the first nine, subscribe this 1 and 8 in their places, together with the Cyphers between 1 and 8, and the nines beyond 8, so is the Product obtained. *Example*, 9999 by 9999, produceth 99980001. Thus,

		9999	
		9999	Common way.
	9999.0000	89991	
abated	1 1 added	89991	
	99980001 Product	89991	
		89991	
		99980001	

15. To multiply without charging the memory.

Example.

To multiply without charging the memory by carrying the Articles. Multiply as before figure by figure, and what amounteth thereof set down if an article or mixt number, but if a digit subscribe the digit and before it a cypher in the place of an article, then multiply the next figure of the *Multiplicand*, and what amounteth subscribe the digit under the Article or Cypher before set down, and if there be an article, place it in the 100 place of the Sum, if no article, place a cypher as before instead thereof, and so going forward for every figure in the *Multiplier* make two lines of production. As to multiply 5142 by 43, because 3 times 2 is but 6 before it is a cypher placed, then 3 times 4 is 12, which is set down according to the former directions, the other like process of the work may be plainer discerned by the little lines drawn between the Numbers arising upon *Multiplication*.

5 1 4 2		5142	
4 3		43	Common way.
0‚1‚0-6		15426	
1-5 3 2		20568	
0‚1‚0-8		221106	
2-0 4 6			
2 2 1 1 0 6	Product.		

To

16. To multiply and bring the Total into the last line of Production.

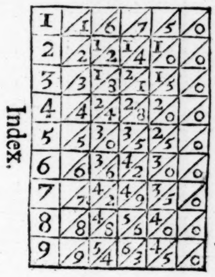
To multiply and bring the total production in the last line of the work, without the ordinary manner of *Addition*, or charging the memory, when the numbers are placed as usual, multiply figure by figure, as before, and what ariseth thereof set down the digit under the line, and the article above, or a Cypher instead thereof, then multiply the next figure, and there add the article before set down, cancelling him with a dash of the Pen, and this amounting Sum place as before, the digit under the line one place nearer to the left hand, and the article if any above, and so do till all the figures of the *Multiplicand* be run through, and the product of the last figure of the *Multiplicand*, set down the article beneath the line as well as the digit; and when the first line is done, draw a rectangle line toward the left hand, cutting off the right hand figure, then take the second *Multiplier*, and begin to multiply therewith, and add in their several places the uncancelled figures within the rectangle line, as well as the articles newly set down, and thus renew the rectangle lines, and proceed according to the number of the multiplying figures: So at last will all the figures be cancelled except those below the last rectangle line, and such right hand figures as were cut off by them, which being pulled down as they stand yield the desired product. *Example.* As to multiply 8546 by 423 when 6 is increased by 3 the amounting 18 is placed 8 beneath the line, and 1 above, then 3 times 4 which is 12, and 1 above the line is 13, of which 3 is set beneath the line, and 1 above, and the first 1 cancelled, then 3 times 5 is 15, and 1 above the line uncancelled is 16, whereof the 6 subscribed and the 1 is set at top; again 3 times 8 is 24, and 1 at top make 25, which is subscribed, so is the first multiplying figure done with, and the work standeth as at Q. Then 8 cut off by the rectangle line, and 2 the second multiplying digit proceedeth with, beginneth with twice 6, which is 12 and 3 above the rectangle make 15, of which 5 subscribed in his place leaveth 1. to be set above the Rectangle, so twice 4 is 8, with 1 last set down, and 6 over him in all make 15, which is placed as before, and so proceeding to the end of the *Multiplicand* by this digit 2, the work appeareth as at R, and in like manner proceeding with the multiplying 4, the finishing Paragraph is represented at S, where are found under the line 36149, and cut off by the Rectangle, 5 and 8, which together make the product 3614958.

```
   Q          R            S
  8546       8546         8546         8546
   423        423          423          423   Common way.
   xxx        xxx          xxx        --------
 ------     -------      -------       25638
 25638      2563|8       2563|8       17092
             xxx|         xxx|       34184
            ------       -------     --------
            19655        1965|5      3614958
                          322|
                        -------
                         36149
```

17. To multiply by Nepaires Bones.

To multiply by *Nepair's* Bones: Alwayes Tabulate or Place the *Multiplicand* on the Bones, and by the help of the *Index* taking the Sum of every *Diagonal Square*, or *Rhomboide*, answering to the several multiplying figures in the *Multiplier* you have the several lines of production which dispose as before taught, *viz.* each line one place nearer to the left hand, then the other according to the multiplying figures, and add them into one total, and you have the product desired. *Example.* As to multiply 16750 by 258, the *Multiplicand* 16750 being tabulated on the Bones, and the *Multiplees* answering to 2. 5 and 8 in the *Index*, taken out are 33500. 83750. and 134000. which added make the product 4321500. See the *Bones* and *Index* at T. the Productions added at V. and as some set them at W.

Bones.

T 

33500

83750

134000

```
      16750            16750
        258              258
   ---------        ---------
     134000            33500
V     83750      W     83750
      33500           134000
   ---------        ---------
    4321500          4321500
```

To

18. To multiply by the Back-side of the Bones.

To multiply by the back-side of the Bones, which some use when they would conceal the numbers they multiply from the present Spectators. First tabulate the *Multiplicand* on the Bones as before, then turn up the Bones just upside down, laying that Face which was underneath upward, and work with the number which here appeareth tabulated, as if it were the true number, to the product found thereby add the *Multiplier*, then substract this total from the *Multiplier*, adding a sufficient number of Cyphers thereto, and the Remain shall be the Product desired.

Example.

As in the former instance 16750 tabulated on the back-sides, will be tabulated 83249, which multiplyed into 258 produceth 21478242, whereto 258 added, the product and total together are 21478500, this number substracted from 258, and so many Cyphers adjoyned as the *Multiplicand* had Figures, *viz.* 5, leaves the former Product 4321500.

$$\begin{array}{r} 25800000 \\ \hline 21478500 \\ \hline 4321500 \\ \hline \end{array}$$

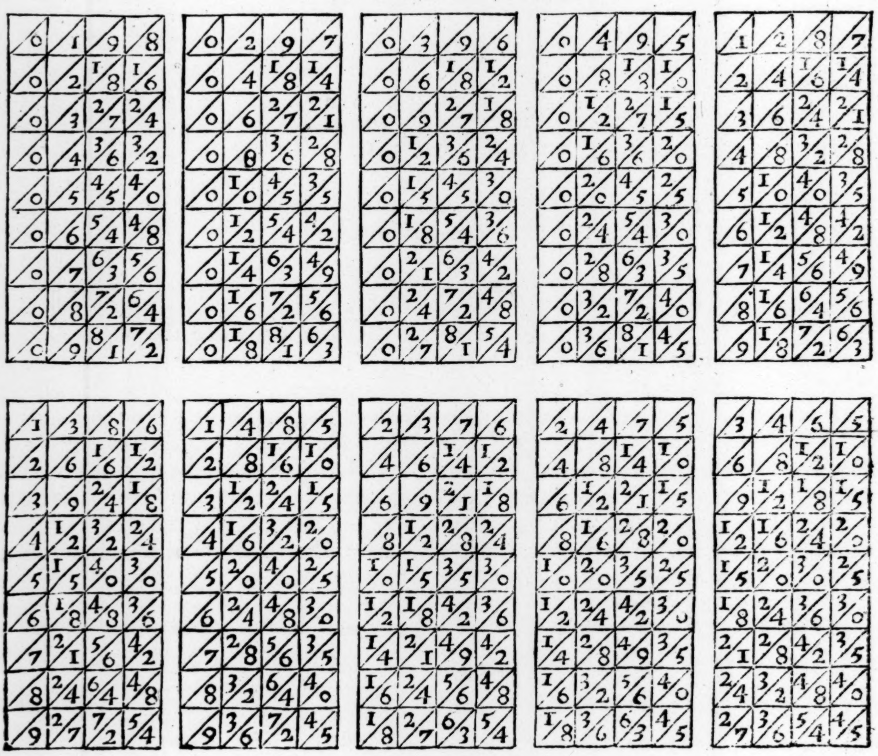
Bones were invented by Nepair.

The use of these Bones or Rods have been sufficiently explained by others, it were but *actum agere* to insist upon it here: Only in brief, They are called *Nepair's* Bones from the Inventer the Honourable Lord of *Marchiston*, they are best made of *Ivory*, or *Wood* 4 square, of an equal thickness, about one fifth part of an Inch square, their length about 9 times their breadth, they are in number 10, but to have two or three sets of them will be convenient, because with one set can be tabulated but only 4 figures of one and the same *Species*; as 4 Cyphers, 4 Units, &c. but 20 or 30 Rods can tabulate 8 or 12. They have all the digits on them, and their *Multiplees* to 9, being only *Pythagoras*'s Table cut in pieces. There is also an *Index* belonging to the Rods, of the same length and breadth. See more in the following Table.

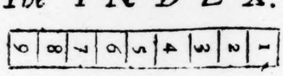
They are Multiplication Table cut in pieces.

The Description of all the four faces of every one of the Ten Rods, and the Index.

Figures of the Bones.



The INDEX.



Probation is here as before, to try the truth or falshood of the work. Particularly *Multiplication* performed by any of the Select Rules aforegoing, may be proved by the Common way. But generally the proof of all sorts of *Multiplication* may be had, either by *Addition*, Casting away Nines, or *Division*. *Proof of Multiplication.*

Addition, (especially in great *Multipliers*,) is a most tiresome and tedious way; yet it is thus set down the *Multiplicand* so many times as there be Units in the *Multiplier*, and add all together, the total will be the same with the Product if the work be right; As 1343 multiplyed by 3, the Product is 4029, and so is the total of that number set down 3 times. *By Addition, most tedious.*

1343		1343	
1343		3	
1343		4029	Product
4029	Total.		

The truth of this dependeth on that *Theoreme*, which is the very Basis of *Multiplication*, *viz.* as an Unit to the *Multiplier*, so is the *Multiplicand* to the *Product*, and *contra*, for if 6 times 4 be 24, then 1 to 4 shall be as 6 to 24, and 1 to 6, as 4 to 24. But this way of Proof makes the remedy worse than the Disease, and therefore is rejected. *Geometry a Theorem that is the Basis of Multiplication.*

The Proof by casting away 9, is to cast 9 as oft as may be from the *Multiplicand* and *Multiplier* severally, and set the two remains at the opposite Angles of a Cross; then multiply these remains one into another, and if the Product thereof be under 9, set it down at another of the Angles of the Cross, if above 9, cast away 9, and set down the residue; Lastly cast away 9 from the *Product*, and the Remain thereof shall alwayes be equal to the Remain last before put down, if the work be right. As in the former Example proved by *Addition*, the nines cast from 1343 leave 2, which set opposite to 3 the *Multiplier*, and multiplyed thereby produce 6, and so much will be left when 9 is cast from the *Product* 4029 standing on the Cross thus, *Proof by 9 uncertain.*

$$\begin{array}{ccc} & 6 & \\ 2 & \times & 3 \\ & 6 & \end{array}$$

This kind of Proof upon the same accompt it was found faulty before in *Addition*, is fallible also here. But the most certain and infallible proof of *Multiplication*, is by *Division*; of which more in the next Chapter, wherefore to conclude this; take the two following Observations. *Proof by Division is best.*

Observations.

1. If any Number be multiplyed, it is so often added to its self as there be Units in the *Multiplier*.
2. If a Number be compounded of two Numbers, and that Number multiply another Number, the *Product* shall be equal to the *Product* of that Number multiplyed by those two Numbers. As 6 is compounded of 2 and 3, let 6 then multiply 9, the Product is 54, which is equal to 9, multiplyed into 2, that is 18, and 18 multiplyed into 3, which is 54.

CHAP. VIII.

Division of Integers.

Division, called by some *Partition*, is the Compound Analysis of Numbers, and that part of Conjunct Numeration, whereby one Number is substracted from another so often as it is contained in it, and by that means is found how many of the one Number are contained in the other: So that the third Number, or Number artificially obtained by the two propounded Numbers shall so often contain an Unit, as the greater of the two given Numbers contains the lesser, and it serveth instead of many *Substractions*, of which and *Multiplication*, it consists, and is composed. *Division what.* *Serveth instead of many Substractions.*

Division observeth *Collocation*, proceedeth to *Operation*, and concludeth with *Probation*.

How to place the Numbers. *Dividend what.* *Divisor what.* *Collocation* respecteth the *Nomenclature*, and duely placeth every number in manner following. First set down the Number to be divided, which is sometime called the *Dividuum*, but commonly the *Dividend*; and under it at the left hand (contrary to all the former Operations of *Arithmetick*) place the Number by which it is to be parted or divided, called the *Divisor*, observing to set the foremost figure to the left hand of the *Divisor*, under the foremost left hand figure of the *Dividend*, if the *Divisor* may be substracted from those figures of the *Dividend*, which stand above over them; but if not, then let the first left hand figure of the *Divisor* stand under the second left hand figure of the *Dividend*, and so in order proceeding to the right hand place all the other figures of the *Divisor*, then draw the Decrescent Lunular, or Seperatrix to distinguish these given Numbers from the Number to be found by *Division*, which is called the *Quotient*, and sometime *Parabola*; *Quotient* is derived from *Quoties*, because the number is gotten by enquiring how often the *Divisor* is contained in the *Dividend*. *Parabola* ariseth, from the application of a plain number, to a given Longitude, that a congruous Latitude may be found out; as will be more demonstrable in Figural numbers. If after *Division* be ended, any figures be left uncancelled, these are called the *Remain*, and denote that the *Dividend* could not be exactly parted by that *Divisor*; but there will be a number remaining, unless the *Dividend* be reduced to another Denomination: As in denominate numbers: And this Remain is alwayes to be less than the *Divisor*; and distinguished from the other figures by little Lunular or Rectangle Lines, like the Quotient Line; or set over the *Divisor* with a little Line between; as a *Fraction*. Lastly may be noted, That to part, measure or compare a number, are termes used synonimically, and signifie but to divide that number.

Quotient what and whence it comes. *Parabola what.* *Remain if any to be left uncancelled.* *To be less than the Divisor.* *How to be set.* *Synonimical termes.*

Operation hath both *Induction*, and *Perfection*; The one sometime called *Simple Division*; and the other *Compound*; and both presuppose the *Dividend* to be greater than the *Divisor*.

Induction called Simple Division. *Induction* by the help of *Multiplication Table*, sheweth how often any digit is contained in the *Multiplee* of any two digits: And consequently by suffrage of the same Table findeth how often the same digit is contained in all the intermediate Numbers that happen between such *Multiplees*.

Use of Multiplication Table for Induction. To find how oft any digit is contained in the *Multiplee*, enter the Table with the *Multiplee*, among the *Areal* Numbers just against the given digit at the left side, and the number desired shall be the digit just over the *Areal* Number at top; but if the given digit be found in the head of the Table, the desired number shall be the digit at the left side just against the *Areal* number. As to find how often 2 is contained in 10, entring with 10, in the collateral Columne against 2 in the side, 5 is found at top over 10, as also against 2 in the head, and 10 in the perpendicular Collumne, which sheweth that 2 is contained 5 times in 10, and in like manner is found how often any of the other *Multiplees* below or above 10 do contain 2, as 4 containeth 2 twice, 6 three times, &c.

By consequence also may be found how often 2 is contained in all numbers intermediate, between those *Multiplees*, for 3 intermediate between 2 and 4 shall contain 2, but once and a half part more, and 5 intermediate between 4 and 6 shall contain 2. but twice and a part more, all intermediate numbers never exceeding the precedent *Multiplees* by an Unit, but only by a part or parts: for where there is but one number intermediate there will an Unit remain after the integral content is substracted; where two intermediate Numbers are, 1. will remain over the first intermediate number, and 2 over the second, and so accordingly in the rest.

Compound Division how wrought commonly. *By Digits.* *Complete Operation* proceedeth first to divide by digits and Articles, and afterwards by mixt numbers; and in either the common and select way of proceeding is considerable.

The Common way to divide by digits is, after the given numbers are placed as before, begin at the left hand, and see how many times the Divising digit can be taken out of the figure of the *Dividend* standing over him, or to the left hand thereof, if the Divising digit stand not under the first left hand figure of the *Dividend*, and having found how often this *Divisor* is contained in such figure or figures of the *Dividend*, set beyond the *Quotient* line a figure signifying how often; as 1 for once, 2 for twice, &c. then substract the content of the *Divisor* so many times as is set down in the *Quotient* from the *Dividend*, and set the *Remain* if any be over the figure of the *Dividend*, where it was left remaining, alwayes cancelling with a dash of the Pen the *Divisor* when done with, and those figures of the *Dividend* whence any thing was taken, then set down the *Divisor* one place nearer to the right hand, and thereby get another *Quotient* figure, and so proceed

proceed as before to the end of the *Dividend*; and so many times as the *Divisor* may be set down in the *Division*; so many figures or cyphers will be in the *Quotient*.

Example. To divide 3762 into 3 equal parts, the numbers being set as at A. inquire how many times the *Divisor* may be had out of 3. the *Dividend* figure over him, and finding it may be taken once, place 1 in the *Quotient*, and substracting once 3 out of 3 there is 0 remaining; then cancelling the 3 at top, and the *Divisor*, the work stands as at B. Then placing the *Divisor* under 7, and inquiring as before, 2 is gotten and set in the *Quotient*, and twice 3 which is 6 substracted from 7 leaves 1 remaining, as at C. Again placing 3 the *Divisor* under 6, by like inquiry is found 5 times 3, which is 15 to be contained in 16, therefore 5 is placed in the *Quotient*, and abating 15 from 16. leaves 1 to be set over 6, and the work appears as at D. Lastly, 3 placed under 2 by inquiry as before, 4 is obtained for the *Quotient*, which multiplying the *Divisor* 3 produceth 12 to be taken from the 12 before left of the *Dividend*. So there remains 0. And the *Quotient* is found to be 1254, which is the third part of the given *Dividend* 3762, and the Compleat work stands as at E. *Example.*

	E	D	C	B	A
	11(0 Remainer	11	1		
Dividend	3762(1254 Quotient.	3762(125	3762(12	3762(1	3762(
Divisor	3333	333	33	3	3

In like manner procede to divide by Articles, dividing the *Dividend* by the signifying figure of the Article, for the Cyphers do but only keep place, and are best set as in the third *Section* of this *Chapter*; though with young Beginners they are set down as in the Example following to divide 1762 by 30 where the *Quotient* is 58, and 22 remain, as at F. *By Articles.*

Example.

	F
	2(2
Dividend	1762 (58 Quotient or $58\frac{22}{30}$
Divisor	300
	3

The Common way to divide by a mixt number is somewhat more difficult, yet is the former Method still observed, in placing the numbers and finding out the first *Quotient* figure only with this difference, that when the *Divisor* was a single digit or article, the same *Quotient* figure is to be as big, as could be substracted from the figures of the *Dividend*, to the left hand thereof; now the first left hand divising figure is to be taken no oftner from the figure or figures standing over him than that also every following figure of the *Divisor*, may be taken so often out of the figure or figures that stand over them, or are to the left hand of them, by borrowing as in *Substraction*, to supply the deficiency thereof; and though sometimes the first figures of the *Divisor* will seem to leave too much behind, yet the 2^d or 3^d figure may want: So as it will be convenient to reckon in mind a little before Operation whether the just figure be taken, which must never be above 9 at most, nor under so many times as the *Divisor* is contained, for then will the *Remain* be greater than the *Divisor*, which is not to be suffered. *By mixt Numbers.*

Example. If 34633 were to be divided by 12, the numbers placed as before, and seeking how often 1 is contained in 3 there is found 3, if 1 were single, because 1 divideth not, but being joyned with 2, if 3 should be cancelled there would be but 4 left, and then thrice 2 which is 6 could not be taken from 4, therefore 1 is to be taken but twice out of 3, and the remaining 1 set over 3, and 2 is set in the *Quotient* as at G. Then seeing 1 is taken but twice out of 3 the other figure of the *Divisor*, 2 must be taken but twice, which is 4, from 4, at top resteth 0, to be set down because of 1 to the left hand as at H. Then setting the *Divisor* down again, and enquiring how many times 1 may be had out of 10 is found but 8, because there will not else be enough left to take the other figure of the *Divisor* so often out of the remaining figures to the left hand; therefore taking 8 for the *Quotient* out of 10 at top, resteth 2 to be set over 0, as at I. And 8 times 2 is 16 in like manner taken out of the figure over him, that is out of 6 cannot, but out of 16, borrowing 10 there remains 0, and the borrowed 1 out of 2 rests 1 as at K, moreover removing the *Divisor*, and by inquiry as before, 8 is found again for the *Quotient*, and twice 8 taken from 23 leaves 7, as at L. Lastly, Removing the *Divisor*, and seeking thereby as before, 6 is found for the *Quotient*, taken from 7 leaves 1, and twice 6 is 12 from 13 is left 1 remaining, when the *Division* is ended at M. where the work is compleat. *Example.*

```
   G          H          I          K          L           M
                         2        1 2        1 22        1 221
   1          10         10         100        1007        1007( 1
   34633(2    34633(2    34633(28   34633(28   34633(288   34633(2886.
   12         12         122        122        1222        12222
                         1          1          11          111
```

If the Dividend be less than the Divisor.

When the *Dividend* is lesser than the *Divisor* set the numbers in manner of a *Fraction*, with a right line between them; For as no *Substraction* in Abstract Integral Numbers can be made of a greater Number from a lesser; so can no lesser *Integer* be divided by a greater in Abstract Numbers, but the further practise therewith must be referred to Fractionary or Contract Operations; As 15 cannot be divided by 16 in whole Numbers, but may be set thus $\frac{15}{16}$(.

Select waies of Division.

The Select way of *Division* consisteth either in certain brief Rules, or other choice Methods of proceeding, bettering, or abbreviating the Common way; comprehend in the 18 *Sections* following.

1. If the Divisor be 1. with Cyphers.

Example.

The Ground thereof.

First, To divide by an Unit with Cyphers, as 10. 100. 1000. *&c.* Cut off from the *Dividend* to the right hand, so many figures as there be Cyphers in the *Divisor*, and place the figures so cut off over the *Divisor* in form of a *Fraction*; as 3423 divided by 10, the *Quotient* is $342\frac{3}{10}$, and by 100. $34\frac{23}{100}$.

The Reason of this Rule, and several others, is grounded upon that *Theorem.* That an Unit neither multiplyeth, nor divideth, for once 2 is but 2 still; and if application be made of 1 to 2, it may be had out thereof twice; and the like it is with any other Number besides 2.

2. If alike Number of Cyphers be in the Data.

Example.

If there be a like Number of Cyphers in the *Dividend*, and *Divisor*, and an Unit the only figure of the *Divisor* cut off the Cyphers by a perpendicular stroke of the Pen: and the residue of the *Dividend* to the left hand shall be the *Quotient*. But if the *Divisor* be more than 1, divide the remaining figures of the one by the Remains of the other given number, as 3400 divided by 100 gives 34 in the *Quotient* thus $\frac{34}{1}|\frac{00}{00}$. and 34800 divided by 12 gives 29 thus.

```
  1   |
  10  |
  348 | 00(29
 -----|
  122 | 00
  1   |
```

3. If Cyphers be at the right hand of the Divisor.

Example.

If the *Divisor* only have one Cypher to the right hand or more, as it happeneth in Articles. Place the Cypher or Cyphers under the right hand figure or figures of the *Dividend*, and divide all the way by the signifying figure or figures of the Article unto the Cypher or Cyphers before set down, and the *Division* is done striking away the Remainer, as in the following Examples, 20 being *Divisor* to 8401, the Cypher of 20 is set under 1, and 840 is divided by 2, and the *Quotient* is $420\frac{1}{20}$, as at N. 800 dividing the same *Dividend*, *Quotient* is $10\frac{401}{800}$ as at O, and 150 dividing the same *Dividend*, the *Quotient* $56\frac{1}{150}$ as at P.

```
                                            3
                                            39
N   840|1(420      O   8|401(10      P   840|1(56
    2220               8800              1550
                                          1
```

4. If a Cypher be taken in the Quotient.

Example.

In *Division*, If a Cypher at any time be taken in the *Quotient* (which happeneth when the *Divisor* being set down cannot be taken once from the numbers standing over him) then forthwith cancel the *Divisor*, and remove him one place nearer to the right hand, and meddle with none of the Dividend figures, and if this be foreseen the setting down the *Divisor* may also be spared by placing the *Divisor* two places nearer to the right hand: Neverthelesse 0 must be set in the *Quotient*. As to divide 1340 by 13 at the second setting down the *Divisor*, 13 cannot be taken out of 4, wherefore 0 being set in the *Quotient* the *Divisor* is cancelled, and set down again, without altering the *Dividend*, because nought was taken therefrom.

```
                                    1(1
1340(1          1340(10          1340(103 Quotient
13              133              1333
                1                11
```

When

When it happeneth in *Division* that by multiplying two digits together, there ariseth an Article, meddle not with the digit of the *Dividend*, standing directly over head, but go to the place of the Article. As in dividing 853 by 24, in the second enquiry twice 5 will be 10, wherefore cancelling the 1 of the 13, the 3 is left, likewise 4 times 5 is 20, which taken from the left hand 3 of the 33, leaveth 13 remaining. *5. If an Article arise in multiplying 2 digits. Example.*

```
 1|1
 23|
 853(35 12/24
 244
  2
```

When in dividing Numbers at the right hand, it happeneth that the *Divisor* hath cut off all the figures of the *Dividend*, so that there remaineth nothing on the *Dividend* but Cyphers; then to the right hand of the *Quotient* adjoyn so many Cyphers as are yet remaining to the *Dividend*, without any part of the *Divisor* standing under them, and the work is finished. As 3654000 divided by 180 or 18, will declare by the following Operations. *6. If Cyphers remain on the Dividend uncancelled. Example.*

```
3654000(20300        3654000(203000
1888    0            1888
 11                   11
```

To take the half of any Number called *Mediation*, *Bipartition*, or *Division* by 2, if the given Number consist of even figures begin at the left, or right hand, it matters not which, and take of every figure respectively the half, and subscribe under the same figures. But if any figure be odd, the best way is to begin at the left hand, and take the least part next the half of the odd figure, and augment the succeeding figure by 10. As to substract the half of 42983, thus the half of 4 is 2, of 2 is 1, of 9 is 4, of 18 is 9, and of 3 being the last 1 $\frac{1}{2}$. So is the full half thereof 21491 $\frac{1}{2}$. *7. To half a Number, or to divide by 2. Example.*

In like sort it may be accustomed to take the third, fourth, fifth, sixth, seventh, eighth, and ninth parts, of any number, and some of these also may be otherwise obtained sooner than to set down the *Divisor* so often as in the Common way of *Division*, for *Tripartition* is but *Division* by 3. *Quadripartition* to divide by 4, *&c. Example*, to take the third part of 3687, the fourth part of 5460. *To take the 3, 4, &c. part of a number. Example.*

```
3)3687                 2                  4)5460              222
  ----               3687(1229              ----              5460(1365 Com way.
1229 Quotient.       3333    Common way   1365 Quotient       4444
```

Quadripartition, or to divide by 4, may also be thus performed by half the half of the given number, as in the former instance 5460, half is 2730, from whence half abated, leaves 1365 the quarter part as before: *8. To divide by 4. Example.*

```
4) 5460
                        222
2) 2730 half.           5460(1365 Com. way.
                        4444
2) 1365 Quotient.
```

Quintipartition, or to divide by 5 may likewise be effected thus, double the number given, and cut off the right hand figure or cypher; if any figure be cut off, take half that figure. So 2340 doubled is 4680, from whence 0 cut off, leaves 468 the fifth part thereof. So also 4569 parted by 5 gives 913$\frac{4}{5}$. *9. To divide by 5. Example.*

```
                                        5)4569
5) 2340          2340(468 Com. way.     913|8          14
Quotient. 468|0  555                                  4569(913 Com. way.
                             Quotient. 913|4/5         555
```

10. *To divide by 6. Example.* *Sextipartition*, or to divide by 6, alſo may be thus done; take half the third part of the given number. As to take the ſixth part of 3684, the third part is 1228, half which is 614 the number ſought.

6) 3684

3) 1228 Third part.

2) 614 Half. Quotient.

2
3684(614 Com. way.
666

11. *To divide by 8. Example.* *Octipartition*, Or to divide by 8, is but to take half the quarter part of the given Number, as to get the eight part of 5460, the quarter part is 1365, whereof the half is 682, and one half the queſited number.

8) 5460

4) 1365 Quarter part.

2) $682\frac{1}{2}$ Half. Quotient.

62(4
5460(682 Com. way.
888

12. *To divide by 9. Example.* *Nonupartition*, or to take the ninth part of a Number, is to take the third part of the third part of the given Number. As to divide 5463 by 9, the *Quotient* will be 607. So will the remain, if the third part of 1821 be taken, which is the third part of 5463.

9) 5463

3) 1821 Third part.

3) 607 Third Part. Quotient.

5463(607 Com. way.
999

13. *To divide by 20, &c. Example.* From hence it follows, that to divide by 20. 30. 40. *&c.* 200, 300, 400, *&c.* after the Cyphers are placed, as *Section* 3. above, the half, third, fourth part, *&c.* of the remaining figures of the *Dividend* may be taken. As to divide 45156 by 50, when 0 is ſet under 6, the fifth part of 4515 is taken, which is 903, and ſix fifties are left remaining.

50) 4515|6
|0

$903\frac{6}{50}$ Quotient.

4515(6 (903 Com. way.
555 0

14. *To divide by Nines. Example.* When any ſquared number of nines is given to be divided by the number of nines whereof it was produced, place the *Diviſor* under the right hand places of the *Dividend*, add them together, and from the total cut off all the Cyphers; the Remain ſhall be the *Quotient*. As to return 99980001 being the ſquare of 9999 into the Root, I add 9999 to 99980001, and the total is 99990000, which 4 Cyphers cut off, the Remain is the Root.

99980001
9999

Radix 99990000

8
198
979
080
18988
99799
008888
1887999
99980001(9999 Common way.
9999999
99999
999
9

To

To cut ſhort the borrowing work in the top figures in *Diviſors* conſiſting of many figures (a moſt commendable practiſe) do thus, after the numbers are placed, and the firſt Quotient figure found out as before, then begin with the right hand figure of the *Diviſor*, and take him ſo many times as the quotient figure denotes from the figure that ſtandeth over him if it may be, if not, borrow in imagination One, or more Tens, as occaſion is, that *Subſtraction* may be made, and place the remain at top as before, then take the next figure of the *Diviſor* and do the like with him, adding in for every 10 borrowed before 1, and ſo proceed toward the left hand with all the diviſing figures, then ſet down the *Diviſor* again, and reiterate this manner of work till the *Diviſion* be ended. *15. To cut ſhort the borrowing, a good and uſeful way.*

As to divide 10816010 by 1234 when the numbers are placed, and 8 found for the firſt quotient figure, then beginning at 4, and taking him 8 times, which is 32, becauſe but 6 ſtandeth over 4 cannot wholly be taken without borrowing 3 tens, but 2 may be taken out of 6, and leave 4 at top, or reckoning entire 32 may be taken out of 36 and leave 4, either way 3 is reſerved to be added in the next place. Then coming to 3 in the *Diviſor*, and taking 3 times 8 which is 24, with the former reſerved 3, make 27, which are to be taken out of 1, but cannot without borrowing 3 tens again, and then 27 out of 31 leaves 4 to be ſet at top. Again 2 in the *Diviſor* multiplyed into 8 in the quotient, produce 16, and 3 before borrowed is 19, where but 2 tens need be borrowed, and out of them and 8 in the *Dividend* which are 28 if 19 be taken there will remain 9. Laſtly 1 taken 8 times, and 2 laſt borrowed added make 10, which abated from 10 at top leaves 0, not ſet down becauſe no figure ſtandeth to the left hand thereof. So is the work with the firſt ſetting down the *Diviſor* ended, Then the *Diviſor* removed one place nearer to the right hand, and the like work reiterated till the whole *Diviſion* be ended, the *Quotient* is 8765, and this work thus performed hath 13 figures leſs at top than that wrought the Common way, and much more will it ſhorten great *Diviſions*, and after a little practiſe be as ready and facil as the other. The ſeveral Paragraphs of the work, at Q. R. S. T. are further Exemplary. *Example.*

Q

```
   944
10816010|8
  1234
```

R

```
    80
   9442
10816010|87
  12344
   123
```

S

```
     6
    801
   94427
10816010|876
  123444
   1233
    12
```

T

```
     6
    801
   94427
10816010|8765
  1234444
   12333
    122
     1
```

```
      1
     26
    188
   2001
  194342
  227427
10816010(8765
  1234444
   12333
    122
     1
```

Some to favour the memory practiſe the *Italian* way, to multiply the *Diviſor* by the ſeveral quotient figures as they are found, and accordingly ſubſtract the ſeveral *Multiplees* from the *Dividend* one after another: And they place the numbers moſt conveniently in this form. *viz.* The *Dividend* between two Parallel Lines, to the right hand whereof the *Quotient* beyond the Decreſcent Lunular, as before; and to the left hand beyond an Increſcent Lunular the *Diviſor*; and to repreſent the place of the firſt figure of the *Diviſor*, under the *Dividend* a Cypher, and when a quotient figure is gotten, and the *Diviſor* multiplyed thereby, this *Multiplee* is firſt ſet down under the *Dividend*, and ſubſtracted, and the remainder ſet at top, then another Cypher is placed under the *Dividend* to denote the remove of the *Diviſor*, and ſo another quotient figure obtained, the proceſs is as before, only in ſetting down the ſeveral *Multiplees*, there are two formes; the one as at V. and the other as at W. The firſt of which is the foregoing Example, wherein the ſeveral Paragraphs of the work are diſtinct. The other in dividing 144980099 by 1798 containeth all the difficulties this kind of *Diviſion* can have, and is ſet entire. *16. The way called the Italian way not chargeable to the memory.*

Numbers ready for Work.

Diviſor. Dividend. Quotient.

```
--------------------------
1234) 10816010 (
        0
--------------------------
```

Firſt Paragraph.

```
        944
--------------------------
1234) 10816010 (8
        0
--------------------------
   8    9-8-72
  ---
9872
```

Second Paragraph. *Examples:*

```
        80
       9442
--------------------------
1234) 10816010 (87
        00
--------------------------
   78   9-8-7-28
  ----   8-6-3
 9872
8638
```

Third

Third | and Fourth | Paragraphs.

```
Third      6                     and Fourth    6
          8 0 1                               8 0 1
          9 4 4 2 7                           9 4 4 2 7
1234) 1 0 8 1 6 0 1 0 (876       1234) 1 0 8 1 6 0 1 0 (8765
        0 0 0                               0 0 0 0

 678   9-8-7-2 8 4               5678   9-8-7-2 8 4 0
         8-6-3'0'                          8-6-3'0'7'
 9872     7-4'                    9872      7-4'1'
8638                             8638         6'
7404                             7404
                                 6170    1 0 8 1 6 0 1 0  Proof.
```

V

In this Example the figures of each *Multiplee*, are marked with a little line for the plainer evidence, though not needful, the following Example is plain without.

```
                          (1
                         673(6  Remain
                        11412 5(7

Divisor.        1798) 144980099 (80634  Quotient.
                       00000

              43608   14384
                       0000
              14384   10788
              0000     5394
Multiplees   10788      7192
             5394
            7192      144979932  Total of the Multiplees.
                            167  Remainer added.

         Proof.       144980099  Dividend Returned.
```

W

17. *To divide and set the Remains below.* Some not only when they have enquired, and obtained a quotient figure, multiply the *Divisor* thereby, and place the *Multiplee* underneath for *Substraction*, as before, but also set what remaineth after *Substraction* beneath the same *Multiplee* without cancelling any figures, and then having the *Divisor* in a moveable piece of paper to apply to the *Dividend* at pleasure, they enquire from those remains with the residue of the *Dividend*, for another quotient figure, and so continue the work till the *Division* be perfected.

Example. As to divide 34636 by 12, the first quotient figure is found to be 2, whereby 12 multiplyed, the *Multiplee* is 24, which substracted from 34, leaves 10 to be subscribed, then apply the *Divisor* in the moveable paper to 10 the remain, and 6 in the *Dividend* next to it, and 8 may be found for the quotient, and so the like is done with the several remains after the substraction of 96, 96 and 72 the Products of the *Divisor* multiplyed by the Quotient figures, and the last remaining 4 is left at the bottom alone thus,

```
Divisor.  Dividend.  Quotient.
                                          Proof.
     12)  34636  (2886
                                         24
   6882   24                              96
                                           96
     24   10                                72
    96    96
   96                                  34632  Total of the Multiplees.
  72      10                               4  Remain added.
          96
                                       34636  Dividend returned.
           7
          72

 Remain    4
```

In

In this manner of *Division* there be divers, for the more ready finding the quotient figures multiply the *Divisor* by all the 9 digits, and set a Prick or Cypher under that figure of the *Dividend* which the right hand figure of the *Divisor* ought to stand under; so it may be presently discerned which of the several Multiplications of the *Divisor* may be substracted from those figures of the *Dividend* standing from the prick or cypher toward the left hand; and the digit producing that *Multiplee* is to be set in the Quotient. *Variety of this work.* As to divide 58688 by 24, the first pricked figure is 8 next to 5, because there should stand the right hand figure of the *Divisor*, and amongst the multiplyed numbers is found 48 next lesser to 58 the pricked number, which is to be substracted, and 2 which begot the same 48 set in the Quotient, the remaining 10 is subscribed, then pricking the next figure of the *Dividend*, and applying the *Divisor* in the moveable paper thereto, with the remaining 10, which is 106 the next lesser number thereto among the multiplyed numbers is 96, which therefore is to be substracted, and 4 set in the Quotient, because 96 is 4 times 24 the *Divisor*, and so proceeding the quotient is found to be 2445, the *Remain* 8, and the work standeth thus. *Example.*

24	1
48	2
72	3
96	4
120	5
144	6
168	7
192	8
216	9

```
Divisor Dividend Quotient
-------------------------
 24)    58688 (2445            Proof.
          ....
-------------------------
         48                    48
        ----                    96
         10                      96
          96                     120
        ----                  ---------
         10                   58680 Total of the Multiplees.
          96                      8 Remain added.
        ----                  ---------
         12                   58688 Dividend returned
         120                  ------
        ----
Remain.    8
```

Others in subscribing the remaines after *Substraction*, pull down and adjoyn the remaining figures of the *Dividend* to the right hand: and others never underwrite the multiplyed Products, but having them in a loose paper, apply them as they do the *Divisor*, and subscribe the Remains only. Examples of both formes appear in the *Division* last before mentioned. *Another Variety.*

Examples.

```
Remaining numb. of the Dividend        Remains only
        pulled down.                    subscribed.
---------------------------       -------------------------
 24 )  58688  (2445                24)  58688  (2445
        ....                              ....
---------------------------       -------------------------
      48                                10
     ----                              ----
      10.688                            10
       96                              ----
     ----                               12
       10.88                           ----
       96                         Remain    8
     ----
        12.8
        120
     ----
Remain     8
```

To divide by *Nepair's* Bones, Set the *Divisor* on the Top of the Bones, then shall you have the *Divisor* multiplyed by all the 9 digits, out of which you may chuse such convenient numbers to substract from the *Dividend* (according to the several imagined settings down of the *Divisor*, noted by pricks on the Number as was mentioned in the last variety) which will be the next lesser to the Numbers so pricked, and the Indices of *18. To divide by Nepair's Bones.*

Example.

such substracted numbers, are to be set in the *Quotient*. As to divide 1396788 by 5678, the *Divisor* 5678 is tabulated on the Bones, and the several *Multiplees* taken out, and 7 in the *Dividend* pricked, where the right hand place of the *Divisor* should stand; the next lesser number to 13967 in the *Dividend*, among the *Multiplees* is 11356, which hath 2 for the *Index*, therefore 2 placed in the *Quotient*, and substraction made, 8 is pricked in the *Dividend*, and among the *Multiplees* 22712 is found to be the next lesser to 26118, which remain to the last prick of the *Dividend*, and the *Index* of 22712 is 4 to be set in the *Quotient*. Lastly the right hand 8 in the *Dividend* pricked, and finding the next lesser number to 34068 to be the *Multiplee* of 6, this is therefore set in the *Quotient*; and after Substraction nothing remaineth. As here appeareth.

1	5	6	7	8	5678
2	1/0	1/2	1/4	1/6	11356
3	1/5	1/8	2/1	2/4	17034
4	2/0	2/4	2/8	3/2	22712
5	2/5	3/0	3/5	4/0	28390
6	3/0	3/6	4/2	4/8	34068
7	3/5	4/2	4/9	5/6	39746
8	4/0	4/8	5/6	6/4	45424
9	4/5	5/4	6/3	7/2	51102

```
5678)  1396788  (246
        11356
         2611
         22712
          3406
          34068
```

General Proof of Division.

Probation is the Proof whether the *Operation* be true or false, and is either general or particular.

By Substraction most tedious.

General Proof is for any kind of *Division*, and this may be effected three wayes.

First by *Substraction*. For if the *Quotient* be substracted from the *Dividend*, so many times as there be Units in the *Divisor*, there will remain at last no more than the remainer of the *Division*, if the work be well wrought. As in the former instance, *pag.* 31. where 3762 is divided by 3, the *Quotient* is 1254. If therefore 1254 be taken 3 times from 3762, there will 0 remain, as on the *Division*.

3762	Dividend.
1254	Quotient abated
2508	Remain.
1254	Second Substraction.
1254	Second Remain.
1254	Third Substraction.
0	Last Remain.

```
 11(0
3762(1254
 3
```

Grounded on a Theorem that is the basis of Division.

This is a most laborious, and therefore an useless way of trial, yet the truth of this is grounded on the *Theorem*, which is the foundation of *Division*, *viz.* The *Dividend* to the *Divisor* is as the *Quotient* to an *Unit*, and *contra*. For if 20 divided by 5 give 4 in the *Quotient*; then 20 to 5 is as 4 to 1, and 1 to 4 is as 5 to 20.

Proof by 9 uncertain.

Secondly, *Division* is commonly proved by casting away nines thus. Cast away 9 from the *Divisor* as oft as may be, and place the remain at one Angle of a Cross, and the remain, after 9 in like sort is cast from the *Quotient*, place at the opposite Angle, then multiply both these remains together, and to the amounting number add the remain of the *Division* if any, and if the total be under 9, place it at another Angle of the Cross, but if above 9, cast away 9 as before, and set down the residue. Lastly, cast away all the nines from the *Dividend*, and set this last remain opposite to the remain last before set down; which if the work be right will be equal thereto, otherwise not. As in the second Example before, the *Divisor* 30 leaving 3, and the *Quotient* 58 leaving 4 when the nines are rejected; these 3 and 4 multiplyed produce 12, which added to 2 and 2 the remain of the *Division* maketh 16, from whence 9 cast resteth 7, and so much will be left when the nines are cast from 1762 the *Dividend*.

```
   7
 3 X 4
   7
```

```
 2(2
1762(2(58
 33 0
```

This

This kind of Proof for the uncertainty before noted in *Addition* is laid by.

Thirdly, as *Addition* and *Substraction*; So are *Multiplication* and *Division*, mutually the most excellent and infallible proofs of each other. *Proof of Multiplication by Division.*

Multiplication is proved by *Division* thus. Divide the *Product* by the *Multiplier*, and the *Quotient* will return the *Multiplicand*; or contrarily the *Product* divided by the *Multiplicand*, the *Quotient* shall return the *Multiplier* when the work is duely wrought. As for instance in the Example of the 16 Section of the precedent Chapter where the *Product* is 3614958, the *Multiplier* 423, the *Multiplicand* 8546.

	2 195 23043				256 19653		
Product, Dividend	3614958	(8546	Quotient.	Product, Dividend	3614958	(423	Quotient.
Multiplier, Divisor	423333 4222 44		Multiplicand.	Multiplicand, Divisor	854666 8544 85		Multiplier.

Division on the contrary is proved by *Multiplication* thus. Multiply the *Quotient* by the *Divisor*, and to the *Multiplees* add the Remain of the *Division*, if any be, the total *Product* shall be equal to the *Dividend*, if the *Division* be right. As for instance in the last Example of the 16 Section of this Chapter where the *Dividend* is 144980099, the *Divisor* 1798, the *Quotient* 80634, and the *Remain* 167. *Proof of Division by Multiplication.*

Multiplicand.	80634	Quotient.
Multiplier.	1798	Divisor.
	645072 7257067 5644386 806341	Remain added
Product.	144980099	Dividend.

From hence flow these two Consectaries. First the *Multiplicand* may be known if the *Multiplier*, and *Product* be given; or the *Multiplier* if the *Multiplicand* and *Product*: By dividing the *Product* by the other given Number. As above in the Proof of *Multiplication*. *Consectaries from hence.* 1.

Secondly, If the *Quotient*, and *Divisor* be given, the *Dividend* may be known by *Multiplication* of the one given number into the other; as above in the proof of *Division*. 2.

And if such numbers were quesited, the questions may in like manner be resolved.

Examples in Questions of Multiplication *and* Division.

1. What number was that which multiplyed by 15, produced 1380? *Questions in Multiplication and Division.*
Answer, 92. for 1380 divided by 15 giveth 92 in the *Quotient*.
2. What number being divided by 15 will give in the *Quotient* 92?
Answer, 1380. for 92 multiplyed by 15 will produce 1380.

	3		92
Operations.	1380(92	Quotient.	15
	255 1		460 92
			1380 Product.

Particular Proof of *Division* is peculiar to some particular methods of Operation; and will not serve for all sorts of *Division*. And so the *Divisions* wrought according to the 16, 17, and 18. Sections of this Chapter may be proved by *Addition*; for the several *Multiplees* added together with the Remain of the *Division* if any be, will infallibly return the *Dividend*; and all the select wayes of *Division* may be tryed by the Common way; as in the Sections before may be seen, and therefore needless here to be repeated, only for a Close to this Chapter, and also to this first part of this Book, concerning *Integers*, take these few Observations and Theorems following. *Particular Proof of Division.*

1. If

Observations and Theorems.

1. 1. If a Quantity, either Magnitude or Number, be made of other two Quantities by their Multiplication, the one of them being the *Factor*, will measure (that is, evenly divide) the same Quantity made by the other, (being the other *Factor.*) As 20 made of 4 into 5, the one of them will measure 20 by the other 4, and so will 4 measure the same 20 by 5. For the measure of 5 is 4 times in 20, and the measure of 4 is 5 times in 20.

2. 2. If a Quantity, either Magnitude or Number, be made of two other Quantities by their Multiplication, it is all one whether any other Number be divided by that one Quantity, or by those other two Quantities. As if 20 made of 4 and 5 as before, divide 60, the Quotient is 3, so is it all one if 60 be divided first by 4, and the Quotient thereof 15 by 5, for this last Quotient will be 3.

3. 3. *Fractions* arise from *Division*, when the *Dividend* is lesser than the *Divisor*, or cannot be evenly measured thereby, a *Fraction* will remain. As several Examples of *Division*, *pag.* 31. and elsewhere demonstrate.

4. 4. When the *Quotient* and *Divisor* are equal, and nothing remain on the *Division*, the *Dividend* is a square number, and the *Divisor* the Root thereof. As in the instance, Sect. 14. of this Chapter 99980001 is a Square Number, and 9999 the Root, *De quibus in suo loco.*

Partis primæ Libri primi

FINIS.

THE

THE SECOND PART OF THE FIRST BOOK.

CHAP. I.

Of FRACTIONS.

Fractions next to Integers to be dealt with.

INtegers dispatched in the former part of this Book. The next sort of *Homogeneal* Numbers to be examined are *Fractions*, sometime called *Fragments*, sometime *Parts*, sometime *Broken Numbers*. This shall be done generally and particularly.

Fractions what they are.

In general, it is requisite to know what they are, whence they come, and how to express them.

By what passed before *Chap.* 2. *pag.* 5, and 6, *Fractions* may be understood to be *Broken Numbers*, , or broken parts of a *Whole Number*, extending infinitely under an Unit as *Integers* do above.

Whence they arise.

Abstract Fractions first treated of.

They arise from the Division of their *Integers*, as was observed in the last Chapter before, and in summ are expressed with the same Notes or Characters, as the *Integers* from whence they come. Those arising from *Abstract Integers* are first to be treated of; and the proper subject of this Part, where *Fractions* without any other note of distinction are to be taken for such. To proceed then,

Broken Numbers brokenly expressed by 2. Terms.

Fractions as they are broken numbers, so are they brokenly expressed, verbally, or in writing, by two termes or numbers, disected as it were the one from the other, but *Integers* are uniforme.

The two termes or numbers expressing a *Fraction*, are by some set one before or over the other, with a *Colon* between them. As Two thirds thus, 2 : 3, or thus $\frac{2}{3}$, but the most usual form is to set them one over another, with an interjacent line. As Two Thirds thus, $\frac{2}{3}$.

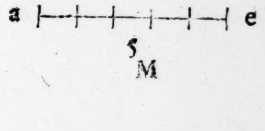
Which of them Numerator, and which Denominator.

Of these two termes, the neathermost under the line, or to the right hand of the *Colon*, denoteth the Unit to be divided into so many equal divisible parts, and is called the *Denominator*. As if it be 2, it noteth the Unit is parted into halfs, if 3 into thirds, if 4 into fourths, *&c.*

The uppermost, or foremost to the left hand, sheweth how many of those parts into which the Unit is broken, are signified to be contained in, or taken from the content or value of the *Fraction*, and this is called the *Numerator*, and sometime *Nominator*. So that the *Numerator* alway containeth the number of the parts signifyed by the *Fraction*; and the *Denominator* the greatness of those parts.

Example.

As in $\frac{3}{5}$, the *Numerator* is 3, the *Denominator* 5, and the *Fraction* signifieth three parts of any one *Integer* divided or broke into five equal parts, which content may be demonstrated by supposing the *Unit* or *Integer* to be the line a. e. equally divided into 5 parts, the Division made by o upon the line toward a. shall be equal to the *Fraction* $\frac{3}{5}$ of that line, and the residue of the line marked with o. e. shall be $\frac{2}{5}$, and make up the whole line, *&c.*



Numerator.

Denominator.

M

From

Inlarging the Denominator or Numerator what effect.

From hence it appears. The more the *Denominator* is increased, the farther is the value of the *Fraction* from the whole Number or *Integer*, and the more the *Numerator* is augmented, the nearer is the value to the whole. Again the *Numerator* may be inlarged till the value be more than the whole, and contrarywise the *Denominator* may be diminished till the *Fraction* become an Unite or more; but the *Denominator* though never so great, yet will ever import a part of the *Integer*, though never so small. For every *Fraction* if the *Denominator* be increased is less, and if decreased is greater in quantity. All this dependeth on that fundamental principle. Such proportion as the *Numerator* beareth to the *Denominator*, the same beareth the parts signified by the *Fraction* to an Unite as in $\frac{3}{4}$ of a Shilling, it is evident that 3 to 4 is as $\frac{3}{4}$ to 1 shilling, or 12 pence, three quarters of which being 9 pence 9 to 12, shall in proportion be as 3 to 4.

The Principle on which the effect depends.

Which Termes are Heterologal or Heterogeneal and the contrary.

Numerators, and *Denominators*, compared one to another, are called *Heterologal*, and sometime *Heterogeneal* Termes; but either of them by themselves, are called *Homologal*, and sometime *Homogeneal*.

Numerator is pronounced before the Denominator.

As in *Integers* the Numeration of their Quantities precedeth their *Denomination*, so in *Fractions*, the *Numerator* is alwayes pronounced before the *Denominator*.

That by Cardinal, and this by Ordinal Numbers.

The *Numerators* are verbally expressed by *Cardinal Numbers*, as One, Two, Three, Four, *&c.* The *Denominators* are best pronounced by the Ordinals, as halves, thirds, fourths or quarters, fifths, *&c.* As $\frac{1}{2}$. $\frac{3}{4}$. $\frac{6}{7}$. $\frac{17}{20}$. thus, one half, three quarters, six sevenths, seventeen twentieths, *&c.*

Fractions diversly divided.

In particular, *Fractions* (as well as *Integers*) are capable of being diversly considered, in reference both to their quantity and quality; as they are single or plural, abstract or contract; generally or specially. A short narrative whereof followeth in the ensuing Table.

A Table of the Nature of Fractions.

- Fractions considered in
  - Quantity
    - Numerator
      - Single.
      - Plural
        - Commensurable.
        - Incommensurable.
    - Denominator
      - Certain.
      - Uncertain.
  - Quality
    - Abstract
      - Naturally. Less than an Unit.
      - Nominally. Arithmetical.
    - Contract
      - Generally.
        - Geodætical.
        - Figural.
      - Specially.
        - Decimal.
        - Astronomical.
        - Logarithmical.
        - Cossical.
        - Surde.
        - Speciosal.

Quantity of a Fraction how signified.

The Quantity of every *Fraction*, as before noted, being signified by his *Numerator*, and *Denominator*, both Termes are to be considered distinctly for the particular knowledge of the *Fraction*.

Numerator notes the content, if Single or Plural.

The *Numerator* denoting the content of the *Fraction*, shews whether it be a single, or a plural *Fraction*; that is, whether it contain only a part of the *Integer*. As $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$, *&c.* or many parts. As $\frac{2}{3}$. $\frac{3}{4}$. $\frac{4}{5}$, *&c.*

These Plural *Fractions* are again distinguished into Commensurable and Incommensurable.

Plural are Commensurable or Incommensurable.

Commensurable, called *Compositi inter se*, or Numbers compound among themselves, are such as besides an Unit have some Number that will divide exactly both the Termes without leaving any Remain. As $\frac{5}{15}$ is Commensurable, for both the Termes may be evenly divided by 5.

Common and Greatest Common Divisor what.

The Number so exactly dividing the Termes, is called the Common Divisor, or Measurer, and because some Termes have more Numbers than one that will thus divide them, the greatest of these Numbers is called the greatest Common Measure, or Divisor, and in Latine *Communis mensura maxima*, or *Communis Divisor maximus*. As $\frac{6}{12}$ may be exactly divided by 2, but the Greatest Common Measure is 6.

Incommensurable,

Incommensurable, called *Primi inter se*, or prime among themselves, are they that cannot be measured by any Number but an Unit, which not dividing properly, they have no number that will serve for a Common Divisor evenly to divide both the Terms without leaving some remaining number upon the Division of one of them. As $\frac{9}{20}$, where 9 may be exactly divided by no number but 3, and 3 cannot exactly divide 20, but 2 will remain. Such *Fractions* are said to be in their least Terms.

Incommensurable in their last Terms.

The Product of any Incommensurable Terms multiplied one into another, is called in Latine *Dividuus communis minimus*, and in *English* the least common Dividend, of which little use is made, but the Common Divisor is very useful, as may be further seen in the next Chapter of *Reduction*. Here by the way may be seen the Difference between Prime and Compound Integers and Prime and Compound Fractions. Uncompound Integers can be measured by no number of multitude exactly. As 3. 5. 7. 11. 13, *&c.* But *Fractions* may have either of the termes taken singly, or apart, a Compound Integer; yet may be *Primi inter se*, considered joyntly. As $\frac{8}{9}$ is Incommensurable, though both 8 and 9 are Compound Integers.

Common Divisor is useful.

Difference between Integers and Fractions.

Fractions are again considered in respect to their Denominators, which are either Certain or Uncertain.

Denominator certain or uncertain.

The Denominators are certain in *Decimals*, *Astronomicals*, and *Logarithmes*, the first alwayes increasing or decreasing by 10. The second by 60. The third according to the *Radius*, and are omitted; because certainly known.

Where certain, and omitted,

Geodæticals certain in regard their Denominators are known by their Denominations, but various according to such Denominations, and the Denominators of *Fractions* sometime used with them are uncertain.

Geodæticals how certain and uncertain.

All other Denominations besides these are uncertain, that is may be any absolute Number or Quantity whatsoever; and therefore of necessity must be expressed.

Where uncertain and expressed.

As *Fractions* are considerable in their quantity, so in their quality. As whether they are Abstract or Contract. Abstract are free from any restraint, but may be a part or parts of any quantity or magnitude. Contract on the contrary have some annexed Denomination, or special Denominator.

Quality of a Fraction to be Abstract or Contract.

Abstract absolute *Fractions*, have community in their nature with others of what sort soever that is to be inferior to an Unit be they single or plural in their Numerators; but they have a name proper to them for distinction sake to difference them from other sorts, that is *Arithmetical Fractions* some time called *Abstract*, *Vulgar*, or *Common Fractions*. These are such as before in this Chapter are instanced, and hereafter in this part at large handled.

Abstract what.

The names of Abstract Fractions.

Contract Fractions, like *Integers*, are more generally contract in *Geodæticals* and *Figural Numbers*, and more especially in *Decimals*, *Astronomicals*, *Logarithmes*, *Cossicks*, *Surdes*, and *Species*. In the two first, and three last principally by reason of their Denominations: In the other three by their special Denominators, as well as in respect of their Denominations. The Nature and Operations of which were briefly before touched, *Chap.* 2. but are particularly to be unfolded in the several parts of the two next Books.

Contract Fractions General or Special.

Denominator and *Denomination*, though sometime used promiscuously, differ in respect of Quantity and Quality, the Denominator sheweth as before the greatness or smalness of the parts the *Integer* is understood to be divided into, the *Denomination* declareth the Nature of the *Integer*, of what kind or quality it was, whither Pounds, Ounces, Yards, Ells, Men, Moneths, *&c.*

How Denominator and Denomination differ.

The *Denomination* is sometime called the Sirname, this annexed makes any abstract Fraction contract, and thereby makes the number seem to be doubly denominate, *Viz.* As to the *Denominator* and *Denomination*, therefore are *Fractions* accompted before less absolute than *Integers*.

Sirname of a Fraction what.

Arithmetical, or *Vulgar Fractions*, in this part dealt with as they arise from *Integers*, so in some things are like them, but in others different. As a greater *Integer* cannot be substracted from, nor divide a lesser: But a lesser *Fraction* may be divided by a greater, though in *Substraction* they agree, that the *Subtrahend* shall be the lesser number. They agree to be composed, and dissolved, but their Operations therein are different, and the most easie in the one, are most difficult in the other, though in the prime and compound parts of Original Numeration they take part with *Integers*, to be added, substracted, multiplyed, and divided, yet because of their *Denominators*, they resemble the mode of Contract Numbers when their *Denominators* are unlike, and will first be reduced before they can be added, or substracted, and so compared to *Integers* in their Simple Elements go in a Retrograde motion, placing Ortive Numeration before Original. They mix well enough with *Integers*, and are as hand-maids to them, whence arise the first sort of *Heterogeneal* Numbers, who in this part with them receive their Operations and Resolutions.

How Abstract Fractions agree and differ with Integers.

Whence Improper Fractions come.

Resolutions. In Fine from the mixture of *Integers* and *Fractions*, issue *Improper Fractions* in reference to which others are called *Proper Fractions*, between both which by increasing the Numerators of *Proper Fractions*, or the Denominators of *Improper Fractions*, *Equal Fractions* spring forth, and are all easily known thus.

Proper Fraction what.

Proper Fractions alwayes have the Numerator less than the Denominator, for then the parts signified are less than an *Unit* or *Integer*, though the Terms be never so great. As $\frac{4}{5}$. $\frac{21}{37}$. *&c.*

Equal Fraction what.

Equal Fractions have the Numerator, and Denominator alwayes equal, and then the *Fraction* is equal to an *Unit*. As $\frac{4}{4}$. $\frac{5}{5}$. $\frac{21}{21}$. *&c.* and it were better to express such by $\frac{1}{1}$ seeing they are but 1. These are improperly called *Fractions*, because they contain one *Integer*, but are thus represented for convenience in work.

Improper Fraction what.

Improper Fractions have alwayes the Numerator greater than the Denominator, for then is the *Fraction* greater than the whole. As $\frac{5}{4}$. $\frac{37}{27}$. $\frac{7}{2}$. $\frac{9}{4}$. *&c.*

Proper Fraction

Proper Fractions are of two kinds, Conjunct, and Divided.

Conjunct.

Conjunct, sometimes called *Fractions of Integers*, are such as are conjoyned together by the Copulative [*and*]. As $\frac{2}{3}$. and $\frac{3}{4}$. *&c.* implying more broken parts than one.

Divided. These are Fractions of Fractions.

Disjunct or divided, are frequently called *Fraction of Fractions* differenced from the other by the intervening [*of*] instead of [*and*]. As $\frac{2}{3}$ of $\frac{3}{4}$. *&c.* denoting only a part of a *Fraction*,

Improper Fractions of 2 sorts.

Improper Fractions also are of two sorts, *Viz.* either they include several *Integers*. As $\frac{9}{3}$. $\frac{20}{4}$. *&c.* or else one or more *Integers* with some part or parts of the *Integer*. As $\frac{5}{4}$. $\frac{13}{5}$. *&c.*

Elements of Fractions.

The Simple Elements of *Fractions* in sum may be concluded under two heads; either to increase or decrease, their Terms or their Value, these as more essential are comprehended under *Original Numeration.* Those as accidental under that part of *Ortive Numeration*, called *Reduction.*

CHAP. II.

Reduction of Fractions.

Reduction what and how useful.

REduction in general, is that part of *Ortive Numeration*, whereby one Number, or Magnitude is reduced to another. Useful in *Fractions* and *Contract Numbers*, that they may receive a more apt form of Operation, in those to bring them from one Term to another; in these from one Denomination to another, yet in both retaining the same value or content.

What it consists in.

Reduction consisteth principally in *Multiplication*, and *Division*, yet occasionally converseth with *Addition* and *Substraction*, as necessity requireth.

The work thereof to be seen in the Sections following.

Reduction so far as concerns *Arithmetical Fractions* now in hand will be understood in two things, *Operation* and *Probation*.

Operation may be thus methodized.

Reduction of
- Proper Fractions
  - To their least Termes
    - by the great Common Measure. §. 1.
    - by Bipartition, Tripartition, *&c.* §. 2.
  - To like Denominators.
    - Conjunct by Multiplication. §. 3.
    - Disjunct by Multiplication. §. 4.
- Improper Fractions
  - To Integers, or mixt Numbers, by Division. §. 5.
  - From mixt Numbers, by Multiplication and Addition. §. 6.
- Integers.
  - To the form of an Improper Fraction, by an Unite. §. 7.
  - To any desired Denominator, by Multiplication. §. 8.
- Proper or Improper Fractions to any desired Denominator by Multiplication and Division. §. 9.
- Integers with Proper, and Improper Fractions. §. 10.

§. Abbreviation by the Common Divisor.

The first sort of *Reduction* belonging to *Proper Fractions* is to reduce them to their least termes. This is commonly called Abbreviation, because it doth abbreviate or cut short the terms of a *Fraction*, and so consequently the work therewith. And because by *Euclide, lib.* 7. *Prop.* 17. if one Number multiply two Numbers their Products will retain the same proportion

proportion either to other the Numbers did before *Multiplication*; of necessity if one Number divide two Numbers, the Quotients must be still proportional. And hence it is evident that the termes of the same *Fraction* may be infinite, and yet bear the same proportion one to another. As $\frac{4}{5}$ may be expressed by $\frac{8}{10}$. $\frac{16}{20}$. $\frac{32}{40}$, &c. multiplying both terms by 2, and by $\frac{12}{15}$. $\frac{36}{45}$ &c. multiplying by 3, every of which *Fractions* is still in value, but $\frac{4}{5}$. And as in denominate numbers it is not proper to speak of 100 pence, 500 shillings, or such like, when there are other Denominations to abbreviate the numbers. So neither in *Fractions* is it proper to mention any termes save the least that they may be reduced into. Wherefore then upon the whole matter, because Abbreviation is but brief Division, it appears that if the Greatest, or Commensurable termes of any *Fraction* be divided by any number that will equally measure or divide them without leaving any remain, then shall the *Fraction* be expressed in lesser terms by help of this Common Divisor; that is to say, by dividing both *Numerator* and *Denominator*, thereby observing to let the Numerator of the new Fraction, be the Quotient of the old Numerators Division: And the Denominator his Quotient likewise.

Most proper to set Fractions in their least Termes.

Commensurable terms sometimes admit several Common Divisors, among which some one is the greatest. As in the foregoing Chapter was observed; wherefore it is necessary to know how to find this great Common Divisor, because thereby the *Reduction* is perfected at once, when the lesser Common Divisors repeat their operations several times.

How to find the Greatest Common Divisor.

The Greatest Common Measure of two Numbers, by *Euclide, lib. 7. prop. 2.* is found by *Substraction*, but the far better way is by continual *Division* of the greater by the lesser, and of the Divisor by the Remainer till either a Cypher or an Unite remain. For that Divisor which first divideth the Dividend without leaving any remain, is the Greatest Common Measure of both the numbers, and both the numbers thereby are found to be Commensurable or Compound among themselves. But if no such Divisor be found till an Unit remain, the numbers are Incommensurable, and will not be reduced to lesser terms.

Example. If $\frac{4899}{5888}$ were the Fraction given, and it were desired to know if the Numbers were Commensurable, and if so, what is the greatest Common measure, and consequently their least terms. Then dividing 5888 by 4899, and 4899 by 989 the remain of the first Division, and again 989 by 943 the second remain, and so continuing this manner of *Division* 23 is found to be the first Divisor, that leaveth 0 remaining on the *Division*, wherefore the Fraction is Commensurable, and the Great common Divisor is 23. by which both terms divided, the new Fraction is found to be $\frac{213}{256}$ which are the least terms of $\frac{4899}{5888}$ and numbers Incommensurable. For dividing $\frac{213}{256}$ as before, no Divisor will be found evenly to divide them till an Unit remain which as aforesaid neither multiplyeth nor divideth.

Example.

	(989	(943	(46	(2	(0	26	
$\frac{4899}{5888}$	5888(1	4899(4	989(1	943(20	46(2	4899(213	Numerator.
	4899	989	943	466	23	2333	
				4		22	

						1	
	(43	(41	(2			123	
$\frac{213}{256}$	256(1	213(4	43(1	$\frac{4(1}{2}$(20	$\frac{2}{1}$(2	5888(256	Denom.
	213	43	41			2333	
						22	

By frequent practise some Common Divisor or other if not the greatest may quickly be espied. To spare therefore such multifarious *Division* in getting the Greatest Common Divisor. It is common to use *Bipartition*, *Tripartition*, or any such select *Division* by the Digits, as in the Chapter of *Division* before was set forth, if the given terms of the Fraction will exactly be divided by any of them; placing the Numerators along upon a line, and the Denominators beneath, and separating the several Quotients in the work, with a down right line or dash of the Pen. As if $\frac{288}{576}$ were to be reduced into its least termes; first mediating, or taking half the Numerator and Denominator, As below till $\frac{9}{18}$ be brought forth, and then the third part of both numbers accordingly. At last the Fraction is reduced to $\frac{1}{2}$ thus.

§. 2. How to abbreviate the Common way.

Example.

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2

Where Cyphers are at the right hand.

Also where Cyphers are in the terms of the Fraction to the right hand as before in *Division*, a like number of them from both Numerator and Denominator may be cut off by a dash of the Pen, and the residue left, or reduced lower if it may be. As $\frac{300}{400}$ thus $\frac{3|00}{4|00}$ will thereby appear to be but $\frac{3}{4}$. Also if from $\frac{2500}{7500}$ be cut off the two right hand Cyphers, the residue $\frac{25}{75}$ straight way is seen to be but $\frac{1}{3}$.

But if no such Common Divisor can be found among the Digits, then the great Common Measure, if any is to be sought, as before; and the Fraction reduced to its least terms thereby.

§. 3. To bring Conjunct Fractions into one Denominator.

The next sort of *Reduction* belonging to proper Fractions is to reduce them from Different to like Denominators, that they may be expressed in more Commodious numbers. For as in Integers, Units must be added to, or substracted from Units, Tens to or from Tens, &c. And as in Denominate numbers Pounds must be added with, or taken from Pounds, Shillings, with or from Shillings, &c. So in Fractions, must halfs be added to halfs, thirds to thirds, &c. and substracted accordingly, or else if the Denominators be unlike, both must be reduced to like Denominators before *Addition* or *Substraction*.

The Reduction of different Denominators into like Denominators is sometime called Reduction to one Denomination, and the Denominators reduced being all one, the same need be set down but once to all the given Fractions, which Denominator so placed is called the Common Denominator.

Common Denominator what.

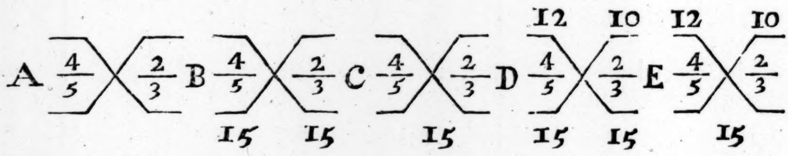
This Reduction differs as the Fractions to be reduced are Conjunct, or Disjunct. Proper Fractions conjunct are two, or more.

When only 2 Fractions are given, and the Denominator Incommensurable.

When two Fractions are given to be thus reduced, dispose them for form sake on either side of a Cross, and if the Denominators be Incommensurable multiply both the Denominators together, and that Product shall be the New or Common Denominator to both the given Fractions; Then for new Numerators multiply Crosswise the Numerator of the one Fraction by the Denominator of the other, and place the Product respectively overt that Fraction whose Numerator was in the Composition.

Example.

As to reduce $\frac{4}{5}$ and $\frac{2}{3}$ to one Denomination. They are set as at A. and then multiply 5 by 3, the Product 15 is the Common Denominator, set as at B or C. also multiplying 4 by 3 the Product 12 is the Numerator to be set over 4, and multiplying 2 by 5 the Product 10 is the other Numerator to be set over 2, as at D. or E. So are the two reduced Fractions $\frac{12}{15}$ and $\frac{10}{15}$.

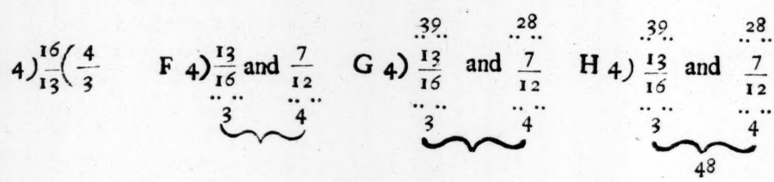


When the Denominators are Commensurable.

If the Denominators of the given Fractions be Commensurable, then first reduce them to their least terms, and alternately place the least term of the one under the other Fraction, and thereby multiply the Numerators respectively for new Numerators, and either of the Denominators multiplyed by the least terms under him shall be the Common Denominator,

Example.

As to reduce $\frac{16}{12}$ and $\frac{7}{12}$ to one Denominator 16 and 12, being Commensurable reduced by 4 their greatest Common Measure, make their least terms 4 and 3, which 3 coming of 12 placed under 16, and 4 coming of 16 under 12, the work appears as at F. then multiplying 13 by 3, and 7 by 4, the Numerators 39 and 28 are produced at G. and 16 by 3, or 12 by 4 the Common Denominator is found to be 48, and the work stands compleat as at H. where is found $\frac{13}{16} = \frac{39}{48}$ and $\frac{7}{12} = \frac{28}{48}$ being reduced.



When

When more than two Conjunct Fractions are given to be reduced to like Denominators multiply all the Denominators together for the Common Denominator, and to find new Numerators, multiply each Fractions Numerator, into the Denominators of all the other Fractions except its own Denominator. *When more than 2 are given, and the Denominators Incommensurable.*

As to reduce $\frac{2}{3}$ and $\frac{4}{5}$ and $\frac{6}{7}$ into like Denominators, multiplying 2 the Numerator of the first by 5 the Denominator of the second Fraction, and the Product 10 by 7 the Denominator of the third, the amounting Product 70 is the Numerator of the first Fraction. For the second Multiply 4 the Numerator of the second by 3 the Denominator of the first, and the Product 12 by 7 the Denominator of the third. So the Product 84 is the second Fractions Numerator. Then multiply 6 the Numerator of the third by 5 the Denominator of the second, and the Product 30 by 3 the Denominator of the first. So is 90 the Numerator of the third Fraction, unto whom the Common Denominator shall be 105. for $3 \times 5 = 15 \times 7 = 105$, and the three reduced Fractions set as at I. or K. *Example.*

Denominators.	Numerators.			I	K
3	2	4	6		
5	5	3	5		
15	10	12	30	$\frac{70}{105}$ and $\frac{84}{105}$ and $\frac{90}{105}$	$\frac{70 \text{ and } 84 \text{ and } 90}{105}$
7	7	7	3		
105	70	84	90		

Some deliver the Rule thus. Multiply all the Denominators together for a Common Denominator, and divide that Common Denominator by each several Denominator, and multiply the several Quotients by their respect Numerators. As in the last Example. *Variety of work.*

3	105(35	105(21	105(15
5	3 .2	5 4	7 6
15	New 70	84	90 Numerators.
7			
Common 105 Denominator.			

Either of these wayes will serve if the Denominators be Incommensurable, but if the Denominators, or any of them be Commensurable, the Fractions thus reduced will not be in their least terms. Therefore to reduce Conjunct Fractions of unlike Commensurable Denominators to one Common Denominator, and yet keep the Fractions in the same value, and least terms, do thus. Reduce all the Denominators to their least termes, if they be all Commensurable, and by these least terms get a Common Denominator, and thereby new Numerators, as in the Operation last before. *Example.* To reduce $\frac{3}{4}$ and $\frac{5}{6}$ and $\frac{7}{8}$. Thus the Denominators abbreviated to their least terms are 2. 3. 4. the Common Denominator gotten thereby is 24, and the several Numerators 18. 20. 21. As by the Opperation appears. *When the Denominators are Commensurable.* *Example.*

$\frac{3}{4}$ and $\frac{5}{6}$ and $\frac{7}{8}$	2	24(6	24(4	24(3	$\frac{18 \text{ and } 20 \text{ and } 21}{24}$
	3	4 3	6 5	8 7	
	6	18	20	21	
	4				
2. 3. 4.	24				

When all the Denominators are not Commensurable but only some of them, then either first reduce those Commensurable to one Denominator, and after work with this reduced, and the other remaining Fraction or Fractions, or else reject those lesser Denominators which are even parts of the Greater Compound Denominator, and work with the other Denominators, and the greater Compounds. *When not all the Denominators are Commensurable.*

As to reduce $\frac{3}{4}$ and $\frac{5}{8}$ and $\frac{2}{5}$ to like Denominators; because 4 is a part of 8 it may be rejected, and only 8 and 5 multiplyed for the Common Denominator. *Example.*

Or

Or else first reducing $\frac{3}{4}$ and $\frac{5}{8}$ the new Fractions are $\frac{6 \text{ and } 5}{8}$ which reduced with $\frac{2}{5}$ make $\frac{30 \text{ and } 25 \text{ and } 16}{40}$, as in the Operations following.

$$4)\ \underbrace{\begin{array}{c} 6 \\ \frac{3}{4} \\ 2 \end{array} \text{ and } \begin{array}{c} 5 \\ \frac{5}{8} \\ 1 \end{array}}_{8} \qquad \begin{array}{r} 8 \\ 5 \\ \hline 40 \end{array} \qquad \begin{array}{r} 40(5 \\ 8\ \ 6 \\ \hline 30 \end{array} \qquad \begin{array}{r} 40(5 \\ 8\ \ 5 \\ \hline 25 \end{array} \qquad \begin{array}{r} 40(8 \\ 5\ \ 2 \\ \hline 16 \end{array} \qquad \begin{array}{c} 30 \text{ and } 25 \text{ and } 16 \\ \hline \frac{3}{4} \text{ and } \frac{5}{8} \text{ and } \frac{2}{5} \\ \hline 40 \end{array}$$

§. 4. Fractions of Fractions reduced. Example.

To reduce Proper Disjunct Fractions; Multiply all the Numerators one into another for a new Numerator, and in like manner all the Denominators together, and the new Denominator is produced. So $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{8}$ reduced will be $\frac{15}{64}$.

$$\text{Numerators.}\ \begin{array}{r} 1 \\ 3 \\ \hline 3 \\ 5 \\ \hline 15 \end{array} \qquad \begin{array}{c} 15 \\ \hline \frac{1}{2} \text{ of } \frac{3}{4} \text{ of } \frac{5}{8} \\ \hline 64 \end{array} \qquad \begin{array}{r} 2 \\ 4 \\ \hline 8 \\ 8 \\ \hline 64 \end{array}\ \text{Denominators.}$$

When some of the terms be Commensurable. Example.

If any of the Alternate Heterologal Terms in the Fractions given to be reduced be Commensurable, the new Fraction gotten as above, will not be in its least terms, unless such Heterologal termes be first abbreviated to their lowest, and operation made therewith as before. So $\frac{1}{4}$ of $\frac{2}{5}$ reduced without abbreviation will be $\frac{2}{20}$, but if the Heterologal terms 2 and 4 be first reduced to their least terms, and then *Multiplication* made of these new Homologal terms as before, the new Fraction will be $\frac{1}{10}$ in its least terms.

$$\begin{array}{c} 2 \\ \hline \frac{1}{4} \text{ of } \frac{2}{5} \\ \hline 20 \end{array} \qquad \begin{array}{c} 1 \\ \hline 2)\ \begin{array}{c} \\ \frac{1}{4} \\ 2 \end{array} \text{ of } \begin{array}{c} 1 \\ \frac{2}{5} \\ \\ \end{array} \\ \hline 10 \end{array}$$

§. 5. Improper Fractions reduced to Integers. Example.

The next kind of *Reduction* belongeth to Improper Fractions, and is double. First, To reduce Improper Fractions into Integers or mixt numbers. Divide the Numerator by the Denominator, and if any thing remain after Division adjoyn it to the Quotient in form of a Fraction by setting the Remain over the old Denominator. As $\frac{15}{3}$ shall give 5 Integers, and $\frac{15}{4}$ the mixt number $3\frac{3}{4}$.

Improper Fraction $\frac{15}{3}(5$ Integers $\qquad \frac{15}{3}(3\ \frac{3}{4}$ Mixt Numbers.

§. 6. Integers and Fractions reduced to Improper Fractions. Example.

This *Reduction* is oft-times needful at the end of Operation, that the Resolution of the Question in hand might be more plain.

Secondly on the contrary, when any mixt number is to be reduced into an Improper Fraction. Multiply the Integer by the Denominator of the Fraction, and to the Product add the Numerator of the Fraction, and this Product shall be the Numerator of the Improper Fraction, the old Denominator shall serve still. As $13\frac{3}{4}$ reduced into an Improper Fraction shall be $\frac{55}{4}$. For 13 multiplyed by 4 produceth 52, to which 3 added the Numerator is 55 to the Denominator 4.

This

This *Reduction* in many times neceſſary is the time of Operation, when mixt Numbers either happen, or are given.

When any Integer, together with *Fractions* are found among the given Numbers, or happen in the work that the terms may be kept diſtinct, it is convenient to ſet ſuch Integers in the form of a Fraction. This is eaſily and ſpeedily done, by placing 1 under the ſame for a Denominator. As 3. 4. &c. thus, $\frac{3}{1}$. $\frac{4}{1}$. &c.

§. 7. Integers ſet as Fractions.

Example.

Sometime it may be requiſite to reduce an *Integer* into a given Denomination, though if it be not of abſolute neceſſity, the former way to place an Unit under it be better for brevity in Operation; but when occaſion requires, it may be thus performed Multiply the Integer given to be reduced by the given Denominator, and this Product ſhall be the Numerator. As to bring 10 into fourths; multiply 10 by 4, the Product 40 ſhall be the Numerator to 4 the Denominator; and ſtand thus; $\frac{40}{4}$.

§. 8. Integers reduced into a Denomination given.

Example.

Sometime it is deſired to reduce a *Fraction* to ſome given Denomination which is thus effected. Firſt let the *Fractions* given, be they of Integers, or of Fractions, Proper or Improper, be brought to conſiſt but of two termes only as a Simple Fraction doth, Then by the Numerator thereof multiply the deſirred Denominator, and divide this Product by the Denominator of the given Fraction, and the Quotient ſhall be the Numerator to the propounded Denominator; if any thing remain after Diviſion, it ſhall be a Fraction of that Fraction, as to bring $\frac{3}{5}$ into 60 parts, 3 multiplyed by 60, and the Product 180 divided by 5 gives 36 in the Quotient for the Numerator to 60, and nothing remain. But if $\frac{2}{3}$ were to be brought into thouſandth parts; beſides $\frac{666}{1000}$ gotten as before, there will remain $\frac{2}{3}$ which ſhall be a Fragment of a 1000 the given Denominator, *Viz.* $\frac{2}{3}$ of $\frac{1}{1000}$. Or by *Reduction* $\frac{1}{1500}$.

§. 9. Fractions reduced into a Denomination given.

Example.

$$\begin{array}{r} 60 \\ 3 \\ \hline 180 \end{array} \qquad \frac{\overset{3}{180}}{5}\Big(\frac{36}{60} \qquad \begin{array}{r} 1000 \\ 2 \\ \hline 2000 \end{array} \qquad \frac{22(2}{\underset{3}{2000}}\Big(\frac{666}{1000} \text{ and } \frac{2}{3} \text{ of } \frac{1}{1000}.$$

In like manner, if *Integers*, *Proper Fractions* conjunct or divided or both, be mixed with *Improper Fractions*, or any other waies, and it be neceſſary to reduce them to one Denominator: The preceeding Rules obſerved, according to the nature of the given *Fractions*, their Reduction will be facil. *Example.* If $\frac{1}{7}$ a proper ſingle Fraction with $\frac{2}{3}$ of $\frac{1}{5}$ be given to be reduced with 2 Integers, and $\frac{7}{2}$ the Improper Fraction, to one Denomination. After $\frac{2}{3}$ of $\frac{1}{5}$ are reduced to one ſingle Fraction, and an Unit placed under the 2 Integers, the Common Denominator is found out as before to be 210, and the ſeveral Numerators 30. 28. 420. 735.

§. 10. Integers mixed with Fractions reduced to one Deminator.

Example.

$$\underbrace{\overset{30}{\frac{1}{7}} \text{ with } \overset{\overset{28}{2}}{\underset{15}{\frac{2}{3} \text{ of } \frac{1}{5}}} \text{ and } \overset{420}{\frac{2}{1}} \text{ and } \overset{735}{\frac{7}{2}}}_{210} \qquad \begin{array}{r} 7 \\ 15 \\ \hline 105 \\ 2 \\ \hline 210 \end{array} \qquad \begin{array}{r} 210(30 \\ 7 \quad 1 \\ \hline 30 \end{array} \qquad \begin{array}{r} \overset{6}{210}(14 \\ 15 \quad 2 \\ \hline 28 \end{array} \qquad \begin{array}{r} 210(210 \\ 1 \quad 2 \\ \hline 420 \end{array} \qquad \begin{array}{r} 210(105 \\ 2 \quad 7 \\ \hline 735 \end{array}$$

Reductional Operation ended, Probation follows, that the ſeveral operations may be proved true.

Proof of Reduction of Fractions.

This Proof of one kind of Reduction is alternately by another; becauſe the *Fractions* are to keep the ſame equality in value, how ever differently they are expreſſed in greater or leſſer terms; Whence it is that Fractions duely reduced to one Denominator, may be returned to their leaſt terms again by Abbreviation, and Fractions abbreviated

Proper.

viated to their least terms may be converted to their former greater terms; that by *Division* with the great Common Measure, and this by *Multiplication* therewith. As in the last Example $\frac{1}{7}$ was reduced with others to $\frac{30}{210}$ which as *termini convertibiles*, may by 30 the great common measure or any other common *Divisor* be reduced from the one to the other. For $\frac{1}{7}=\frac{30}{210}$.

$$30)\ \frac{30(1}{210(7} \qquad \frac{3|0 \mid 1}{21|0 \mid 7}$$

Improper.

Likewise Reduction of Improper Fractions reciprocally prove one sort the other: As *Multiplication*, and *Division* in Integers mutually do, they being performed thereby, as above may be seen. For $3\frac{3}{4}$ is but the Quotient of $\frac{15}{4}$, and $\frac{15}{4}$ the Product of $3 \times 4 + 3$. Wherefore $3\frac{3}{4}=\frac{15}{4}$.

Integers set like Fractions by subjecting 1. As in the 7 *Sect.* above are soon reverted into their old form by taking away the subjected Unite. Integers also reduced to given Denominators are returned back by *Division* of the Numerators by the Denominators, like Improper Fractions: And so $\frac{40}{4}$ shall return 10.

In like manner, *Fractions* reduced to a given Denominator may be abbreviated by Common *Divisors* till the first terms be returned. In the Instance above $\frac{3}{5}$ made $\frac{36}{60}$ therefore by the great Common measure 12 shall $\frac{3}{5}$ be returned.

But if a *Fraction* remained; as in turning $\frac{2}{5}$ into thousandths, the Proof is somewhat more difficult than the work because of the divided *Fraction*. Yet neither such nor *Fractions* of *Fractions* are destitute of trial. For in those if the Fragment be added to the Quotient, and reduced to their least terms, the former given *Fraction* will be returned. And in *Fractions* of *Fractions* if the *Fraction* of one Denomination be divided by either of the parts multiplyed, the other will be returned in the Quotient, because *Multiplication* and *Division* prove each other. And besides, in *Contract Fractions* another kind of proof may be had; by finding the value denominate. As in the next Book of *Geodaticks* may be seen. But forasmuch as *Addition* and *Division* of *Fractions* are not yet taught, and both these *Reductions* are performed by *Multiplication* or *Division*, or both, as if they were *Integers*: It may satisfie as to the truth of these *Reductions*, if the *Multiplications*, and *Divisions* be found right.

Proof of the Reduction of Fractions of Fractions, *See pag. 53.*
Of Contract Fractions, See Geodæticals.

CHAP. III.

Addition of Fractions.

Proper Fractions are increased by Addition. and Division diminished by Substraction & Multiplication.

HOW to increase or decrease the terms of a *Fraction* hath been seen in *Reduction*; it remains now to see how to increase or decrease their value.

Proper Fractions like to *Integers* have their value increased by *Addition* and diminished by *Substraction*.

But contrary to *Integers* are lessened by *Multiplication*, and increased by *Division*.

Improper, how they increase, or decrease.

Improper Fractions by *Addition* increase, and *Substraction* decrease their value according to *Integers*, and *Proper Fractions*. In *Multiplication* they are redundant by their value, and increase as *Integers*; defective by their *Fractions*, and decrease as *Fractions*. But in *Division* are altogether like *Integers*.

Mixt, how they increase, or decrease.

Proper Fractions mixt with *Improper*, or *Integers*; are augmented or diminished in *Addition* or *Substraction*, , as before. In *Multiplication* they follow the manner of

Improper Fractions; in *Division* if the *Fraction* be the *Dividend* the Quotient is decreased, if *Divisor* the contrary.

Addition of *Fractions*, and mixt Numbers contains Operation and Probation. *Addition of Fractions included under 4 Cases.*

In Operation are four Cases. The two first general, the two last special.

1. Case. When the Denominators are alike, add the Numerators together, as *Integers*, and beneath the total subscribe the Common Denominator. As $\frac{2}{7}$ and $\frac{4}{7}$ added together make $\frac{6}{7}$, And so $\frac{3}{20}$ and $\frac{13}{20}$ make $\frac{16}{20}$, and by Abbreviation $\frac{4}{5}$ by some set as at A. by others as at B. *1. Denominators alike. Examples.*

$$\text{A}\ \frac{2}{7}\overset{6}{\underset{7}{\mathbf{X}}}\frac{4}{7} \qquad \overset{\text{B}}{\frac{2}{7}+\frac{4}{7}=\frac{6}{7}} \qquad \text{A}\ \frac{3}{20}\overset{16}{\underset{20}{\mathbf{X}}}\frac{13}{20} \qquad \overset{\text{B}}{\frac{3}{20}+\frac{13}{20}=\frac{16}{20}} \text{ or } \frac{4}{5}$$

2. Case. When the Denominators are unlike, first reduce them to one Denomination, then as before add the Numerators, and under the total subscribe the Common Denominator. As $\frac{3}{4}$ and $\frac{4}{5}$ reduced make $\frac{15 \text{ and } 16}{20}$ then added make $\frac{31}{20}$ the Improper Fraction, or $1\frac{11}{20}$ set as at C. or D. So $\frac{2}{3}$ & $\frac{3}{4}$ & $\frac{2}{3}$ first reduced make $\frac{1\ \&\ 9\ \&\ 18}{12}$, then added are $2\frac{11}{12}$. As at E. or F. *2. Denominators unlike. Examples.*

$$\text{C}\ \overset{\overline{31}}{\overline{15\quad 16}}\ \frac{3}{5}\underset{20}{\mathbf{X}}\frac{4}{5} \qquad \overset{\overline{31}}{\overline{15\quad 16}}\ \frac{3}{4}+\frac{4}{5}=\frac{31}{20} \text{ or } 1\frac{11}{20}\ \text{D}$$

$$\text{E}\ \overset{35}{\overline{\frac{8}{\frac{2}{3}}\text{ and }\frac{9}{\frac{3}{4}}\text{ and }\frac{18}{\frac{3}{2}}}}{}_{12} \qquad \overset{\text{F}}{\frac{2}{3}+\frac{3}{4}+\frac{3}{2}=\frac{35}{12}} \text{ or } 2\frac{11}{12}$$

3. Case. If Integers or mixt Numbers are to be added with Fractions, either add the Integers after the manner of Integers, and Fractions after the manner of Fractions severally by themselves; or else reduce the mixt numbers into Improper Fractions, and then proceed as above. As if $3\frac{1}{3}$ and $6\frac{1}{4}$ were to be added with $\frac{2}{5}$ either 3 and 6 the Integers added make 9, and the Fractions $\frac{1}{3}\frac{1}{4}\frac{2}{5}$ make $\frac{59}{60}$ or reducing the mixt numbers $3\frac{1}{3}$ and $6\frac{1}{4}$ into Improper Fractions, and then proceeding the same total is at last resulting to $9\frac{59}{60}$ as at G. or H. *3. Integers or mixt Numbers with Fractions. Examples.*

$$\text{G}\ \left.\begin{matrix}3\\6\end{matrix}\right\}_{9} + \overset{59}{\overline{\overset{20}{\frac{1}{3}}+\overset{15}{\frac{1}{4}}+\overset{24}{\frac{2}{5}}}}{}_{60}\Big\} = 9\frac{59}{60} \qquad \text{H}\ \overset{599}{\overline{\overset{200}{\frac{10}{3}}+\overset{375}{\frac{25}{4}}+\overset{24}{\frac{2}{5}}}}{}_{60}\Big\} = \frac{599}{60} \text{ or } 9\frac{59}{60}.$$

4. Case. If Integers, mixt Numbers, Fractions, and Fractions of Fractions, are to be added together, either they may first be severally added, and afterwards their several Totals into one total; or else reduced to one Denomination, and then added. *Example.* To add 2 Integers, $3\frac{1}{5}$ a mixt Number, $\frac{1}{3}$ a Fraction, and $\frac{1}{2}$ of $\frac{1}{4}$ a Fraction of a Fraction. As at I. or K. *4. Integers and mixt Numbers with Fractions. Examples.*

$$\text{I}\ \left.\begin{matrix}2\\3\end{matrix}\right\}_{5} + \overset{79}{\overline{\underbrace{\overset{24}{\frac{1}{5}}+\overset{40}{\frac{1}{3}}+\overset{15}{\frac{\overset{1}{\overline{1}}}{2}\text{ of }\frac{1}{4}}_{8}}_{120}}} \qquad \text{K}\ \overset{679}{\overline{\underbrace{\overset{240}{\frac{2}{1}}+\overset{384}{3\overset{16}{\frac{1}{5}}}+\overset{40}{\frac{1}{3}}+\overset{15}{\frac{\overset{1}{\overline{1}}}{2}\text{ of }\frac{1}{4}}_{8}}_{120}}}$$

Addition

Proof of Addition of Fractions.

Addition of Fractions like absolute Integers is proved by *Substraction* and reserved to the next Chapter. But *Addition* being learned: The Proof of *Reduction* in the 9. *Sect.* of the last *Chapter* may be remembered, and exemplified, where after the first *Division* remained a Fraction of a Fraction, in reducing of $\frac{2}{3}$ to 1000ths there was $\frac{666}{1000}$ and $\frac{2}{3000}$ both which added by the second case above will return $\frac{2}{3}$. As before noted, and prove the *Reduction* right.

$$\begin{array}{c} 2|000 \\ 1998 \quad\quad 2 \\ 1000)\ \dfrac{666}{1000} + \dfrac{2}{3000} = \dfrac{2}{3} \\ \underbrace{3 \quad\quad\quad 1}_{3|000} \end{array}$$

CHAP. IV.

Substraction of Fractions.

To find which of 2 Fractions given is the greatest.

Examples.

BEcause a greater Fraction cannot be taken out of a lesser, it is convenient first to know how to find which of any two propounded Fractions is the Greater, that so the Propositions may not be impossible. Which to do, Multiply the Numerator of the one into the Denominator of the other, and the Product which is the greatest shall demonstrate that Fraction biggest, whose Numerator was one of the Factors. As to know which is the biggest Fraction of $\frac{3}{4}$ or $\frac{4}{5}$ multiplying 3 by 5, the Product is 15, and 4, by 4 yieldeth 16, which sheweth $\frac{4}{5}$ to be greater than $\frac{3}{4}$ and import a bigger part of the Integer. So likewise $\frac{2}{3}$ is more than $\frac{3}{5}$, because 10 exceeds 9, and in like manner Equal Fractions may be found, as $\frac{2}{5}$ and $\frac{4}{10}$.

$$\text{Minor}\ \overset{15}{\tfrac{3}{4}} \mathbf{X} \overset{16}{\tfrac{4}{5}}\ \text{Major.} \qquad \overset{10}{\tfrac{2}{3}} \mathbf{X} \overset{9}{\tfrac{3}{5}}\ \text{Minor} \qquad \text{both}\ \overset{20}{\tfrac{2}{5}} \mathbf{X} \overset{20}{\tfrac{4}{10}}\ \text{Equal.}$$

Substraction of Fractions included under 8. Cases.

Substraction of Fractions, and mixt Numbers, includes Operation, and Probation. In Operation may happen Eight Cases the two first general, the six last particular.

1. Denominators alike. Examples.

1. Case. When the Denominators are alike, then substract the lesser Numerator from the greater, as in Integers, and place the remain over the Common Denominator. As $\frac{2}{5}$ from $\frac{3}{5}$ leaves $\frac{1}{5}$. So $\frac{3}{8}$ out of $\frac{7}{8}$ there resteth $\frac{4}{8}$, and by Abbreviation $\frac{1}{2}$ set as at A by some, as at B by others.

$$\text{A}\quad \frac{2}{5} \underset{5}{\overset{1}{\mathbf{X}}} \frac{3}{5} \qquad \overset{\text{B}}{\frac{3}{5} - \frac{2}{5} = \frac{1}{5}} \qquad\qquad \text{A}\quad \frac{3}{8} \underset{8}{\overset{4}{\mathbf{X}}} \frac{7}{8} \qquad \overset{\text{B}}{\frac{7}{8} - \frac{3}{8} = \frac{4}{8}\ \text{or}\ \frac{1}{2}}$$

2. Denominators unlike. Examples.

2. Case. When the Denominators are unlike, first reduce them to one Denomination; then abate the lesser Numerator out of the greater, and under the Remain subscribe the Common Denominator. As in substracting $\frac{5}{7}$ from $\frac{7}{9}$ being reduced they are $\frac{45 \quad 49}{63}$ then

then $\frac{45}{63}$ substracted from $\frac{49}{63}$ leaves $\frac{4}{63}$ set as at C. or D. So $\frac{13}{20}$ out of $\frac{4}{5}$ leaveth remaining $\frac{3}{20}$. As at E. or F. *Examples.*

$$\text{C}\quad \overset{4}{\overline{\begin{matrix}45 & & 49\\ \frac{5}{7} & \times & \frac{7}{9}\end{matrix}}}\ \underset{63}{} \qquad \text{D}\quad \frac{7}{9}-\frac{5}{7}=\frac{4}{63}$$

$$\text{E}\ 5)\ \overset{3}{\overline{\begin{matrix}16 & 13\\ \frac{4}{5} & \frac{13}{20}\\ 4 & 1\end{matrix}}}\ \underbrace{\qquad}_{20} \qquad \text{F}\quad \frac{4}{5}-\frac{13}{20}=\frac{3}{20}$$

3. Case. If mixt Numbers consisting of Integers and Fractions are given, or happen in the work, then first reduce them into Improper Fractions; and afterwards proceed as before. So $1\frac{5}{12}$ taken from $1\frac{13}{16}$ shall leave $\frac{19}{48}$ as at G. and $3\frac{1}{4}$ from $7\frac{3}{4}$ leaves $4\frac{1}{2}$ as at H. *3. Mixt Numbers. Examples.*

$$\text{G}\ 4)\ \overset{19}{\overline{\begin{matrix}87 & & 68\\ 29 & & 17\\ 1\frac{13}{16} & - & 1\frac{5}{12}\\ 3 & & 4\end{matrix}}} = \frac{19}{48} \qquad \underbrace{\qquad}_{48}$$

$$\text{H}\quad \overset{18}{\overline{\begin{matrix}31 & & 13\\ 7\frac{3}{4} & - & 3\frac{1}{4}\end{matrix}}} = \frac{18}{4} \text{ or } 4\frac{1}{2} \qquad \underbrace{\qquad}_{4}$$

4. Case. If many Fractions are to be substracted from one, or one from many, then first add them that are to be substracted together, if more than one, into one total, and likewise those Substraction is to be made from, and afterwards substract the total of the Subtrahend from the other total, as before. As to take $\frac{7}{8}$ and $\frac{9}{16}$ from $\frac{3}{4}$ and $\frac{5}{6}$, first $\frac{7}{8}$ and $\frac{9}{16}$ reduced and added, their Total is $\frac{23}{16}$, then $\frac{3}{4}$ and $\frac{5}{6}$ reduced and added, their Total is $\frac{19}{12}$. Lastly, $\frac{23}{16}$ subducted from $\frac{19}{12}$, there remaineth $\frac{7}{48}$, the Operations appear at I. K. L. *4. Many Fractions given. Examples.*

$$\text{I}\ 8)\ \overset{23}{\overline{\begin{matrix}14 & & 9\\ \frac{7}{8} & + & \frac{9}{16}\\ 2 & & 1\end{matrix}}} = \frac{23}{16} \quad \underbrace{\qquad}_{16} \qquad \text{K}\ 2)\ \overset{19}{\overline{\begin{matrix}9 & & 10\\ \frac{3}{4} & + & \frac{5}{6}\\ 3 & & 2\end{matrix}}} = \frac{19}{12} \quad \underbrace{\qquad}_{12} \qquad \text{L}\ 4)\ \overset{7}{\overline{\begin{matrix}76 & & 69\\ \frac{19}{12} & - & \frac{23}{16}\\ 4 & & 3\end{matrix}}} = \frac{7}{48} \quad \underbrace{\qquad}_{48}$$

5. Case. If in two mixt given numbers the lesser Fraction belong to the Subtrahend, then to work with the Integers severally after the manner of Integers, and with the Fractions by themselves after their manner, is the best way for brevity, because it saves many times Reduction and great Multiplications. As to abate $19\frac{1}{5}$ from $48\frac{3}{5}$, the Integers 19 from 48 leave 29, and $\frac{1}{5}$ from $\frac{3}{5}$ leaves $\frac{2}{5}$. So is the whole Remain $29\frac{2}{5}$ as at M. Also $40\frac{1}{4}$ from $63\frac{1}{2}$ leave $23\frac{1}{4}$ for 40 withdrawn from 63 leaveth 23, and $\frac{1}{4}$ from $\frac{1}{2}$ leaveth $\frac{1}{4}$ as at N, which as many others sometime happen are so commonly known, that upon sight, without further work may be discerned, but if need be, Operation may be made for the Fraction as before; yet will the work be far shorter than if all the Numbers were turned into Improper Fractions. *5. Mixt Numbers and the Lesser Fraction be in the Subtrahend. Examples.*

$$\begin{array}{lll} & 48\frac{3}{5} & \text{Greater Homogeneal.} \\ & 19\frac{1}{5} & \text{Subtrahend.} \\ M & 29\frac{2}{5} & \text{Remain.} \end{array} \qquad \begin{array}{ll} & 63\frac{1}{2} \\ & 40\frac{1}{4} \\ N & 23\frac{1}{4} \end{array} \qquad 2)\ \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

6. *Mixt Numbers and the Greater Fraction be in the Subtrahend. Example.*

6. Caſe. If the Fractions of the two given mixt Numbers be of one Denomination, and the Fraction belonging to the Subtrahend be greater than the other Fraction, then add the Denominator of the Number from which *Subſtraction* is to be made to the Numerator, and abate an Unit from his Integer, and afterward make *Subſtraction* as before. So if $3\frac{5}{6}$ were to be ſubducted from $9\frac{1}{6}$, becauſe $\frac{5}{6}$ is the major Fraction, 6 is added to 1, and 1 taken from 9. So is the number thus altered $8\frac{7}{6}$, from whence $3\frac{5}{6}$ ſubſtracted there is left $5\frac{2}{6}$ or $5\frac{1}{3}$. For $9\frac{1}{6} = 8\frac{7}{6} - 3\frac{5}{6} = 5\frac{2}{6}$, or $5\frac{1}{3}$

7. *One Fraction, and that in the Subtrahend. Examples.*

7. Caſe. If there be but one Proper Fraction in the two given Numbers, and that belong to the *Subtrahend*, then abate the Numerator from the Denominator, and ſubſcribe under the remain the Denominator, and accompt the Integers in the Subtrahend 1 more, or thoſe in the Greater number 1 leſs. As to take $\frac{2}{3}$ from 4 Integers; firſt 2 taken from 3 leaves 1 to be ſet over 3, then 1 from 4 leaveth the whole remain $3\frac{1}{3}$ As at O. So if $6\frac{3}{7}$ be abated from 9 Integers, 3 taken out of 7 leaves $\frac{4}{7}$, and 6 and 1 out of 9, or 6 out of 8, there remaineth $2\frac{4}{7}$, as at P.

$$\begin{array}{lll} & 4 & \text{Integers.} \\ & 0\frac{2}{3} & \text{Subtrahend.} \\ O & 3\frac{1}{3} & \text{Remain.} \end{array} \qquad \begin{array}{lll} & 9 & \text{Integers.} \\ & 6\frac{3}{7} & \text{Subtrahend.} \\ P & 2\frac{4}{7} & \text{Remain.} \end{array}$$

8. *Subtrahend an Integer. Example.*

8. Caſe. If of the two given Numbers, the Subtrahend be Integral, then keep the Fraction intire to the Remain, and make Subſtraction as in Integers. As to take 3 Integers from $5\frac{1}{4}$, take 3 from 5, and the remain ſhall be $2\frac{1}{4}$.

Proof of Subſtraction of Fractions.

Probation of *Subſtraction* and *Addition*, as in Integers, ſo in Fractions is reciprocal, the one by the other; wherefore if one of the Addends be ſubſtracted from the Total, the Additionary work will be proved by the Remain equal to the other Addends; ſo if the Subtrahend and Remain be added, the Subſtractionary work will be proved, for the Sum ſhall amount to the greater Number, from which *Subſtraction* was made, and as in *Subſtraction* of *Integers*, if the Remain be ſubducted from the Greater Number, this Remain ſhall be the Subtrahend *vice-verſâ*. Examples in the Anſwer of theſe 2 Queſtions.

Queſtions in Addition and Subſtraction of Fractions.

1. What Number is that from which if $\frac{1}{20}$ be ſubſtracted, the Remain will be $\frac{3}{4}$?

Anſwer, $\frac{4}{5}$, for ſo much is the Total of $\frac{1}{20}$ and $\frac{3}{4}$ added together.

2. What Number was that to which $\frac{1}{20}$ added the Total was $\frac{4}{5}$?

Anſwer, $\frac{3}{4}$, for ſuch is the Remain after $\frac{1}{20}$ is ſubſtracted from $\frac{4}{5}$.

Proof

Proof of Subſtraction.

$$4)\ \overbrace{\underbrace{\overset{1}{\frac{1}{20}} + \overset{15}{\frac{3}{4}}}_{\underset{20}{1 \quad 5}}}^{16} = \frac{16}{20} \text{ or } \frac{4}{5}$$

Proof of Addition.

$$5)\ \overbrace{\underbrace{\overset{16}{\frac{4}{5}} - \overset{1}{\frac{1}{20}}}_{\underset{20}{4 \quad 1}}}^{15} = \frac{15}{20} \text{ or } \frac{3}{4}$$

CHAP. V.

Multiplication of Fractions.

BOTH the prime parts of *Fractionary Numeration* ended in *Addition* and *Subſtraction*, their Compound Compoſition and Diſſolution are to follow, in *Multiplication* and *Diviſion*.

Multiplication of *Proper Fractions*, may increaſe the termes but leſſeneth the value, as before obſerved, in this part, *Chap.* 3. and of neceſſity can do no leſs, becauſe being leſs than one, and making another Number ſo many times leſs alſo, muſt needs produce a number as many times leſs as the multiplying Fraction containeth parts in it, wherefore the Elements here may change the names they had in *Integers*, and *Multiplication of Fractions* may be called their Diſſolution, and *Diviſion* their Compoſition, they being increaſed thereby for the moſt part both in termes, and value. Nevertheleſs in *Improper Fractions* it is otherwiſe. As was noted in the ſame 3[d] *Chap.* before.

The reaſon why the value is leſſened, though the Terms increaſe by Multiplication.

Multiplication of Fractions may be called Diſſolution.

Multiplication of *Fractions* and *mixt Numbers*, retains no difficulty in *Operation* and *Probation*.

Operation comprehendeth Six Caſes, The firſt three *Eſſential*, the laſt 3 *Accidental*.

Multiplication of Fractions included under 6. Caſes.

1.

1. Caſe. If the two Numbers given be *Proper* or *Improper Fractions*, or the one *Proper*, and the other an *Improper*, and the Alternate Heterologal termes Incommenſurable, then multiply as in *Integers* the Homologal Terms, that is Numerator by Numerator, for the Numerator of the Product, and Denominator by Denominator for the Denominator of the Product. As to multiply $\frac{2}{3}$ by $\frac{4}{5}$ the Product is $\frac{8}{15}$ and $\frac{3}{2}$ by $\frac{5}{4}$ produce $\frac{15}{8}$ or $1\frac{7}{8}$. Alſo $\frac{2}{3}$ by $\frac{4}{3}$ will produce $\frac{8}{9}$, every of which have their various Collocations. As at A, or B. C, or D. E, or F.

Alternate Terms Incommenſurable.

Examples.

$$\text{A}\ \overbrace{\underbrace{\frac{2}{3} \times \frac{4}{5}}_{15}}^{8} \qquad \overset{\text{B}}{\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}} \qquad \text{C}\ \overbrace{\underbrace{\frac{3}{2} \times \frac{5}{4}}_{8}}^{15} \qquad \overset{\text{D}}{\frac{3}{2} \times \frac{5}{4} = 1\frac{7}{8}} \qquad \text{E}\ \overbrace{\underbrace{\frac{2}{3} \times \frac{4}{3}}_{9}}^{8} \qquad \overset{\text{F}}{\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}}$$

2. Caſe. If the Alternate Heterologal Terms, or either of them be Commenſurable firſt abbreviate them, and then multiply the new Homologal Terms as before; ſo ſhall the Product be alſo in its leaſt terms. As to multiply $\frac{4}{15}$ by $\frac{5}{6}$, here 4 and 6 may be abbreviated to 2 and 3. Alſo 5, and 15. to 1, and 3. then multiplying 2 by 1, the new Numerator is 2 and 3, by 3 the new Denominator is 9. So is $\frac{2}{9}$ the Product Incommenſurable. So $\frac{3}{2}$ by $\frac{6}{5}$ produce $\frac{9}{5}$. And $\frac{5}{7}$ by $\frac{9}{5}$ $\frac{9}{7}$, as at G. H. I.

2.

Alternate Terms Commenſurable.

Examples.

$$G\quad \begin{matrix}2)\\5)\end{matrix}\ \overbrace{\frac{\overset{2}{4}}{\underset{3}{15}} \times \frac{\overset{1}{5}}{\underset{3}{6}}}^{2}_{\underbrace{\qquad\qquad}_{9}} = \frac{2}{9} \qquad H\ 2)\ \overbrace{\frac{3}{\underset{1}{2}} \times \frac{\overset{3}{6}}{5}}^{9}_{\underbrace{\qquad\qquad}_{5}} = 1\frac{4}{5} \qquad I\ 5)\ \overbrace{\frac{\overset{1}{5}}{7} \times \frac{9}{\underset{1}{5}}}^{9}_{\underbrace{\qquad\qquad}_{7}} = 1\frac{2}{7}$$

3. *Mixt Numbers both, or one a Fraction or Integer. Examples.*

3. Case. If Integers or mixt Numbers with a Fraction, or both mixt Numbers be given to be multiplyed, subject an Unit as in *Reduction* under the Integers, and reduce the mixt Numbers into Improper Fractions, and then proceed as above. So 3 by $2\frac{2}{7}$ will produce $\frac{48}{7}$, as at K. and 4 by $\frac{2}{3}$ produce $\frac{8}{3}$, as at L. Also $4\frac{1}{3}$ by $3\frac{1}{2}$ produce $\frac{91}{6}$, as at M.

$$K\quad \overbrace{\frac{\overset{16}{3}}{1} \times 2\frac{2}{7}}^{48}_{7} = 6\frac{6}{7} \qquad L\quad \overbrace{\frac{4}{1} \times \frac{2}{3}}^{8}_{3} = 2\frac{2}{3} \qquad M\quad \overbrace{\frac{13}{3} \times \frac{7}{2}}^{91}_{6} = 15\frac{1}{6}\ \ \left(4\frac{1}{3}\ \ 3\frac{1}{2}\right)$$

4. *Integer and Fraction or mixt Number.*

4. Case. When an Integer and a Single Fraction, or an Integer and a mixt Number whose Fraction is single, the Denominators of the Fractions being digits, be the two given Numbers; then the Integer may be made *Multiplicand*, and the other Number the *Multiplier*, which if a mixt Number, multiply the *Multiplicand* by the Integers of the *Multiplier*, and for the Fraction of the mixt Number take $\frac{1}{2}$. $\frac{1}{3}$. $\frac{1}{4}$, &c. as the Fraction is, of the *Multiplicand*, and add to the former Product, and the total product shall be the desired Number. But if the *Multiplier* be only a single Fraction, then either take the half, third part, fourth part, &c. of the Integer according to the given Fraction, or else duplicate, triplicate, quadruplicate, &c. the Fraction according to the given Integer, for the Product desired: For to multiply any Integer Fraction-wise by $\frac{1}{2}$ is Bipartition, by $\frac{1}{3}$ Tripartition, &c. And contrary-wise to multiply any Fraction by 2 is but to double the Numerator, or take half the Denominator; by 3 is to triple the Numerator, or take the third part of the Denominator, &c. And hence sometimes in like sort *Multiplication* by *Plural Fractions*, if their Denominators be Digits is made use of, as sooner accomplished then by *Reduction* into *Improper Fractions*.

Examples. Examples of all these varieties, at N. O. P. in the one 48 is multiplyed by $9\frac{1}{3}$ in the other by $\frac{1}{5}$ and in the third by $12\frac{3}{4}$; so plainly they need no illustration.

N		O		P	
48	Multiplicand.	48	Multiplicand.	48	Multiplicand.
$9\frac{1}{3}$	Multiplier	$\frac{1}{5}$	Multiplier.	$12\frac{3}{4}$	Multiplier.
432	Product of the Integers.	$9\frac{3}{5}$	Product.	96	Product of 12.
16	Third Part added.			48	
				36	$\frac{3}{4}$ of 48 added.
448	Total Product.				
				612	Total Product.

5. *Both mixt Numbers with Single Fractions, &c.*

5. Case. When two mixt Numbers whose Fractions are single, and their Denominators Digits, are given to be multiplyed; after *Multiplication* by their Integers, and the the Fraction of the *Multiplier*, as last above-mentioned, then multiply the Numerator of the *Multiplicand Fraction* by the Integers in the *Multiplier*, and the Product divide by the Denominator of that Fraction; and add this Quotient to the numbers before set down with the Product of the two Fractions multiplyed, the Sum of all these shall be the

the total Product. As to multiply $48\frac{1}{2}$ by $12\frac{3}{4}$, first 48 by 12 produce 576, then $\frac{3}{4}$ of 48 is 36, then 12 halfs is 6 whole ones, or 12 multiplying 1, and dividing by 2 is all alike. Wherefore 6 added to the other, and $\frac{3}{8}$ the Product of $\frac{1}{2}$ into $\frac{3}{4}$ make together $618\frac{3}{8}$ for the Total Product; as at Q. examined by the Common way at R. and found alike. *Example.*

Q

$48\frac{1}{2}$	Multiplicand.	
$12\frac{3}{4}$	Multiplier.	
96 48	Product of 12	added
36	$\frac{3}{4}$ of 48.	added
6	12 halfs.	added
$\frac{3}{8}$	$\frac{1}{2}$ of $\frac{3}{4}$.	added
$618\frac{3}{8}$	Total Product.	

$$\begin{array}{r} 97 \\ 51 \\ \hline 97 \\ 485 \\ \hline 4947 \end{array}$$

R

$$48\tfrac{1}{2} \times 12\tfrac{3}{4} = \frac{\overset{97}{} \quad \overset{51}{}}{} \frac{4947}{8} = 618\tfrac{3}{8} \qquad \begin{array}{l} \;\;\cancel{1}6(3 \\ \cancel{4947}(618 \\ \;\;8 \end{array}$$

6\. Case. When the Heterologal Terms either way are equal, cancel them, and let the other Terms stand as they are for the Product. But when they are equal both wayes take 1, for the Product shall alwayes be an Unit or equal Fraction. As in multiplying $\frac{1}{3}$ by $\frac{3}{4}$, the Product shall be $\frac{1}{4}$ cancelling 3, and 3. And in multiplying $\frac{3}{4}$ by $\frac{4}{3}$, the Product shall be $\frac{1}{1}$, or 1. *6. Alternate Terms equal. Examples.*

S

$$3)\ \frac{\cancel{1}}{\cancel{3}} \times \frac{\cancel{3}}{4} = \frac{1}{4}$$

T

$$\begin{array}{l}3)\\4)\end{array} \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{3}} = 1$$

Besides the particular Proof of the different *Multiplications* one by another in General; The Proof of *Multiplication* as in *Integers* so in *Fractions*; is by *Division*, and to be sought in the next Chapter. *Proof of Multiplication of Fractions.*

CHAP. VI.

Division of Fractions.

D*Ivision of Fractions* may be called their Composition as increasing both their terms and value if the *Fractions* be proper, and Homologal terms Incommensurable; but Improper and mixt Numbers partly increase and partly decrease their Quotients as before noted in *Addition of Fractions. Chap.* 3. *Division of Fractions when their Composition.*

Fractions and *mixt Numbers*, are divided with much facility, both *Operation* and *Probation* being performed by *Multiplication*; that after the manner of *Integers*; this of *Fractions*. *How wrought.*

Operation concludeth with Six Cases, the three first Essential, and three last Accidental. *Division of Fractions included under 6. Cases.*

1\. Case. If the two given Numbers be Proper or Improper *Fractions*, or the one Proper, and the other Improper, and the Homologal terms Incommensurable; then *1. Homologal Termes Incommensurable.*

multiply as in *Integers* the Heterologal Terms, that is to say the Numerator of the Dividend by the Denominator of the Divisor, for the Numerator of the Quotient, and the Denominator of the Dividend by the Numerator of the Divisor, for the Denominator of the Quotient. *Example.* As to divide $\frac{1}{2}$ by $\frac{2}{3}$ and $\frac{3}{2}$ by $\frac{5}{3}$, and $\frac{1}{3}$ by $\frac{3}{2}$, the several Quotients are $\frac{3}{4}$, $\frac{9}{10}$, $\frac{2}{9}$ variously set as at A. or B. C. or D. E. or F.

A $\frac{2}{3}\overset{3}{\underset{4}{\mathbf{X}}}\frac{1}{2}$ B $\frac{2}{3})\frac{1}{2}(\frac{3}{4}$ C $\frac{5}{3}\overset{9}{\underset{10}{\mathbf{X}}}\frac{3}{2}$ D $\frac{5}{3})\frac{3}{2}(\frac{9}{10}$ E $\frac{3}{2}\overset{2}{\underset{9}{\mathbf{X}}}\frac{1}{3}$ F $\frac{3}{2})\frac{1}{3}(\frac{2}{9}$

2. Homologal Termes Commensurable. Examples.

2. Case. If the Homologal Terms or either of them be Commensurable; first reduce them to their least Terms, and then multiply the new Heterologal Terms as before, and so the Quotient shall be also kept in its least Terms. As $\frac{2}{3}$ divided by $\frac{4}{5}$, because 2 and 4 may be abbreviated to 1 and 2 the Product of 2 into 3 shall be the Denominator of the Quotient; and the Product of 1 into 5 the Numerator, as at G. So the Quotient of $\frac{2}{3}$ divided by $\frac{5}{6}$ shall be $\frac{4}{5}$, as at H. and the Quotient of $\frac{5}{8}$ by $\frac{15}{16}$ shall be $\frac{2}{3}$, As at I.

G $2)\ \frac{\overset{2}{4}}{5}\ )\ \frac{\overset{1}{2}}{3}\ (\frac{5}{6}$ H $3)\ \frac{5}{\underset{2}{6}}\ )\ \frac{2}{\underset{1}{3}}\ (\frac{4}{5}$ I $\begin{matrix}5)\\8)\end{matrix}\ \frac{\overset{3}{15}}{\underset{2}{16}}\ )\ \frac{\overset{1}{5}}{\underset{1}{8}}\ (\frac{2}{3}$

3. Mixt Numbers both or one a Fraction or Integer. Examples.

3. Case. If *Integers*, or *mixt Numbers*; with *Fractions*, or two mixt Numbers, are given to be divided; Subscribe an Unit as in *Reduction* under the Integers, and reduce the mixt Numbers into *Improper Fractions*, and then proceed as above. So 4 divided by $\frac{3}{4}$ shall give in the Quotient $\frac{16}{3}$ or $5\frac{1}{3}$ as at K. and 4 divided by $\frac{9}{2}$ as at L. shall give $\frac{8}{9}$, also $4\frac{1}{3}$ dividing $3\frac{1}{2}$ the Quotient shall be $\frac{21}{26}$, as at M.

K $\frac{3}{4})\frac{4}{1}(\frac{16}{3}$ or $5\frac{1}{3}$ L $\frac{9}{2})\frac{4}{1}(\frac{8}{9}$ M $4\frac{\overset{13}{1}}{3})3\frac{\overset{7}{1}}{2}(\frac{21}{26}$

4. Integer and Fraction or mixt Number.

4. Case. When an *Integer* is given to divide an Improper *Fraction*, or a Single *Fraction*, whose Denominators are digits, or such a single *Fraction* given to divide an *Integer*, then if the *Integer* be Dividend, double, triple, quadruple, &c. the Dividend according to the Denominator of the *Fraction*. But if the *Integer* be Divisor, and the Improper *Fraction* or other *Fraction* be Dividend, then either double, triple, quadruple, &c. the Denominator of the Dividend according to the given *Integer*, or else accordingly take the half, third part, quarter, &c. of the Numerator, which may best be done. *Examples.* As to divide 3 by $\frac{1}{2}$ the 3 doubled shall make the Quotient $\frac{6}{1}$. But $\frac{1}{2}$ divided by 3, because the third part of 1 cannot be had, 2 shall be tripled, and make the Quotient $\frac{1}{6}$. And in dividing $\frac{3}{2}$ by 3 either 2 may be tripled, or the third part of the Numerator which is 1 taken, and this is best, because the Quotient $\frac{1}{2}$ will be in its least Termes, otherwise it would be $\frac{3}{6}$, and need Abbreviation. See the Common Operations at N. O. P.

N $\frac{1}{2})\frac{3}{1}(\frac{6}{1}$ O $\frac{3}{1})\frac{1}{2}(\frac{1}{6}$ P $3)\ \frac{\overset{1}{3}}{1}\ )\ \frac{\overset{1}{3}}{2}\ (\frac{1}{2}$

5. Case

5. Case. When an Improper *Fraction* is given to divide an *Integer* greater in quantity than the Numerator of the *Fraction*, *Division* may be made after the manner of *Integers* thus; Divide the Dividend by the Numerator of the *Fraction*, and substract this Quotient from the Dividend, and lastly divide the Remain by the *Integers* contained in the *Fraction*, and this last Quotient shall be the quesited Number. As to divide 1480 by $3\frac{1}{12}$, or $\frac{37}{12}$, after 1480 is divided by 37, and the Quotient 40 substracted, the Remain 1440 is to be divided by 3, and this Quotient 480 is the Number sought. As at Q. agreeing with the Common way at R.

5. Improper Fraction, and Integer, &c.

Example.

$$Q\ 3\frac{1}{12}\ \frac{37}{\ }\quad \frac{1480}{37}\Big(40\quad \frac{1480}{1440}\quad \frac{1440}{3}\Big(480\qquad R\ 37)\ 3\frac{\overset{1}{37}}{\frac{1}{12}}\Big)\frac{\overset{40}{1480}}{1}\Big(\frac{480}{1}$$

6. Case. When the Numerators are equal, cancel them, and place the Denominator of the Divisor for the Numerator of the Quotient, over the Denominator of the Dividend. But when the Denominators are equal reject them, and *vice versa* place the Numerator of the Divisor under the Numerator of the Dividend for the Denominator of the Quotient. And if both the given *Fractions* be equal, for the Quotient take 1, for the new *Fraction* in such case shall alwayes be equal; and generally may be observed, if the Dividend be the greater of the two propounded *Fractions*, the Numerator of the Quotient will be greater than the Denominator, but if the Divisor be the *major Fraction* the contrary. As to divide $\frac{2}{5}$ by $\frac{2}{3}$, the Quotient will be $\frac{3}{5}$. So $\frac{1}{3}$ divided by $\frac{2}{3}$ gives $\frac{1}{2}$ in the Quotient, and $\frac{1}{3}$ by $\frac{1}{3}$ makes the Quotient an Unit. As at S. T. V.

6. Homologal Terms equal.

Examples.

Numerators Equal.	Denominators equal.	Fractions equal.
$S\ \frac{2}{3}\Big)\frac{2}{5}\Big(\frac{3}{5}$	$T\ \frac{2}{3}\Big)\frac{1}{3}\Big(\frac{1}{2}$	$V\ \frac{1}{3}\Big)\frac{1}{3}\Big(\frac{1}{1}$

As in *Integers*, so in *Fractions*, *Division* and *Multiplication* alternately prove each other, and though particularly one sort of work may be tried by another sort, yet regular and general *Probation* is by dissolving the Numbers compounded, and compounding the Numbers dissolved. Wherefore if the Product of any *Fraction* be divided by either of the *Factors*, the other *Factor* shall be found in the Quotient, and if the Quotient of any *Fraction* be multiplyed by the Divisor, the Dividend shall be returned. *Example* in the Answer of these two following Questions.

Proof of Division of Fractions.

1. What Number is that which being multiplyed by $\frac{1}{5}$ shall produce $\frac{3}{10}$?

Answer. $1\frac{1}{2}$, for dividing $\frac{3}{10}$ by $\frac{1}{5}$, the Quotient is $\frac{3}{2}$ or $1\frac{1}{2}$.

2. What Number being divided by $\frac{1}{5}$ will give in the Quotient $\frac{3}{2}$?

Answer. $\frac{3}{10}$, for such is the Product of $\frac{1}{5}$ multiplyed by $\frac{3}{2}$.

Questions in Multiplication and Division of Fractions.

Proof of Division.	Proof of Multiplication.
$\dfrac{3}{\dfrac{1}{5}\times\dfrac{3}{2}=\dfrac{3}{10}}\over 10$	$5)\ \frac{1}{\underset{1}{5}}\Big)\ \frac{3}{\underset{2}{10}}\Big(\frac{3}{2}$

Hence it is evident that *Reduction of Fractions of Fractions* may be proved; if the reduced *Fraction*, being but the Product of their *Multiplication*, be divided by any one of the Fragments, for then will the Quotient be the other Fragment, or the Sum of the other Fragments, if more than two were in the Composition. As in the former Instances,

Proof of the Reduction of Fractions of Fractions in *p. 48.*

Instances, 2. *Chap.* 4. *Sect.* of *Reduction,* $\frac{1}{4}$ of $\frac{2}{5}$ reduced became $\frac{1}{10}$, if therefore $\frac{1}{10}$ be divided by $\frac{1}{4}$, the Quotient will be $\frac{2}{5}$, or by $\frac{2}{5}$ it will be $\frac{1}{4}$, as at U. and W.

And the Reduction of $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{8}$ unto $\frac{15}{64}$ will be found true; for if $\frac{15}{64}$ be divided by any of the three Fragments, the Sum of the other two will appear in the Quotient, as at X. Y. Z.

$$\overset{\text{U}}{2)\frac{1}{\underset{2}{4}}\Big)\frac{1}{\underset{5}{10}}\Big(\frac{2}{5}} \qquad \overset{\text{W}}{5)\frac{2}{\underset{1}{5}}\Big)\frac{1}{\underset{2}{10}}\Big(\frac{1}{4}} \qquad \overset{\text{X}}{2)\frac{1}{\underset{1}{2}}\Big)\frac{15}{\underset{32}{64}}\Big(\frac{15}{32} = \frac{15}{\frac{3}{4}\text{ of }\frac{5}{8}}\Big/32}$$

$$\overset{\text{Y}}{\begin{matrix}3)\\4)\end{matrix}\frac{\overset{1}{3}}{\underset{1}{4}}\Big)\frac{\overset{5}{15}}{\underset{16}{64}}\Big(\frac{5}{16} = \frac{5}{\frac{1}{2}\text{ of }\frac{5}{8}}\Big/16} \qquad \overset{\text{Z}}{\begin{matrix}5)\\8)\end{matrix}\frac{\overset{1}{5}}{\underset{1}{8}}\Big)\frac{\overset{3}{15}}{\underset{8}{64}}\Big(\frac{3}{8} = \frac{3}{\frac{1}{2}\text{ of }\frac{3}{4}}\Big/8}$$

This kind of Proof was mentioned before in *Reduction*, but reserved till after *Division* was taught; as not probable before to be understood.

Partis secundæ, & Libri primi

FINIS.

THE

ARITHMETICK.

The Second BOOK,

CONCERNING

Numbers generally contract;

In Two PARTS.

WHEREIN

GEODÆTICALS } are { Declared.
FIGURALS } are { Demonstrated.

AND THEIR

SIMPLE ELEMENTS.

CHAP. I.

Of GEODÆTICALS.

Sufficient hath been said of *Absolute Integers* and *Abstract Fractions* in the former Book, it is necessary now to proceed to *Contract Numbers.* *After Abstract Numbers the Contract treated of.*

Contract Numbers were before declared to be Numbers restrained by some annexed Denomination, or special Denominator, and *Book* 1. *Part* 1. *Chap.* 2. divided into two sorts, *General* and *Special.* *Contract of 2 sorts.*

General are such whose Denominations are generally known in most Nations, and usual not only in *Mathematical Sciences*, but also in *Mechanical Arts*, and of Common and Vulgar Frequentment in Traffique, Merchandizing, Buying, Selling, *&c.* *General are*

Numbers generally Contract are *Geodætical* or *Figural.* *Geodæticals and Figurals.*

Geodæticals include all Numbers contracted by Vulgar Names or Denominations according to the common and usual distinctions, divisions, dimensions, or legal institutions, customs, or usages of Nature or Nations, As Men, Women, Horses, Sheep, Weights, Measures, *&c.* *Geodæticals what.*

Whence the Word.

The word *Geodætical* comes from *Geodæsia*, and this from the two *Greek* words, γῆ and δαίω, signifying a Division, Measure or Dimension of the Earth, and may seem too short to denote what is intended thereby; yet because all Denominations are truly but measures of quantities or gravities, accompted according to the Standart of earthly dimensions instituted by the Law of Nature or Nations upon the Terrestrial Globe; the Term *Geodætical* may fitly stand for the purpose here used till another more fit be found out.

Geodæticals distinguished.

Numbers thus concluded may be distinguished as in the Order following.

Geodæticals
- Greater
  - Integral.
  - Fracted.
- Smaller.
  - English
    - Measure.
    - Weight.
  - Forrain
    - Antient
      - Hebrew.
      - Greek.
      - Latin, and Roman.
    - Modern, *Vide. pag.* 110. to 157.

Greater Geodæticals.

The greater *Geodæticals* are such Denominate Numbers whose Names or Denominations admit not smaller artificial divisions, or are the highest denominations of that kind or quality, and if they may be divided into smaller parts yet cannot be heightned into greater. Of the first sort are Angels, Men, Women, Cities, Towns, Horses, Sheep, Ships, and a multitude of such other Nounes. Of the other sort are Leagues, Years, Circles, Bales, Butts, Tons and such like, which are capable of being divided into smaller parts or denominations.

Integral or Fracted.

These greater *Geodæticks* are *Integral* or *Fracted*.

Integral what.

Integral, When such denomination is annexed to any *Abstract Integer*, as 10000 Angels, 36 Men, 800 Ships, *&c.*

Fracted what.

Fracted, When such denomination is annexed to any *Arithmetical Fraction*, as $\frac{1}{2}$ of a Ship, $\frac{3}{4}$ of a Ton, *&c.*

Geodæticals how they come to be reckoned among Integers denominate.

And because several of these greater denominations may be parted into lower and subtiller parts, every of which parts having Proper Names, and being thereby known, those parts so named become reckoned for *Integers* denominate, and the greater denominations omitted as before, *Chap.* 2. *Of the Nature of Numbers* was declared. For $\frac{3}{4}$ of a Ton, because a Ton is divided into 20 Hundreds shall be called 15 Hundreds, which is $\frac{3}{4}$ of 20, and pass for a denominate *Integer*, and the great denomination [*Ton*] omitted: So because a Pound of Money is divided into Shillings, and Shillings into Pence, parts of a Pound shall be accompted by Shillings, and parts of a Shilling by pence, *&c.*

Smaller Geodæticals.

The smaller *Geodæticks* arise from such of the greater as admit of subdivisions and lesser parts under particular names and denominations which being abbreviated with the greater denomination may be reduced back again from a smaller denominate *Integer* to a fracted *Geodætick* of the greater Contraction; As 10 Shillings abbreviated with 20, the Shillings in one pound shall be reduced to $\frac{1}{2}$ *l.*

English or Foreign.

Geodæticals of the smaller sort may be comprehended under two heads, *English* and *Foreign*.

English take in Measure and Weight.

English, Include both *Measure* and *Weight*.

Measure what.

Measure is the accompt of any Quantity or Magnitude taken either in length or breadth, or according to its solid thickness, or hollow capacity, by some Standart approved as a known Measure.

Weight what.

Weight is the accompt taken of the gravity of any Quantity or Magnitude according to some approved Standart or known Measure.

Both doubly considered.

Both these, *Viz. Weight* and *Measure* referring to some denominations are pure as considered *per se*, to others impure as considered *inter se*, but in Authours found somewhat confusedly, as Money and Bread passing only as numbred or accompted, and yet notwithstanding to have their due proportion of Weight, and Woollen Cloth generally passing by Measure in length, ought also to have due breadth and weight.

English Measures.

Under *English* Denominations that refer to measures of length may be contained not only things once measured by the Inch, Foot, Yard, Ell, Rod, *&c.* but also several things accompted by Number, or as commonly called Tale, as Fish, Skins, Paper,

per, &c. for it is sufficiently evident that every Number in some sense is but a lineary Measure or Multitude of Units supposed to stand in length one by another.

Long Measures therefore comprise the Denominations used in *Astronomy* of *Time* and *Motion*. In *Cosmography* and *Geography* from the measure of Leagues down to the length of Barly-Corns. In Merchandise the denominations of many sorts of Silks, Stuffs, Linnen Cloth, Fish, Skins, Paper, Parchment, &c. *Long.*

Broad Measures imply things twice measured by a line, once in respect to length and again to breadth, and contain the Denominations used in *Geometry* from *Acres* downward, and in the square measures of Glass, Plank, Pavements, &c. In Merchandize Woollen Cloth, which also is to have its proportion of Weight. *Broad.*

Measures of Thickness or *Bodily Capacity* are either Solid, as the Sizes of Fuel, Measure of Timber, Stone, &c. or Concave which have hollow Capacity. Concave Measure is divided into dry and liquid; Dry take in the Measures of Corn, Salt, Coals, Lime, &c. Liquid, Ale, Beer, Honey, Butter, Fish, Oyl, Sope, Wine, &c. *Thick.*

Among *English Weights* but two are considerable, *Viz. Troy* and *Avoirdupois*. *English Weights 2. considerable.*

The Book Intituled *The Path-way to Knowledge* Translated out of *Dutch* into *English* by *W. P.* 1596. makes mention of three other Weights. *Dutch Book mentions.*

1. The *Pound Tower* usual for Mintage, in the names of its divisions like the *Pound Troy*, but in quantity less, for 16 *Pounds Tower* make but 15 *Pounds Troy*. *Pound Tower.*

2. The *Pound Subtil* or *Suttle* so termed, for that in small quantity it may be made ratable to represent any other greater Weight whatsoever, as *Four Penny weight Troy*, or less, to answer in due proportion unto the whole *Pound Troy* with all his parts, every part sensible and severally to be handled. This Weight is private to Assay-Masters and such as can make trial of *Minerals*, and not known to many other, neither in ordinary Accompts is there any use thereof. *Pound Subtil.*

3. The *Pound Foile*, is lesser than the *Pound Troy* by [illegible] part of the *Pound Troy*, and hath small use, save only amongst those that make *Gold Foil* and *Wyre*. *Pound Foile.*

But the *Troy* and *Avoirdupois* Weights are of common use, the first of force by Statute, the other by Custom, yet confirmed by Statute. Why so called I could never satisfie my self, but crave leave to conjecture that *Troy Weight* was the Weight anciently used in Towns and Cities, and *Verstigan* tells us that *Troy Novant*, is as much as to say *New Town*; and *Avoirdupoies, Averdupois, Averdepois,* or *Avoirdupoys* however written seems to sound like an Over-Weight, from the old *Norman* Language, and was either allowed upon drossy and coarse Goods, or probably sold in the Countrey and the Over-Weight allowed in lieu of Carriage to a Market. *Troy and Avoirdupois Weights of common use in* England. *Why so called probably. Troy weight* quasi *Town weight. Avoirdupois* quasi *to have Over weight of* 12. *on* 100.

By the *Troy-Weight* is accompted the Assize of Bread and some Liquid Measures constituted, also Gold and Silver Plate and Bullion, Rings, Jewels, Precious Stones, Musk, Civit, &c. are weighed thereby. *What weighed by Troy weight.*

By the *Avoirdupois-Weight* are bought and sold all base Mettals, as Copper, Tinn, Steel, Iron, Lead, and several other Merchandizes, as Allom, Rozin, Pitch, Tar, Wax, Tallow, Hemp, Flax, Beef, Cheese, Meal, Corants, Raisins, Prunes, Figgs, Almonds, Spices, Druggs, Sugar, Tobacco, Wooll, &c. So Sope and Butter when sold by Retail, and many other Retailed Commodities whereof any Garble, Refuse or Waste comes; But Butter, Sope, and some other things when sold in gross, pass by the Barrel or Firkin where the Content or Measure of the Vessel is also considerable *What weighed by Avoirdupois weight.*

Coyne respecting both Weight and Number or Tale which is but long Measure as before noted may be placed under either of them, but though vulgarly passing by Tale, yet properly belongs to *Troy-weight* according to the divisions whereof, for the most part, *English Money* is divided, and ought to be valued in Weight and Fineness. *Coyne respects Weight and Number.*

Of all these in order, it is convenient something be said, and their Dimensions and Divisions seen, before Foreign Measures and Weights be spoken of.

English Measures.

The Measures of Time and Motion being properly *Astronomicals*, having certain Denominators and special Operations belonging to them are referred to the second part of the next Book. *Measures of Time and Motion. See Book 3. Par. 2.*

Cosmographical and *Geographical* Measures of length begin at a Barly-Corn, and increase upward to a League, in Latin called *Leuca*, which is the greatest denomination principally in use with Mariners, for with Geographers commonly the greatest denomination *Barley-Corn the beginning of Long Measures.*

League from the Latin Leuca, *Mile from* Mille.

mination is a Mile, generally thought to be derived from the Latin *Mille*, signifying 1000, because a Mile with the Latins contained just so many Paces, but an English Mile containeth 1056 Paces.

A Table of English Long Measures for Land.

In 1 League are 3 Miles. In 1 Mile 8 Furlongs, &c. As in the following Table.

League.	1	Mile.					
Miles. *. 1.	3	1	Furlong.				
Furlongs. *. 2.	24	8	1	Perch.			
Perches. *. 3.	960	320	40	1	Foot.		
Feet. *. 4.	15840	5280	660	16½	1	Inch.	
Inches.	190080	63360	7920	198	12	1	
Barley-Corns *. 5.	570240	190080	23760	594	36	3	Barley-Corns.

Length of an English Mile.

*. 1. That an *English* Mile shall contain so many Furlongs, Perches and Feet may be seen in the Statute. *Anno* 35. *Eliz. Cap.* 6. Intituled, *An Act for the Restraint of New Building*, &c. and long before the making of that Statute was a Mile of the same content.

Pulton *misprinted in the number of Furlongs.*

*. 2 The Collection of the Statutes at large by *Ferdinando Pulton*, Printed 1640. *pag.* 1191. in the Statute of 35. *Eliz.* last mentioned deserves Correction, who there makes 1 Mile but 5 Furlongs, whereas it should be 8, as by another Printed Copy by me, and other good Authours may appear.

Perch, several names thereof. Less than a Rood. The Sorts.

*. 3. A Perch or Pearch hath other names, as Pole, Rod, and Lugge, but they are grosly mistaken who call it a Rood, for a Rood is a quarter of an Acre, a far greater quantity than a Rod.

Perch for the Measure of Woodland. Church-land. Forests.

The Pole, Rod or Perch, for they are the most usual names, is by the Statute aforesaid to be but 16½ Feet, yet by the usage of some Countreys the Pole doth vary, for in some places it is 18 Feet, in some 21 Feet, and in others 24. And Mr. *Osborne* (as witnesseth *Dalton* in his *Countrey-Justice, Chap.* 65.) writeth that the measure of 18 Feet to the Perch is commonly called Woodland Measure, 21 Feet to the Pole is called Church Measure (*Scilt.* of Land which doth or did belong to some Church), and 24 Feet to the Pole is called, (and that rightly) Forest Measure.

Foot, the length thereof. Often used Plurally.

*. 4. That a Foot shall contain 12 Inches, and one Inch 3 Barley-Corns laid end to end, (or as some say 4 in thickness being dry and round and taken out of the midst of the Ear) is evident by the Statute made *Anno* 33. *Edw.* 1. *An. Dom.* 1306. *De terris Mensurandis, & De compositione Ulnarum & Perticarum.* Foot is often used in the Plural as well as Singular; as 2 Foot, 3 Foot, &c. for 2 Feet, 3 Feet.

Barley-Corn no measure in it self.

*. 5. A Barley-Corn is in it self no Measure, but the least thing in a Measure, whereof as it were Measure is made, and whereby it is rectified by the Ordinance, Intituled, *Compositio Ulnarum & Perticarum.* 3 Barley-Corns laid end to end make an Inch, 12 Inches a Foot, &c.

Measures for depths, &c. Yard, the length Ell, the length.

Besides these, some measure lengths whether depths, heights, or distances by Scores, Goads, Fathoms, Paces, Ells, and Yards, which are thus divided.

In 1 Yard are 3 Feet, or 36 Inches.

In 1 Ell are 3 Feet 9 Inches, or 45 Inches.

Both these ordained by Statute, and of general use in *England*, for the measure of Silks, Stuffs, Cloth, Lace, Ribband, and several other Wares and Merchandize; commonly Woollen Cloth by the Yard, and Linnen Cloth, Silks, &c. some by the one, and some by the other, as is well known to Tradesmen, but seldom used for Land. Both Yard and Ell divided equally into halfs, quarters, half quarters and Nails, or Neils. So 1 Yard or Ell containing 16 Nails.

Measures of Cloth how much in the

To some Cloth and Silks belong greater denominations than Ells and Yards, as Chefs, Bolts, Pieces, Hundreds, Roules, &c.

Chef. In 1 Chef of { Fine Linnen, Silks —— 10 ; Fustian —— 14 } Ells.

Bolt. In 1 Bolt of { Pole Davies ; Spruce Elbing } 28 Ells.

In

In 1 Piece of	Barbers Aprons, or Checks	10 Yards	
	Curle Sipers		
	Borratoes or Bombasines, Buffins	15 Yards.	
	Moccadoes and Lile Grograins, Bustians		
	Carrels, Dornix, Strip't or Tufted Canvas		
	Rashes, Flanders Serges, &c.		*Piece.*
	Beaupurs, Frizado, Hounscot Say	24 Yards.	
	English Tufted Canvas	30 Yards.	
	Ribband	36 Yards.	
In 1 Piece of	Cambrick	13 Ells.	
	Lawn		
	Lockram called Treagers, Broad Dowlass	106 Ells.	
In 1 Hundred of	Canvas	120 Ells.	
	Sackcloth		
	Soulthwitch		*Hundred.*
	Tiking and Twill of Scotland		
In 1. Roul of	Minsters	1500 Ells.	
	Ozenbrigs		*Rowle.*

In 1 Pace or rather Pass, from the Latin *Passus*, are 5 Feet, called a Pace *Geometrical*, to difference it from other Paces of greater or lesser content, and is properly a Foreign Measure for Land. *Pace or Pass how much.*

In 1 Fathom are 6 Feet, used in measuring Depths, and Sounding at Sea. *Fathom.*

In 1 Goad 1 ½ Yard, or 4 ½ Feet, a Measure in some places for Land and Cloth received by Custom. *Goad.*

In 1 Score 20 Yards or 60 Feet, a Measure also not ordained by Statute. *Score.*

Some Denominations frequent in *English* Books, or Digits, Palms, Spans and Cubits great and small, are no usual *English* Measures, but came from other Nations hither, and retaining their Foreign Dimensions and Divisions, are to be sought among the Foreign Accompts. *Digits, Palms, Spans, and Cubits, Foreign not English Measures.*

The Measure called a Handful used in measuring the height of Horses, by 27. *Hen.* 8. *Chap.* 6. is ordained to be 4 Inches. *Handful how much.*

In Merchandize under Long Measures as before noted, fall sundry Lengths, commonly called Number or Tale

Many small Wares called Habberdashery, and some other Commodities are sold by Dozens, Scores, Shocks, Hundreds, Thousands, Lasts, Grosses, &c. *Small Wares the Measure.*

Every dozen contains 12, and of some things in some places 13. *Dozen.*

Every Score 20, and in some places 21 for 20. *Score.*

Every Shock 60. *Shock.*

The Hundred is more or less according as the Commodity is, and the Thousand and Last greater or lesser as the Hundred. *Hundred, 1000 and Last.*

The Gross is small or great, the small Gross is 12 dozen, the great Gross is 12 small Gross. *Gross great and small.*

A Breviat of some Merchandizes accompted by these and such like Denominations.

Alphabets. In 1 Set—24. *Set.*

Balkes, Barrel board, Bome spars. In 1 Hundred—120. *Hundred.*

Bookes. In 1. Maund. 2. Fats. In 1 Fat 4 Bales. *Maund, Fat &c.*

Bowstaves. In 1 Hundred 120. *Hundred.*

Boxes called Sope-Boxes. In 1 Shock 60. *Shock.*

Bracelets or Necklaces of Glass. In 1 small Gross 12 Bundles or Dickers. *Gross, Bundle.*

Bread, in several places. In 1 dozen 13 Penny Loaves. *Dozen.*

Buttons. In 1 great Gross 12 small Gross. In 1 small 12 ordinary dozen of Buttons. *Grosses, &c.*

Canes. In 1 Shock 60. *Shock.*

Cantspars, Capravens. In 1 Hundred 120. *Hundred.*

Clapholt or Clapboard. In 1 Great Hundred 12 Rings. In 1 Ring 2 small Hundred. In 1 small Hundred 120 Boards. So that 1 great Hundred contains 24 small Hundred, or 2880 Boards. *Hundred, &c.*

Deales. In 1 Hundred, 120.

Length of Fish sold by Tale.

Fish, Of the greater sort barrelled called Countable or Tale-Fish, ought to contain in length from the Bone in the Finn to the third joynt of the Tale, 26 Inches at the least, by the 22. *Edw.* 4. *Cap.* 2.

Fresh Fish how sold. Maund thereof. Mary.

Most fresh Fish is sold by the Common Dozen or Score, Sometimes by the Maund, if the Fish be small; the Maund or Moane, holdeth about a Gallon; and Six Maunds full set together, and an heap on them all at top is called a Mary.

Besides these some quantities of Fish fresh and Salt sold out of Cask have Common Denominations.

Cod, &c. the 100.

Cod, Also Haberdine Ling, and Newland Fish. In 1 Hundred 124.
Yet the Book of Rates reckons but 120 to the Hundred.

Eeles the Bind, Strike.

Eeles, In 1 Bind, 10 Strikes, In 1 Strike 25 Eeles. So is 1 Bind 250 Eeles.

Herrings.

Herrings Fresh or Salted at Sea, called carn'd or corbed by the Last in some places thus divided.

A Table of the Last of Herrings.

Last.	1	Thousand			
Thousands.	10	1	Hundred.		
Hundreds.	100	10	1	Warpe.	
Warpes.	3200	320	32	1	
Herrings.	12800	1280	128	4	Herrings.

But by the Statute of Herrings made 31. of *Edw.* 3. *An. Dom.* 1357. *Cap.* 2. there is appointed but Six Score to the Hundred, so one Last shall contain but 12000. Yet at *Yarmouth* they sell 33. Warpe to the Hundred.

White Herrings. Red Herrings.

Accordingly White Herrings that is Salted in Barrels is sold by Retail, and Red Herrings that is dryed in the Smoak, in some places are accompted by 120 to the Hundred.

A Cade or Carde of Red Herrings ought to contain in 1 Cade 5 Hundred, that is 600 Herrings, and one Last 20 Cades.

Hundred. Shrimpes. In 1 Hundred, 120.

Turn. Soles. In 1 Turn 4.

Hundreds. Sprats. The Hundred as Herrings. The Cade of Red Sprats 5000. Yet by the Book of Rates Outwards but 1000.

Stock-fish. In 1 Hundred, 120.

Rastal. 8 Title, Weights and Measures, saith the Hundred of hard Fish must be 8 Score.

Ropes. Garlick. In 1 Hundred 15 Ropes. In 1 Rope 15 Heads. So is 1 Hundred, 225 Heads.

Dickers. Gloves. In 1 Dicker, 10 Pair.

Horshooes. In 1 Dicker, 10 Shooes.

Dozen. Iron. In 1 Dozen 6 pieces. Rastal 8 Weights and Measures.

Bundle, Dicker. Knives. In 1 Bundle 6 Dickers. In 1 Dicker, 10 Knives.

Last, Dicker. Leather. In 1 Last 20 Dickers. In 1 Dicker 10 Hides. So the Last is 200 Hides.

Hundred. Oars. In 1 Hundred, 120.

Paper and Parchment how reckoned.

Paper.

Bale.	1	Ream.		
Reams.	10	1	Quire.	
Quires.	200	20	1	
Sheets.	5000	500	25	Sheets.

Parchment.

Rowle.	1	Dozen.	
Dozens.	5	1	
Skins.	60	12	Skins.

Bind of Skins. Skins. In 1 Bind, 33 Skins.

Timber of Skins.

Skins, or Furrs of
- Ermines. Fitches. Grayes. Jennets. Letwis. Martrons. Minkes. Sables. } In 1 Timber 40 Skins.
- Budge. Cat. Coney. } In 1 Hundred 5 Score Skins.

Skins

Skins of Calves.	In 1 Dozen 12.	Skins.
Goats.	In 1 Kippe. 50.	
Kidds. Lambs. Sheep.	In 1 Hundred, 5 Score. Yet by the Book of Rates Outward 6 Score.	

Dozen, Kippe, Hundred of ins.

Trayes of Wood. In 1 Shock, 60.

Shock.

Measures long and broad reach the Denomination used about Land Measuring, called Acres. It is sufficiently manifest by the Statute *An.* 33. *Edw.* 1. aforesaid, that an Acre of Land is to contain in length 10 Perches, and in breadth 16. So if the breadth be 1 Perch, the length shall be 160; if the breadth be 2, the length shall be 80; and so proportionably, that is to say alwaies 160 Square Perches. The other proportions of the length and breadth of one Acre of Land mentioned in that Statute were observed by Record in his Book of *Arithmetick* long since, not to be exact. The Acre is thus divided.

Broad or Square Measures for Land in England. *Acre how much.* *Statute thereof not exact.*

A Table of English *Square Measures.*

Acre.	1	Rood.		
Roods. *.	4	1	Daies work.	
Daies Works.	40	10	1	
Square Perches.	160	40	4	Square Perches.

*. A Rood is sometime called a Farthendele, and sometime a Yardland, but as to the latter very corruptly, for a Yardland containeth much more than an Acre.

Rood how called Yardland, more than an Acre.

Several Denominations about Land Measure besides a Yardland are found in the Law Books, as Hides, Plowlands, Carves, Carucates, and Oxeganges, but are grown so obsolete, That the Lawyers themselves can hardly agree about the Content thereof; the first 4 seem to be all one, and are reckoned to contain by some 85, by most, 100 Acres, yet *Norden* in his *Surveyors Dialogue*, and others, make a difference between a Hide of Land and the other three, and say that a Hide of Land containeth 4 Plowlands, and every Plowland, Carve or Carucate, which are all one, 4 Yardlands, and every Yardland 30 Acres. So shall one Plowland contain 120 Acres, and one Hide of Land 480 Acres, *Cambden* and *Hollingshead* will have one Hide of Land to contain 100 Acres, and others say, 8 Oxeganges make a Hide or Plowland, and every Oxegange containeth 15 Acres. *Dalton* in his *Countrey-Justice* saith, that the Common Account in the East part of *Cambridge-shire* of a Yardland is but 24 Acres. And Sir *Edward Coke* in the first part of his Institutes under the Title *Escuage* (perhaps prudently foreseeing these differences irreconcileable) is of Opinion that a Plowland is of no certain Content, but is rather to be reckoned by the Value than Content, and that more in one place and lesser in another shall be a Plowland according to the quantity that one Plow may till in a Year. But all agree that a Yardland called in Latin *Quatrona terra* is much more than an Acre, and therefore ought not to be used for a Rood, which is but a quarter of an Acre.

Difference in the Accompts of Hides, Plowlands. &c. *Sir* Edward Coke*'s Opinion of a Plowland.* *Yardland how called.*

Among long and broad Measures fall in next, Glass, Plank, Pavements, of which Glass-Windows are commonly measured by the Foot Rule of 12 Inches to the Foot. So one Square Foot shall contain 12 Inches in length, and as many in breadth, that is 144 Square Inches. Unwrought-Glass sometime sold by Weight. *See among Weights.*

Glass, &c. *how measured.*

Pavements are sometimes measured by the Yard, commonly by the Foot-Rule, and a parcel of Pavement or Tiling of 10 Foot long and 10 Foot broad is ordinarily in these parts called One Square. And equivalent to such a Square shall be the laying of 100 Gutter Tiles, or Redge-Tiles, though of the latter some count but 50.

Pavements, Roofs, how measured. *The Square thereof.*

Plank or Board is commonly accounted among Square Measures, yet more properly belongeth to Timber Measure, for that besides the length and breadth respect is had to the thickness of the Plank, whether it be Inch, Inch and half, Two Inch, Three Inch Plank or more, all commonly measured by the Foot of 12 Inches. The Accompt of the Load of each is referred to Timber Measure.

Plank or Board how measured.

All Square Measures, whether of Acres, Glass, Pavements, Plank, *&c.* do properly belong to Figural Numbers treated of in the next part of this Book

Square Measures belong to Figurals.

But Woollen Cloth though as before noted, must have length, breadth and weight, yet being commonly accompted only by length, and accordingly by Retail sold by the Yard keeps place among *Geodæticals*, and as to the making, and Wholesale thereof duely placed here.

Woollen Cloth.

The

Statute for the due make thereof.

The due making of Woollen Cloths is declared in a Statute made *An.* 4. *Jac. Chap.* 2. being an Epitome of all former Acts to that purpose, in which may be seen their Weight, Breadth and Length, Workmens Orders, with the Viewing, Searching and Forfeitures or Abatements of and for the same.

One Sack of Wooll (the Content whereof is found among Weights) is accompted to make 4 Standard Clothes of clean Wooll called sorting Clothes, weighing 60 ℔ the Cloth, and being 24 yards long and 6 quarters ½ broad or thereabouts within the remedy or allowance of 2 ℔ weight upon a Cloth.

Retailed by the Yard and Inch.

The length is prescribed by the Statute to be measured wet within the list of the Cloth, by the Yard and Inch, instead of which Inch is the allowance of the Thumb in Retailing usual.

Cloth to be weighed.

In Weighing, which is to be by the *Avoirdupois* Weight is to be observed that the Clothes be well scoured, thickned, milled, and fully dryed, the Weight of a Cloth seems more to be regarded than the measure, because the Weight containeth substance which may be abused by stretching into Measure.

Assize of Cloth by Statute.

The Assizes of Woollen Cloths by the Statute An. 4. Jacobi. Cap. 2.

	Length. Yards.		Breadth. Quarters.	Weight. Pounds.
Long Broad Cloth, and Clothes of Died Woolls and mingled Colours of *Kent*, *Yorkshire* and *Reading*, between	30 &	34	6½	86
Whites of *Worcester*, *Coventry* and *Hereford*	30	33	7	78
Plunkets, Azures, Blews, and long Whites of *Suffolk*, *Norfolk*, and *Essex*.	29	32	6½	80
Sorting Clothes, *Suffolk*, *Norfolk*, and *Essex*	23	26	6	64
Fine short *Suffolks*	23	26	6½	64
Handiwarpes	29	32	7	76
Plunkets, Azures, &c. of *Wilts*, *Somerset*, &c.	26	28	6½	68
Yorkshire short Clothes	23	25	6½	66
Broad listed Whites and Reds of *Wilts*, *Glocester*, *Oxford* and East part of *Somerset*	26	28	6½	64
Narrow listed White and Red	26	28	6½	61 White 60 Red
Fine plain listed Clothes of the *Shires* last mentioned	29	32	6½	72
Tauntons, *Bridgwaters*, and *Dunsters*	12	13	7	30
Short Cloths of Died Wooll, &c.	23	25	6½	66
Narrow of *Somerset*, &c.	24	25	4	30
Devon Kersies called Dozens	12	13	4	13
Check Kersies, Straights and Plain Graies	17	18	4	24
Ordinary Pennistone or Forest Whites	12	13	5½	28
Sorting Pennistones	13	14	6½	35
Whashers of *Lancashire* and others	17	18		17
Cogware, Kendal, and Karptmeales at pleasure	20	at least.		

Cottons by the Goad.

Some accompt *Manchester*, *Cheshire*, and *Welch* Cottons by the Goad, allotting the *Lancashire* Cotton to be in length between 20 and 21 Goads, in breadth ¾ within the list, and in weight 21 pound. The *Manchester* and *Cheshire* 22 Goads in length, in breadth as the *Lancashire*, and in weight 30 Pound.

Measures Bodily for Bodies thick, solid, or concave.

The next sort of Measures are bodily, whither Solid or Concave, and properly belong to Figural Numbers handled in the next part, yet because many of them are not Rooted Numbers, and in Common Commerce reckoned by Number or Tale rather than in respect to their Capacity, they may as to their Denominations stand among *Geodaticals*.

Among Solid Measures are the Assizes of Fuel, Plank, as before noted, Timber, Stone, Laths, Tiles, &c.

Fuel the Measure.

Fuel contains Billets, Cordwood, Faggots, Talwood, and Coals. But Coals are sold by Bushel, and therefore placed among Concave Measures.

Billets the Assize.

Every Billet by the 7. *Edw.* 6. *Chap.* 7. must be 3 Feet 4 Inches in length, and is accompted for 1 or more, according to the bigness thereof. For if it be but 7½ Inches about, it shall be but a single Billet. If it were 10 Inches about it was called a Cast, and was marked with 1 notch within 4 Inches of the end, and to pass for 2 Billets. If the two Billets were 14 Inches about it was called a Cast of two, and marked with 2 notches, within 6 Inches of the middle, and to pass in Tale for 4 Billets.

But

But by the 43. *Eliz. Chap.* 14. the Assize was altered as to the Casts, Cleft Billets, assized, and provision made that no single Billet should be cleft: Thus,

	Round.	Half Round.	Quarter cleft.	Length.
A Single Billet.	$7\frac{1}{2}$			40 Inches.
A Cast marked 1.	11	13	$12\frac{1}{2}$	40 Inches.
A Cast of 2 marked 2.	16	19	$18\frac{1}{2}$	40 Inches.

Billets marked with 3, 5, or 7 notches are to pass for so many single Billets, and to be proportional. Billets are commonly sold by the Hundred, 5 Score to the Hundred.

Cordwood the Load or Cord.

Cordwood is Wood of the bigger sort of Firewood, measured by a Cord or Line, of which there are two Measures. That called the Fourteen Foot Cord is to be 14 Feet in length, 3 Feet in breadth, and 3 Feet in height.

The other Cord is to be 8 Feet long, 4 Feet high, and in breadth 4 Feet, yet in some places 3 of the 4 Feet high is 4 Foot Wood, and the other Foot but 3 Foot Wood.

Faggots the Load.

Faggots called Two bands by the last mentioned Statutes are to be 3 Feet in length, and the band 24 Inches about, besides the knot. Of such Faggots 50 go to one Load.

Faggots of smaller Wood called Bavin and Spray are sold by the Hundred, and 100 accompted for a Load.

Talshide the Assize.

The Assize of Round Talshide Ordained by 7. *Edw. 6. Chap.* 7. is confirmed by 43. *Eliz. Chap.* 14. and Talshides half round and quarter cleft assized thus.

	Number of Notches or Marks.		Round.	Half Round.	Quarter cleft.	
Every Talshide named	1	must be	16	19	$18\frac{1}{2}$	Inches about within a Foot of the middle.
	2		23	27	26	
	3		28	33	32	
	4		33	39	38	
	5		38	44	43	

Board or Plank.

Plank or Board customarily is accompted by the Load according to the thickness of the Plank, and to be measured by the Foot Rule; the Load thus reckoned.

The Load how much.

	Feet long.	Feet broad.	Inches thick.	
Plank or Board	600	1	1	make 1 Load.
	400	1	$1\frac{1}{2}$	
	300	1	2	
	240	1	$2\frac{1}{2}$	
	200	1	3	
	$171\frac{3}{7}$	1	$3\frac{1}{2}$	
	150	1	4	

If the breadth be more than 1 Foot, the length must be less proportionably.

Timber the Load the Ton.

Timber well hewn and perfectly squared, *viz.* 1 Foot broad, and 1 Foot thick 40 Feet long make 1 Ton or Tun. And 50 such Feet 1 Load. If the breadth or thickness be more, the length must be less, if less, the length must be more.

Stone how measured.

Stone sometimes is measured by the Foot after the manner of Timber, and sometime reckoned by the Ton Weight.

Lath, Tann, and Tile, because in them respect is had to their length, breadth and thickness may fitly be placed here.

Laths the Load and Bundle, how much.

Lathes are sold in gross by the Load bound up in Bundles, every Load 30 Bundles. In Retail by the Bundle, every Bundle 100 Laths. Every Lath ought to be 5 Feet long, 2 Inches broad, and $\frac{1}{2}$ Inch thick, if the Lath be but 4 Feet long, then there must be 6 Score to the Hundred.

Tann, the Load how much.

Tann, 1 Load must be 60 yards long, 1 yard high, 3 Rinds thick set up on each side of a Pole laid along to rest against, and 2 Rinds at top. Yet 45 yards thus set is a good Waggon Load.

Tiles the Assize.

Tiles are of 3 sorts, 1st Plain Tile or Thack Tile. 2ly Gutter Tile, or Corner-Tile. 3dly Roof Tile, Creastile or Ridge Tile, commonly sold by the Hundred in some places of 6 Score, and some 5 Score to the Hundred, all assized by the Statute *An.* 17. *Ed.* 4. *Chap.* 4. thus.

		Inches long.	Thick.	Broad.
Plain	Tile	$10\frac{1}{2}$	$\frac{1}{2}$ and $\frac{1}{2}$ of $\frac{1}{4}$	$6\frac{1}{4}$
Gutter	Tile	$10\frac{1}{2}$	with convenient Thickness and Breadth.	
Ridge	Tile	13	$\frac{1}{2}$ and $\frac{1}{2}$ of $\frac{1}{4}$	with convenient Breadth.

Concave Meaſures.

Concave Meaſures whether Dry or Liquid had their Original from Weight, and therefore ſome place them among Weights, but being formally Bodies, and common uſe reſpecting rather their Content or Quantity than their Weight, they may fitly ſtand here.

Dry.

Dry Meaſures are thoſe by which Dry Goods are meaſured, as Corn, Salt, Coals, Lime, *&c.* And may be diſtinguiſhed into *Wincheſter* Meaſure and Water Meaſure. *Wincheſter* Meaſure is the Standart Meaſure. Water Meaſure greater.

Corn Meaſure.

Corn is meaſured by Troy Weight, and weighed by *Averdupois*, that is the Meaſures for Corn are according to Troy Weight, but Corn ſold by Weight ſhall be weighed by *Averdupois* Weights.

The Aſſize thereof. Wincheſter *Meaſure.* *The Weight.*

By 51 *Hen.* 3. *An.* 1266. 31 *Edw.* 1. *&* 12. *Hen.* 7. *Chap.* 5. The Content of a Gallon of Wheat is to be 8 Pounds or Pints Troy, 8 Gallons 1 Buſhel *London* Meaſure, and 8 Buſhel 1 Quarter. This is called *Wincheſter* Meaſure.

By the Book of Aſſize of Bread ſet forth by *John Powel*, the Buſhel is to contain 56 Pounds or Pints of *Averdupois* Weight, and ſo proportionably for half Buſhels, Pecks, *&c.*

Buſhel above 8 Gallon in ſome places, and by heap.

Cuſtom hath begotten in ſome places greater Buſhels, then 8 Gallons, as 9, 10, *&c.* Alſo greater quantities than the Quarter have their denominations, yet all are to be reckoned according to the meaſure of the Buſhel and Gallon ordained by the Statutes.

Uſage in ſome places hath continued Meaſure by heap, although ſome Statutes order it by Strike, and allowance in ſome places is 21 for 20.

The Diviſions and Subdiviſions of Corn Meaſure called *Wincheſter* Meaſure, may be inſpected in the Table following.

A Table of the Laſt of Corn Meaſure.

Laſt.	1	Load.										
Loads.	2	1	Quarter.									
Quarters or Seams.	10	5	1	Coomb.								
Cornoocks or Coombs.	20	10	2	1	Strike.							
Striks or half Coombs.	40	20	4	2	1	Buſhel.						
Buſhels.	80	40	8	4	2	1	Tovit.					
Tovits or half Buſhels.	160	80	16	8	4	2	1	Peck.				
Pecks.	320	160	32	16	8	4	2	1	Gallon.			
Gallons.	640	320	64	32	16	8	4	2	1	Pottle.		
Pottles.	1280	640	128	64	32	16	8	4	2	1	Quart.	
Quarts.	2560	1280	256	128	64	32	16	8	4	2	1	
Pints or Pounds Troy.	5120	2560	512	256	128	64	32	16	8	4	2	Pints.

Apples, &c. the Laſt, Barrel.

Apples, Nuts, Oatmeal, In 1 Laſt 12 Barrels. 1 Barrel 3 Buſhels.

Charcoal the Load, Sack.

Charcoal is ſold ſometimes by the Load. In 1 Load 80 Buſhels. Sometime by the Sack. In 1 Sack 4 Buſhels, 7. *Edw.* 6. *Cap.* 7. and ſometime they reckon 88 Buſhels, or 22 Sacks *Wincheſter* Meaſure to a Load.

Meal ſold by Meaſure.

Meal in ſome places ſold by Meaſure. In 1 Buſhel 12 Gallons ſtriked.

Lime by Water Meaſure, how much the Buſhel

Lime, Salt, Seacoal are meaſured by Water Meaſure, the Buſhel whereof by the 11. *Hen.* 7. *Cap.* 4. is to contain 10 Gallons of *Wincheſter* Meaſure, nevertheleſs in ſome places is 12. 14. *&c.* Gallons.

Salt, the Hundred Wey.

Salt is reckoned by the Hundred and Wey. In 1 Hundred of Salt $10\frac{1}{2}$ Weyes, in 1 Wey 40 Buſhels. So 1 Hundred contains 420 Buſhels, Water-Meaſure.

Seacoale the Laſt Chaulder.

Seacoale is accompted by the Laſt and Chaulder. In 1 Laſt of Seacoal *Newcaſtle* Meaſure $7\frac{1}{2}$ Chaulders. The Chaulder generally 32 Buſhells, but differs at ſeveral places according to the quantity of the Buſhel. In 1 Chaulder Rye-Meaſure 32 Buſhels, in 1 Buſhel 12 *Wincheſter* Gallons by heap, and the Cop on the Buſhel was equal to 4 Gallons more: Whereupon of late, upon new making the Buſhel, order was given to make a Buſhel that ſhould hold the old Buſhel with the Cop, and this new Buſhel is ſtill in uſe filled up to the brim, but not by heap.

Chaulder at Rye.

Concave Meaſures Liquid.

Liquid Meaſures are Cask that contain Moiſt or Liquid Commodities; As Ale, Beer, Butter, Fiſh, Honey, Oyl, Soap, Wine, *&c.*

In Measures of this Nature two things are to be heeded, *viz.* the Content of the Vessel which is alwaies by the *Winchester* Measure, and the weight of the empty Cask when the Commodities are sold by weight.

Ale 23. *Hen.* 8 *Cap.* 4.

A Table of Ale Measures.

Last.	1	Barrel.						
Barrels.	12	1	Kilderkin.					
Kilderkins.	24	2	1	Firkin.				
Firkins.	48	4	2	1	Gallon.			
Gallons.	384	32	16	8	1	Pottle.		
Pottles.	768	64	32	16	2	1	Quart.	
Quarts.	1536	128	64	32	4	2	1	
Pints.	3072	256	128	64	8	4	2	Pints.

Beer 23. *Hen.* 8. *Cap.* 4.

A Table of Beer Measures.

Last.	1	Ton											
Tons.	2	1	Pipe.										
Pipes.	4	2	1	Hogshead.									
Hogsheads.	8	4	2	1	Bunn.								
Barrels or Bunns.	12	6	3	1½	1	Barrel.							
Herring Barrels.	16	8	4	2	1⅓	1	Kilderkin.						
Kilderkins.	24	12	6	3	2	1½	1	Firkin.					
Firkins.	48	24	12	6	4	3	2	1	Gallon.				
Gallons.	432	216	108	54	36	27	18	9	1	Pottle.			
Pottles.	864	432	216	108	72	54	36	18	2	1	Quart.		
Quarts.	1728	864	432	216	144	108	72	36	4	2	1		
Pints.	3456	1728	864	432	288	216	144	72	8	4	2	Pints.	

Butter and Soap.

Butter and Soap are of like measure, and the Content of the Barrel, Kilderkin, and Firkin of Sope is ordained by 23. *Hen.* 8. *Cap.* 4. to be as the Ale Measure, that is the Barrel 32 Gallons, *&c.* And the Weight of the empty Cask as followeth.

Weight of the Cask.

	lb.	
Barrel————————	26	Averdupois.
Kilderkin or half Barrel————	13	
Firkin————————	6½	

So that of Butter and Sope that is besides the Weight of the Cask should be { lb. 56 Butter. 60 Soap.

Fish, as Eeles and Salmon by the 22. *Edw.* 4. *Cap.* 2. and other Statutes have one and the same Measure, *Viz.*

A Table of the Measure of Cask for Eeles and Salmon.

Last.	1	Butt.				
Butts.	6	1	Barrel.			
Barrels.	12	2	1	Half Barrel.		
Half Barrels.	24	4	2	1	Firkin.	
Firkins.	48	8	4	2	1	
Gallons.	504	84	42	21	10½	Gallons.

Herrings

Herrings in Cask, the Last.

Herrings by the same Statutes are to be as Ale-Measure, the Barrel 32 Gallons, Half Barrel 16, Firkin 8, and 12 Barrels are usually accompted for a Last. But unpacked Herrings by the Book of Rates Outwards, 1 Last shall be 18 Barrels.

Sturgeon the Firkin.

Sturgeon. In 1 Firkin 2 Caggs.

Other Fish, the Last.

Other Fish barrelled. In 1 Last 12 Barrels.

Honey, Oyl, and Wine, the Assize.

Honey, Oyl and Wine Measures are alike, and in 2 *Hen.* 6. *Cap.* 11. 18. *Hen.* 6. *Cap.* 17. 1 *Ric.* 3. *Cap.* 13. and 28. *Hen.* 8. *Cap.* 14. may be seen to contain as in the Table following, but for Honey the Assize is altered by 23. *Eliz. Cap.* 8. to 32 Gallons the Barrel, 16 the Kilderkin, *&c.* like the Measure for Ale.

A Table of the Measure of Wine and Oyl.

Tun or Ton.	1	Pipe.									
Butts or Pipes.	2	1	Puncheon.								
Puncheons or Tertians of a Tun.	3	$1\frac{1}{2}$	1	Hogshead.							
Hogsheads.	4	2	$1\frac{1}{3}$	1	Tierce.						
Tierces of a But. *. 1.	6	3	2	$1\frac{1}{2}$	1	Barrel.					
Barrels.	8	4	$2\frac{2}{3}$	2	$1\frac{1}{3}$	1	Rundlet.				
Rundlets or Rondlets. *. 2.	14 *ferè.*	7 *ferè.*	$4\frac{20}{37}$	$3\frac{15}{37}$	$2\frac{10}{37}$	$1\frac{26}{37}$	1	Gallon.			
Gallons.	252	126	84	63	42	$31\frac{1}{2}$	$18\frac{1}{2}$	1	Pottle.		
Pottles.	504	252	168	126	84	63	37	2	1	Quart.	
Quarts.	1008	504	336	252	168	126	74	4	2	1	
Pints or Pounds. *. 3.	2016	1008	672	504	336	252	148	8	4	2	Pints.

Tierce reckoned wrong in Pulton.

*. 1. Either an *Erratum* of the Press, or an Error of the *Amanuensis* seems to be the Cause why *Pulton* in his Collection of the Statutes at large, Imprinted 1640, *pag.* 640. makes the Tierce to contain but 41 Gallons, since it is contrary to the Path-way. *John Legat* at the end of *Thomas* his Dictionary, and other good Authours, nor can 41, but 42 be the third part of 126, the Gallons in a Butt.

Rundlet how generally taken.

*. 2. Rundlet is now grown a general name to any small Cask not gage.

Honey the Pound, not a Pint.

*. 3. The Pound of Honey is not a Pint, but the Pound of Wine Troy Weight is reckoned for a Pint 31. *Edw.* 1. and 12. *Hen.* 7. *Cap.* 5, and accordingly the Gallon and other Measures ordained.

English Weights.

English Weights.

Balances to be exact.

Because Weights cannot be tryed without true Ballances it may not be unprofitable here to consider the requisites to the framing of exact Ballances, though it may be thought a digression. Balances require a like proportion between the parts of the Beam, or else two unequal Weights may counterpoize one another: For if the Beam be 23 Inches long, and the handle or axle-tree so placed that it be distant from one end of the Beam 12 Inches, and the other but 11, conditionally that the shorter end be as heavy as the longer, then putting in two unequal Weights in such proportion as the parts of the Beam one to another, which is 12 to 11. So that the greater Weight be put into the Scale which hangs on the shorter part of the Beam, and the lesser Weight in the contrary Scale, yet will the Balances hang in *æquilibrio*, as *Aristotle* in his *Mechanical Questions* fully proves.

How to be made.

Wherefore in making of Balances let the Beam be of any convenient length, the Tongue or Point half so long as the Beam, and standing upright in the midst, equidistant from both the ends, the Axle-tree three square and straight set at right Angles in the midst of the Beam, so that it may alwayes bear upon the edge when the Balances shall be charged, the Cheeks very straight and somewhat longer than the Tongue, with a pendant point in the middle between them, the Plates or Scales of equal Weight, the Cords of the bigness, and equal in length to the Beam.

Troy Weight how much by Statute.

The Statutes aforementioned 51. *Hen.* 3. 31. *Edw.* 1. 12. *Hen.* 7. do ordain that an *English* Penny round without clipping shall weigh 32 Grains of Wheat well dried and taken out of the midst of the Ear, 20 pence 1 Ounce; 12 Ounces 1 pound Troy, 8 pounds 1 Gallon of Wine, Wheat, *&c.* as before noted, so that the pound Troy by the Statutes should contain 7680 Grains. But since the making of the Statutes the Penny-weight hath been divided into 24 Grains, and the Grain into smaller Divisions, as in the following Table.

Troy Weight.

A Table of Troy-Weight

Pound.	1	Ounce.						
Ounces.	12	1	Penny-weight.					
Penny-weights.	240	20	1	Grain.				
Grains.	5760	480	24	1	Mite.			
Mites.	115200	9600	480	20	1	Droite.		
Droites.	2764800	230400	11520	480	24	1	Peroite.	
Peroites.	55296000	4608000	230400	9600	480	20	1	
Blanks.	1327104000	110592000	5529600	230400	11520	480	24	Blanks.

A Mast how much.

Troy Weight hath seldom any greater denomination than the pound, yet sometime 2½ ℔. thereof is called a Mast allowed for Amber and Gold and Silver Thread of Collen by the Book of Rates.

A Mark how much.

Eight Ounces of the Pound *Troy* make the Weight, called sometime *Bes*, sometime the Mark Weight in *Latin*, *Marchus & Marca*. This is beyond Sea more than in *England* used to weigh Silver; every Ounce divided into 20 *English*, and every *English* into 32 Grains, called by some Asses or Azes.

Karacts by what Names called, how much, and for what use.

The Goldsmiths divide the Ounce *Troy* into 24 parts, which they call Carats, Carrats, Caracts, Characts, Karacts or Karects, and every Karect into 4 Grains. So the pound shall have 1152 Grains Karect, and the Ounce 96. They also divide the Ounce into 150 Karects, and every of these Carrats into 4 Grains. By the first sort of Carrats they try the fineness of Bullion, buy and sell Gold and Silver; by the latter Pearls and Diamonds. An old Authour I have seen that parts each of the 24 Karects into 12 Grains, but these seem *Exotick*, and not of *English* Extract. *Legat* at the end of *Thomas* his *Dictionary* divides the Ounce *Troy* into 16 parts, which he calls Farthing Gold Weights, and allows to every Charact 20 grains, From hence appear Grains of 3 sorts, *Viz.*

Farthing Gold-Weights.

Grains the sorts.

Grains by the Statute. } in 1 ℔. *Troy* { 7680.
Grains of Assize. } in 1 ℔. *Troy* { 5760.

Grains Caract in 1 ℔. *Troy*. { 7200. 3456. 1152.

The Grains of Assize, and the last of the Grains Caract are most in use with us. Why called Caract, see after in the Notes on the *Greek Weight Siliqua*, *pag.* 96.

What to be weighed by Troy Weight.

The Constitution of Measures by *Troy Weight* hath been already touched, besides which, the use thereof is to weigh Bread, Bullion, Money, Pearles and Precious Stones, and choice Physical Druggs, as Ambergrease, Bezoar, Civet, Musk, Unicorns-horn, *&c.* to all which except Bread and Money, nothing need more to be said.

Bread.

Bread the old Assize.

The Statute 51. *Hen.* 3. Intituled the Assize of Bread and Ale, Enacteth, That when the price of a Quarter of Wheat is 1 *s.* the Wastel Bread of a Farthing shall weigh 6 *l.* 16 *s.* 00 *d.*

Bread Cocket of the same Corn and Bultel shall weigh more than the Wastel by 2 *s.*
Bread Cocket of lower price Corn more than the Wastel by 5 *s.*
Bread made into a Simnel less than the Wastel by 2 *s.*
Bread made of the Whole Wheat shall weigh a Cocket and a half;
So that the Cocket weigh more than a Wastel by 5 *s.*
Bread of Treet shall weigh 2 Wastels.

	l.	s.	d.
Bread of common Wheat shall weigh 2 great Cockets.			
When a Quarter of Wheat is sold for 1 *s.* 6 *d.* then Wastel-Bread of a Farthing white and well baked shall weigh———	4	10	08
When for 2 *s.*———	3	08	00

and so proportionably decreasing at the increase of 6 *d.* in every Quarter of Wheat.

Not exact. But whither through want of knowledge in reversed proportions, or by the mistake of some *Amanuensis* is not certain, yet sure it is that the Assize of Bread set forth in that Statute for some higher prices than 2 *s.* the quarter of Wheat in all the Printed *Latin*, *French* and *English* Copies extant is erronious, as was long since rightly observed by Record in his Book of *Arithmetick* called *The Ground of Arts.* Wherefore and for that at the making of the Statute Money by which the Assize of Bread is there reckoned was at the old rate, *Viz.* One pound of Silver Money weighed One pound *Troy-Weight*, which is since much altered, this old Assize grew intricate, and later times have appointed the Assize by the *Troy-Weight*, and not by Money, yet grounded on the Assize in the Statute, accounting 6 *l.* 16 *s.* Money according to the Standart then to answer to 6 ℔. 9 ℥. 12 *pwts. Troy*, as indeed it doth.

Old Assize of Bread.

How altered.

Sorts of Bread, in the Statute. The Statute allows of seven sorts of Bread, though those mostly now in use is White, Wheaten and Houshold.

Wastel Bread what. Waster Bread seems to be Rowles or fine Manchet Bread used principally in Victualling Houses to drink with, perhaps from the old word *Wassail*, to drink or banquet, as it were a drinking Bread.

Bread Cocket what. Bread Cocket is White Bread of the best sort drawn through the finest Boultel or Laun-Sieve, or of the best Wheat; This is the first in the latter Assize Books.

Fine and Course Bread Cocket of the lower price Corn is the ordinary Wheat-Bread now usual, drawn through a courser Cocket or Boultel than the former, or made of worse Wheat.

Simnel Bread what. Simnel Bread is a kind of Cake or Bisket-Bread made with some Butter and Spice, still in use in some places.

Bread of the whole Wheat. Bread made of the whole Wheat is sometime called Cribble or fine Ravel Bread, but in the Assize Books, and in most places called Wheaten Bread, made of Flour courser than the White, but not so course as the Houshold Bread.

Bread of Treet. Bread of Treet seems to be Houshold-Bread of the best Wheat unravelled, or ravelled through the coursest Boultel, that is only the Husk or Bran taken out of the Meal.

Houshold Bread Houshold-Bread in the Assize-Books is assized like that in the Statute called Bread of Common Wheat, as being common for the whole Houshold, which then seemed either to be made of the common sort of VVheat or baked as it came from Mill.

Horse Bread. Besides these Horse Bread though not ordained by Statute, yet hath been of long continuance and allowed, every loaf thereof is alway, at what price soever Corn be sold, to be of the full weight of the Penny white Loaf, and the Baker is to sell three such Loaves for a Penny, and 13 to the dozen, that is 39 loaves for 1 *s.*

Sea-Bisket. Sea-Bisket of excellent use for the Sea, because baked without Salt, and well dried, is not assessed by the Statute, therefore weighed and sold by the hundred of *Averdupois* weight.

Allowance to Bakers. In the Book of Assize of Bread published towards the latter end of Q. *Eliz.* Countrey Bakers were allowed 4 *s.* upon a Quarter above the middle price of VVheat: But Bakers that dwell in Cities and Towns were allowed 6 *s.* in regard they were subject to more Scot and Lot than the Countrey-Bakers were, which 6 *s.* is still generally allowed to Townish Bakers; but by the Orders of the Councel Board Dated the last of *January*, 1604. the Countrey-Bakers Penny white-Loaf shall weigh 2 Ounces more than the Bakers of the Towns, their Penny wheaten loaf 3 Ounces, and their Penny houshold Loaf 6 Ounces.

Corn Cheap when. Former Times had Corn very cheap, either by reason of the Plenty thereof, or scarcity of Money, for in a Book of Presidents remaining among the Records of this Town of *Rye*, *pag.* 131. is a *Memorandum* entred, That the Year Old *Winchelsey* was drowned, which is there said to be *An. Dom.* 1287. Corn was at 2 *s.* the Quarter, which at the Rate Money was then, is but 6 *s.* of our Money now. VVhereas the cheapest it hath been known of late years was *Annis Dom.* 1654. and 1655, when good VVheat was sold in some places in *England* for 12 *s.* the Quarter.

Old Winchelsey *drowned when.*

Farthing Loaves formerly. Anciently by reason of the cheapness of Corn, Farthing Loaves of all sorts of Bread assized by the Statute were made; but now none less than half-penny Loaves, and these but of white and wheaten-bread, for of houshold bread no Loaf is to be made under a Penny. Neverthelesse by the Assize of the Farthing Loaf is the Assize of the half-penny and penny Loaf easie to be had by doubling the weight of the lesser Loaf for the Assize of the greater, and contrarywise by halving the weight of the greater the assize of the lesser Loaf is had, and having the assize of the penny Loaf of any sort of bread at 1 *s.* per Quarter according to the Statute, the Assize of the same Loaf by the Reversed Rule of Three, as hereafter in the 4th Book, *Of Proportions*, may be seen, is obtained at any other Rate, but by the Law the Assize is not to be altered, but when there is 6 *d.* increasing or decreasing in the Price of the Quarter of VVheat.

How the Assize at any Rate may be had.

The

The Statute Assize of Farthing, Half-Penny, and Penny-Bread at the price of Twelve pence the Quarter of Wheat, reduced from the Old Standard of Money to Troy-weight, accompting 20 s. *of Money then to be a Pound Troy.*

The Assize of Bread by Statute.

Bread.	Loaves.	Money. l. s. d.	VVeight. ℔. ℥. ptws.
Wastell	Farthing	06—16—00	06—09—12
	Half-penny	13—12—00	13—07—04
	Penny	27—04—00	27—02—08
Fine Cocket.	Farthing	06—18—00	06—10—16
	Half-penny	13—16—00	13—09—12
	Penny	27—12—00	27—07—04
Course Cocket	Farthing	07—01—00	07—00—12
	Half-penny	14—02—00	14—01—04
	Penny	28—04—00	28—02—08
Simnel	Farthing	06—14—00	06—08—08
	Half-penny	13—08—00	13—04—16
	Penny	26—16—00	26—09—12
Wheaten	Farthing	10—11—06	10—06—18
	Half-penny	21—03—00	21—01—16
	Penny	42—06—00	42—03—12
Treet	Farthing	13—12—00	13—07—04
	Half-penny	27—04—00	27—02—08
	Penny	54—08—00	54—04—16
Houshold	Farthing	14—02—00	14—01—04
	Half penny	28—04—00	28—02—08
	Penny	56—08—00	56—04—16

From this Legal Basis is deduced the following Table containing the Assize by the *Troy Weight* of the Penny white, wheaten, and houshold Loaves being the Bread still in use from 12*d.* the Quarter of VVheat unto 2 *l.* the Quarter, omitting some of the small Fractions.

The Assize of Bread now in use.

Price of Wheat. l s d	Penny White. ℔ ℥ pwts	Penny Wheaten. ℔ ℥ pwts	Penny Houshold. ℔ ℥ pwts	Price of Wheat. l s d	Penny White. ℔ ℥ pwts	Penny Wheaten. ℔ ℥ pwts	Penny Houshold. ℔ ℥ pwts
— 1—0	28—02—08	42—03—12	56—04—16	1—00—6	1—04—10	2—00—15	2—09—00
— 1—6	18—09—12	28—02—08	37—07—04	1—01—0	1—04—02 1/4	2—00—03	2—08—04 1/2
— 2—0	14—01—04	21—01—16	28—02—08	1—01—6	1—03—14 3/4	1—11—12	2—07—09 1/2
— 2—6	11—03—07 1/5	16—11—00 4/5	22—06—14 2/5	1—02—0	1—03—07 1/2	1—11—01 1/4	2—06—15
— 3—0	9—04—16	14—01—04	18—09—12	1—02—6	1—03—00 3/4	1—10—11	2—06—01 1/2
— 3—6	8—00—13 5/7	12—01—00 4/7	16—01—07 3/7	1—03—0	1—02—14 1/4	1—10—01	2—05—08 1/2
— 4—0	7—00—12	10—06—18	14—01—04	1—03—6	1—02—08	1—09—12	2—04—16
— 4—6	6—03—04	9—04—16	12—06—08	1—04—0	1—02—02	1—09—03	2—04—04
— 5—0	5—07—13 3/5	8—05—10 2/5	11—03—07 1/5	1—04—6	1—01—16	1—08—14	2—03—12
— 5—6	5—01—10 6/11	7—08—05 9/11	10—03—01 1/11	1—05—0	1—01—10 1/2	1—08—05 3/4	2—03—01 1/2
— 6—0	4—08—08	7—00—12	9—04—16	1—05—6	1—01—05 1/3	1—07—18	2—02—10 1/2
— 6—6	4—04—01	6—06—01 1/2	8—08—02	1—06—0	1—01—00 1/4	1—07—10	2—02—00 1/2
— 7—0	4—00—06 6/7	6—00—10 2/7	8—00—13 5/7	1—06—6	1—00—15 1/4	1—07—03	2—01—10 1/2
— 7—6	3—09—02	5—07—13	7—06—04	1—07—0	1—00—10 1/2	1—06—16	2—01—01
— 8—0	3—06—06	5—03—09	7—00—12	1—07—6	1—00—06	1—06—09	2—80—12
— 8—6	3—03—16	4—11—14	6—07—12	1—08—0	1—00—01 1/2	1—06—02 1/4	2—00—03
— 9—0	3—01—12	4—08—08	6—03—04	1—08—6	—11—17 1/4	1—05—15 3/4	1—11—14 1/2
— 9—6	2—11—12	4—05—08	5—11—04	1—09—0	—11—13 1/4	1—05—10	1—11—06 1/2
—10—0	2—09—16 4/5	4—02—15 1/5	5—07—13 3/5	1—09—6	—11—09 1/4	1—05—04	1—10—18 1/2
—10—6	2—08—04 1/2	4—00—06	5—04—09	1—10—0	—11—05 1/2	1—04—18 1/4	1—10—11
—11—0	2—06—15 3/11	3—10—02 3/4	5—01—10 6/11	1—10—6	—11—01 3/4	1—04—12	1—10—03 1/2
—11—6	2—05—08 1/2	3—08—02 3/4	4—10—17	1—11—0	—10—18 1/4	1—04—07 1/4	1—09—16 1/2
—12—0	2—04—04	3—06—06	4—08—08	1—11—6	—10—14 3/4	1—04—02	1—09—09 1/2
—12—6	2—03—01	3—04—11 1/2	4—06—02	1—12—0	—10—11 1/2	1—03—17 1/4	1—09—03
—13—0	2—02—00 1/2	3—03—00 3/4	4—04—00	1—12—6	—10—08	1—03—12	1—08—16
—13—6	2—01—01	3—01—11 1/4	4—02—02	1—13—0	—10—05	1—03—07 1/2	1—08—10
—14—0	2—00—03 3/7	3—00—05 1/7	4—00—06 6/7	1—13—6	—10—02	1—03—03	1—08—04
—14—6	1—11—06 3/4	2—11—00 1/8	3—10—12	1—14—0	—09—19	1—02—18 1/2	1—07—18
—15—0	1—[illegible]—11	2—09—16 1/2	3—09—02	1—14—6	—09—16 1/4	1—02—14	1—07—12 1/2
—15—6	1—[illegible]—16 1/2	2—08—14 3/4	3—07—13	1—15—0	—09—13 1/4	1—02—09 3/4	1—07—06 1/2
—16—0	1—09—03	2—07—14 1/2	3—06—06	1—15—6	—09—10 1/2	1—02—05 3/4	1—07—01
—16—6	1—08—10	2—06—15	3—05—00	1—16—0	—09—08	1—02—02	1—06—15
—17—0	1—[illegible]—18	2—05—17	3—03—16	1—16—6	—09—05 1/4	1—01—17 3/4	1—06—10 1/2
—17—6	1—07—06 1/2	2—04—19 3/4	3—02—13	1—17—0	—09—02 3/4	1—01—14	1—06—05 1/2
—18—0	1—06—16	2—04—04	3—01—12	1—17—6	—09—00 4	1—01—10	1—06—00 1/2
—18—6	1—05—05 3/4	2—03—08 1/2	3—00—11 1/2	1—18—0	—08—18	1—01—07	1—05—16
—19—0	1—05—16	2—02—14	2—11—12	1—18—6	—08—15 3/4	1—01—03	1—05—11 1/2
—19—6	1—05—07	2—02—00 1/2	2—10—14	1—19—0	—08—13 1/2	1—01—00 1/4	1—05—07
1—00—0	1—04—18 2/5	2—01—07 1/2	2—09—16 4/5	1—19—6	—08—11 1/3	1—00—16	1—05—02 2/3
				2—00—0	—08—09 1/5	1—00—13 3/4	1—04—18 2/5

Money.

Money.

English *Money why called* Sterling.

English Coine is often called *Sterling* Money, as *Legat* in the end of *Thomas* his *Dictionary* conceives, because there was anciently stamped upon it one quarter a little Bird called a *Star* or *Starling*; although others think it was because Coined first at a place so called, or rather by the *Esterlings*, which Name in the time of the *Saxon Heptarchy* in *England* was given to those *Saxons* which inhabited in the *Eastern* parts, as *Cambridgeshire*, *&c.*

Sterling *Standard.*

The *Sterling* Standard for Money long before the making of the last forementioned Statutes even from the time of the *Saxons* had been observed to be 20 pence for an Ounce of Silver 11℥. 2 p^{wts} fine, and about 22 *s.* and 3 *d.* for an Ounce of Gold of 22 Carats fine, and so continued till the Reign of *Henry* 6th and then the Ounce of Silver of the same fineness was raised from 20 pence to 30 pence. *Edw.* 4th raised it to 40 pence. *Hen.* 8th to 45 pence, and *Eliz.* to 60 pence, at which rate it yet stands; all which was done by Proclamations according to the Exigencies of Affairs in the Kingdom at the times when it was altered, so that though both pounds *Sterling* and *Troy* were divided somewhat like, that into 20 and 12, this into 12 and 20, yet were both pounds then equal, but now three pound of Money in Accompt is but one pound *Troy*, and scarce that, and consequently our Shilling and Penny is but the third part of the old Shilling and Penny, and in like manner was Gold advanced.

Advanced by Proclamation.

Touching Money, Two things are to be noted. 1. The Accompt. 2. The Coins. In which latter the Weight and Fineness are also considerable.

The common Accompt of Money is by Pounds, Shillings and Pence. But there are several other Denominations and Subdivisions of the Pound *Sterling*, as in the Table following.

Sterling Money.

A Table of English Money.

	*1	*2	*3	*4	*5											
Pound.	1															
Mark.	1½	1														
Angels.	2	1⅓	1													
Nobles.	3	2	1½	1												
Crowns.	4	2⅔	2	1⅓	1											
Half Crowns.	8	5⅓	4	2⅔	2	1										
Shillings.	20	13⅓	10	6⅔	5	2½	1									
Sixpences.	40	26⅔	20	13⅓	10	5	2	1								
Groats.	60	40	30	20	15	1½	3	1½	1							
Threepences.	80	53⅓	40	26⅔	20	10	4	2	1⅓	1						
Twopences.	120	80	60	40	30	15	6	3	2	1½	1					
Three half pences.	160	106⅔	80	53⅓	40	20	8	4	2⅔	2	1⅓	1				
Pence.	240	160	120	80	60	30	12	6	4	3	2	1½	1			
Three Farthings.	320	213⅓	160	106⅔	80	40	16	8	5⅓	4	2⅔	2	1⅓	1		
Half-pence.	480	320	240	160	120	60	24	12	8	6	4	3	2	1½	1	
Farthings.	960	640	480	320	240	120	48	24	16	12	8	6	4	3	2	1
Mites.	5760	2840	2880	1920	1440	720	288	144	96	72	48	36	24	18	12	6

A Pound why called a Piece.

* 1. A Pound or Twenty Shillings is sometime called a piece in reference to *xx s.* pieces of Gold; for it is ordinary to say 1 piece, 2 pieces, 3 pieces, for 1. 2. or 3. Pounds.

A Mark why so called.

* 2. A Mark which is ⅔ of a Pound, and 13 *s.* 4 *d.* deriveth its name from the Mark Weight, because at the old Rate a Mark of Silver Money weighed 8 Ounces *Troy*.

* 3. An

* 3. An Angel is 10 *s.* and so called from the Impression of an Angel upon pieces of Gold of that Value. Yet are there Gold Coins called Ship-Angels of more worth; because since they were Coined Gold is raised. *An Angel why so called.*

* 4. A Noble is 6 *s* 8 *d.* half a Mark, or the third part of a Pound, and for the reason last before-mentioned several pieces of Gold bearing the name of Nobles, are worth more than 6 *s.* 8 *d.* as the Angel Noble, first Coined for a Noble, then advanced to an Angel, and now worth more. *Nobles of old Coine worth more now.*

* 5. A Crown is 5 *s.* the Quarter of a Pound, and the greatest piece of *English* Silver Coin, sometime called the *English* Dollar, from whence down to Farthings are Silver Coins, and besides these some Harpers in value 9 *d.* a piece, and half Harpers worth 4 *d.* a piece were Coined by Q. *Elizabeth*, but seem to be *Irish* Money, now almost worn out as well as her Three Farthing pieces. *Crown the greatest Silver Coine in England called sometime Dollar*

Farthings before the advance of the Ounce of Silver to 60 pence were Silver Coins, as appeareth by the Statutes 4 *Hen.* 4. *Cap.* 10. & 14. *Hen.* 8. *Cap.* 12. but since grew inconvenient by their smallness. *Farthings sometime Silver Coines.*

Mites are no pieces of Coines, but a lesser division used about reduction and finding out the Value of Foreign Coines by the *Sterling* Standard; and some for their private use and more curiosity divide the Farthing into 2 *Ques*, the Q into 2 *Cees*, the C into 2 *Dodkins*, the D into 2 *Mites*, that is 16 Mites in one Farthing. *Mites no Coines.* *Farthings how divided by some.*

The Coines of *England* are of Gold and Silver, some of greater value, some lesser, many elder, others later, and so different in Fineness and Weight, that it is hard for any but those whose common Converse is thereabouts, as Mintmasters, Goldsmiths, &c. to give any perfect accompt thereof; and the rather, because they proceed not by the strict Rules of *Arithmetick*, besides any such accompt will be subject to future alteration by addition of new Coins or advancing the Value of the old.

Mr. *Gerard Malynes* sometime a Commissioner about the Mint Affairs, and a Man expert in Coinage in his Book Intituled *Lex Mercatoria*, hath Calculated the Weight and Fineness of several both *English* and *Outlandish* Coines, and from thence in the succedent Tables much light is borrowed. Nevertheless whether because he hath (as he saith) omitted the small Fractions as unnecessary, or because he hath rather inserted what Weight the Coins ought to be according to the Royal Orders at the times when they first issued forth then what they are, I know not, but sure I am, his accompt in several of the Gold Coines doth not agree with the Book called *Perfect Directions for all English Gold*, Imprinted 1663, nor others, nor yet with the Common Weights, as by often experience trying such as have come to my hands I have found *English Coines of both sorts different in value and fineness.*

Wherefore in the following Table for *English Gold*, besides the Weight after *Malynes* and others according to the more regular proportions of *Arithmetick*, is another Collumn containing the Common weight as the pieces were when Coined (for many by use are worne much lighter, and must have allowance) according to which Weight is the value reckoned; and because the value of Old Coines have since the time of their Coinage been increased not only by Proclamation, but beyond the Rates limited thereby, to wit, even as Merchants have found them valued in Foreign Exchange, or as Goldsmiths have found them worth to melt down: Let therefore the *Sterling* Silver Coines be understood as they anciently were and still are Currant in *England*, but the Gold Coines are differently valued in their respective Collumns, *viz.* those of K. *James* and K. *Charles* the First, as in their Proclamations, those elder as they were currant with the Goldsmiths *Anno* 1640, and are now worth since the Proclamation of K. *Charles* the Second *Anno* 1660. accompting in the one Penny weight of Gold 22 Carats fine worth 3 *s.* 4 *d.* and in the other 3 *s.* 6 *d.* 2 *q.* 4 *m.* omitting in some of the small pieces the odd Farthings and Mites, and proportionably for Gold of other fineness: Nevertheless according to the Proclamations and vulgar currant Exchange, most of the Old Coines were not, nor yet are valued so high, for the Old Spurre Royal by the Proclamation of King *James* 1611. is rated at 16 *s.* 6 *d.* yet, with the Goldsmiths long before the Proclamation 1660. was worth 18 *s.* which is 1 *s.* 6 *d.* more than that Proclamation values it, and the like may be observed in others. As to the pieces to the *Pound Troy* where the *Fractions* hapned to be small and inconsiderable in stead thereof the next nearest is taken and marked with — or + according as it is too little or too much; other things are perspicuous enough by the Tables themselves, and need no explanation. *Old Coinage advanced since their Coinage by Proclamation and otherwise.*

English *Silver Coines their Finenes, Weight and Value.*

The Table of Sterling-Silver-Coines *now Currant.*

Names of the Pieces.	Kings and Queens.	Finenefs. Ounces	pwts. ℥.	Weight. pwts. gr. m. dr.	Worth s.	d.
Crown of	*Edw. 6. & Eliz.*	11	2	1 or 20 00 00 00	5	0
	James, Charles 1. & 2.	11	0	19 08 10 $07\frac{23}{31}$	5	0
Half Crown of	*Edw. 6. & Eliz.*	11	2	10 00 00 00	2	6
	James, Charles 1. & 2.	11	0	9 16 05 $03\frac{13}{31}$	2	6
Shilling of	*Edw. 6. Phil. & Mary, & Eliz.*	11	2	4 00 00 00	1	0
	James, Charles 1. & 2.	11	0	3 20 18 $01\frac{17}{31}$	1	0
Six pence of	*Edw. 6. Phil. & Mary, & Eliz.*	11	2	2 00 00 00	0	6
	James, Charles 1. & 2.	11	0	1 22 09 $00\frac{24}{31}$	0	6
Old Groat of *Henry* 8.		11	2	1 20 13 $00\frac{24}{43}$	0	4+
Last Groat of *Henry* 8.		11	2	1 12 00 00	0	4+
Groat of	*Mary & Eliz.*	11	2	1 08 00 00	0	4
	Charles 1.	11	0	1 06 19 $08\frac{16}{31}$	0	4
Three pence of	*Elizabeth.*	11	2	1 00 00 00	0	3
	Charles 1.	11	0	0 23 04 $12\frac{12}{31}$	0	3
Two pence of	*Henry* 8.	11	2	0 18 00 00	0	2+
	Elizabeth	11	2	0 16 00 00	0	2
	James, Charles 1. & 2.	11	0	0 15 09 $16\frac{8}{31}$	0	2
Three half pence of *Elizabeth.*		11	2	0 12 00 00	0	$1\frac{1}{2}$
Penny of	*Henry* 8. & *Edw.* 6.	11	2	0 09 00 00	0	1+
	Mary, & Eliz.	11	2	0 08 00 00	0	1
	James, Charles 1. & 2.	11	0	0 07 14 $20\frac{4}{31}$	0	1
Three Farthings of *Elizabeth.*		11	2	0 06 00 00	0	$0\frac{3}{4}$
Half penny of	*Elizabeth.*	11	2	0 04 00 00	0	$0\frac{1}{2}$
	James, Charles 1. & 2.	11	0	0 03 17 $10\frac{2}{31}$	0	$0\frac{1}{2}$

English *Gold Coines, their Fineness Weight and Value.*

The Table of most Sterling Gold Coines *yet Currant.*

Names of the Pieces.	Pieces to the lb *Troy.*	Weight by *Malynes, &c.* pwts. gr. m. dr.	Common Weight. pwts. gr. m. dr.	Pieces to the lb *Troy.*	Fine Car. gr.	Value 1640. l.	s.	d.	Value 1660. l.	s.	d.	Allow. gr.
Old Double Rose Noble.	$23\frac{3}{8}$	10 06 08 $08\frac{40}{187}$	10 00 00 00	24	23 $3\frac{3}{4}$	1	16	4	1	18	8	5
Double Rose Noble of *Henry* 8. *Edw.* 6. *Phil. & Mary* *Elizabeth.*	24	10 00 00 00	9 22 00 00	$24\frac{1}{5}$ +	23 $3\frac{3}{4}$	1	16	0	1	18	4	5
Great Soveraign of K. *James.*	24	10 00 00 00	9 16 05 $03\frac{37}{31}$	$24\frac{4}{5}$	22 0	1	13	0	1	15	3	$4\frac{1}{2}$
Double Rose Noble of K. *James.*	$26\frac{2}{3}$	9 00 00 00	8 21 06 16	27	23 $3\frac{3}{4}$	1	13	0	1	15	3	$4\frac{1}{2}$
Double Rose Royal or Real.	—	—	8 02 03 03	$29\frac{2}{3}$ +	23 $3\frac{1}{2}$	1	10	0	1	12	0	$4\frac{1}{2}$
Double Old Soveraign.	$27\frac{3}{8}$	8 18 08 $05\frac{19}{73}$	8 00 00 00	30	22 0	1	06	8	1	08	5	4
Best Double Soveraign of *Henry*; Double Soveraign of *Edw.* 6. *Elizabeth.*	30	8 00 00 00	7 04 00 00	$33\frac{1}{2}$ —	22 0	1	03	10	1	05	5	4
Double Soveraign of K. *James* called *Unite* or *Jacobus.*	36	6 16 00 00	6 10 16 $18\frac{18}{31}$	$37\frac{1}{8}$	22 0	1	02	0	1	03	10	3
Laureat or *xx s.* piece of *James.*	$39\frac{3}{5}$	6 01 09 $02\frac{2}{11}$	5 20 09 $18\frac{6}{41}$	41	22 0	1	00	0	1	01	4	3
Twenty Shillings piece of *Charles* 1.	40	6 00 00 00	5 20 09 $18\frac{6}{41}$	41	22 0	1	00	0	1	01	4	3
Old Rose Noble.	$46\frac{3}{4}$	5 03 04 $04\frac{20}{187}$	5 00 00 00	48	23 $3\frac{3}{4}$	0	18	2	0	19	4	$2\frac{1}{2}$
Spurre Royal of *Henry* 8. *Edward* 6. *Philip & Mary* *Elizabeth.*	48	5 00 00 00	4 23 00 00	$48\frac{2}{5}$ +	23 $3\frac{3}{4}$	0	18	0	0	19	2	$2\frac{1}{2}$
Spurre Royal of *James*	$53\frac{1}{3}$	4 12 00 00	4 10 13 08	54	23 $3\frac{3}{4}$	0	16	6	0	17	7	$2\frac{1}{2}$
Double Noble of *Elizabeth*	—	—	4 10 06 16	$54\frac{1}{5}$ —	23 $3\frac{1}{2}$	0	16	0	0	17	1	$2\frac{1}{2}$
Old Noble or Noble of *Henry*	$53\frac{3}{4}$	4 11 03 $06\frac{6}{43}$	4 10 00 00	$54\frac{18}{53}$	23 $3\frac{1}{2}$	0	16	0	0	17	1	$2\frac{1}{2}$
Rose Royal.	—	—	4 01 01 $13\frac{1}{2}$	$59\frac{1}{3}$ +	23 $3\frac{1}{2}$	0	15	0	0	16	0	$2\frac{1}{2}$
Old Soveraign.	$54\frac{3}{4}$	4 09 04 $02\frac{46}{73}$	4 00 00 00	60	22 0	0	13	4	0	14	2	2
Best Soveraign of *Henry*; Soveraign of *Edward* 6. *Elizabeth.*	60	4 00 00 00	3 14 00 00	$66\frac{42}{43}$	22 0	0	11	11	0	12	8	2
Old Angel Noble or Angel of *Henry*	69	3 11 09 $13\frac{13}{27}$	3 08 00 00	72	23 $3\frac{1}{2}$	0	12	1	0	12	10	2

Names of the Pieces.	Pieces to the ℔ *Troy.*	Weight by *Malynes, &c.* pwts. gr. m. dr.	Common Weight. pwts. gr. m. dr.	Pieces to the ℔ *Troy.*	Fine Car. gr.	Value 1640. l. s. d.	Value 1660. l. s. d.	Allowance. gr.
Last Angel Noble of *Henry*, *Edw. 6.*, *Phil. & Mary.*, *Elizabeth.*, First Angel of *James.*	72	3 08 00 00	3 07 05 00	$72\frac{2}{3}$ +	23 $3\frac{1}{2}$	0 11 11	0 12 8	2
Soveraign of K. *James* called Double-Brittain Crown.	72	3 08 00 00	3 05 08 $09\frac{9}{31}$	$74\frac{2}{5}$	22 0	0 11 0	0 11 9	2
George Noble.	—	—	3 00 00 00	80	23 $3\frac{1}{2}$	0 10 10	0 11 6	2
Last Angel of *James.*	80	3 00 00 00	2 23 02 $05\frac{1}{3}$	81	23 $3\frac{1}{2}$	0 11 0	0 11 9	2
Half Laureat of *James.*	$79\frac{1}{5}$	3 00 14 $13\frac{1}{11}$	2 22 04 $21\frac{3}{41}$	82	22 0	0 10 0	0 10 8	2
Ten Shilling Piece of *Charles* 1.	30	3 00 00 00	2 22 04 $21\frac{3}{41}$	82	22 0	0 10 0	0 10 8	2
Angel of *Charles.*	—	—	2 16 14 09	89 +	23 $3\frac{1}{2}$	0 10 0	0 10 8	2
Half Spurre Royal.	96	2 12 00 00	2 11 10 00	$96\frac{4}{5}$ +	23 $3\frac{3}{4}$	0 09 0	0 09 7	2
First Crown of K. *Henry.*	$100\frac{1}{2}$	2 09 06 $06\frac{30}{67}$	2 09 00 00	$101\frac{3}{57}$	22 +	0 08 0	0 08 5	2
Single Noble of *Elizabeth.*	—	—	2 05 03 08	$108\frac{2}{5}$	23 $3\frac{1}{2}$	0 08 0	0 08 6	2
Half Old Noble.	$107\frac{1}{2}$	2 05 11 $15\frac{3}{43}$	2 05 00 00	$108\frac{36}{53}$	23 $3\frac{1}{2}$	0 08 0	0 08 6	2
Salute.	108	2 05 06 16	2 05 00 00	$108\frac{36}{53}$	23 3	0 07 11	0 08 5	2
Base Crown of K. *Henry*, called Rose-Crown.	120	2 00 00 00	1 23 00 00	$122\frac{26}{47}$	20 0	0 05 11	0 06 4	1
Crown of *Edward 6.*, *Elizabeth.*	120	2 00 00 00	1 19 00 00	$133\frac{41}{43}$	22 0	0 05 11	0 06 4	1
Half Angel Noble of *Henry*	138	1 17 14 $18\frac{18}{23}$	1 16 00 00	144	23 $3\frac{1}{2}$	0 06 00	0 06 5	1
Half Last Angel of *Henry.*, Half Angel of *Edw. 6.*, *Phil. & Mary*, *Elizabeth.*, Half first Angel of *James.*	144	1 16 00 00	1 15 12 12	$145\frac{1}{3}$ +	23 $3\frac{1}{2}$	0 05 11	0 06 4	1
Brittain Crown of *James.*	144	1 16 00 00	1 14 14 $04\frac{20}{31}$	$148\frac{4}{5}$	22 0	0 05 6	0 05 10	1
Half George Noble.	—	—	1 12 00 00	160	23 $3\frac{1}{2}$	0 05 5	0 05 9	1
Half Last Angel of *James.*	160	1 12 00 00	1 11 11 $02\frac{2}{3}$	162	23 $3\frac{1}{2}$	0 05 6	0 05 10	1
New Crown of *James.*	$158\frac{2}{5}$	1 12 07 $06\frac{6}{11}$	1 11 02 $10\frac{22}{41}$	164	22 0	0 05 0	0 05 4	1
Crown of *Charles* 1.	160	1 12 00 00	1 11 02 $10\frac{22}{41}$	164	22 0	0 05 0	0 05 4	1
Two parts of Salute	162	1 11 11 $02\frac{2}{3}$	1 11 00 00	$164\frac{4}{7}$	23 3	0 05 3	0 05 7	1
Half *Henry* first Crown	201	1 04 13 $03\frac{15}{67}$	1 04 10 00	$202\frac{6}{57}$	22 +	0 04 0	0 04 2	1
Half Salute	216	1 02 13 08	1 02 10 00	$217\frac{19}{53}$	23 3	0 03 11	0 04 2	1
Half Rose Crown	240	1 00 00 00	0 23 10 00	$245\frac{5}{47}$	20 0	0 02 11	0 03 2	$\frac{1}{2}$
Half Crown of *Edward 6.*, *Elizabeth.*	240	1 00 00 00	0 21 10 00	$267\frac{39}{43}$	22 0	0 02 11	0 03 2	$\frac{1}{2}$
Quarter Old Angel Noble.	276	0 20 17 $09\frac{9}{23}$	0 20 00 00	288	23 $3\frac{1}{2}$	0 03 00	0 03 2	$\frac{1}{2}$
Quarter Last Angel of *Henry.*, Quart. Angel of *Edw. 6.*, *Phil. & Mary.*, *Elizabeth.*, Quarter First Angel of *James.*	288	0 20 00 00	0 19 16 06	$290\frac{2}{3}$ +	23 $3\frac{1}{2}$	0 02 11	0 03 2	$\frac{1}{2}$
Half Brittain Crown of *James.*	288	0 20 00 00	0 19 07 $02\frac{10}{31}$	$297\frac{3}{5}$	22 0	0 02 09	0 02 11	$\frac{1}{2}$
Quarter Last Angel of *James.*	—	—	0 17 15 $13\frac{1}{3}$	324	23 $3\frac{1}{2}$	0 02 09	0 02 11	$\frac{1}{2}$

Physical Doses how weighed.

Many of the Physical Doses are weighed by the Pound of 12 Ounces, and every Ounce is divided into 8 Drams, 1 Dram into 3 Scruples, and 1 Scruple into 20 Graines.

Course Druggs and base Mettals how weighed. Avoirdupois and Troy-weight what proportion each beareth to the other.

Mettals more base than Gold or Silver with Course Druggs and divers sorts of Goods and Merchandizes as before noted are bought and sold by *Avoirdupois* Weight, the ordinary Hundred whereof contains 112 Pounds, and the Pound 16 Ounces, which Ounces are less than the Ounce *Troy*, though the Pound be bigger; because that Pound is divided but into 12 Ounces, each of which bear proportion to the Ounce *Avoirdupois* as 1 to $1\frac{3}{29}$ for 1 Ounce *Troy* makes $1\frac{3}{29}$ Ounce *Avoirdupois*, and 1 Ounce *Avoirdupois* is but $\frac{29}{32}$ of 1 Ounce *Troy*, as saith my *Dutch* Authour, and accordingly he makes the Pound *Troy* to be but $\frac{24}{29}$ of the other Pound, and 1 ℔ *Avoirdupois* to equal $1\frac{5}{24}$ ℔. *Troy*.

Herewith also agreeth *Dalton* and *Malynes* before named in their making 56 ℔ *Avoirdupois*, and 67 ℔ 8 ℥ *Troy* justly accord, though elsewhere both of them unhappily mistake to count 7 ℔ *Avoirdupois* equal to 102 ℥ *Troy*, which is $8\frac{1}{2}$ ℔, for then should 56 ℔ *Avoirdupois* be 68 ℔ *Troy*. Others affirm 16 ℥ *Avoirdupois* equal to 14 ℥ 12 *pwts. Troy*, and then shall 56 ℔ *Avoirdupois* be equal to 68 ℔ 1 ℥ 12 *pwts. Troy*. Nevertheless some think anciently the Pounds admitting the like Number of Graines differed no more than the weight of Wheat and Barley one to another, seeing 1 ℔. *Avoirdupois* contains 7680 Grains or Barley-Corns, and so many Grains of Wheat are found in 1 ℔. *Troy*, if every Pennyweight be multiplyed by 32 Grains, or Wheat Cornes according to the Old Statutes.

By some thought to differ no more than Wheat and Barley.

Common

Common greater and smaller Divisions than the Hundred of Avoirdupois *Weight.*

A Table of Avoirdupois-Weight.

	Tonn.	C. Hundred.	½ C. Half Hundred.	qr. Quarter.	℔. Pound.	℥. Ounce.	ʒ. Dram.	℈. Scruple.	
Tonn.	1								
Hundreds.	20	1							
Half Hundreds.	40	2	1						
Quarters.	80	4	2	1					
Pounds.	2240	112	56	28	1				
Ounces.	35840	1792	896	448	16	1			
Drams.	286720	14336	7168	3584	128	8	1		
Scruples.	860160	43008	21504	10752	384	24	3	1	
Grains.	17203200	860160	430080	215040	7680	480	60	20	Grains.

Custom hath made familiar the use and knowledge of Stones, Nails, Cloves, Tods, and such like denominations, though frequented but to weigh some sort of Commodities, and to which of them to allot more Allowance or Tare than 12 on the 100 or less, the Experienced Merchant well knows. However for satisfaction of the Curious Inquisitor, take a Breviat of such as have come to hand in perusal of the Statutes, the Book of Rates (according to which the King receiveth his Customs), and other approved Authours and Experience.

By Avoirdupois Weight.

Allom the Hundred, Stone.

Allom, 1 Hundred, 13½ Stones, 1 Stone 8 Pounds, by the Ordinance *Compositio de Ponderibus*, which allows but 108 ℔. to the Hundred.

Ashes the Last Barrel.

Ashes, Called *Pot-Ashes*, also *Soap-Ashes*, 1 Last 12 Barrels, 1 Barrel 2 Hundred, by the Rates Inwards.

Barillia the Barrel.

Barillia, Or *Saphora*, 1 Barrel 2 Hundred weight, by the same Rates.

Beef, the Nail, Score.

Beef, 1 Nail 8 Pounds of Common use. Some places sell by the Score, each Score 20 Pounds.

Butter the Wey.

Butter in Cask is assized by Statute, as is seen before in Measures, but besides of *Suffolk* and *Essex Butter*, the Wey is usually reckoned alike to their Wey of *Cheese*.

Cheese the Wey, Clove.

Cheese by the Statute of 9. K. *Hen.* 6. *Chap.* 8. One Wey is to contain 32 Cloves, and 1. Clove 7 Pounds. *Suffolk-Cheese* by the Assize Book 1597, and usage ever since, notwithstanding 8 Pounds to the Clove, and *Essex-Cheese* 10½ Pounds to the Clove, and both 32 Cloves to the Wey. But by some the Wey of *Essex-Cheese* doth contain 42 Cloves, and the Clove but 8 Pounds. Both which agree to make the Wey of *Essex-Cheese* 336 ℔ of *Suffolk* 256 ℔ whereas by the Statute a Wey is but 224 ℔. as before.

Cinnamon as Allom.

Cinnamon, by the Ordinance *Compositio de Ponderibus* is to have the same weight as before noted of *Allom*.

Glass the Seam, Stone.

Glass, by the same Ordinance containeth 1 Seam, 24 Stones. 1 Stone 5 Pounds.

Hay, the Load, Truss.

Hay, by Custom 1 Load 40 Trusses, 1 Truss 56 Pounds, which make 20 Hundred weight to the Load, yet most times it passeth with 18 Hundred.

Hemp the Stone.

Hemp, is commonly sold by the Stone, which by the Statute of 21. *Hen.* 8. *Cap.* 12. is especially ordained to contain 20 Pounds. Nevertheless in *Rye* a Stone of *Hemp* is 32 ℔. and so hath been time out of mind.

Lead the Fodder.

The Old Weight

Lead, the Common Accompt 1 Fodder 19½ Hundreds, 1 Hundred 112 Pounds. By the Book of Rates Outwards to 1 Fodder is allowed 20 Hundreds. By the Ordinance abovesaid, 1 Load 30 Formells, 1 Formel 6 Stones wanting 2 ℔. every Stone 12 ℔. and 1 Pound 25 Shillings *Sterling*. So was the Formel then 70 Pounds, a Weight now grown obsolete.

Meal commonly sold by Weight, 1 Bushel 2 Tovits or Half Bushels, 1 Tovit 2 Pecks, 1 Peck 2 Gallons, 1 Gallon 7 Pounds. So the Bushel must weigh 56 ℔. and hereto agree the old and later Books of Assize, yet thereby is the Bushel more than the Bushel by Statute, for 56 ℔. or Pints of *Avoirdupois* Weight exactly answers to 67 ℔. 8 ℥. *Troy* Weight, whereas the Bushel by Statute is to contain but 64 ℔. or Pints *Troy*.

Meal how sold by Weight.

Nutmegs, *Pepper* and *Spice*, as *Allom* and *Cinnamon*, by the Ordinance above mentioned.

Nutmegs, &c. the Old Weight:

Raw Silk, of *China*, *Morea*, Long and Short, 1 Pound, 24 Ounces

Raw Silk the Pound.

Silk Nubs, or *Husks of Silk*, 1 Pound 21 Ounces, both by the Book of Rates Inwards.

Silk Nubs the Pound.

Wooll, hath the Weight established by 31. *Edw.* 3. *Cap.* 8. and other Statutes, acding to the *Lunar* Year of 13 Moneths, and 28 Dayes to the Moneth, making one Sack 26 Stones, and 1 Stone 14 Pounds, which makes the Sack 364 ℔. other denominations may be seen in the following Table.

Wooll the Weight

The Sack how much.

A Table of Wooll Weight.

Last.	1	Sack.					
Sacks.	12	1	Wey.				
Weyes.	24	2	1	Todd.			
Todds.	156	13	6½	1	Stone.		
Stones.	312	26	13	2	1	Clove.	
Cloves.	624	52	26	4	2	1	
Pounds.	4368	364	182	28	14	7	Pounds.

A Pack of *Wooll* contains but 240 ℔. that is, 2 Hundred Weight, and 16 ℔. over, less by 124 ℔. than the Sack of *Wooll* by the Statute.

Difference between the Pack and Sack of Wooll.

Yarne, called *Irish Yarne*, by the Book of Rates Inwards is accompted, 1 Pack 4 Hundreds, 1 Hundred 120 ℔.

Yarn the Pack Hundred.

Besides the Weights allowed as before-mentioned, there hath been allowed at the Kings Custom House for Tare (which is the Weight of the Cask or Wrappers, wherein Goods are packed up) as followeth.

Tare, What Allowance for it at the Custom-House.

		℔.
Upon	Butts of Currance *per Cent.*	14
	Caritels of Currance *per Cent.*	16
	Quarter Rowles of Currance *per Cent.*	18
	Prunes, 6, or 7 C.	84
	Prunes 10 C. and upwards	112
	Raisins Solis, *per Cent.*	12
	Malaga Raisins, *per Piece*	3½
	Figgs the Barrel	10
	The ¾ Barrel	8
	The ½ Barrel.	6
	The ¼ Barrel.	4
	Mather the Bale	28
	Bales of Raw-Silk from	
	Aleppo, with Cotton Legee, the Bale	34½
	Ardus, the Bale	32
	Smirna, the Bale.	14
	Messina, the Bale	8
	Bales of Grogran-Yarne from	
	Aleppo, the Bale	28
	Smirna, the Bale	16
	Bales of Silk from	
	Naples, the Bale	14
	Bologna, the Bale	30
	Spanish Tobacco the Barrel.	28
	The Half Barrel	20
	Sugar Chests	⅕ part.
	Sugar in Fatts, 6 C. weight.	84
	Goods packed up in Paper.	
	For Paper and Packthread, *per Cent.*	2

What allowed by Merchants. *Cloff what.* *Hundred at* Londonderry. *Forraign Geodæticals.*

Moreover there is an Overweight allowed by Merchants called Tret, which is 4 ℔. upon every Hundred of 112 ℔. And also 2 ℔. upon every Scale of 3 C. weight, which is called *Cloff*, but in many Places if not conditioned for, will not be allowed. At *Londonderry* in *Ireland* 140 ℔. is reckoned for 1 Hundred weight.

The *English* Accompt of Measures and Weights passed, an Eye may be cast now on the Forrain Accompt of smaller *Geodæticals*, whether Ancient, or Modern, and the Credit of both must depend on the respective Authours out of which they are here Collected.

Of the Hebrews Greeks *and* Latins *to what compared.* Jews *Value of their Gold.*

Ancient Measures and Weights to avoid prolixity are referred only to *Hebrew*, *Greek*, and *Latine* as before, most of which are here compared to our *Winchester* Measure, and *Troy* Weight, and the Money valued by our *Sterling* Coine at the rate of 5 *s.* the Ounce of Silver, and 3 *l.* the Ounce of Gold, though some say the *Jews* valued their Gold but 10 times as much as their Silver. And this is one cause why some Authours differ in the value they put on their *Hebrew* Coines.

A Table of the Hebrew *Measures.*

Hebrew *Measures.*

Hebrew *Measures.*

- Long
  - (*a*) *Etsbang*, A Fingers Breadth, an Inch.
  - (*b*) *Tophach*, A Palm or Hands Breadth, 4 Fingers, or Inches.
  - (*c*) *Zereth*, A Span.
  - (*d*) *Pagnam*, A Foot, or 12 Inches.
  - (*e*) *Ammah*, A Cubit
    - Common, Half a Yard.
    - Holy, A Yard.
    - Kings, Half a Yard and 3 Fingers.
    - Geometrical, Three Yards.
  - (*f*) *Tsugad*, A Pace, Five Feet.
  - (*g*) *Orgyia*, A Fathom, Six Feet.
  - (*h*) *Chebel*, A Cord, Line, or Rope to measure Land with.
  - (*i*) *Kaneh*, A Reed, common 6 Cubits, 6 Cubits and a Palme.
  - (*k*) *Stadium*, A Furlong, 125 Paces.
  - (*l*) *Cibrath haarets*, Half a Dayes Journey, *&c.*
  - (*m*) *Milliarium*, A Mile, 1000 Paces.
  - (*n*) *Parasanga*, 30 Furlongs.
- Broad
  - (*o*) *Noph*, A Clime, or Tract of Land 60 Feet every way.
  - (*p*) *Maanath*, An Acre, in length 240, in breadth 120 Feet.
- Dry.
  - (*q*) *Kab*, A Quart, *&c.*
  - (*r*) *Omer*, Three Pints and an half.
  - (*s*) *Seah*, A Gallon and an half.
  - (*t*) *Ephah*, Four Gallons and a Pottle.
  - (*u*) *Lethec*, Two Bushels, 6 Gallons and a Pottle.
  - (*w*) *Homer*, *Cor*, Five Bushels and Five Gallons.
- Liquid
  - (*x*) *Log*, Half a Pint.
  - (*y*) *Hin*, Three Quarts, *&c.*
  - (*z*) *Bath*, Four Gallons, and an Half.

Hebrews *begin their Dry Measures with Barley, and Wet with Eggs.* Shiur *used for an Accompt.* Estbang, *the Opinions thereof* Zithe *how taken.* Tophach *how reckoned.* Cith *used for what.* Zereth *how much.* Pagnam *the length.*

(*a*) *Weemse* in his *Christian Synagogue*, tells us, the beginning of their Dry Measure was Barley, and their Wet Eggs, and therefore saith he, An Accompt is called *Shiur*, from *Shiur* Barley. And one *Estbang* by him and others, contained the breadth of 6 Barley-Cornes in their greatest thickness. Others again but the space that 2 laid end to end, or 4 laid close side by side, will lye in. And in round reckoning (though not exactly) passed for an Inch. For 4 Fingers make 3 of our Inches, as most generally accompt. *Junius* on *Ezek.* 40. 5. and *Jer.* 52. 21. *Holyoke* on the *Latine* word *Pollex*, saies an Inch is 1 $\frac{1}{3}$ Fingers breadth, if so, then should 3 Inches be 4 $\frac{1}{2}$ digits, or Fingers breadths. Some of the Rabins call a Fingers breadth *Zithe*.

(*b*) The lesser Palm or Hands breadth, 4 *Estbangs*, *Exod.* 28. 16. and 37. 12. *Ezek.* 40. 5. 2 *Chron.* 4. 5. may be accompted 3 Inches *English* Measure. With some of the *Rabins*, *Cith*, is used for the Measure of the Wrist, to the Roots of the Fingers, which is somewhat more than 4 *Estbangs*.

(*c*) The greater Palme 3 *Tophachs*, *Exod.* 28. 16. *Isa.* 40. 12. properly a Span, and by the 70 rendred Σπιθαμὴ, in *Ezek.* 43. 13. containing the length between the Thumb and the Top of the little Finger stretched out.

(*d*) *Pagnam*, 4 *Tophachs*, or 16 *Estbangs*, *Peter Martyr* in 1 *Kings* 6.

(*e*) A Cubit,

Ammah *the Sorts of Cubits.*

(e) A Cubit, some say, is the length from the Elbow to the Wrist, others to the top of the longest Finger, some making it the 4th, others the 6th part of a Man, allowing some 2 Foot, most 1 ½ Foot, to the Common Cubit or Cubit of a Man so called, *Deut.* 3. 11. to which the Great Cubit is reckoned double, 1 *King.* 7. 15. with 2 *Chron.* 3. 15. See it called the Great Cubit, *Ezek.* 41. 8. and the Cubit of the first Measure, 2 *Chron.* 3. 3. The Kings Cubit in *Herod. lib.* 2. in *Descrip. Babyl.* mentioned to be 3 Fingers longer than the Common Cubit. The Geometrical Cubit, *Origen Hom.* 2. *in Gen. Augustine de Civit. Dei, lib.* 15. *cap.* 27. take for the Measure used in Building *Noah's* Ark, And to this Rabbi *Cambi* in his Comment on *Ezek.* 46. 2. cited by *Arias Montanus de Mensuris Sacris*, comes near assigning 1000 *Emoth* or Cubits to make a Mile, And some say, this is the Cubit used in *Egypt*. *Gomed*, *Judg.* 3. 16. taken by Interpreters for the same with *Ammah* a Cubit.

Gomed *how rendred.*

Tsagad, *uncertain.*

Two sorts of Paces.

(f) *Tsagad*, or *Tsaad* mentioned often, but because never measured in the Text, as to the certainty must remain unknown. Several Authours make Two sorts of Paces, the *Minor* of 2 ½ Feet, a Step, or Half a Remove of the Body; the *Major* of 5 Feet, a Stride or a Pace by removing both Leggs from the Heele at the first Stand to the Toe at the last.

Fathom *how much.*

(g) A *Fathom* in *Greek* Ὀργυιὰ, greatly questionable if ever any Measure with the *Hebrews*, because not once mentioned in the *Old Testament*, and but twice in the *New*, in one Verse, *Viz. Acts* 27. 28. is as much of a Rope or Line as a Man can include between the tops of his longest Figures, when the Arms are stretched out at length in a right line, and so uncertain according to the length of the Armes fathoming, but generally taken for 2 Yards or 6 Feet *English* Measure. Some promiscuously taking it for the Pace give it but 5 Feet. Others render it in the *Latine*, *Ulna*, but then must not be taken for our Ell, which is but 3 Feet 9 Inches. The *Fathom* is mostly used at Sea to measure their Ropes and Soundings, wherein they do not strictly take the length from Finger to Finger, but so much as shall be included holding the Rope in the Hands extended between the Thumbs and Fore-Fingers.

When rendred Ulna *must not be taken for our Ell.*

Fathom *used at Sea.*

Chebel, *uncertain.*

(h) Most confess their Ignorance in the length of the *Chebel*, *Psal.* 16. 6. taken Metonymically for the Inheritance it self.

Kaneh *of what use.*

The length.

Salel *and* Rus *how taken.*

(i.) Used to measure Buildings, as the *Chebel*, Lands, expressed *Ezek.* 40. 5. to contain 6 Cubits, and an Handsbreadth, but *Tremelius* on the place takes the Reed to contain so much of the Kings Cubits, though the Common Reed, by the *Targum* and several others is accompted but 6 Cubits just. *Salel* a *Rabinical* word for a Reed of 6 Cubits, and *Rus* for 70 Reeds, to be found in their Writings.

Stadium *the Furlong from whence.*

The Length.

(k) A *Furlong*, (*quasi*, *Furrowlong*, because in Champion Countreys their Furrows were usually very long) not mentioned in the *Old Testament*, a Measure brought in with the *Græcian* Monarchy as seemeth, because first met with in 2 *Mac.* 12. 9. continued till the *New Testament* Times, and there often the compute of distances at Sea and Land, with *Pliny*, *lib.* 2. *cap.* 23. *Isidore* and others made to be 125 Paces, and so must be the Eighth part of an *Italian* Mile, which contains 1000 Paces, though the same *Pliny* will have 7 ½ Furlongs make a Mile. There is no doubt but *Stadium* hath admitted of different acceptations, as the Controversie between *Pliny* and *Diodorus Siclus* testifies, So also *Stadium* because of the different content thereof in several Countreys with Authours is respectively to be taken, besides the *Italian* which contained as before 125 Paces, or 625 Feet, sometime for the Olympique at 120 Paces, or 600 Feet, and sometime for the *Pirhique Stadium*, at 200 Paces, or 1000 Feet.

Difference *thereof.*

Cibrath, &c. *differently taken.*

(l) Half a Dayes Journey, a Dayes Journey, a Sabbath-Dayes Journey, and a Space less than any of them, a Bow-shot, all left indeterminate in the Old Testament, the New *Acts* 1. 12. *John* 11. 18. accompts the Sabbath-Dayes Journey about 15 Furlongs, which doubled in a Journey forward and backward made about 4 Miles. So *Fuller*, *Pis. Sight of Palestine*, *Book* 1. *p.* 43. 44.

Miles *of different length.*

(m) The *Italian* Mile consisting of 1000 Paces, hath set the Name *Mile* from the Latine *Mille* in *English* 1000. Nevertheless in several Countreys more than 1000 Paces go to make up a Mile, and the *German* Mile is 4 times as much as the *Italian*, and though *Buxtorf* renders the word *Cibrath*, *Miliare*, yet we find no Mile mentioned by Translators of the Old Testament, *Mat.* 5. 41. argues it rather of *Roman* Extraction than *Jewish*, and *Kimchi lib. Rad.* thinketh that כ in *Cibrath* is but *Servile*, and that the Root is *Barah*, or as some write *Berah*, which *Tremelius*, *Gen.* 35. 16. and 48. 7. and 2 *King* 5. 19. readeth it *Exiguum terræ Spatium*, and our Translation, a little way. Nevertheless some Learned Men conceive *Berah* answers to the proportion of a *Roman* Mile, but *Barah* properly signifying a Dinner or Meale. Others will when applyed to Journeys take it for such a space of Ground, as usually is travelled or conveniently may be gone in Half a Day, between Meal and Meal, or Bait and Bait.

Barah Berah *what.*

(n) This

What allowed by Merchants. *Cloff what.* *Hundred at* Londonderry. *Forraign Geodæticals.*

Moreover there is an Overweight allowed by Merchants called Tret, which is 4 ℔. upon every Hundred of 112 ℔. And also 2 ℔. upon every Scale of 3 C. weight, which is called *Cloff*, but in many Places if not conditioned for, will not be allowed. At *Londonderry* in *Ireland* 140 ℔. is reckoned for 1 Hundred weight.

The *English* Accompt of Measures and Weights passed, an Eye may be cast now on the Forrain Accompt of smaller *Geodæticals*, whether Ancient, or Modern, and the Credit of both must depend on the respective Authours out of which they are here Collected.

Of the Hebrews Greeks *and* Latins *to what compared.* Jews *Value of their Gold.*

Ancient Measures and Weights to avoid prolixity are referred only to *Hebrew*, *Greek*, and *Latine* as before, most of which are here compared to our *Winchester* Measure, and *Troy* Weight, and the Money valued by our *Sterling* Coine at the rate of 5 *s.* the Ounce of Silver, and 3 *l.* the Ounce of Gold, though some say the *Jews* valued their Gold but 10 times as much as their Silver. And this is one cause why some Authours differ in the value they put on their *Hebrew* Coines.

A Table of the Hebrew *Measures.*

Hebrew *Measures.*

Hebrew *Measures.*

- Long
  - (a) *Etsbang*, A Fingers Breadth, an Inch.
  - (b) *Tophach*, A Palm or Hands Breadth, 4 Fingers, or Inches.
  - (c) *Zereth*, A Span.
  - (d) *Pagnam*, A Foot, or 12 Inches.
  - (e) *Ammah*, A Cubit
    - Common, Half a Yard.
    - Holy, A Yard.
    - Kings, Half a Yard and 3 Fingers.
    - Geometrical, Three Yards.
  - (f) *Tsugad*, A Pace, Five Feet.
  - (g) *Orgyia*, A Fathom, Six Feet.
  - (h) *Chebel*, A Cord, Line, or Rope to measure Land with.
  - (i) *Kaneh*, A Reed, common 6 Cubits, 6 Cubits and a Palme.
  - (k) *Stadium*, A Furlong, 125 Paces.
  - (l) *Cibrath haarets*, Half a Dayes Journey, *&c.*
  - (m) *Milliarium*, A Mile, 1000 Paces.
  - (n) *Parasanga*, 30 Furlongs.
- Broad
  - (o) *Noph*, A Clime, or Tract of Land 60 Feet every way.
  - (p) *Maanath*, An Acre, in length 240, in breadth 120 Feet.
- Dry.
  - (q) *Kab*, A Quart, *&c.*
  - (r) *Omer*, Three Pints and an half.
  - (s) *Seah*, A Gallon and an half.
  - (t) *Ephah*, Four Gallons and a Pottle.
  - (u) *Lethec*, Two Bushels, 6 Gallons and a Pottle.
  - (w) *Homer*, *Cor*, Five Bushels and Five Gallons.
- Liquid
  - (x) *Log*, Half a Pint.
  - (y) *Hin*, Three Quarts, *&c.*
  - (z) *Bath*, Four Gallons, and an Half.

Hebrews *begin their Dry Measures with Barley, and Wet with Eggs.* Shiur *used for an Accompt.* Estbang, *the Opinions thereof* Zithe *how taken.* Tophach *how reckoned.* Cith *used for what.* Zereth *how much.*

(a) *Weemse* in his *Christian Synagogue*, tells us, the beginning of their Dry Measure was Barley, and their Wet Eggs, and therefore saith he, An Accompt is called *Shiur*, from *Shiur* Barley. And one *Estbang* by him and others, contained the breadth of 6 Barley-Cornes in their greatest thickness. Others again but the space that 2 laid end to end, or 4 laid close side by side, will lye in. And in round reckoning (though not exactly) passed for an Inch. For 4 Fingers make 3 of our Inches, as most generally accompt. *Junius* on *Ezek.* 40. 5. and *Jer.* 52. 21. *Holyoke* on the *Latine* word *Pollex*, saies an Inch is $1\frac{1}{3}$ Fingers breadth, if so, then should 3 Inches be $4\frac{1}{2}$ digits, or Fingers breadths. Some of the Rabins call a Fingers breadth *Zithe*.

(b) The lesser Palm or Hands breadth, 4 *Estbangs*, *Exod.* 28. 16. and 37. 12. *Ezek.* 40. 5. 2 *Chron.* 4. 5. may be accompted 3 Inches *English* Measure. With some of the *Rabins*, *Cith*, is used for the Measure of the Wrist, to the Roots of the Fingers, which is somewhat more than 4 *Estbangs*.

(c) The greater Palme 3 *Tophachs*, *Exod.* 28. 16. *Isa.* 40. 12. properly a Span, and by the 70 rendred Σπιθαμὴν, in *Ezek.* 43. 13. containing the length between the Thumb and the Top of the little Finger stretched out.

Pagnam *the length.*

(d) *Pagnam*, 4 *Tophachs*, or 16 *Estbangs*, *Peter Martyr* in 1 *Kings* 6.

(e) A Cubit,

(e) A Cubit, some say, is the length from the Elbow to the Wrist, others to the top of the longest Finger, some making it the 4th, others the 6th part of a Man, allowing some 2 Foot, most 1 ½ Foot, to the Common Cubit or Cubit of a Man so called, *Deut.* 3. 11. to which the Great Cubit is reckoned double, 1 *King.* 7. 15. with 2 *Chron.* 3. 15. See it called the Great Cubit, *Ezek.* 41. 8. and the Cubit of the first Measure, 2 *Chron.* 3. 3. The Kings Cubit in *Herod. lib.* 2. in *Descrip. Babyl.* mentioned to be 3 Fingers longer than the Common Cubit. The Geometrical Cubit, *Origen Hom.* 2. *in Gen. Augustine de Civit. Dei, lib.* 15. *cap.* 27. take for the Measure used in Building *Noah*'s Ark, And to this Rabbi *Cambi* in his Comment on *Ezek.* 46. 2. cited by *Arias Montanus de Mensuris Sacris*, comes near assigning 1000 *Emoth* or Cubits to make a Mile, And some say, this is the Cubit used in *Egypt. Gomed*, *Judg.* 3. 16. taken by Interpreters for the same with *Ammah* a Cubit.

Ammah *the Sorts of Cubits.*

Gomed *how rendred.*

(f) *Tsagad*, or *Tsaad* mentioned often, but because never measured in the Text, as to the certainty must remain unknown. Several Authours make Two sorts of Paces, the *Minor* of 2 ½ Feet, a Step, or Half a Remove of the Body; the *Major* of 5 Feet, a Stride or a Pace by removing both Leggs from the Heele at the first Stand to the Toe at the last.

Tsagad, *uncertain.* *Two sorts of Paces.*

(g) A *Fathom* in *Greek* 'Οργυιά, greatly questionable if ever any Measure with the *Hebrews*, because not once mentioned in the *Old Testament*, and but twice in the *New*, in one Verse, *Viz. Acts* 27. 28. is as much of a Rope or Line as a Man can include between the tops of his longest Figures, when the Arms are stretched out at length in a right line, and so uncertain according to the length of the Armes fathoming, but generally taken for 2 Yards or 6 Feet *English* Measure. Some promiscuously taking it for the Pace give it but 5 Feet. Others render it in the *Latine*, *Ulna*, but then must not be taken for our Ell, which is but 3 Feet 9 Inches. The *Fathom* is mostly used at Sea to measure their Ropes and Soundings, wherein they do not strictly take the length from Finger to Finger, but so much as shall be included holding the Rope in the Hands extended between the Thumbs and Fore-Fingers.

Fathom *how much.* *When rendred* Ulna *must not be taken for our Ell.* Fathom *used at Sea.*

(h) Most confess their Ignorance in the length of the *Chebel*, *Psal.* 16. 6. taken Metonymically for the Inheritance it self.

Chebel, *uncertain.*

(i.) Used to measure Buildings, as the *Chebel*, Lands, expressed *Ezek.* 40. 5. to contain 6 Cubits, and an Handsbreadth, but *Tremelius* on the place takes the Reed to contain so much of the Kings Cubits, though the Common Reed, by the *Targum* and several others is accompted but 6 Cubits just. *Salel* a *Rabinical* word for a Reed of 6 Cubits, and *Rus* for 70 Reeds, to be found in their Writings.

Kaneh *of what use.* *The length.* Salel *and* Rus *how taken.*

(k) A *Furlong*, (*quasi*, *Furrowlong*, because in Champion Countreys their Furrows were usually very long) not mentioned in the *Old Testament*, a Measure brought in with the *Græcian* Monarchy as seemeth, because first met with in 2 *Mac.* 12. 9. continued till the *New Testament* Times, and there often the compute of distances at Sea and Land, with *Pliny*, *lib.* 2. *cap.* 23. *Isidore* and others made to be 125 Paces, and so must be the Eighth part of an *Italian* Mile, which contains 1000 Paces, though the same *Pliny* will have 7 ½ Furlongs make a Mile. There is no doubt but *Stadium* hath admitted of different acceptations, as the Controversie between *Pliny* and *Diodorus Siclus* testifies, So also *Stadium* because of the different content thereof in several Countreys with Authours is respectively to be taken, besides the *Italian* which contained as before 125 Paces, or 625 Feet, sometime for the Olympique at 120 Paces, or 600 Feet, and sometime for the *Pirhique Stadium*, at 200 Paces, or 1000 Feet.

Stadium *the Furlong from whence.* *The Length.* *Difference thereof.*

(l) Half a Dayes Journey, a Dayes Journey, a Sabbath-Dayes Journey, and a Space less than any of them, a Bow-shot, all left indeterminate in the Old Testament, the New *Acts* 1. 12. *John* 11. 18. accompts the Sabbath-Dayes Journey about 15 Furlongs, which doubled in a Journey forward and backward made about 4 Miles. So *Fuller*, *Pis. Sight of Palestine*, *Book* 1. *p.* 43. 44.

Cibrath, &c. *differently taken.*

(m) The *Italian* Mile consisting of 1000 Paces, hath set the Name *Mile* from the Latine *Mille* in *English* 1000. Nevertheless in several Countreys more than 1000 Paces go to make up a Mile, and the *German* Mile is 4 times as much as the *Italian*, and though *Buxtorf* renders the word *Cibrath*, *Miliare*, yet we find no Mile mentioned by Translators of the Old Testament, *Mat.* 5. 41. argues it rather of *Roman* Extraction than *Jewish*, and *Kimchi lib. Rad.* thinketh that כ in *Cibrath* is but *Servile*, and that the Root is *Barah*, or as some write *Berah*, which *Tremelius*, *Gen.* 35. 16. and 48. 7. and 2 *King* 5. 19. readeth it *Exiguum terræ Spatium*, and our Translation, a little way. Nevertheless some Learned Men conceive *Berah* answers to the proportion of a *Roman* Mile, but *Barah* properly signifying a Dinner or Meale. Others will when applyed to Journeys take it for such a space of Ground, as usually is travelled or conveniently may be gone in Half a Day, between Meal and Meal, or Bait and Bait.

Miles *of different length.* Barah Berah *what.*

(n) This

Parasang *how taken.* (*n*) This seems to be a *Persian* Word and Measure, taken for 30 Furlongs, *Herodot. lib.* 2. *Ramus Geometry* set forth by *Bedwell* in *English*, by others for a *German* Mile. *Elias* in *Thisbi* mentioneth a *Persah*, which he saith was the Great Mile, and contained 4 lesser.

Noph *how rendred.* (*o*) *Noph*, rendred by *Buxtorfe*, A Clime, or Tract of Land, *Psal.* 48 2. *Noph*, also the name of a City in Scripture, but the content thereof not there found, other Writings, as *Holyokes Dictionary*, *&c.* reckons it 60 Feet every way, yet may be a Plot of ground big enough for a small Town.

Maanath *diversly taken.* (*p*) Used 1 *Sam.* 14. 14. and made by several fitly to correspond with *Jugerum* in the *Latine*, but differ in the quantity, some reckon it 200 Foot every way, others 240, in length, and 120 in breadth. *Quint.* 1. 28. *Isidor.* 15. 15.

Kab *what it contained.* (*q*) The fourth part of a *Kab*, 2 *Kings* 6. 25. for want of more accurate Correspondencies may be taken for our Half Pint, and so the *Kab* for our Quart. *Buxtorf* out of *Rab. Alphes. Tract. de Paschate. cap.* 5. *fol.* 176. with whom divers conclude, that the fourth part of a *Kab*, and a Log are of a like quantity, and that each contained 6 Eggs, *viz.* as much as will fill Six Hen Egg-shells of the ordinary, or middle size, and thence called by some a *Sextary*, and so accordingly the *Kab*, 24 Eggs, or 4 Logs.

Called a Sextary. Omer *how much.* (*r*) An *Omer*, was the Dayly Ordinary of a Man, and the tenth part of an *Ephah*, *Exod.* 16. 16. 36. more than 3 $\frac{1}{2}$ Pints our Measure, and not full a Pottle, 43 $\frac{1}{5}$ Eggs.

Scah *the Content.* (*s*) A *Seah*, noted *Gen.* 18. 6. and 2 *King.* 7. 1. commonly estimated by Writers at 6 *Kabs*, that is 2 Hins, or 144 Eggs, or about a Gallon and an Half our measure. *Godwin* in *Moses* and *Aaron*, *lib.* 6. *cap.* 9.

Ephah *the Content.* (*t*) An *Ephah* contained 3 *Seahs* or 18 *Kabs*, in Eggs 432 about 9 Pottles *English* Measure, it was the tenth part of an *Homer*, *Ruth* 2. 17. *Ezek.* 45. 11.

Lether *its content.* (*u*) *Lether*, in *Hosea*, 3. 2. is reckoned to be the Half of an *Homer* or 5 *Ephahs*, and consequently 90 *Kabs*.

Homer *and* Cor *how much.* (*w*) An *Homer*, being 10 *Ephahs*, *Ezek.* 45. 11. is thought by some to be the ordinary Burden of an Ass, but with us a good Horseload, at the Rates aforesaid 5 Bushels and 5 Gallons, our Measure. The *Cor* was equal to the *Homer*, common to measure both Liquids and Dry. *Ezek.* 45. 14. *Luke* 16. 7.

Log *the Content.* (*x*) The *Log* was the smallest Measure for Liquids, we find mentioned, see *Levit.* 14. 12. and before in the *Kab*.

Hin *how much.* (*y*) The *Hin* often in Scripture, in quantity Three Quarts our Measure or thereabouts, 3 *Kabs*, or 72 Eggs. This Measure was divided into the Half, Third, Fourth, and Sixth parts, *Numb.* 15. 6, 9. and 28. 5. *Ezek.* 4. 11.

Bath *and* Solomons Sea *their Contents.* (*z*) The *Bath*, *Ezek.* 45. 14. is made alike to the *Ephah*, so must the Molten Sea, 2 *Chron* 4. 5. holding 3000 Baths, contains 210 Quarters, 7 $\frac{1}{2}$ Bushels, or 13500 Gallons our Measure.

Nebel *how translated.* As for *Nebel*, translated *Jer.* 13. 11. a Bottle, few reckon it a Measure except *Epiphanius* who saies it contains 150 Sextaries.

A Brief

A Brief view of most of the Hebrew *Measures aforesaid, with their* English *Content may be taken in the following Table.*

A Table of Hebrew Long Measures compared with the English.

The Table of *Hebrew* Long Measures.

	1	8	o	$833\frac{1}{3}$	1000		5000		60000	Milliarium
	Mile.	1	n	$104\frac{1}{6}$	125	h	625		7500	Stadium
	Milliariū.	Furlongs	i	$1\frac{13}{24}$	$1\frac{17}{20}$	lis	$9\frac{1}{4}$	s.	111	Kaneh
Mil.	1	Stadium	Reeds	1	$1\frac{1}{5}$	ng	6	re	72	Orgya
Stadium.	8	1	Kaneh.	Fathoms	1	E	5	su	60	Tsagad
Kaneh.	$54\frac{20}{37}$	$67\frac{21}{37}$	1	Orgya	Paces.	1	$1\frac{1}{2}$	ea	18	Ammah c.
Orgya.	$833\frac{1}{3}$	$104\frac{1}{6}$	$1\frac{13}{24}$	1	Tsagad	Cubits	1	M	12	Pagnam
Tsagad.	1000	125	$1\frac{17}{20}$	$1\frac{1}{5}$	1	Ammah c.	Foot	1	9	Zereth
Ammah Com.	$3333\frac{1}{3}$	$416\frac{2}{3}$	$6\frac{1}{6}$	4	$3\frac{1}{3}$	1	Pangam	Palmes	3	Tophach
Pagnam.	5000	625	$9\frac{1}{4}$	6	5	$1\frac{1}{2}$	1	Zereth.	Inches	
Zereth.	$6666\frac{2}{3}$	$833\frac{1}{3}$	$12\frac{1}{3}$	8	$6\frac{2}{3}$	2	$1\frac{1}{3}$	1	Tophach	
Tophach.	20000	2500	37	24	20	6	4	3	1	
Estbang.	80000	10000	148	96	80	24	16	12	4	

A Table of Hebrew Concave Measures compared with the English.

The Table of *Hebrew* Dry and Liquid Measures.

	$5\frac{5}{8}$	$11\frac{1}{4}$	$22\frac{1}{2}$	45	90	180	360	720	Homer, Cor.
	Bushels	$5\frac{5}{8}$	$11\frac{1}{4}$	$22\frac{1}{2}$	45	90	180	360	Lethec.
	HomerCor	Tovits	$2\frac{1}{4}$	$4\frac{1}{2}$	9	18	36	72	Ephah, Bath.
Homer, Cor.	1	Lethec	Pecks	$1\frac{1}{2}$	3	6	12	18	Seah.
Lethec.	2	1	Ephah, Bath	Gallons	$1\frac{1}{2}$	3	6	12	Hin.
Ephah, Bath.	10	5	1	Seah	Pottles	$1\frac{4}{5}$	$3\frac{3}{5}$	$7\frac{1}{5}$	Omer.
Seah.	30	15	3	1	Hin	Quarts	2	4	Kab
Hin.	60	30	6	2	1	Omer	Pints	1	Log, $\frac{1}{4}$ Kab.
Omer.	100	50	10	$3\frac{1}{3}$	$1\frac{2}{3}$	1	Kab	Half Pints	
Kab.	180	90	18	6	3	$1\frac{4}{5}$	1	Log	
Log, $\frac{1}{4}$ Kab.	720	360	72	24	12	$7\frac{1}{5}$	4	1	
Eggs.	4320	2160	432	144	72	$43\frac{1}{5}$	24	6	

Hebrew *Weights.*

The principal Weights in use among the *Jews* were Talents, Pounds, Shekels, Drams, and other small divisions of the Shekel, all which may be further seen in the Accompt of their Money. Hebrew *Weights.*

A Table of the Hebrew *Coines* and their Value.

Hebrew Coines.

				l.	s.	d.
Hebrew	Money	Silver	(*a*) *Shekel*, of the *Sanctuary* in Weight ½ ℥ *Troy.* Value to *Sterling* Money.	00	02	06
			(*b*) *King's Shekel*, Half the *Sanctuary Shekel*, called *Bekah.*	00	01	03
			(*c*) Third part of a *Shekel.*	00	00	10
			(*d*) *Zuz*, Fourth part of a *Shekel.*	00	00	07½
			(*e*) *Gerah*, *Agorah*, *Keshitah.*	00	00	01½
		Gold	(*f*) *Zahab*, *Golden Siclus*, or *Shekel* — each ½ ℥ *Troy* in value 12 *Silver Shekles* of the *Sanctuary*	01	10	00
			(*g*) *Adarkon.* *Dracmon*, or *Darcon.* — each half so much	00	15	00
	Sums of Money		(*h*) *Maneh*, or *Mina*, or a Pound, valuable in Gold 100 *Shekels.* 25 ℥ *Troy.*	75	00	00
			Silver, 60 *Shekels.* 30 ℥ *Troy.*	07	10	00
			(*i*) *Chichar*, or *Talent*, in Weight 3000 *Shekels* 125 ℔. *Troy* valuable in Gold	4500	00	00
			Silver	375	00	00

Shekel how inscribed and stamped. Whence the word.

(*a*) This piece of Coin (is often mentioned in Sacred Writ, and looks like the Standard of all the rest) on the one side shewed the Vessel in which the *Manna* was, inscribed with this Perigraphe *Shekel Israel*, on the other side, the Rod of *Aaron* that budded, with this Inscription, *Jerusalem Kodshash*, in *English*, *The Holy Jerusalem*, called *Shekel* from *Shakal* to weigh, for Money at the first seems to be a Merchandise exchanged or given for other Commodities, as *Gen.* 23. 16. after the *Chaldee* called *Silgha*, and commonly with the *Hebrews*, *Keseph*, *i. e. Silver*, and being put absolutely rendred a *Silverling*, or piece of Silver, by Expositors; as usually as a *Shekel* when the quality is not mentioned is taken for a Silver *Shekel*.

Silverling what.

Bekah how much, and for what paid.

(*b*) First mentioned, *Gen.* 24. 22. paid by all as a Yearly tribute, *Exod.* 30. 13. 15. 2 *Chron.* 24. 6. 9. towards the Repair of the Tabernacle first, and after of the Temple, between this and the *Sanctuary Shekel*, some mention a third sort of *Shekel*, called the Common *Shekel*, valued at 20 *d.* of our Money, of which I find nothing but uncertainty.

Third part of a Shekel.

(*c*) In *Nehem.* 10. 32. noted as a yearly tribute given of the *Jews* by a Civil Decree to the second Temple.

Zuz, how much.

(*d*) 1 *Sam.* 9. 8. spoken of, and equal to the Old *Attick* Dram, and *Roman* Peny, as several say, and by some called *Zuz*, *Zuza*, *Zur*, and *Zura*.

Gerah, or Agorah the Image of a Lamb thereon.

(*e*) *Agorah*, rendred *Gerah* by the *Chaldee* Paraphrase, and also *Megna*, or *Megha*, in that Tongue, by the *Arabians*, *Megah*, *Greeks*, *Obolus*, and in *English*, *a Piece of Silver*, 1 *Sam.* 2. 36. called also in the *Hebrew*, *Keshitah*, because signed with the Image of a Lamb, See *Gen.* 33. 19. *Joshua* 24. 32. Twenty of these made a *Shekel. Exod.* 30. 13.

Zahab a Gold Coine how reckoned.

(*f*) The *Zahab*, and Golden *Shekel* in *Rider*'s Dictionary undervalued at 15 *s.* for though *Hunt* in his *Handmaid to Arithmetick* out of *Breerwood de nummis*, mentioneth the Weight but 2 *Attick* Drams, yet he valueth it at 30 *s.* *Alsted* in his *Encyclopædia of Arithmetick* makes it 4 Drams, which accompted ½ ℥ *Troy*, must yield double that value of 15 *s.* and most Authours agree that the Gold *Shekel* was equal in weight with the Silver *Shekel*, so the difference must be only in the value of the Mettal. Some to keep up the Credit of differing Authours conceive there were 2 sorts of Golden *Shekels* as well as Silver, the one double to the other.

A Darkon or Drakmon seems a Persian Coin.

(*g*) These Golden *Adarkons*, and *Drakmons* seem to be *Persian*, or Coines of some other *Nations*, and Currant, not Coined in *Judea*, not read of till after the Captivity, for though the word be used 1 *Chron.* 29. 7. yet both Books of the Chronicles, as most take it were penned by *Ezra*, after his return from *Babylon*, in whose Book mention is made thereof, as in *Ezra* 2. 69. and 8. 27. *&c.* in *Greek* called *Drachme*, and in *English*, rendred *Drams*. *Alsted*, in his *Encyclopædia* of *Arithmetick* makes the value equal to a Ducat of *Hungary*. But *Hunt* who throughout his Book hath much lost himself in his Method; forgot himself in valuing 2 *Attick* Drams in the *Zahab* at 30 *s.* and 2 *Attick* Drams in the *Adarkon* (which he saith is the weight thereof) at 15 *s.*

Maneh, Mina, how much. Difference about it.

(*h*) The *Maneh* in Gold by comparing 1 *King* 10. 17. with 2 *Chron.* 9. 16. is found to be 100 *Shekels*, which *Buxtorf* and others understand not of the Holy but the Royal *Shekel*. The *Maneh* in *Ezek.* 45. 12. seems to be 60 *Shekels*, and hereto several agree, but some think it was now increased 10 *Shekels* more than of old, and call it, The *New Maneh*,

valued neverthelefs with moft after the *Holy Shekel*, in Silver at 7 ℔. 10 *s. Buxtorfe* tells us alfo of a *Maneh* of 25 *Holy Shekels.* The *Affemblies Annotations* on *Ezek.* 45. 12. make 3 forts of *Manehs, viz.* Common 15, the Kings 20, and the Holy 25 *Shekels*, but the word *Maneh* is Singular and not Plural in the *Hebrew* Text there, as noting all thofe divifions to make but one *Maneh.*

(*i*) The *Chichar*, by fome wrote *Kichar*, and commonly tranflated *Talent*, contained 3000 *Shekels*, as may be collected from *Exod.* 38. 25, 26. where the Silver Collected is expreffed to be 100 Talents, and 1775 Shekels, *v.* 25. and the Perfons that paid it at half a Shekel a piece, *v.* 26. numbred to be 603550, whereof the half is 301775, which divided by 3000, the Shekels in one Talent, makes 100 Talents, and leaves the odd Shekels remaining 30|1775(100 — *Talent, how much.*

3 000

The Talent thus valued after the Holy Shekel, makes the Number of Talents mentioned in fome Texts of Scripture, efpecially 1 *Chron.* 22, and 29 *Chapters* amount to fuch Maffy Sums, that fome think the Talents are to be reckoned at the rate of the other Shekel; and others, not improbably that the *Jews* had a piece of Money or Plate of Gold of fmall value (as may be obferved anciently in *Homer Iliad, lib.* 23.) called a Talent. And *Fuller* in his *Pifgah Sight of Paleftine, Book* 3. *p.* 356, 357. fhews whereon fuch an Opinion may be ftrengthened, and that the Talent mentioned in fome Scriptures may be rather this than the other. — *Doubts thereof.* — Talent, *a piece of Coine.*

The Accompt both of the Hebrew Weight *and* Money, *with their Value in* Sterling-Money *and* Troy-Weight, *is contracted into the following Table.* — *A Table of* Hebrew Gold *and Weights.*

Hebrew Gold *and* Weights.

Chichar, *or* Talent.	1	Maneh.			
Maneh, *or* Pound.	old 60 | new 50	1 new	old	Zahab.	
Zahab, *or* Shekel.	3000	60	50	1	Adarkon.
Adarkon, *or* Dram.	6000	120	100	2	1
Troy-Weight.	125 ℔.	2½ ℔	2℔ 1℥	½ ℥	¼℥
Sterling-Money.	4500 l.	90 l.	75 l.	1l. 10s.	15 s.

Hebrew Silver *and* Weights.

A Table of Hebrew Silver *and Weights.*

Chichar, *or* Talent.	1	Maneh.						
Maneh, *or* pound.	old 60 | new 50	1 new	old	Shekel.				
Shekel.	3000	60	50	1	Bekah.			
Bekah.	6000	120	100	2	1	Third part.		
Third parts.	9000	180	150	3	1½	1	Zuz.	
Fourth Parts, Zuzims, *or* Drams.	12000	240	200	4	2	1⅓	1	Gerah.
Gerahs.	60000	1200	1000	20	10	6⅔	5	1
Troy-Weight.	125 ℔	2½ ℔	2℔ 1℥,	½ ℥	¼ ℥.	1/6 ℥	⅛ ℥	1/40 ℥
Sterling-Money.	375 ℔.	7l 10s.	6l. 5s.	2s. 6d.	1s. 3d.	10d.	7½ d.	1½d.

The like Method may be obferved in viewing the Meafures, Weights and Monies of the old *Greeks*, who imitating the *Hebrews* kept fome like theirs, altered and added others. — Greeks, *their* Geodæticals.

Gracian

Græcian Long Measures.

A Table of Long Measures of the Greeks*, and the* English *Inches therein.*

		Parasang.	Dolich.	Mile.	Hippacon.	Diaulus.	Furlong.	Plethron.	Fathome.	Pace.	Cubit.	Pygon.	Pygme.	Foot.	Span	Orthodoron.	Licha.	Paleft.
Schoenes	1																	
Parasangs	2	1																
Dolichs	5	$2\frac{1}{2}$	1															
Miles	$7\frac{1}{2}$	$3\frac{3}{4}$	$1\frac{1}{2}$	1														
Hippicas	15	$7\frac{1}{2}$	3	2	1													
Diaulus	30	15	6	4	2	1												
Furlongs	60	30	12	8	4	2	1											
Plethras	375	$187\frac{1}{2}$	75	50	25	$12\frac{1}{2}$	$6\frac{1}{4}$	1										
Fathoms	6250	3125	1250	$833\frac{1}{3}$	$416\frac{2}{3}$	$208\frac{1}{3}$	$104\frac{1}{6}$	$16\frac{2}{3}$	1									
Paces	7500	3750	1500	1000	500	250	125	20	$1\frac{1}{5}$	1								
Cubits	25000	12500	5000	$3333\frac{1}{3}$	$1666\frac{2}{3}$	$833\frac{1}{3}$	$416\frac{2}{3}$	$66\frac{2}{3}$	4	$3\frac{1}{3}$	1							
Pygons	30000	15000	6000	4000	2000	1000	500	80	$4\frac{4}{5}$	4	$1\frac{1}{5}$	1						
Pygmes	$33333\frac{1}{3}$	$16666\frac{2}{3}$	$6666\frac{2}{3}$	$4444\frac{4}{9}$	$2222\frac{2}{9}$	$1111\frac{1}{9}$	$555\frac{5}{9}$	$88\frac{8}{9}$	$5\frac{1}{3}$	$4\frac{4}{9}$	$1\frac{1}{3}$	$1\frac{1}{9}$	1					
Feet	37500	18750	7500	5000	2500	1250	625	100	6	5	$1\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{8}$	1				
Spans	50000	25000	10000	$6666\frac{2}{3}$	$3333\frac{1}{3}$	$1666\frac{2}{3}$	$833\frac{1}{3}$	$133\frac{1}{3}$	8	$6\frac{2}{3}$	2	$1\frac{1}{3}$	$1\frac{1}{2}$	$1\frac{1}{3}$	1			
Orthodorons	$54545\frac{5}{11}$	$27272\frac{8}{11}$	$10909\frac{1}{11}$	$7272\frac{8}{11}$	$3636\frac{4}{11}$	$1818\frac{2}{11}$	$909\frac{1}{11}$	$145\frac{5}{11}$	$8\frac{8}{11}$	$7\frac{3}{11}$	$2\frac{2}{11}$	$1\frac{9}{11}$	$1\frac{7}{11}$	$1\frac{5}{11}$	$1\frac{1}{11}$	1		
Lichas	60000	30000	12000	8000	4000	2000	1000	160	$9\frac{3}{5}$	8	$2\frac{2}{5}$	2	$1\frac{4}{5}$	$1\frac{3}{5}$	$1\frac{1}{5}$	$1\frac{1}{10}$	1	
Palestes	150000	75000	30000	20000	10000	5000	2500	400	24	20	6	5	$4\frac{1}{2}$	4	3	$2\frac{3}{4}$	$2\frac{1}{2}$	1
Dactyls.	600000	300000	120000	80000	40000	20000	10000	1600	96	80	24	20	18	16	12	11	10	4
Inches *Eng.*	450000	225000	90000	60000	30000	15000	7500	1200	72	60	18	15	$13\frac{1}{2}$	12	9	$8\frac{1}{4}$	$7\frac{1}{2}$	3

Schoen *how different.*

A *Schoen*, in *Greek* Σχοῖνος, generally taken with *Herodot. lib.* 2. to contain 60 Furlongs, *Pliny lib.* 12. *cap.* 14, reckons it 40, some say 32, others but 30, an *Egyptiack* Measure as some think, *Scapula in Verb.*

Parasang. *Vide antea.*

A *Parasang*, See before in the *Hebrew* Measures *p.* 84.

Dolich *how taken.*

A *Dolich*, some will have 24 Furlongs, but the Common Accompt is but 12, *Scapula in Verb.*

Mile, *Vide antea.*

A *Mile*, called μίλιον, *Mat.* 5. 41. by birth a *Latine*, though in use a Word and Measure with the *Greeks*, *Hebrews*, *Syrians*, *&c.* See before in *Hebrew* Measures, *p.* 84. also *Leigh. Crit. Sacra gr.*

Hippicon *the Length, and* Diaulus $\frac{1}{2}$.

An *Hippicon*, commonly taken for 4 Furlongs.

A *Diaulus*, for half an *Hippicon*.

A *Furlong*, See before in the *Hebrew* Measures, *Stadium*, *p.* 83.

Plethron *uncertain.*

A *Plethron*, *Scapula* from *Plutarch* reports an Acre from *Suidas* $\frac{1}{6}$ of a Furlong, or an 100 Feet, from *Hesychius*, a Measure of 10000 Feet. of others 100 Furlongs, such uncertainty there is in the Measure or discrepancy in the Authours. In the Table I followed *Suidas*, at 100 Feet, which occasion the following Numbers in the Table to differ from those in *Alsted* and some other Authours.

A *Fathome*,

A *Fathome*, See before in the *Hebrew* Measures, *Orgyia*, *p.* 83.

A *Pace*, in *Greek* βῆμα, See before in the *Hebrew* Measures, *Tsagad*, *p.* 83.

A *Cubit*, after the measure of the Common Cubit with the *Hebrews*. See before in their Measures *Ammah*, in *Greek* πῆχυς.

Fathome, Pace *and* Cubit, *Vide, antea.*

A *Pygon*, taken with *Hesychius* to be a Measure containing the space from the Elbow to the Fingers bent, called by some *Palmipes*, of a Foot, and a Palm, being 20 Fingers breadth, *Scapul. in verb.* and others.

Pygon, *how taken.*

A *Pygme*, used *Mark* 7 3. in measure taken for the length from the Elbow to the Fingers closed, as the Hand is contracted when it is called a Fist, *Leigh Crit. Sacra. Greek* 2 Fingers breadth shorter than the *Pygon*.

Pygme, *the length.*

A *Foot*, in *Greek* πȣ̃ς, the same with the *Hebrew Pagnam*, though *Hunt*, upon what Authority I know not, will have the *Greek* πȣ̃ς less than the *Roman* Foot, $\frac{1}{4}$ Inch, and greater than the *Hebrew Pagnam* almost $_4$ Inch. But *James Capel* a Man of far more exactness in his Treatise *De Mensuris intervallorum*, makes the *Attick* Foot and the *English* to agree as near as 75 to 76, and the *Roman* and *English* Foot as 18 to 19. And the Learned *Willebrand*, *Snellius* of *Leiden* in his *Eratosthenes Batavus* (who some think comes nearer the Truth) makes the

Pous, *a Foot, like* Pagnam.

English	Foot agree as	484.
Roman		500.
Old *Greek*		521.

A *Span*, in *Greek* σπιθαμὴ, in *Latine*, *Palmus major*, and *Dodrans*, by *Poll. lib.* 2. *Hesych* and others alwaies accompted 12 Fingers breadth, like the *Hebrew Zereth*, answering to 9 of our Inches. One calls this Measure a *Græciary*.

Spithama *for how much taken.*

An *Orthodoron*, *Poll. lib.* 2. calls a *Palm*, some others a *Span*, shorter by a Fingers breadth, than the *Span*, or greater *Palme*.

Orthodoron, *the length.*

A *Lichas* is generally reckoned for the length between the Thumb and the Extent of the Fore-finger, shorter than *Orthodoron* by a Fingers breadth Some make it the same with *Dichas*, but *Cooper* in his *Dictionary* makes *Dichas* but 8 Fingers breadths, when most agree *Lichas* is a *Span* with the Thumb and Fore-Finger as before.

Lichas, *how taken.*

A *Palest*, in *Greek* Παλαίςη, also Δῶρον, is the less *Palme*, agreeing to the *Hebrew Tophach*, 4 Fingers breadths, answering to 3 of our Inches.

Palest, *the Length.*

A *Dactyl*, Digit or Fingers breadth: See *Etsbang* in the *Hebrew* Measures.

Dactyl, *what.*

The *Greeks* had few if any Land Measures of length and breadth notable, save what they borrowed of the *Hebrews* and *Latines*, *Plethron* before spoken of, is often rendred *Jugerum* the Old *Latine* word for an *Acre*, which gives occasion to some to think, that the Measures aforesaid, or many of them were considered both in breadth and length, as necessity served to make use of them. Wherefore passing over what might be further said of those, the next that come in order to be seen are Measures of length, breadth, and depth.

Plethron, *oft rendred in Latine* Jugerum.

Whether by confounding the *Attick* and *Roman* Sextaries, or the Pounds Mensural or Ponderal, or the *Attick* and *Georgick* Measures, or by what other occasions, I know not; but sure I am, it is hard to reconcile Authors one to another, and some to themselves about the capacious Measures of the *Greeks*, and being not willing to spare so much time, or tumifie these Papers, I have given much credit to the Accompt set down by *Alsted* out of *Daniel Angelocrator. lib. de Ponderibus.* and from him and others wherein most general agreement is to be found, have collected what follows.

Authors hardly reconciled about the Græcian *Measures.*

Greek Measures of capacity may be considered, as *Indigenital* or of most use and chief note among them. Or, 2. *Exotick*, or used but in some particular places. Or Thirdly, *Hippiatrical*, or used about the Cure of Beasts.

Capacious Measures of the Greeks *of* 3. *sorts.*

Indigenital, or Proper *Græcian* Measures are again considered as useful for things dry only, or liquid, or both, and those either *Attick* or *Georgick*.

Indigenital, Attick, *or* Georgick.

Græcian *Indigenital Capacious Measures.*

Dry	Common	Liquid.
Kypsele.	*Sextary.*	*Metretes.*
Medimnus.	*Kotyle.*	*Amphora.*
Modios.	*Oxybaph.*	*Chous.*
Choenix.	*Kyath.*	*Tetarton.*
	Concha.	
	Mystrum.	
	Chemes.	
	Cochlear.	

A Table of the Attick *Measures of the* Greeks *compared with the* English.

The Table of Attick Measures *compared with the* English.

Kypsele.	1	Medimnus Metretes												
Medimnus Metretes	6	1	Modion.											
Modios.	36	6	1	Chous.										
Chous.	72	12	2	1	Choenix.									
Choenices.	288	48	8	4	1	Sextary.								
Sextaries.	432	72	12	6	$1\frac{1}{2}$	1	Kotyle.							
Kotyles.	864	144	24	12	3	2	1	Tetarton.						
Tetartons.	1728	288	48	24	6	4	2	1	Oxybaph.					
Oxybaphs.	3456	576	96	48	12	8	4	2	1	Kyath.				
Kyaths.	5184	864	144	72	18	12	6	3	$1\frac{1}{2}$	1	Concha.			
Conchas.	10368	1728	288	144	36	24	12	6	3	2	1	Mystrum.		
Mystras.	20736	3456	576	288	72	48	24	12	6	4	2	1	Cheme.	
Chemes.	25920	4320	720	360	90	60	30	15	$7\frac{1}{2}$	5	$2\frac{1}{2}$	$1\frac{1}{4}$	1	
Cochlears.	51840	8640	1440	720	180	120	60	30	15	10	5	$2\frac{1}{2}$	2	
Pints *or* Pounds *Troy.*	648	108	18	9	$2\frac{1}{4}$	$1\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{40}$	

A Table of the Georgick *Measures of the* Greeks *compared with the* Attick.

The Table of Georgick Measures *compared with the* Attick.

Medimnus. Metretes.	1	Amphora.											
Amphoras.	2	1	Chous.										
Chous.	8	4	1	Choenix.									
Choenices.	48	24	6	1	Sextary.								
Sextaries.	72	36	9	$1\frac{1}{2}$	1	Kotyle.							
Kotyles.	96	48	12	2	$1\frac{1}{3}$	1	Tetarton.						
Tetartons.	192	96	24	4	$2\frac{2}{3}$	2	1	Oxybaph.					
Oxybaphs.	384	192	48	8	$5\frac{1}{3}$	4	2	1	Kyath.				
Kyaths.	576	288	72	12	8	6	3	$1\frac{1}{2}$	1	Concha.			
Conchas.	1152	576	144	24	16	12	6	3	2	1	Mystrum.		
Mystras.	2304	1152	288	48	32	24	12	6	4	2	1	Cheme.	
Chemes.	2880	1440	360	60	40	30	15	$7\frac{1}{2}$	5	$2\frac{1}{2}$	$1\frac{1}{4}$	1	
Cochlears.	5760	2880	720	120	80	60	30	15	10	5	$2\frac{1}{2}$	2	
Atticke Cochlears.	12960	6480	1620	270	180	135	$67\frac{1}{2}$	$33\frac{3}{4}$	$22\frac{1}{2}$	$11\frac{1}{4}$	$5\frac{5}{8}$	$4\frac{1}{2}$	

Kypsele *the content.*

Kypsele, or after the *Latine*, *Cypsele*, by *Scap.* out of the *Annotat. Schol. Aristoph.* is reckoned for a Corn-Measure, and by *Hunt* to contain 6 *Atticke Medimnos.*

Medimnus *how reckoned.*

Medimnos, or *Medimnus* both *Attick* and *Georgick*, *Suiadas*, *Scapula*, *Legat*, and several others agree to contain 48 *Choenices* or 72 *Sextaries*, and *Pollybius lib.* 4. affirms the

the *Hemimedimnus* to be 24, *Choenices* accordingly; but because the *Georgick Coenix* was bigger than the *Atticke*, the *Medimnus* was alike proportional.

Metretes, or *Metreta*, a Liquid Measure, rendred in Latine sometime *Cadus*, sometime *Amphora*, (but corruptly *Amphora* being another Measure) *Legat* and some others make equal with the *Atticke Medimnus*, but that all other *Georgick* Measures should be greater than the *Atticke*, and only the *Metretes* equal, seems unlikely, used *John* 2. 6. in *English* there rendred a Firkin, but how justly, *quære*.

Metretes *how rendred.*

Amphora, or *Amphoreus*, properly a *Georgick* Liquid Measure, and was half the *Georgick Medimnus* or *Metretes*, by *Alsted*, though *Schrevelius* mentions it as an *Atticke* Measure, and saies it contained 3 Urnes, 20 whereof made a *Roman* Cule.

Amphora *the Content.*

Modios, or *Modion*, both Name and Measure seem borrowed from the Latine *Modius*, used in *Mat.* 5. 15. *Mark* 4. 21. *Luke* 11. 33. ordinarily Englished a Bushel, but in quantity far less than the *English* Bushel. Neither may the Latine *Modius* and Greek *Modios* agree exactly, though each should contain a like number of *Sextaries*, if the *Sextaries* be different. *Alsted* reckons it but 8 *Atticke Coenices*, that is 12 *Sextaries*, as in the Table above. Great Annotations 16 *Sectaries*, and by some, a Pint less than our Peck.

Modios *whence.*
Different from the English *Bushel.*

Chous, wrote often *Chus*, sometime *Choas*, and confusedly *Congius* for a *Roman* Measure of that Name, was both *Atticke* and *Georgicke*, that contained 6 *Atticke Sextaries*, and this 9 *Georgicke Sextaries*, as most agree.

Chous *is corruptly taken for* Congius.

Choenix, mentioned *Rev.* 6. 6. taken for a Measure serving a Servant with Food enough for a day, whence that saying of *Pythagoras*, *Super Choenice non Sedendum*, intending the provident care that should be taken for the future. The *Attick Choenix* contained by several $1\frac{1}{2}$ *Atticke Sextaries*, the *Georgicke Choenix* $2\frac{1}{4}$ *Atticke Sextaries*, whereby the *Georgicke Sextary*, and by consequence, all the lesser *Georgicke* Measures are proportionally greater than the *Atticke*, and accordingly the former Tables are computed. But there are others, and of good note too, that reckon the *Choenix Georgicke* to contain but 2 *Sextaries Atticke*. Some mention a Triple *Choenix*, as *Bilibral*, *Quadrilibral*, and *Quinquelibral*.

Choenix *how much.*
By some is made 3 fold.

Sextary, Sometime *Xesta*, from the *Greek* ξεστης, the sixth part of an *Atticke Chous*, made 2 *Kotyles*, but the *Georgicke Sextarie* though bigger made but $1\frac{1}{3}$ *Georgicke Kotyle*, a Measure translated a Pot, *Mark* 7. 4. and though usurped sometimes for the *Roman* Measure of that Name, yet upon more exquisite search, questionable; since some affirm 11 *Atticke Sextaries* made 12 *Roman*. *Alsted* and several others make the *Attick Sextarie* contain half the *Augustane* Measure and 2 Ounces over, that is 18 ℥ Mensural. Some reckon it 20 ℥, others 24 ℥, that is $1\frac{1}{2}$ Pound, at 16 ℥ to the Pound, as 18 ℥ at 12 ℥ to the Pound. Others will have it 13 ℥ 7 *pwts.* 18 *gr. Troy*. Some $1\frac{1}{2}$ Pint our Measure. Others but half a Pint, and equal to the *Hebrew Log*.

Sextary *of what Content how Translated.*

Kotyle Atticke is half the *Sextary*. *Scap.* out of *Dioscorides & Heraclitus* make it equal with the *Roman Hemina*, and if so, then must the *Roman* and *Atticke Sextaries* be equal. And with *Thuc. apud. Athen. lib.* 11. is made to contain 9 Mensural Ounces. Of the *George Kotyle* see above, with the *Latines* wrote *Cotyle*.

Kotyle *the Content.*

Tetarton, was properly a Liquid Measure in Latine *Quartarius*, being the Quarter of the *Atticke Sextary*, but the *Georgicke Sextary* is $2\frac{2}{3}$ *Georgicke Tetartons*.

Tetarton *what and how much.*

Oxybaph, was a Vessel to pour Vinegar in to dip Meat into at the Table, as the Latine *Acetabulum*, if *Georgicke*, was the Eight part of the *Choenix*, but if *Atticke*, the Twelfth.

Oxybaph, *what the Content.*

Kyath, in Latine *Cyathus* $1\frac{1}{2}$ whereof whether *Atticke*, or *Georgicke* made 1 *Oxybaph*, nevertheless in quantity proportioned to the respective *Sextaries*. A *Kyath* was used at *Athens* for a little Drinking Cup.

Kyath *how much.*

As for the *Concha*, *Mystrum* and *Cheme*, (measures more Minute than the *Kyath*) the Tables follow *Alsted*, *Scapulus*, and others, yet there are not wanting that speak of the uncertainty of them as Measures, make 2 sorts of them a greater and a less, and divide the *Kyath* otherwise than above.

Differences about the Concha, Mystrum *and* Cheme.

As *Malines*, in his *Lex Mercatoria*, thus.		*Legat*, at the end of *Thomas* his *Dictionary*, thus,	
1 *Kyath*	$\frac{2}{3}$ *Great Concha.*	1 *Oxybagh.*	= *Greater Concha.*
	2 *Small Conchas.*	1 *Kyath.*	= *Lesser Concha.*
	3 *Great Mystras.*	1 *Kotyle* =	16 *Greater Mystras.*
	4 *Small Mystras.*		20 *Lesser Mystras.*
	5 *Chemes.*	1 *Kotyle.* =	20 *Greater Chemes.*
	10 *Dragma*, *Cochlears.*		30 *Lesser Chemes.*

The

The loweſt Rank in the Table of *Georgick* Meaſures accompts the Number of *Atticke Cochlears* or Spoonfuls in every of the ſaid *Georgick* Meaſures at the rate of $2\frac{1}{4}$ of the one for $1\frac{1}{2}$ of the other. And the loweſt rank in the Table of *Atticke* Meaſures, values their Content with the *Engliſh* Pint or Pound of 12 ℥ proportionally to $1\frac{1}{2}$ ℔ for the *Atticke Sextary*, wherein at preſent I am beſt ſatisfied.

A Table of Exotick *Meaſures of the* Greeks *compared with their* Atticke.

Græcian Exoticke Meaſures *compared with the* Atticke.

		Kypſele.	Med.	Mod.	Chous.	Choen.	Sext.	Kot.	Tetar.	Oxyb.
1	*Achana Perſica*	7	3							
2	*Metreta Syria*	0	1	4						
3	*Artaba Perſica*	0	1	0	0	3				
4	*Kypros*	0	1	0	0	0				
5	*Ægyptia Artaba*	0	0	5	0	0				
6	*Medimnus Kyprius* { *Salam*	0	0	5	0	0				
	Medimnus Kyprius { *Papho*	0	0	4	1	0				
7	{ *Collathum Syrium* / *Modios Pondicus* }	0	0	2	0	0	1			
8	*Ponticus Cyprus*	0	0	2	0	0	0			
9	*Sabitha Syria*	0	0	1	1	2	1			
10	*Mares Ponticus*	0	0	1	1	1	½			
11	*Kophinus*	0	0	1	1	0	0			
12	*Modios Kyprios*	0	0	1	0	3	½			
13	{ *Kampſaces* / *Tetarpe Laconicé* }	0	0	1	0	0	0			
14	*Dadix*	0	0	0	1	2	0			
15	{ *Aphin* / *Topium* }	0	0	0	0	4	0			
16	*Choenix Syria*	0	0	0	0	2	1			
17	{ *Capitha* / *Mares* }	0	0	0	0	2	0			
18	*Inion*	0	0	0	0	0	1			
19	*Elenius*	0	0	0	0	0	0	0	1	
20	*Gabenon*	0	0	0	0	0	0	0	0	1
21	*Alabaſtron*									

Achana-Perſica *what.* Metreta *of* Syria *the content.* Artaba-Perſica *the content.*

1. A *Perſian* Corn Meaſure, as *Heſychius* teſtifieth.

2. Expreſſed by *Legat* to contain 120 *Sextaries*, which is all one with 1 *Medimnus* 4 *Modios.*

3. Moſt from the Authority of *Herodotus*, *lib.* 1. *pag.* 49. agree the *Perſian Artaba* was 3 *Choenices* greater than the *Atticke Medimnus*, And whence *Hunt* in his Table of *Græcian* Liquid Meaſures ſhould make it leſs then the *Metreta* by 3 *Chous* is to me unknown. *Hierom* on *Iſaiah* 5. *cap.* writes the *Ægyptian Artab* was 20 *Modios*.

Artaba *of* Ægypt *the content by* Hierome. Kypros *the content.* Artaba *of* Ægypt *the content by* Fannius. Medimnus *the ſorts and content.*

4. Equal to the *Atticke Medimnus* ſaith *Legat*, after the Latine wrote *Cyprus*.

5. The ſame Author tells us with *Fannius* this Meaſure is but $3\frac{1}{3}$ *Modios*, with *Epiphanius* equal to the *Atticke Medimnus*, as alſo the *Median Artab*.

6. The *Medimnus* at *Papho* is leſs by half a *Modion* than that at *Salaminca*, though both *Kyprian* Meaſures.

Collathum *and* Pontick Modios *how much.* Pontick, Cyprus, Sabitha, Pontick Mares *the content.* Kophinus *the content.*

7. Both the *Collathum Syrium* and the *Pontick Modios* are counted to contain alike 25 *Atticke Sextaries* in our Meaſure, as ſay the Great Annotations a Peck and a Pottle, others 12 Ounces a quarter and a half more.

8. The *Pontick Cyprus* after *Epiphanius* is 2 *Modios* as above.

9. *Sabbitha Syria*, is accompted to contain 22 *Attick Sextaries* all one as above

10. *Mares Ponticus*, *Epiphanius* delivers to contain 20 *Alexandrian Sextaries*, which if different from the *Atticke*, the Content above muſt be corrected accordingly.

11. *Kophinus*, a Boetick Meaſure both of Liquid and Dry, according to *Legat*, contains 3 *Congios*, and in our Meaſure 1 Gallon, half a Pint, 3 Ounces ¾, but if the *Congius* be 5 Pints $1\frac{1}{4}$ ℥ *Engliſh* as he ſaith *Kophinus* muſt contain more than a Peck by ½ a Pint 3 ¾ Ounces, and near a Peck, if by *Congius* the *Attick Chous* be underſtood, Wherefore of the certainty, *quære*.

Modios Kyprios *the content.*

12. *Modios Kyprios*, or after the Latine *Modius Cyprius*, is reckoned to contain 17 *Atticke Sextaries* ; of our Meaſure by the Great Annotations, a Peck and a Pint ; by *Legat*, but 14 Pints, 5 Ounces, a quarter and half.

13. *Kampſaces*

13. *Kampsaces*, and the *Tetarpe Laconice* are equal, the one being 12 *Sextaries*, the other 24 *Kotyles*, seeing 2 *Kotyles* make but 1 *Sextary*, but some make the *Kampsaces* but 4 *Sextaries*. The *Tetarpe Laconice* seems to be the quarter of the *Laconian Metretes*.

Kampsaces *and* Tetarpe, *their Contents.*

14. *Dadix*, by *Pollybius* and others containeth 6 *Choenices*. *Malines* calls it a *Boetick* Measure.

Dadix *the Content.*

15. *Aphin*, an *Ægyptian* Measure, containing 4 *Choenices*, and of the same capacity doth *Hesychius* account the *Topium*, but with whom in use he saith not.

Aphin *and* Topium *their Contents.*

16. The *Syrian Choenix*, is supposed to be the same which with *Fannius* is set down at 4 *Sextaries*.

Choenix Syria *the Content.*

17. The *Capitha*, a *Persian* Measure contained 2 *Attick Choenices*, and the *Mares* was equal thereto, containing 6 *Kotyles*, and a *Boetick* Measure, as some say.

Capitha *and* Mares *their Contents.*

18. *Inion*, with the *Ægyptians*, as *Legat* saith, was the name of a *Sextarie*, which with the *Alexandrians* contained 2 ℔ of Oyl, as *Epiphanius* hath it. But if by Pound he intend, the *Roman Libra*, or the *Greek Mna*, it must not be taken for our Pound; since some affirm neither of them weighed 11 Ounces *Troy*.

Inion *how taken.*

19. *Elenius*, being the quarter of the *Sextarie* seems only another name for the *Tetarton*.

Elenius *the Content.*

20. *Gabenon* was all one with the *Oxybaph*, or *Aretabule*.

21. *Alabastron*, contained one of their Pounds of Oyle.

Gabenon *and* Alabastron *the Contents.*

Gracian Hippiatrical Measures seem for the most part to keep the Names of the *Atticke* Measures, though the Divisions and quantities differ, in *Alsted* thus found.

Hippiatrical *Measures of the* Greeks.

12 Ounces hath 2 *Oxybaphs*, 1 *Oxybaph*, 3. *Kyaths*. 1 *Kyath* 4 *Mystras*. 1 *Mystrum* 2 *Cochlears*.

In one *Ounce* 8 *Drams*, 1 *Dram* 3 *Scruples*, &c.

Legat, out of *Absyrtus*, *pag*. 34. and *Hierocles pag*. 35. mentions the *Choe* to contain 10 *Ounces* of Liquid Measure, which if Mensural, then was the *Choe* lesser than the *Hippiatrical Kotyle*, but if Ponderal equal; for he saith the *Roman* Mensural Pound (to which the *Hippiatrical Kotyle* was equal) contained so much Oyle, as 10 Ponderal Ounces weighed.

Choe *mentioned by* Legat.

In like manner as the Measures, so the Weights among the *Greeks* are differently to be taken; as they are *Attick*, *Physical*, *Hippiatrick*, *Indigenital*, or *Exotick*. Of which see further the following Tables and Notes on the same.

Weights of the Greeks *as their Measures of divers sorts.*

Græcian Atticke Weights.

A Table of the Atticke Weights.

	Pounds. Minas.	Ounces. Uncias.	Drams. Drachmas.	Scruples. Grammata.	Obolos.	Lupines. Thermos.	Keratias. Siliquas.	Aereolos. Chalkos.	Graines. Sitars.	Minutes. Leptas.
Talent Greater.	80	1000	8000	24000	48000	72000	144000	288000	576000	2016000
Talent Lesser.	60	750	6000	18000	36000	54000	108000	216000	432000	1512000
Mina. New		$12\frac{1}{2}$	100	300	600	900	1800	3600	7200	25200
Mina. Old. *a*		$9\frac{3}{8}$	75	225	450	675	1350	2700	5400	18900
Uncia. Ounce. *b*			8	24	48	72	144	288	576	2016
Drachm. Dram. *c*				3	6	9	18	36	72	252
Gramma. Scruple. *d*					2	3	6	12	24	84
Obolus. *e*						$1\frac{1}{2}$	3	6	12	42
Therme. Lupine. *f*							2	4	8	28
Siliqua. Keration. *g*								2	4	14
Chalkus. Aereolus. *h*									2	7
Sitar. Graine. *i*										$3\frac{1}{2}$ *k*

A Table of Weights used by the Græcian *Physitians.*

Græcian Physical Weights.

	Ounces. Uncias.	Drams. Drachmas.	Scruples. Grammata.	Obolos.	Lupines	Carobseeds. Keratias. Siliquas.	Aereola. Chalkos.	Graines, Sitars.	Minutes. Leptas.
Mina.	16	128	384	768	1152	2304	4608	9216	32256
Litra.	12	96	288	576	864	1728	3456	6912	24192
aa Uncia. Ounce.		8	24	48	72	144	288	576	2016
Drachm. Dram.			3	6	9	18	36	72	252
Gramma. Scruple.				2	3	6	12	24	84
Obolus.					$1\frac{1}{2}$	3	6	12	42
Lupine.						2	4	8	28
Siliqua. Keration.							2	4	14
Carobseed. Chalkus. gg Æreolum.								2	7
Sitar. Graine.									$3\frac{1}{2}$
									l

A Table of Weights used by the Græcian *Farriers.*

Græcian Hippiatrical Weights.

	Ounces.	Denarions.	Drams.	Scruples.	Obolos.
Mina.	15	$84\frac{3}{8}$	$112\frac{1}{2}$	$337\frac{1}{2}$	675
Litra.	12	$67\frac{1}{2}$	90	270	540
aaa Ounce.		$5\frac{5}{8}$	$7\frac{1}{2}$	$22\frac{1}{2}$	45
Denarion.			$1\frac{1}{3}$	4	8
bb Dram.				3	6
cc Scruple.					2
					ll

Notes on the Table.

Mna *the sorts how much.*

a The *Mna* of 100 Drachms is called *Solons*, *Mna*, because thought to be constituted by him, sometime turned into *Latine* by *Mina*, often by *Libra*, though *Libra* be 4 Drachms lighter; the *Roman Libra* being but 96 *Attick* Drachms. The old *Mna* of 75 Drachms now obsolete for Memory sake hath found room in the Table.

Mna *of the Physitian.*

aa. The Physitians, as by *Dioscorides* and *Galen* appears. used a *Mna*, or Pound of 16 Ounces, and a *Litra* or other Pound of 12 Ounces, conceived all one with the *Roman Libra* consisting of 96 Drachms as this did, and by Interpreters commonly rendred *Libra*, and seldom or never *Mna*, and *Mna*, and *Litra*, as also *Libra*, commonly Englished a Pound.

Mna *of the* Farriers.

aaa. The *Hippiatricks* had a *Mna* of 15 Ounces, and a *Litra* of 12.

Oungia, Uncia *not the* English *Ounce.*

b. Oungia, in Latine *Uncia*, must not be taken for our Ounce, but for one of their Ounces, arising by the division of their Pound into Drams differently according to the quantity of Drams in one Pound.

Denarion *how much.*

bb. Among the *Hippiatrical* Weights there was a *Denarion* of 4 Scruples, $5\frac{5}{8}$ whereof made one of their Ounces.

Drams *the Names and sorts.*

c. Drachme, *Drachma*, and *Dragma*, in *Greek* and *Latine*, in *English* a *Dram*, is the eight part of their Ounce, whether the Pound had 12 or 16 Ounces therein. By *Alsted* made to equal the *German* Weight *Quintlein.* Some call a Dram *Resolus*, some *Holke*, from the *Greek* ὁλκή.

cc. The Ounce *Hippiatrick*, that divided as well the *Mna* of 15 Ounces, as the *Litra* of 12 Ounces; had but $7\frac{1}{2}$ Drams in it.

Scruples *how called.*

d Drams of all sorts were parted into 3 Scruples. A Scruple in *Greek* sometime *Gramma*, sometime *Grammata*, in *Latine Scripulum*, *Scriptulum*, and *Scrupulum.*

e. Obolus

e. *Obolus*, Sometime a Weight, sometime a piece of Money commonly rendred an Half-penny, because alwaies was the half of a Scruple.

Obolus, why our word Half-penny from hence.

f *Lupine*, in *Greek Thermos*, was a Weight equal in poise to the *Lupine*, which is a Seed growing in a Cod like to a Pease, and both Plant and Seed bear that name. And seeing there are many sorts as *Parkinson Theater of Plants*, *pag.* 1073. which sort of *Lupine* is meant is uncertain, probably, the Middle White, which are most in use, bigger than the Yellow, and not so big as the great Blew, and from the nearness in Weight thereto, if not exactness might be so called.

Lupine, *the Weight of a Seed.*

g.gg. Siliqua, in *Greek Keration*, a Weight alike heavy to the *Carobseed* or Sweet Bean, common in many Countreys subject to the *Gracian* Empire. Sometime called *Carat* or *Caract*, from whence the word still in use with us.

Siliqua, *how much.*

Caract, *whence.*

h. Chalkos, in *Latine Aereolus* and *Aereolum*, *Aereolus* was also a piece of Brass Money currant in Antient times among those Countreys of the *Gracian* Dominion.

Chalkos, *both Weight and Coine.*

Sitar, a Grain of Corn from Σῖτος, *Frumentum*, likely to have been the Original of their Weight. 2 whereof made 1 *Chalkos*.

Sitar, *what.*

k. Lepton, from *Leptos*, in *Latine Minutum*, and *Minutia*, supposed to be some small Scale of the Rinde or Bark of some Tree, 3 ½ ballanced the *Sitar*

Lepton, *what.*

l. Besides these in the Table of Physical Weights, some Books mention the *Assarion*, allowed for 2 Drams which is ¼ of an Ounce. Also the *Exagion*, wrote sometime *Stagion*, sometime *Agion*, for brevity, which was the *Roman Sextula*, the Sixth part of their Ounce, whereof 12 made the *Litra*. Likewise *Orobus* which was a graine of a Wild Vetch. And *Phaike* a *Lentill*, but whether Weights or no is not worth the Inquiry.

Other Weights mentioned by some Authors.

ll. As the other Weights are divided into lesser Divisions than the *Obolus*, so no doubt but the *Hippiatrick* also were, and may accordingly be done, when occasion serves. The *Obolus* of all sorts admitting the like smaller Denominations.

Gracian Exotick Weights.

A Table of the Exotick *Weights of the* Greeks.

Talents	Mentioned by *Vitrivius*, supposed to be the *Thracian*, or *Bizantium* Talent	120	*Libras.*
	Several mentioned by *Hesychius*	100	
		125	
		165	
		405	
		1150	
	Old *Sicilian* (*m*)	24	*Minas.*
	New *Sicilian* (*m*)	12	
Talent of	*Alexandria*	12000	*Attick Drams.*
	Aegina, *Corinth*	10000	
	Egypt	8000	
	Babylon	7000	
	Rhodium	4500	
	Euboicum	4000	
	Syria	1500	
Mna	*Alexandria*	20	*Uncias.*
	Ptolemaica	18	
Drachma	*Egyptia*	1	*Obolus.*

(*m*) The *Sicilian* Old and New Talent is thus reckoned by *Legat* before-mentioned, but *Rider* and another Author make them pieces of Money, and of a far smaller Value, set afterward among the *Gracian* Coines.

Gracian

A Table of Græcian *Coines and their Value.*

					l.	s.	d.
Gracian	Money	Brass	Mite		00	00	$00\frac{3}{32}$
			Aereolus		00	00	$00\frac{5}{24}$
			Quadrans	Oboli	00	00	$00\frac{5}{16}$
				Assis	00	00	$00\frac{3}{16}$
			Assarius		00	00	$00\frac{3}{8}$
		Silver	*Semiobolus*		00	00	$00\frac{5}{8}$
			Danaces		00	00	$00\frac{5}{6}$
			Obolos	*Attick*	00	00	$01\frac{1}{4}$
				Aeginean	00	00	$02\frac{1}{12}$
			Diobolus		00	00	$02\frac{1}{2}$
			Triobolus	*Attick*	00	00	$03\frac{3}{4}$
				Aeginean	00	00	$06\frac{1}{4}$
			Cistophorus		00	00	$04\frac{9}{16}$
			Tetrobolus		00	00	05
			Drachma	*Attick*	00	00	$07\frac{1}{2}$
				Aeginean	00	01	$00\frac{1}{2}$
			Siglus, Sardinian and *Persian*		00	00	10
			Didrachma		00	01	03
			Tridrachma		00	01	$10\frac{1}{2}$
			Stater	*Attick*	00	02	06
				Corinthian	00	01	$08\frac{3}{4}$
				Macedonian	00	02	$09\frac{5}{12}$
		Gold	*Semistater*	*Attick*	00	07	06
				of *Darius*	00	07	06
			Stater	*Attick*	00	15	00
				of *Darius*	00	15	00
			Stater Macedonian		00	18	04
			Stater Cizycen		01	01	00
			Tetra-Stater		03	00	00
	Sums of Money.	*Mna Attick*			03	02	06
		Talent	Lesser	*Rhegium*	00	00	$03\frac{3}{4}$
				Sicilian New	00	01	$10\frac{1}{2}$
				Sicilian Old	00	03	09
				Neapolitan	00	03	09
			Greater	*Syrian*	046	17	06
				Euboicum	125	00	00
				Rhodium	140	12	06
				Attick Less	187	10	00
				Attick Greater	250	00	00
				Babylonian	218	15	00
				Egyptian	250	00	00
				Aeginæan / *Corinthian*	312	10	00
				Alexandrian	375	00	00

Mite *how called, the Value.*

A *Mite* in Latine *Minutum* and *Minutia*, in Greek *Lepton* used *Mark* 12. 42. and from thence proved to be half the Quadrant, (but not half of our Farthing) by the *Syriack* Interpreter, and *Alsted* reputed $\frac{1}{8}$ of the *Assarion*. It weighed $\frac{1}{2}$ Barley Corn as some say, and was currant for as much as $\frac{3}{32}$ of our Peny, but some will have it twice as much.

Aereolus *the Value.*

Aereolus, and *Aereolum*, several agree to be the weight of 2 Graines, and call it *Chalkos*, and according to the *Attick* Weight, weighed 7 of their *Leptas*, or *Mites*, and was valuable with $\frac{5}{24}$ of our Peny, being the 36th part of their Dram, worth $7\frac{1}{2}$ *d. Sterling.*

Quadrans *the Sorts and Value. not a Farthing.*

The *Quadrans*, commonly translated a Farthing, *Mat.* 5. 26. *Mark* 12. 42. in *Greek* κοδράντης, by some is taken either for the Fourth part of the *Obolus*, or the *Ass*; and so accordingly valued as before. The latter was the double of the *Mite*. The *Ass* was the 10th part of the *Roman* Peny, which being $7\frac{1}{2}$ *d.* made the *Ass* $\frac{3}{4}$ *d.* and consequently the 4th part thereof $\frac{3}{16}$.

Ass *of the* Romans *how much.*

Assarius *the difference about it how called and translated.*

Assarius, or *Assarium*, *Holyokes* Dictionary makes the 4th part of the *Assis* equal to the *Quadrans*; but *Leigh* makes it equal to the *Assis* it self; for he saith, it is the 10th part of the *Roman* Peny, the 96th part of the *Attick Stater* that is with us but $1\frac{1}{4}$ Farthing. Others make it more, and say it was 1 Farthing and an Half, which I rather

ther incline to, and so have set it down, of old called *Assar*, and by the *Rabins*, *Issor*, *Mat.* 10. 29. translated a Farthing. *Alsted* likewise makes *Assarium* worth 1 *Cruciat* or *Creutzer*, that is 3 Farthings *Sterling*, all one with the *Roman As*, counting 40 *Creutzers* to a *Stater*, which is 2 *s.* 6 *d.* and so 10 to the *Drachmal Denary*.

Semiobolus, is $\frac{1}{12}$ of an *Attick* Dram (of which below) that is 2 $\frac{1}{2}$ Farthings our Money, or $\frac{5}{8}$ of a Peny.

Semiobolus *how much.*

Danaces, in *Greek* Δάνακη Δάνικη, and Δάναῆ, *Charon*'s Ferriage-Piece, which the *Barbarians* used to put into the Mouths of Dead Persons to pay *Charon* for their Carriage over the River *Styx* into the *Elisian Fields*. If it be an *Obolus* as *Lucian* calls it, it is worth 1 $\frac{1}{4}$ *d.* our Money. But if of the same weight which the *Greek* and *Arabian* Writers call the *Arabian Danich*, weighing $\frac{2}{3}$ of an *Obolus*, is $\frac{5}{8}$ of a Peny, that is 3 *q*, and $\frac{1}{3}$ of a Farthing our Money.

Danaces, *used to be put into the Mouthes of Dead Persons.*

Obolus, is twofold, the *Attick* which is $\frac{1}{6}$ of their Dram in Value with us 1 $\frac{1}{4}$ *d.* the *Aeginean* almost double the other, *Viz.* 2 *d.* and $\frac{1}{12}$ of a Peny. *Holyoke* thinks *Obolus* came from *Obelos*, which sometimes signified a *Dart* used in War, as being stamped with the like Form: Or was so called from the Oblong Form thereof, or from the Image of some *Obelisk*, or *Spire* coined thereon.

Obolus, *the value, why so called.*

Diobolus, was $\frac{1}{3}$ of the *Attick Dram*, or double the *Attick Obolus*, and had on the one side *Jupiters* Face, and on the other an *Owle*.

Diobolus, *how stamped, the Value.*

Triobolus, was both *Attické* and *Aeginean*, that just $\frac{1}{2}$ *Dram*, the other 3 *Aeginean Oboli*, that is 6 $\frac{1}{4}$ *d.* in Value.

Triobolus, *the Value.*

Cistophorus, so called from the Form of a *Cofer* or *Chest* thereon, valued in *English* Money 4 $\frac{1}{2}$ *d.* and a quarter of a *Farthing*, by *Holyoke* and others who set not down the weight thereof.

Cistophorus *how stamped, the Value.*

Tetrobolus, had *Jupiters* Face on the one side, and 2 *Owles* on the other, contained 4 *Oboli*, or $\frac{2}{3}$ of the *Attick Dram*, worth with us 5 *d.*

Tetrobolus *the Value, what Print thereon.*

Drachma, or *Drachme*, used *Luke* 15. 8, 9. Sometime *Arguris*, in *English* a *Dram*, *Acts* 19. 19. a *Silverling*, a Piece of Money common with the *Athenians*, bearing the Image of *Minerva*'s Candle burning, in weight $\frac{1}{8}$ of an Ounce, and accordingly valued at 7 $\frac{1}{2}$ *d. Sterling* at the rate of 5 *s. Sterling* the Ounce. This was called the *Attick Drachme*, and was all one as very many conceive with the *Roman* Peny. The *Aeginean Dram* was heavier, and so worth more the weight 1 $\frac{2}{3}$ ʒ *Attick*, Value 12 $\frac{1}{2}$ *d. English*.

Dram, *the Names, Sorts, and Values. The Image thereon.*

Siglus, was of *Exotick* Extract, and weighed 1 $\frac{1}{3}$ ʒ or 4 *Attick Scruples*, may be valued at 10 *d.*

Siglus *the Value.*

Didrachmum, called also by the *Athenians Bous*, or *Boos*, because there was an *Ox* stamped thereon, whence the Proverb, *Bos in Lingua*, as the *English*, *The Angels blind their eyes*, applyed to them that are bribed to speak, or blinded in Judgment, equal to 2 *Drams*, or $\frac{1}{2}$ their *Silver Stater*, and was $\frac{1}{4}$ of our *Ounce Troy*, and worth 1 *s.* 3 *d.* all one with the [Hebrew] *Bekah*, paid by the *Jews* to the Sanctuary, and Temple, till *Cæsar* changed it into Tribute-Money for his own Cofers, *Mat.* 17. 24. and afterwards by Vertue of a Decree made by *Vespasian* paid towards the *Roman* Capitol.

Didrachmum *how stamped and called. Proverbs, whence.*

Didrachmum *Exacted by* Cæsar.

Tridrachmum, was 3 *Drams Attick*, and valued with us accordingly.

Tridrachmum *the Value.*

Stater of Silver, was either *Attick*, having on the one side *Minerva*'s Head, and an *Owle* on the other, worth with us 2 *s.* 6 *d.* weighing $\frac{1}{2}$ Ounce; Or *Corinthian*, which was not full 3 *Attick Drams* and worth but 1 *s.* 8 $\frac{3}{4}$ *d.* Or *Macedonian*, which was bigger than either, and worth 2 *s.* 9 $\frac{3}{4}$ *d.* The *Attick Stater* was double the *Didrachmum*, and so served for Tribute-Money both for *Christ* and *Peter*, *Matth.* 17. 27. and is sometimes called the *Tetradrachmum*, because it contained 4 *Drams*.

Silver Stater *how stamped, the Value and Sorts.*

Tetradrachmum.

The *Semistater of Gold*, both the Common *Attick*, and that of *Darius* Coine are reckoned equal either of them 1 ʒ. valuable with us, 7 *s.* 6 *d.*

Semistater *of* Gold *the Value.*

The *Staters* also in Weight equal 2 *Drams* of our *Troy Weight*, or $\frac{1}{4}$ ʒ worth 15 *s.* That of *Darius* is reported to have the Image of *Sagitarius* thereon.

Staters *the Sorts and Values.*

Of Darius.

The *Macedonian Stater* weighed of *Attick* Weights 2 ʒ. 2 *ob.* 2 *Siliq.* worth proportionally with us 18 *s.* 4 *d.*

Macedonia.

The *Stater of Cizycen* or *Cizycus*, a City in *Greece* 2 $\frac{4}{5}$ ʒ *Attick*, was valuable in *Sterling* Money at 1 *l.* 1 *s.* 0 *d.*

Cizycen.

The *Tetrastater*, seems to some no piece of Coine, but signifies only Four *Staters*, worth 3 *l. Sterling*.

Tetra Stater.

Mna Attick, containing 100 *Drams*, 96 whereof being equal to the *Pound Troy*, make the whole *Mna* in Silver at the rate as worth with us 3 *l.* 2 *s.* 6 *d.*

Attick Mna *how much in Accompt.*

Talents the [illegible] of them.

Talents of the Lesser sort, and improperly so called, seem to me rather Pieces, than Sums of Money. That of *Rhegium* a Town in *Italy*, currant in *Greece*, was in Value but 3 ¾ *d.* of the New and Old *Sicilian Talents*, the Old was double the New, and the biggest worth no more than that of *Naples*, to wit 3 *s.* 9 *d.* *English* Money.

The Sorts and Weight.

Talents of the Greater sort, and indeed deserving that name, were divers as before noted with their respective Value in our Silver Money, according to their weight of the *Attick Drams* to which they are compared, being some of them *Exotick* as here followeth.

Talents	Drams Attick	℔. Troy	℥. Troy
Syrian	1500	15	7½
Euboicum	4000	41	8
Rhodes	4500	46	10½
Attick Less	6000	62	6
Babylon	7000	72	11
Attick Great / *Egyptick*	8000	83	4
Aegina	10000	104	2
Alexandria	12000	125	0

Geodæticals *of the* Latines *and* Romans.

The Third and last sort of the Ancients whose *Geodaticks* are to be seen, are the *Latines*, and their Successors the *Romans*.

Measures of the Latines.

Alsted fits us with Tables for the Long and Superficial Measures, and another which he calls *Geometrical*; wherein the main differences between the other Long Measures, and these are about the Mile and Furlong. A Fourth Table also he hath for division of the Inch, all which here follow.

A Table of the Long Measures of the Latines.

Latine Long Measures.

	Furlongs	Decempedes.	Passes.	Steps.	Cubits.	Palmipedes.	Feet.	Palmes.	Inches.	Digits.
Mile	8	500	1000	2000	3333⅓	4000	5000	20000	60000	80000
a Furlong.		62½	125	250	416⅔	500	625	2500	7500	10000
Decempede.			2	4	6⅔	8	10	40	120	160
b Pass.				2	3⅓	4	5	20	60	80
Step.					1⅔	2	2½	10	30	40
c Cubit.						1⅕	1½	6	18	24
Palmipede.							1¼	5	15	20
d Foot.								4	12	16
e Palme.									3	4
f Inch. *g*										1⅓ *h*

A Table of the Measures for Land used with the Latines.

Latine Superficial Land-Measures.

	Centuries.	Jugera.	Modes.	Verses.	Climes.	Acts.	Feet.
Saltus.	4	400	800	1152	3200	24000	11520000
i Centurie.		100	200	288	800	6000	2880000
Jugerum.			2	$2\frac{22}{25}$	8	60	28800
k Mode.				$1\frac{11}{25}$	4	30	14400
l Verse.					$2\frac{7}{9}$	$20\frac{5}{6}$	10000
m Clime.						7½	3600
n Act.							480

Latine

Latine Geometrical-Measures.

A Table of the Long Measures of the Latines *used on special Occasions.*

	Miles.	Furlongs.	Cubits.	Feet.	Palmes.	Digits.	Graines.
Parasang, *or* Schoone.	3	30	12000	18000	72000	288000	1728000
	Mile.	10	4000	6000	24000	96000	576000
	aa	Furlong.	400	600	2400	9600	57600
			Cubit.	$1\frac{1}{2}$	6	24	144
				Foot.	4	16	96
					Palme.	4	24
						Digit.	6
							bb

The Division of an *Inch.*

A Table of the Division of the Inch *according to* Alsted.

	Drams.	Scruples.	Obolos.	Siliquas.	Points.	Minutes.	Moments.
Inch.	8	24	48	144	288	576	1152
gg	Dram.	3	6	18	36	72	144
		Scruple.	2	6	12	24	48
			Obolus.	3	6	12	24
				Siliqua.	2	4	8
					Point.	2	4
						Minute.	2

a. aa. Hereby it seemeth the *Latines* had 2 sorts of Miles, *viz.* The Common consisting of 8 Furlongs, every Furlong 625 Feet, that is 5000 Feet in the Mile; and a Mile called *Geometrical*, or used in accompt upon Special Occasions, consisting of 10 Furlongs, every Furlong 600 Feet which made the Mile 6000 Feet. In the first reckoning the Mile was shorter, and the Furlong longer than in the second *Milliare*, or *Milliarium* in the *Latine* for a Mile came from *Mille* 1000, as was said before. *Miles of 2 sorts.* *Whence the word.*

Of the *Parasang* or *Schoene*, *Furlong*, *Pace* and *Cubit*, see before in the *Hebrew* and *Greek* Measures. Parasang *and* Schoen. *Vide antea.*

b. A *Decempede*, some call a Perch, but because they agree it was but 10 Feet long, and so signified by the very name; it cannot be taken for our Perch, which is $6\frac{1}{2}$ Feet longer, as before. Some mention a *Decempede* of 12 Feet. Decempede *the length.*

c. A *Step*, in *Latine*, *Gressus*, and *Gradus*, here taken for half a Pass, or $2\frac{1}{2}$ Feet, and not to be Englished a *Degree*, which terme is most proper for the 360th part of a Circle. *Alsted* counts upon 3 sorts of Paces or Passes, each of a double difference, thus. Paces *and* Steps *the sorts and lengths.*

		Feet.		Palmes.	
Simple	of the first difference	2	or	8.	
Double		4		16.	
Simple	of the second difference	$2\frac{1}{2}$	or	10.	This the *Grade*, or *Step*.
Double		5		20.	This the Pace *Geometrical*.
Simple	of the third difference	3	or	12.	
Double		6		24.	

That double of the first difference be called *Ulna Communis*, or the Common Ell, to difference it from the Cubit of $1\frac{1}{2}$ Feet, which he sometime calls *Ulna*. The double of the third difference he calls *Ulna agrestis, seu Orgyia*, the Countrey Ell or Fathom. Ulna *the sorts.*

d. A

Palmipes, the Length.

d. A *Palmipes* may be seen before in the *Greek Pygon*, the *Latine* Name shews the Content thereof.

Foot of the Latines and Romans. how called. The parts thereof.

e. To what hath been said already on the *Hebrew Pagnam* and *Greek Pous*, may be added, that little difference with any certainty being observed the *Roman* or *Latine Pes* may be parallel'd with the *English* Foot. The *Romans* called their Foot sometimes a *Pound*, and 2 Foot *Dupondium*, and divided several of their Land Measures into 12 parts called *Unchiæ*, or *Inches*, of which below at *g*, and such Inch into 24 Scruples, using like Names as for Weights.

Palmes of 2 sorts.

f *Palmes* are of 2 sorts, though but one set in the Tables, a Greater answering to the *Hebrew Zereth*, and *Greek Spithame*, containing 3 Lesser Palmes or 12 Digits, The Lesser Palme which is placed in the Table contains 4 Digits answerable to the *Hebrew Tophach*, and *Greek Paleste*.

Inch how much the names thereof. Uncia diversly rendred.

g. gg. And Inch in *Latine*, *Pollex*, rendred sometimes a Thumb, because many times of the same breadth, equal to a Digit or Fingers breadth, and a third part of a Digit; The parts of which Inch follow in the 4th foregoing Table into Imaginary Moments *Uncia*, when relating to Measure is translated an Inch, when to weight an Ounce, sometime wrote *Ouncia*, but whether corruptly, or that it contains 3 Digits or 2 Thumbs, making thereby 1 Thumb or Inch, 1 ½ Digit, as *Malines* and *Thomas* say, is further to be quæried. *Vide plus* at *k* on the parts of the *Jugerum*.

Digit, the Length.

h. hh. A Digit or Fingers breadth, answering to the *Greek Dactyle*, and *Hebrew Etsbang* is there spoken of, and here in the *Geometrical* Table made to contain 6 Graines; but in the upper Table to be reckoned only the breadth of 4 Graines of Barley.

Saltus how taken.

i. *Saltus*, Sometime taken for a Grove or Forest, here for a piece of Land, 4 *Centuries*, or 400 *Jugera*, every *Centurie* being 100 *Jugers*.

Jugerum, the Content. Greater than the English Acre.

k. *Jugerum*, commonly translated an Acre, must alwaies be understood for the *Roman* and not *English* Acre, being far larger as containing 28800 Square Feet in the *Area* thereof arising from the Multiplication of 240 Feet in length, and 120 in breadth, when as the *English* Acre containeth but 2640 Feet, which is the Product of 160 Pecks multiplyed by 16 ½ the Feet in one Perch as before was declared. *Alsted* divides the *Roman Juger* into 12 parts which he calls Inches, and every Inch into 7 parts, as followeth.

A Table of the Roman Juger divided by Alsted.

	Inches.	1 Semiuncias.	2 Siliquas.	3 Sextulas.	4 Drams.	5 Semisextulas.	6 Scrupl.	7 Obolos.	Longer. Side.	Shorter. Side.	Area. Square Feet.
Juger.	12	24	48	72	96	144	288	576	240	120	28800
Inch.		2	4	6	8	12	24	48	60	40	2400
Semiuncia. ½			2	3	4	6	12	24	40	30	1200
Siliqua. ¼				1½	2	3	6	12	30	20	600
Sextula. 1/6					1½	2	4	8	20	20	400
Dram. 1/8						1½	3	6	20	15	300
Semisextula. 1/12							2	4	20	10	200
Scruple. 1/24								2	10	10	100
Obolus. 1/48									10	5	50

Modus how called the Content.

l. *Modus* is half a *Juger*, called often *Actus quadratus*, containing 14400 Feet Square, and was so called from the Square Form thereof, being every way 120 Feet.

Versus, the Content

m. *Versus*, used by *Pliny* for a Square Plot of Ground 100 Feet every way.

Clima, how taken.

n. *Clima*, for the *Hebrew Noph*.

Act how called the Content.

o. An Act, called *Actus Minimus*, the least or lesser Act, for distinction from *Actus quadratus*, which is 30 times bigger than this lesser Act, that a Square, and this an Oblong or Long Square, one side whereof was 4, and the other 120 Feet, or proportionally 10, that the *Area* might be 480 Feet.

Roman

Roman Capacious Meaſures.

Capacious Meaſures of the Romans.

Dry	Common	Liquid.
Modius.	*Sextary.*	*Cule.*
Modiolus.	*Hemin.*	*Amphora.*
	Acetable.	*Urne.*
	Cyath.	*Congius.*
	Ligula.	*Quartary.*

The Table of *Roman* Dry and Liquid Meaſures.

A Table of the Roman Concave Meaſures.

	Amphoras.	Urnes	Modios	Congies.	Modiolos	Sextaries.	Heminas	Quartaries.	Acetables	Cyaths.	Ligulas.
Cule. *a*	20	40	60	160	240	960	1920	3840	7680	11520	46080
Amphora. *b*		2	3	8	12	48	96	192	384	576	2304
Urne. *c*			1½	4	6	24	48	96	192	288	1152
Modius *d*				2⅔	4	16	32	64	128	192	768
Congius *e*					1½	6	12	24	48	72	288
Modiolus. *f*						4	8	16	32	48	192
Sextarie. *g*							2	4	8	12	48
Hemina *h*								2	4	6	24
Quartarie									2	3	12
Acetable.										1½	6
Cyath.											4

i. k.

a. Culeus, and ſometime *Coleus*, *Culeum* and *Culleum*, in *Latine* taken alſo for a *Sack*, or ſuch like, wherein *Parricides* were wont to be put, and ſo caſt into the River *Tyber*, by the Old Law of the *Romans*. Some mention *Doleum*, and ſay it contained a *Cule* and an Half. *(Cule how taken. Puniſhment of Parricides. Doleum the Content.)*

b. Amphora, ſome ſay was of a Cubick Form, and therefore called *Quadrantal*. *(Amphora how called.)*

c. Urna. Sennertus in his *Inſtitutions of Phyſick, lib.* 5. *par.* 3. *ſect.* 1. *cap.* 4. affirms to be ½ the *Italian*, ⅓ of the *Attick Amphora*, making thereby the *Greek* half as big again as the *Italian Amphora*. *(Urna the Content.)*

d. Modius, Engliſhed a Buſhel, was ſpoken to among the *Greek* Meaſures, but whereas there it was made to contain 12 *Attick Sextaries*, here, upon the Authority of *Holyoke*, *Legat*, and others, it is made of 16 *Roman Sextaries*. *(Modius Vide antea.)*

e. Congius, was of a like number of *Roman Sextaries* as the *Greek Chous*, of *Atticke Sextaries*, which may be the Reaſon why they are ſometime taken the one for the word were needleſs for other. *(Congius the Content.)*

f. Modiolus, mentioned in *Plautus*, but without mention of its capacity, yet *Alſted* makes it the Quarter part of their *Modius*, a diminitive of *Modius*, the very word beſpeaks it, and leſs than the *Semi-Modius*, or Half Buſhel very probably, or elſe another the ſame. *(Modiolus the Content leſs than Semi-Modius.)*

g. The *Sextary*, and ſo downward to the *Cyath* are divided alike to the *Attick* Meaſures. This *Sextary* was called *Italicus Sextarius*, to difference it from the *Greek Sextarie*, and alſo *Urbicus. q. d.* the City *Sextarie*, with reſpect to *Sextarius Caſtrenſis*, which was a *Sextarie* uſed in the Army, and double to the other. *(Sextary the Sorts.)*

h. Hemina is ſometime called *Cotyla*, and *Cotyla Romana*. *Alſted* mentions *Cotyla Italica*, which he ſaith is 12 Menſural Ounces, this ſeems to be ſome New, and not the Old, which himſelf reckons but at 9 *(Hemina how called.)*

i. Between the *Cyath* and *Ligula*, *Sennertus* placeth a *Muſtrum*, which he calls a Common little Spoone, containing half a *Cyath*, as the *Greek Concha*, and may not be confounded with the *Greek Myſtrum*. *(Muſtrum of Sennertus.)*

k. Ligula, a *Lingel*, as ſome *Engliſh* it, rather a *Spoon* or *Cochlear*, of which *Sennertus* makes 4 ſorts. *(Ligula what.)*

The Sorts.

The Leaſt containing—0 ½ ʒ of a thing of a middle Weight.
The next bigger———1 ʒ.
The Great————1 ½ ʒ or 2 ʒ.
The Greateſt————0 ½ ℥.

This *Ligula* then may be reckoned for the *Attick Myſtrum*, for as 4 of them made one *Kyath*, ſo 4 *Ligulas* make one *Roman Cyath*.

Roman *Meaſures their accompt by weight according to* Sennertus.

Sennertus before named accompts the Content of the *Roman* Meaſures by Weight of Oyle, Wine, or Water, and Honey, as followeth, ſave only I have proportioned the Weight of the *Ligula* according to the former Table, at the rate of ¼ of the *Cyath*, and have inſerted the *Modius* and *Modiolus*, which being dry Meaſures, *Sennertus* omitteth. *Malines*, *p.* 29. of his *Lex Mercatoria* ſaies the *Romans* did accompt 10 ℥ Ponderal for 12 ℥ Menſural, and ſo the *Sextarie* at 18 ℥ ſhould be 21 ⅗ and not 21 ½ as is there ſet.

	Oyle.			Wine or Water.				Honey.			
	℔	℥	ʒ	℔	℥	ʒ	℈	℔	℥	ʒ	℈
Cule.	1440	0	0	1600	0	0	0	2160	0	0	0
Amphora.	72	0	0	80	0	0	0	108	0	0	0
Urne.	36	0	0	40	0	0	0	54	0	0	0
Modius.	24	0	0	26	8	0	0	36	0	0	0
Congius.	9	0	0	10	0	0	0	13	6	0	0
Modiolus.	6	0	0	6	8	0	0	9	0	0	0
Sextarie	1	6	0	1	8	0	0	2	3	0	0
Hemina.	0	9	0	0	10	0	0	1	1	4	0
Quartary.	0	4	4	0	5	0	0	0	6	6	0
Acetable.	0	2	2	0	2	4	0	0	3	3	0
Cyath.	0	1	4	0	1	5	1	0	2	2	0
Muſtrum.	0	0	6	0	0	6	2	0	1	1	0
Ligula.	0	0	3	0	0	3	1	0	0	4	1½

℥ at 12 *per* ℔

A Table of Roman Weights.

The Table of *Roman* Weights.

	Minas.	Libras.	Uncias.	Semiun	Duellas.	Sicilicas	Sextulas.	Denarios.	Drams.	Quinars.	Scruples.	Quadr.	Sext.	Obol.	Siliq.	Graines.
Talent.	75	152	1500	3000	4500	6000	9000	10500	12000	21000	36000	42000	63000	72000	216000	864000
Mina.		1⅔	20	40	60	80	120	140	160	280	480	560	840	960	2880	11520
Libra. a			12	24	36	48	72	84	96	168	288	336	504	576	1728	6912
Uncia.				2	3	4	6	7	8	14	24	28	42	48	144	576
Semiuncia. b					1½	2	3	3½	4	7	12	14	21	24	72	288
Duella. c						1⅓	2	2⅓	2⅔	4⅔	8	9⅓	14	16	48	192
Sicilicum. d							1½	1¾	2	3½	6	7	10½	12	36	144
Sextula. e								1⅙	1⅓	2⅓	4	4⅔	7	8	24	96
Denarius. f									1 1/7	2	3 3/7	4	6	6 6/7	20 4/7	82 2/7
Dram.										1¾	3	3½	5¼	6	18	72
Quinar. g											1 5/7	2	3	3 3/7	10 2/7	41 1/7
Scruple. *												1⅙	1¾	2	6	24
Quadrans. h													1½	1 5/7	5 1/7	20 4/7
Sextans. i														1 1/7	3 3/7	13 5/7
Obolus. k															3	12
Siliqua																4

a. Libra

a. Libra called alſo *As*, by Tranſlators commonly rendred a Pound, was divided into 12 Ounces, and for every number of Ounces under 12, a Proper Name uſed, As, | Roman Libra *how called and divided.*

Deunx	11	Ounces.
Dextans and Decunx.	10	
Dodrans	9	
** Bes, Beſſis, and of old Des.	8	
Septunx.	7	
Semis, Semiſſis, Semiſſius, Selibra, and Simbella.	6	
Quincunx.	5	
Triens.	4	
Quadrans, and Triunx.	3	
Sextans	2	
Uncia.	1	

Malines, p. 24. of his *Lex Mercatoria*, divides *Pondus*, which he calls the Old Pound of the *Romans*, into | *The Diviſion of* Pondus *by* Malines.

64 Denarios.
128 Quinarios.
256 Seſtertios.
640 Aſſes.
1280 Semilibella's
2560 Teruncios.

A Reaſon is wanting why *Legat* makes the *Roman Libra* of 12 ℥ but 10 $\frac{1}{2}$ ℥ *Troy*, ſince if he reckon by the Number of Graines (the Original of Weights) at 5760 Graines of Aſſize in the Pound *Troy*; it can be but 10 ℥ juſt; for 10 times 6912 the Graines in a *Roman* Pound, and 12 times 5760 are equal. But if he count the ℔ *Troy* at 7680 Graines according to the Statute at 32 Graines of Wheat to a Penny Weight, the *Troy* Pounds will be 13 $\frac{1}{3}$ ℥ *Roman*. | Legat *queſtioned.*

** *Bes*, is the Mark Weight, two thirds of the Pound, *Malines p.* 24. aforeſaid makes the *Bes*, or old Mark of the *Romans* to be divided into. | Bes, *how divided.*

16 Loot, *or* Tetradrams.
23 $\frac{1}{3}$ Tridrams.
32 Didrams.
64 Drams.
96 Obolos, *or* Treobolas.
128 Triobulos.
384 Obolos.
768 Miobolos.
3840 Moments.

b. Semiuncia, or the Half Ounce is ſometimes called *Aſſarion*, and *Aſſarius*, and by *Alſted*, *Lotho*, anſwering to a *Germaine* Weight of that Name. | Semiuncia *how called.*

c. Duella, being double to the Weight of the *Sextula* is ſometimes called *Bina Sextulæ*. | Duella *how much.*

d. Sicilicum, or *Sicilicus*, and by Abbreviation *Siclus* is $\frac{1}{4}$ of an Ounce. | Sicilicum *how much.*

e. Sextula, uſed promiſcuouſly with *Sextans*, and underſtood by Import of the Name to be the Sixth part. | Sextula *uſed for* Sextans.

f. Denarius, a Peny-weight, the ſeventh part of the Ounce, whether uſed to weigh any thing but Money as other the Diviſions thereof, ſomewhat queſtionable, See among the Money. *Alſted* compares the *Drachmal Denarius* to the *German* Weight *Quintlein*. | Denarius *the Weight.*

g. Quinar was half the Penyweight, and a piece of Money ſet afterward among the *Roman* Coines. | Quinar *both Weight and Coine.*

*. Between the *Quinar* and *Scruple*, ſome mention a Weight called *Tremiſſis*, containing 32 Graines, being the 18th part of the Ounce. | Tremiſſis *how much.*

h. Quadrans, here is $\frac{1}{4}$ of the Peny weight, and ſo called *Quadrans Denarii* to diſtinguiſh it from *Quadrans Libræ*, which was 3 ℥. | Quadrans *what.*

i. Sextans, called *Sextans Denarii* to difference it from *Sextans Libræ*, was the ſixth part of the Peny-weight, and ſometime called *Sextula*. | Sextans *the weight thereof.*

k. Obolus, or Half a *Scruple*, called ſometimes *Simplium*, weigheth 12 Graines. If there be another *Obolus*, as ſome ſay, which was the third part of a *Quinar*, it ſeems: | Obolus *how called, the weight thereof.*

It

It seems to be a Piece of Coine, and must weigh 13 ⅓ Graines, and so is all one with the *Sextans*, according to the *Tabulary* Division; yet this sort of *Obolus*, they make to contain but 10 Graines.

Cerates *how much.*

Between the *Obolus* and the *Siliqua*, some mention a *Cerates*, which they say contains 6 Graines, and so is $\frac{1}{2}$ the *Obolus*, or $\frac{1}{4}$ of the *Scruple*.

A Table of Roman *Coines and their Values, before the translation of the Imperial Seat.*

Roman *Coines after the Imperial Seat translated.*

Roman *Summs of Money.*

Roman *Monies and their* English *Values.*

				l.	s.	d.
Roman — *Money before the translation of the Imperial Seat to* Bizantium.	Brass	*Less than the* As.	Sextula, $\frac{1}{6}$ Unciæ.	00	00	00 $\frac{1}{96}$
			Semiuncia, $\frac{1}{2}$ Unciæ.	00	00	00 $\frac{1}{32}$
			Uncia, $\frac{1}{12}$ Assis	00	00	00 $\frac{1}{16}$
			Sextans, $\frac{1}{6}$ Assis.	00	00	00 $\frac{1}{8}$
			Quadrans. Triunx. Teruntius. $\frac{1}{4}$ Assis.	00	00	00 $\frac{3}{16}$
			Triens, $\frac{1}{3}$ Assis.	00	00	00 $\frac{1}{4}$
			Semissis, $\frac{1}{2}$ Assis	00	00	00 $\frac{3}{8}$
		As, *or* Libra, $\frac{1}{10}$ Denarij.		00	00	00 $\frac{3}{4}$
		Greater than the As.	Decussis, 10 Asses.	00	00	07 $\frac{1}{2}$
			Vicessis, 20	00	01	03
			Tricessis, 30	00	01	10 $\frac{1}{2}$
			Quadracessis, 40	00	02	06
			Quinquacessis, 50	00	03	01 $\frac{1}{2}$
			Sexacessis, 60	00	03	09
			Septuacessis. 70	00	04	04 $\frac{1}{2}$
			Octacessis, 80	00	05	00
			Nonacessis, 90	00	05	07 $\frac{1}{2}$
			Centussis, 100	00	06	03
	Silver	Nummi *or* Nummuli	Teruntius, $\frac{1}{40}$ Denarij	00	00	00 $\frac{3}{16}$
			Sembella, $\frac{1}{2}$ Libellæ	00	00	00 $\frac{3}{8}$
			Libella, $\frac{1}{10}$ Denarij	00	00	00 $\frac{3}{4}$
			Obolus, $\frac{1}{6}$ Denarij	00	00	01 $\frac{1}{4}$
			Sestertius, 2 $\frac{1}{2}$ Asses	00	00	01 $\frac{7}{8}$
			Victoriatus, Quinarius $\frac{1}{2}$ Denarij.	00	00	03 $\frac{3}{4}$
			Bigatus	00	00	07 $\frac{1}{2}$
		Denarius	New, 10 Asses, $\frac{1}{8}$ ℥.	00	00	07 $\frac{1}{2}$
			Old, $\frac{1}{7}$ ℥.	00	00	08 $\frac{4}{7}$
	Gold		Tremissis, *or* Golden Triens	00	05	00
			Semissis, or Golden Drachmal	00	07	06
			Imperatorius	00	15	00
			Amient, *or* Consularis	00	17	01 $\frac{3}{4}$
Money after the translation of the Imperial Seat to Byzantium	Brass		Follis.	00	00	00 $\frac{5}{12}$
	Silver	Siliqua, *or*	Ceratium Simple	00	00	05
			Ceratium Magnum	00	00	07 $\frac{1}{2}$
			Milliarisium	00	01	03
	Gold		Constantines Piece	00	08	06 $\frac{6}{7}$
			Valencinians Piece	00	10	00
			Semissis, *or* Half Piece.	00	05	00
			Triens, *or* $\frac{1}{3}$ of that Piece	00	03	04
			Scruple, *or* $\frac{1}{4}$ of that Piece	00	02	06

		l.	s.	d.
Roman *Sums of Money.*	Sportula, containing 100 Quadrantes, *or* 10 Sestertios.	00	01	06 $\frac{1}{4}$
	Libra, a Pound of 96 Drams	03	00	00
	Sestertium (in the *Neuter Gender*) containing 1000 Sestertios (in the *Masculine Gender.*)	07	16	03
	Talent, containing 24 Sestertias, *or* 6000 Denarios.	187	10	00

The

The Brass *Uncia*, misprinted in *Rider*, at $\frac{1}{7}$ *Assis*, for $\frac{1}{7}$ part of 3 Farthings cannot be ' of our Peny, counting 4 *Cees* to a Farthing as the doth

Rider misprinted.

So also is *As*, at *ob. q.* for *ob. qa.* for *As* being the 10th part of the *Denarius* must be 3 Farthings, 10 times 3 making 30 Farthings which is 7 $\frac{1}{2}$ *d.* the value of the *Denarius.* To the Brass *As* was the Silver *Libella* equal in value.

Obolus, being ' of the *Roman* Peny, is called by *Celsus*, *Sextans.*

Obolus. *how called.*

Sestertius, Englished a *Sestertian* was $\frac{1}{4}$ of the *Roman* Peny, and being of the *Masculine Gender* was differenced from the other being of the *Neuter Gender*, and in Numbring by these *Sestertias* these 3 Rules are to be observed.

Sestertius *the Accompt thereby.*

1. If the Numeral Noun agree in *Case*, *Gender* and *Number* with the *Sestertian*; it signifieth barely just so much as was pronounced, as *Decem Sestertii* is 10 *Sestertians.*
2. If the Numeral Noun of another *Case* be joyned with the *Genitive Case Plural* of *Sestertius*: It noteth so many Thousands, as *Decem Sestertiûm* (for *Sestertiorum*) is Ten Thousand *Sestertians.*
3. If an *Adverb* be put without any Numeral joyned, as *Decies*, *Vigesies*, *&c.* or joyned with *Sestertiûm* the *Genitive Case Plural*; there is understood by it so many Hundred Thousand, as *Decies Sestertiûm*, is Ten Hundred Thousand *Sestertians.*

Alsted delivers it thus.

From 1 *Sestertian* to 1000 in the *Masculine Gender*, as *Unus Sestertius*, *Decem Sestertii*, *&c.* is 1 *Sestertian*, 10 *Sestertians.*

From 1000 to 100000 in the *Neuter Gender* and *Plural Number*, as *Singula Sestertia* 1000 *Sestertians*, *Bina Sestertia*, 2000 *Sestertians*, *&c.*

From 100000 upward, all expressed adverbially and in the *Genitive Plural*, as *Semel Sestertiûm* 100000, *Decies Sestertiûm* 1000000, *&c.*

Victoriatus was so called, because stamped with the Image of *Victory*, and *Quinarius* because equal in value to 5 Brass *Asses*, or Half the *Denarius.*

Victoriatus *how stamped.* Quinar, *how much.*

Bigatus, some call *Quadratus*, had the Print of a *Cart* or *Chariot* on it, and was of value equal with *Denarius.*

Bigatus, *the Print and Value.*

Denarius, *q. s. Dena æris*, because it contained 10 *Asses*, rendred a Peny, *Mat.* 18. 28. and 22. 19. at the old rate was $\frac{1}{7}$ of an Ounce, and at the New $\frac{1}{8}$, and at this rate all the other Coines are valued in the Table. This is sometime called the *Drachmal Denary* for distinction sake. Some make 3 sorts of Pence, the heavier weighing 1 $\frac{1}{7}$ *Attick* Dram, the Meane of 1 Dram, and the Least lighter than 1 Dram by $\frac{1}{24}$ of an Ounce, or thereabouts. Some say one was $\frac{1}{7}$ of the *Roman Uncia*, the Mean $\frac{1}{8}$, and the Lighter $\frac{1}{9}$. *Budæus* makes the *Attick* Dram and *Roman* Peny of the same Weight and worth, wherewith most agree, and accordingly each in the foregoing Tables are valued at 7 $\frac{1}{2}$ *d.* after 5 *s.* the Ounce.

Denarius, *the Value. The Sorts.*

The Golden *Denarius* mentioned in *Holyoke* at 2 *s.* 4 $\frac{1}{2}$ *d. Sterling* I have omitted; as not satisfied in the Weight, nor certain of such a Coine.

Golden Denarius.

The Golden *Amient*, seems the Eldest and Greatest, a Piece Coined by the *Consuls*, therefore called *Consularis*, weighed 2 $\frac{2}{7}$ Drams.

Amient *how called.*

The *Imperatorius*, or Piece of the Emperors Coine 2 Drams.

Imperatorius.

The *Drachmal* 1 Dram, and the *Triens* $\frac{1}{7}$ of the *Imperatorius.*

Drachmal.

After *Constantine* removed his Seat to *Bizantium*, now called *Constantinople*, a City after his own Name; we read of *Follis* in *Eusebius*, a Brass Piece, as *Lamprid*, or of *Iron*, as *Eustathius* saith, so called because it represented a Leaf in Latine *Folium*, and was $\frac{1}{12}$ of the Silver Simple *Siliqua.*

Follis, *what, why so called.*

The Silver *Ciliqua* or *Ceratium* was double. The Simple $\frac{1}{7}$ of the *Milliarisium*, valued 5 *d.* The Great called *Cerates*, 1 Dram equal to the Peny 7 $\frac{1}{2}$ *d.*

Cerates *of 2 sorts.*

Milliarisium, weighed 2 Drams.

Milliarisium *the weight.*

Constantines Piece of Gold was called *Romanus Solidus*, at the proportion of 7 *s.* 6 *d.* for a Dram of Gold must weigh 1 $\frac{1}{3}$ Dram.

Romanus Solidus.

These continued currant till *Valentinian*, who made his Coine somewhat heavier.

Valentinian's Piece of Gold by some is called *Sextula*, and being valued at 10 *s.* *Sterling*, must weigh 1 $\frac{1}{3}$ Dram.

Valentinian's Sextula.

Parts thereof.

Of which the { *Semissis* / *Tremissis*, or *Triens* / *Scruple* } being { $\frac{1}{2}$ / $\frac{1}{3}$ / $\frac{1}{4}$ } was { $\frac{3}{5}$ / $\frac{4}{9}$ / $\frac{1}{3}$ } of a Dram.

Sportula, say some, was a Lawyers Fee, or an Almes distributed by Princes among the People. See Selden's *History of Tythes*, *chap.* 4. *p.* 37, 38.

Sportula *what.*

Moderne Geodæticals.

Geodæticals of the smaller sort, with the Antient *Hebrews*, *Greeks* and *Latines* Emedullated; it is time to say something of others more Moderne. A Perfect List whereof is not yet come to hand, and those that are, very difficult to reconcile one with another to any exact Computation, though sometimes the difference be inconsiderable, and in round reckoning pass with Authours one for another, without much sensible Error. As the *English* Mile is often joyned with the *Italian*, yet as aforesaid this contains but 1000 Paces, and that 1056. So the *Russian Vorst*, wrote also *Verst*, and sometime *Worst*, commonly called a Mile, and counted of *English* or *Italian* Measure; though *Fletcher* in his History of *Russia*, *Chap.* 1. tells us it wants a quarter. And so *Johnson* in his *Atlas* accompts *Heylin* without help of the *Printers Errata's* is irreconcileable to himself in his *Cosmography*, *p.* 511. who counts 2260 *Vorsts* to 3690 *Italian* Miles, and not 4 lines of 4400 *Vorsts* to 3300 Miles.

Russian Vorst.

Some of the Principal, not all of the Modern Geodæticals follow with the Kingdoms and Countries where used.

Out of the Variety and incertainty of Authours to undertake the discovery of the different Measures, Weights and Monies of all places; were endless. Wherefore amongst the Moderne, some of the Principal may suffice. And to spare the often Writing the Names of the Kingdoms and Countreys wherein most of the Provinces and Cities herein mentioned lye: Take the brief Accompt following; which by help of the figures annexed will easily direct the Reader to find them out when he comes at them.

Parts

Parts of the WORLD.

Parts of the World, and some of the Kingdoms and Countries therein.

1. Europe.	2. Asia.	3. Africa.	4. America.
1 *Denmark*	1 *Anatolia*	1 *Ægypt*	1 *Mexico*
2 *England*	2 *Arabia*	2 *Barbary*	2 *New Spain,* or, *Nova Spagnia*
3 *France*	3 *Armenia*	3 *Isles*	3 *Peru*
4 *Germany*	4 *Chaldea*	4 *Terra Nigritay*	
5 *Greece*	5 *China*		
6 *Ireland*	6 *India*		
7 *Italy*	7 *Oriental Isles*		
8 *Low Countries*	8 *Palestina*		
9 *Poland*	9 *Persia*		
10 *Portugal*	10 *Russia*		
11 *Russia*	11 *Syria*		
12 *Scotland*			
13 *Slavonia*			
14 *Spain*			
15 *Sweden*			

Denmark *some Places noted therein.*

1. 1. *Denmarke*
- Heylegerhaven.
- Holstein
  - Hamburgh, *or* Hamborough.
  - Lubecke.
- *Islands*
  - Fameren.
  - Scandia-Elbogen, *or* Nellebogh.
  - Seland
    - Coppenhagen, *or* Haffen.
    - Elsinure, *or* Elsingnore
- Juitland — Ebbeltorfe
- Norway — Bergen, *or* Barrow.

England, *some of the Counties and Cities there.*

1. 2. *England*
- Berkshire — Reading.
- Cambridgeshire — Cambridge
- Chestire — Westchester
- Devonshire
  - Dartmouth.
  - Exceter.
  - Plymouth.
- Essex — Colchester.
- Glocestershire
  - Bristol.
  - Glocester.
- Hamshire — Winchester.
- Herefordshire — Hereford.
- Kent — Canterbury.
- Lancashire
  - Lancaster.
  - Manchester.
- Middlesex
  - London.
  - Westminster.
- Norfolke — Great Yarmouth.
- Northumberland — Newcastle *upon* Tyne.
- Oxfordshire — Oxford.
- Somersetshire
  - Bridgewater.
  - Dunster.
  - Taunton.
- Suffolke — Dunwich.
- Sussex
  - Rye.
  - Winchelsea.
- Warwickshire — Coventry.
- Westmorland — Kendal.
- Wiltshire —
- Worcestershire — Worcester.
- Yorkshire — York.

France
Ireland
Germany
Hungary
Some of the Places in the Book afterward referred to.

- 1. 3. *France.*
  - Aquitaine.
    - Gascony — Coniac, Oleron
    - Guiennæ — Baionne, Bourdeaux, Condet, Libourne, *or* Lisborne
    - Xantoigne *or* Santoine — Bruage, Rochel, Santonum, *or* St. Antoine
  - Anjou — Angiers, Tours
  - Berry — Bourges, Clermont
  - Bourbon — Tureme
  - Bretagne — Morlaix, Nantes, St. Malo
  - Borgoingue *or* Burgundy — Auxere
  - Champagne — Province
  - Daulphine — Valence, Vienna
  - Isle of France — Clermont, Paris
  - Languedoc — Aquismort, *or* Aigues Mort, Montpellier, Narbon, Tholouse
  - La Beausse — Blois, Orleans
  - Lionois — Lyons
  - Normandy — Caen, *or* Cane, Dieppe, Rouen, *or* Roan
  - Nivernois — Nevers
  - Picardy — Abbevile, Amiens, Calais, Boulogne, Roy
  - Perigort — Angolesme, *or* Engoulesme
  - Poictou — Poictiers, Mirebeau, *or* Mirabel
  - Provence — Aix, *or* Ay, Avignon, Aurange, *or* Orange, Marseilles
- 1. 4 *Germany Hungary* or *Switzerland*
  - Alsatia — Colmar, Rufach, Strausburgh, *or* Strasburgh, Thann, Weissenbergh
  - Austria — Vienna
  - Baden — Baden, Durlach
  - Bavaria — Munchen, Passaw, *or* Patavia, Regensburgh, *or* Ratisbone, Saltsburgh
  - Bohemia — Prague
  - Brandenburg — Brandenburgh
  - Carinthia — Frisach
  - Cleveland — Cleue, Wesel, *or* Wisel
  - Collenland — Collen, Rhineburgh
  - East Frisland — Emoden
  - Franconia — Bamberg, *or* Babemberg, Frankford, *on the* Moene, Nurenburgh, Wurtspurg, *or* Wurtzburgh
  - Gulich — Aix, Aken, Aquen, *or* Achon, Gulich, Juliers, *or* Guliers.
  - Hungary — Poosen, *or* Presburgh
  - Lorrein, *or* Lorraine. — Mets, Verdun
  - Lusatia — Bautsen, *or* Botsen
  - Lunenbourgh — Lunenbourgh
  - Mecklenburgh *or* Macklenburgh — Domyn, *or* Dammin, Rostock, Wismar
  - Mark *or* March — Tremone, *or* Dortmond, Werden
  - Mentz — Koningsbergh, Mentz
  - Meydeburgh — Meydeburgh, *or* Magdeburgh, *or* Mageburgh
  - Palatinate — Eisted, *or* Aichstad, Heydelburgh, Norenborgh, *or* Norembergh, *or* Nurenbergh, Spiers, Wormes
  - Pomerania — Costin, Gripswald, Ockermond, Stetin, Straelsondt
- 1. 6. Ireland — Dublin, London-Derry.

Germany *and* Switzerland Greece, Italy, *several Places of Note in them afterward referred to.*

B

- Saxony
  - Turingia.
    - Erdford
    - Halo, *or* Kala
    - Jene
  - Meissen.
    - Friberg
    - Lipsich
    - Meysen
    - Mansfield
- Silesia — Breslaw
- Suevia, *or* Almaine
  - Ausburgh, *or* Auspurgh
  - Bibrach
  - Brisach, *or* Brisgow
  - Constance
  - Friburgh
  - Kempten, *or* Campidona
  - Norlingen
  - Offen, *or* Offner
  - Ravenspurgh
  - Scaffhausen, *a* Canton *of the* Switz
  - Ulme
- Switzeeland, *or* Helvetia
  - Upper Baden
  - Basil
  - Berne
  - Friburg
  - Lausanna
  - Lucern
  - Soloturn
  - Switz
  - Zurich
- Triers
  - Confluentz
  - Triers
  - Upper Wesel
- Tyrol
  - Inspruch
  - Trent
- Westphalia
  - Breme
  - Homberg
  - Osenbrigh, *or* Osenbridg
  - Ravensburgh
- Weteraw
  - Fribergh
  - Nassaw
  - Weilborough, *or* Wisselborough
- Wirtenbergh
  - Tubing
  - Wiberg

1. 5. *Greece.*

- Achaia — Athens
- Albania — Lavalona
- Arcadia
- Archipelago
- Epirus — Larta
- *Islands*
  - Candia, *or* Crete
  - Corfu
  - Euboe — Nigropont
  - Scio
  - Zant
- Laconia — Sparta, *or* Lacedemon
- Lepanto
- Macedon — Salonichi, *or* Salonici, *of* Old Thessalonica
- Thrace
  - Sapy
  - Constantinople.

1. 7. *Italy.*

- *Church Lands.*
  - Compagnia di Roma — Rome
  - Ducato Spoletano
    - Narnia,
    - Negropont
  - Estate of Urbin
    - Pesara
    - Urbin
  - Marca Anconitana
    - Ancona
    - Rechanati
  - Romandiola, *or* Romagnia
    - Boloignia, *or* Bononia
    - Cervia
    - Cesena
    - Faenza
    - Forli, *or* Furli
    - Ravenna
    - Rimini, *or* Rimano
  - Territory of Ferrara
    - Carpi
    - Ferrara
- *Common-Wealths.*
  - Genoa — Genoa, *or* Genes
  - Luca — Luca
  - Venice
    - Friuli — Trieste
    - Histria
      - Cape de Istria
      - Piran
    - Marca-Trevigiana
      - Bergamo
      - Brescia
      - Crema
      - Padua
      - Treviso
      - Verona
      - Vincentia, *or* Vincenza
    - Venice
- *Dukedoms.*
  - Mantua — Mantua
  - Millan
    - Como
    - Cremona
    - Millan
    - Pavia, *or* Papia
  - Modena — Modena
  - Montferrat
    - Alba
    - Casal, *or* St. Vas
  - Parma
    - Mirandula
    - Parma
    - Placentia, *or* Plaissance
  - Tuscaine
    - Florence
    - Ligorne
    - Massa
    - Prato, *or* Perato
- *Islands.*
  - Corsica
  - Sardinia
  - Sicilia
    - Marsala, *or* Masilla
    - Messena, *or* Messana
    - Nicosia
    - Palermo
    - Syracusa
- *Kingdom of Naples.*
  - Abruzzo
    - Aquila
    - Lanciano, *or* Lansan
  - Apulia, *or* Puglia
  - Calabria — Rhegium
  - Terra de Otranto — Otranto
  - Terra di Lavoro
    - Naples
    - Rivelli
    - St. Fumia, *or* Fiume

- *Principality and Provinces of the Allobroges, &c.*
  - Piedmont — Turin
  - Savoy — Chambery, Lunebourgh
  - Geneva, *or* Walsland — Jenser
  - Walsland — Sedun
  - Grisons — Curiensis
- 1. 8. *Low Contries.*
  - Artois — Arras, Perne, St. Omer
  - Brabant *and* Marquisate *of the Empire* — Antwerp; Arschot, *or* Aschot; Barrow, *or* Bergen op Zome; Bolduc, Boiseleduc, Hertogenbosh, *or* Shertogenbosh; Brussels, *or* Bruxels; Diest; Lovaine; Maestricht; Machlyn, *or* Mechlyn; Malines
  - Cambray — Cambray
  - Flanders — Alst, Ailst, *or* Alost; Audenarde, *or* Oudenarde; Axele; Bridges, *or* Bruges; Casel; Cortrycke, *or* Courtray; Damme; Deynse, *or* Deyse; Dixmude; Doway, *or* Douay; Dunkerk; Gaunt; Graveling; Honschotten; Hulst; Ipre, *or* Ypre; Lisle, *or* Ryssel; Loo, *or* Lowe; Meanen, *or* Meenen; Newport; Orchies, *or* Orsies; Ostend; Popering; St. Amand; Scluse, *or* Sluys; Tournay, *or* Dornir; Walsland, *or* Land Van Waes; Winocksborough, *or* Winocksbergh
  - Groeninghen — Dam, *or* Damme; Groeninghen
  - Guelderland — Arnhem; Bomel; Batenborgh; Ghent; Gueldres; Nimmegent, *or* Nimmeghen
  - C
  - Henault — Ach; Halle; Valenciennes; Covyn, *or* Convennes
  - Holland — Amsterdam; Briel; Delfe; Dort; Enckhuysen; Hague; Harlem; Horne; Leiden; Medenblick; Naerden; Rotterdam; Schoonehaven; Texel; Wassenar
  - Limbourg — Carpen; Limbourgh
  - Luickland — Asselt; Hoey, Hoye, *or* Huy; Liege; Looth
  - Luxenbourg, *or* Lutzenberg — Masiers; Sedan, *set by some in* France.
  - Namurs — Namen
  - Overyssel — Campen; Deventer; Swooll, *or* Zwoll
  - Utrech — Utrech
  - West-Frisland, *or* Frisland — Harlingen
  - Zeland
    - Scowen — Zuricza
    - South Beverland — Goes, *or* Tergoes; Romerswal
    - Walcheren — Armuyden, *or* Ramme; Camfere, *or* Vere; Flushing; Middleburgh
  - Zutphen — Groll; Zutphen

Low-Countries. Several of the Places afterward referred to.

- 1. 9. Poland
  - Liefland, *or* Livonia — Riga
  - Lithuania — Vilna, *or* Wilde
  - Poland — Cracow; Warsaw
  - Prussia — Coningsbergh; Dantzick; Melvyn; Thoren

Poland *the like.*

- 1. 10. Portugal
  - Algarve
  - Portugal — Lisbon; St. Tubal, *or* Setubal

Portugal *the like.*

- 1. 11. Russia — Muscovie
- 1. 12. Scotland — Edenburgh

Russia, *and* Scotland *the like.*

- 1. 13. *Slavonia.*
  - Dalmatia — Ragusa, *or* Raguza; Sebenico; Spalato
  - Liburnia — Zara; Zegna, *or* Sequia
- 1. 14. *Spaine.*
  - Andaluzia — St. Lucar.; Sevill
  - Arragon — Saragossa
  - Biscay — Bilboa
  - Castile — Burgos; Madrid; Medinadel Campo; Salamanca
  - Catalonia — Barcelona; Girone
  - Corduba — Xeres, *or* Sherry
  - Gallicia — Baiome
  - Granada — Almeria; Granada; Malaga
  - Guipuscoa — St. Sebastians; Toloso
  - Islands — Gades (Cadiz; St. Mary); Majorca
  - Leon — Leon; Villa Viciosa
  - Murcia — Carthagena
  - Navarre — Viano; Victoria, *or* Vellica
  - Toledo — Toledo; Medina Cœli
  - Valentia — Allicante; Valentia
- 1. 15. *Sweden.*
  - Boden —
  - Finland —
  - Gothland — Ostrogoth; Smalandia; Westrogoth
  - Lapland —
  - Liefland, *or* Livonia — Revell, *or* Rivallia; Narva, *or* Nareca
  - Sweden — Helsingen; Suderman; Westman; Upland, *in which is the City* Stockholme
- 2. 1. Anatolia, *or* Asia *the less*
  - Asia *specially so called* — Ephesus; Smyrna
  - Bithynia — Bursa, *or* Prusa
  - Cilicia — Alexandria, *or* Scanderoone
  - Islands — Cyprus; Rhodes; Scio
- 2. 4. Chaldea — Babylon
- 2. 6. India
  - Decan — Goa
  - Malabar — Calicute
  - Narsinga — Maliapur, *or* St. Thomas
- 2. 7. Orient Isles — Molucco
- 2. 8. Palestine — Judea — Rama
- 2. 11. Syria
  - Comagena — Aleppo; Aman
  - Cœle-Syria — Damascus
  - Phoenicia — Acon, Achri, *or* Acre; Barutti, *or* Berytus; Tripoli
  - Syria — Antioch
- 3. 1. Ægypt
  - Alcario
  - Alexandria
  - Arcadia — Cairo, *or* Caire
  - Forfori
  - Sciba
  - Zaidin
  - Zeroi
- 3. 2. Barbary
  - Algiers — Algiers; Oran
  - Fesse — Fesse — Fesse
  - Moroco — Sus — Capo d' Aguer *or* Capo d' Algier; Morocco — Morocco
  - Tunis — Tripoly — Tripoly; Tunis — Tunis
  - Una
- 3. 3. Isles — Canaries; Madera; St. Thomas
- 3. 4. Terra Nigritarum — Cabo Verde; Guynea

Slavonia, Spain, Sweden, the places of Note in them and in Asia, Africa also in the Book afterward referred to.

The *English* are not alone in the knowledge and use of the Inch, Foot, Ell, Pace, Fathom, Furlong, Mile, *&c.* for besides the *Hebrews*, *Greeks* and *Latines* before spoken of; most Nations especially of *Europe*, use the same as occasion serves. Some nearly correspondent, some vastly different, and some Countries have divers Measures of the same Names. Hence is observable, that although most of the *Europæans* reckon distances of places by Miles, yet there are no agreement between them. And in *Germany* it self are

Long Measures in England *used elsewhere, But different.*

German *Miles the sorts.*

are 3 ſorts. The Common Mile which is 4 *Italian* Miles, and yet the leaſt of the 3. A Mean which is 5, and the Great (called ſometimes the *Great Saxony*) Mile, being 6 *Italian* Miles. And beſides theſe *Alſted* mentions a Mile that containeth 22800 Feet.

Spaniſh Miles. Leagues. *Spain*, by ſeveral hath [illegible] Mile to a *Dutch* Mile; but their Common Accompt is by Leagues called with them *Leguas*, whereof 17 $\frac{1}{2}$ make 60 *Italian* Miles. *Alſted* ſpeaks of a *Spaniſh* Mile to be 21000 Feet.

French *Leagues.* *France*, where they reckon by Leagues, hath no leſs than 4 ſorts, as *Cotgrave* in his *Dictionary* hath well obſerved. As,

The Sorts. *Lieue de Bourgongue*, 50 *Portées*,, every *Porteé*, 12 *Cords.* Every *Cord* 30 *Feet*, moſt in uſe in *Burgundy*.

Grande Lieue, as the common ſort of *German* Miles, every *Lieue* 4 *Italian* Miles.

Moyennes Lieues, as thoſe of *Dauphine* or *Languedoc*, containing 3 *Italian* Miles.

Petites Lieues, as thoſe of *Italy*, and of this ſort, the *Fiery Collumne*, commonly called the *Dutch Waggoner* in his *Sea Charts* often joynes as one with the *Engliſh* Mile.

French *Mile.* A *French* Mile ſaith *Cotgrave* is 2 of ours.

Flemiſh *Mile.* *Malines*, notes the *Flemiſh* Mile of 2 ſorts. One 1000 Rods of 20 Feet to the Rod, which is alike to the Common *Germane* Mile. And the other of 1400 Rods of 15 Feet to the Rod. And the *Holland* Mile he ſaith is 2000 Roads, and that 5 of them are 4 *Flanders* Miles. *Alſted* mentions the *Belgick* Mile to be 18000 Feet, and by this accompt the Mile of *Flanders* muſt be 2500 Roads, but how long the *Holland* Road is he ſaith not.

Swediſh *Leagues* *Leuca Suevica*, after the *Latine*, or the *Swediſh* League called *Scandica* is 5000 Paſſes, or 5 *Italian* Miles.

Florence *Mile.* *Florence*, *Malines* reckons 3000 Braces for a Mile, which at the rate of 122 $\frac{1}{2}$ Braces for 60 *Engliſh* Ells comes 5510 Feet, 2 Inches, and ſomewhat over.

Ægyptian *Schoenes.* In *Ægypt* they reckon by *Schoenes* of various Magnitude, ſome 60, ſome 40, ſome 20 Furlongs. Some will have the *Schoene* equal to a *Spaniſh* Mile. See before in *Greek* Meaſures. The *Ægyptian* Cubit, ſaith *Malines* is 6 of our Cubits.

Ruſſia *Vorſt* vide antea Of the *Ruſſian Vorſt*, and *Perſian Paraſang* ſee before.

Chinian *Stade.* The *Fur long* or *Stade* of *China* is almoſt twice as long as ours, for 12 of ours are but 6 [illegible] of theirs by *Johnſon* in his *Atlas*; who for the moſt part the following Table, follows.

Long Forain Meaſures compared to a Degree.

Long Outlandiſh Long Meaſures compared.

In	Dalacia	$7\frac{2}{3}$	Miles, make 1 Degree of a Great Circle in the Earth.
	Weſtrogoth	$8\frac{1}{2}$	
	Saxony Great	10	
	Almaine, Oſtrogoth, Suderman, Upland, Weſtman	12	
	Germany, Smalandia	15	

In	Spain	$17\frac{1}{2}$	Leagues	make 1 Degree of a Great Circle in the Earth.
	Finland, Cajan, Livonia	18	Miles	
	France	20	Leagues	
	Helſingen, Botnia, Lapland	22	Miles	
	Scotland	50	Miles	
	Italy, England	60	Miles	
	Ruſſia	80	Vorſts	
	China	250	Furlongs	

Grimſtone

Grimstone, *p.* 715 tells us that 69516 *Diez* of *China* Measure make almost 1000 *Spanish* Leagues. And that the *Chinois* have 3 Measures to Survey withal, which they call *Lij*, *Pu*, and *Icham*. The *Lij* comprehends as much Space as may be assigned to a Mans Voice thrust forth with all his force in a Calme Season upon a fair Plain, 10 of these *Lij's* makes 1 *Pu*, which is a great *Spanish League*, and 10 *Pu's* make 1 *Icham*, or a whole Journey. Chinian Diez, *their* Lij, Pu, *and* Icham.

The Modern Furlongs, Fathoms, and Paces of the *Europæans* differ little if at all from the Ancients. The Furlong 125 Paces, the Pace 5 Feet, or two of the lesser Paces. *Moderne Furlongs.*

The Fathom in *France* called *Toise* ordinarily is as ours 6 Feet, the Kings Fathom 7 Feet 4 Inches. For Woods and Forests by the Custom of *Orleans* for 5 $\frac{1}{2}$ Feet. The *Burgonian* Fathom is 7 $\frac{1}{2}$ Feet. French *Fathom*

The Ordinary (called also the Royal) *Pied*, or Foot, there is 12 Inches, that of *Clermont* 11, of *Engoulesme* 1/[illegible] longer than the Ordinary. The Foot used about *Bourdeaux* to measure Land called *Pied de Terra* is longer than the ordinary by 8 in $\frac{3}{4}$. *Pied de Ville* there used for Timber, Stone, *&c.* longer than the Ordinary by $\frac{5}{8}$ of an Inch. The *Petit Pied* is shorter than the Ordinary. French *Foot: The Sorts.*

Several affirm, that the *Foot* used in several Countries is different, as of 10, 12, 16. *&c.* Inches, and they of different bigness. *Capel* and *Snellius* before-mentioned have compared them together, of whom the latter is by some reckoned most exact. The Tables of both follow.

Capel's *Comparison of the Foot to the Foot of* Toledo *in* Spain, *supposed to be divided into 120 parts.*

Capel *his Comparison of the Foot of* Toledo *with others.*

The Foot of	*Heidelberg*	in	*Germany*	137	Parts
	Tuscany		*Italy*	138	
	Sedan		*France*	139	
	Rome		*Italy*	144	
	Athens		*Greece*	150	
	London		*England*	152	
	Paris		*France*	160	
	Syria		*Asia*	166	
	Egypt		*Africa*	171	
	Judea		*Palestine*	180	
	Babylon		*Chaldea*	200	

Snellius *his Comparison of the* Foot, *to the* Foot *of* Rome *in* Italy, *or* Rheinland, *or* Leiden, *which he saith are all one, supposed to be divided into 1000 parts.*

Snellius *his Comparison of the Foot of several Places with others.*

The Foot of	*Toledo*	in	*Spain*	864	Parts.
	Mechlin		*Brabant*	890	
	Strasburgh		*Germany*	891	
	Amsterdam		*Holland*	904	
	Antwerp		*Brabant*	909	
	Lovain		*Brabant*	909	
	Bavaria		*Germany*	924	
	Coppenhagen		*Denmark*	934	
	Goes		*Zeland*	954	
	Middleburgh		*Zeland*	960	
	London		*England*	968	
	Norembergh		*Germany*	974	
	Zurichzee		*Zeland*	980	
	The Ancient *Greek*			1042	
	Dort	in	*Holland*	1050	
	Paris		*France*	1055	
	Briel		*Holland*	1060	
	Venice		*Italy*	1101	
	Babylon		*Chaldea*	1172	
	Alexandria		*Egypt*	1200	
	Antioch		*Syria*	1360	

Palmes, &c. *like the old.* Spanish Xeme.

The Palmes greater and less Inches and Digits, for the most part have the same Divisions as the Ancient. The *Spanish Xeme* is half a Foot, or 6 Inches. Their *Cotus* half of their greater Palme, or $4\frac{1}{2}$ Inches, both *Spain* and *France* allow 16 digits to their 12 Inches, called by that *Pulgada*, by this *Poulcee*.

Inch of Spain *and* France. French *Line.* *Superficial Measures of the Moderne.* Arpentiers *of* France. *The Sorts.*

France hath a little Measure called a *Ligne*, or Line, whereof 12 to 1 Inch.

Superficial Measures of Land having length and breadth are subject also to the different Laws of divers Countries, whether they measure by Lines, Cords, Rods or Perches, *&c.* And accompt their Measures by Acres, Arpentiers, Bunderen, *&c.*

France reckoning by Arpentiers useth not fewer than 10 sorts, as *Cotgrave* accompts, *Viz.*

Arpent, that is ordinary is 100 Perches Square of 18 Feet to the Perch.

Arpent de Bois is $2\frac{1}{2}$ Roods, 1 Rood 40 Perches, 1 Perch 24 Feet, 1 Foot 24 Inches.

Arpent de Bois, de Bourgoigne, is 440 Perches.

Arpent de Clermont is 100 Verges in most places, in some but 70, at 26 Feet to the Verge.

Arpent de Dunois, is 100 Perches at 20 Feet to the Perch.

Arpent de Nevers is 4 quarters square, 1 quarter 10 Fathoms, 1 Fathom 6 Feet.

Arpent de Paris, is 100 Perches Square, at 22 to the Perch, and in some places about *Paris* it contains 25 Feet, and in others at the ordinary rate 18.

Arpent de la Perch, 100 Perches, 1 Perch 24 Feet, 1 Foot 13 Inches.

Arpent de Poictou is 80 Paces Square.

Arpent Romain, is 240 Feet long, and 120 Feet Broad. This is like to the *Latine Juger*, of which before.

French Septier *of* Land. *Their* Muid *and* Mine.

A *Septier* of Land he saith is much about the *Arpent*.

A *Muid* of Land is 12 *Septiers* or *Arpents*.

A *Mine* of Land in *La Chastellenie de Bulles*, contains 50 Verges of 24 Feet to the Verge.

The Sorts. Scruple *of Land.* Spanish Jugada Faneca Stadale.

A *Mine* of Land in *Clermont*, 60 Verges of 22 Feet to the Verge.

A *Mine* of Land in the *Seigneurie de Remy*, 80 Verges of $22\frac{1}{2}$ Feet to the Verge.

A *Scruple* of an *Arpent* is $\frac{1}{48}$ of an *Arpent* or 10 Feet Square every way.

In *Spain*, 1 *Jugada* is 50 *Faneca's*, 1 *Fancca* is of Land sowed with Barley 400 Square *Stadales*, of Land with Wheat 600. And 1 *Stadale* is 11 Feet. A Square *Stadale* is 121 Feet.

Moderne Long Measures.

Merchants have their *Ells*, *Aulnes*, *Braces*, *Cannes* or *Canes*, *Varras* or *Varas*, *Pitchy*, *&c.* for Commodities proper to be measured thereby after the Laws and Usages of several Countries. And by Traders is diligently to be sought out, because in several Countreys, though some Common Measures be of most general use, yet in sundry places in one and the same Countries shall measures of the same Name, Nature and use be different, as in *France*, the Common *Aulne*, or that called *Aulne du Roy* is 3 Feet 7 Inches and 8 Lines. But that of *Bourdeaux* $4\frac{1}{2}$ Feet almost, that of *Dijon* and *Province*, but $2\frac{1}{2}$. That used of *Merchants* for Silks half an Inch shorter than the Common, that of *Paris* but 3 Feet and $\frac{2}{3}$ of an Inch, and about $\frac{1}{50}$ of a Foot. Some Customs bespeak it formerly 3 Feet 8 Inches and 4 Lines long. And allowing 100 Ells of *Antwerp* to agree with 60 Ells or 75 Yards at *London*, the correspondency thereof with other places follow out of *Malines* his *Lex Mercatoria*.

Aulnes *of* France *how different.*

100 *Ells of* Antwerp *compared with the Measures of other places.*

The 100 Ells of *Antwerp* make at

Townes and Cities	Directions as before		Merchandise Measured	Contents *or* Quantities	Names of the Measures.
Abbevile	1	3		84	Ells
Achrie, *or* Acon	2	11		115	Pichy
Acon, *or* Aken	1	8		$104\frac{1}{2}$	Ells
Adler				$33\frac{1}{2}$	Canes
Aleppo	2	11		108	Pichy
Alexandria	2	1		124	Pichy
Aman	2	11	as *Aleppo*		
Amsterdam	1	8		$101\frac{1}{4}$	Ells
Ancona	1	7		$107\frac{1}{2}$	Braces
Andaluzia	1	14		$83\frac{1}{2}$	Varras
Antwerp	1	8	for Silks	$98\frac{2}{3}$	Ells
Aragon	1	14		43	Cannes
Archipelago	1	5		100	Pichy
Artois	1	8	all the Province	$98\frac{1}{3}$	Ells

Asselt

100 *Ells of* Antwerp *compared with the Measure of other Places.*

The 100 Ells of *Antwerp* make at

Asselt	1	8	as *Acon* in the *Netherlands*		
Audenarde	1	8	as *Antwerp* for Silks	$98\frac{2}{3}$	Ells
Avignon	1	3		60 / 36	Ells / Cannes
Ausburgh, *or* Ausborough	1	4	for Linnen / Cloth	125 / 127	Ells
Barrow, *or* Bergen	1	1	Uncertain, for they measure by the bigness of your head with a Rope for		1 Ell
Barcelona	1	14	as *Arragon*		
Basill	1	4		125	Ells
Bautson	1	4	for Cloth / Silks	111 / 120	Ells
Bergamo	1	7		$101\frac{2}{3}$	Braces
Bolduc, *or* Hertogenbosh	1	8		102	Ells
Bologna	1	7	as *Ancona*		
Brabant	1	8	all the Province (except such Places as are herein excepted) like *Antwerp* for Silks.		
Breme	1	4		$122\frac{1}{2}$	Ells
Brescia	1	7	as *Bergamo*		
Breslo	1	4	as *Bautson*		
Bruges, *or* Bridges	1	8	in the Shops / but for Linnen	$98\frac{2}{3}$ / $94\frac{1}{4}$	Ells
Brussels	1	8	as *Bolduc.*		
Bauti, *or* Baruti	2	11		$111\frac{1}{3}$	Pichy
Bursa	2	1		114	Pichy
Cadiz	1	14	for Cloth / Silks	81 / 108	Varras / Ells
Calabria	1	7	as Adler		
Cambray	1	8		96	Ells
Candia	1	5		108	Pichy
Capo d' Algier	3	2		$136\frac{1}{2}$	Covados
			There 1 Cane is 12 Cavados.		
Carpi	1	7	as *Ancona*		
Cassel	1	8	as *Bolduc*		
Castile	1	14		85	Varras
			Some allow but	81	Varras
			There 1 Varra is 4 Quarters, 1 Quarto 2 Palmes.		
Cesena	1	7	as *Ancona*		
Collen	1	4		120	Ells
Conninsbergh	1	9		125	Ells
Constantinople	1	5	/ For Canvers	113 / 80	Pichy
Corfu, *an Island*	1	5		$116\frac{2}{3}$	Braces
Covin	1	8		70	Ells
Crema, Cremona	1	7	as *Bergamo*		
Damascus	2	11	as *Bruti*		
Damme	1	8	as *Antwerp* for Silks		
Dantsick	1	9		122	Ells
Deyse *and* Diest	1	8	as *Bolduc*		
Domyn	1	4	as *Breme*		
Doway	1	8	as *Cambray*		
Dunkirk	1	8	as *Antwerp*	100	Ells
Embden	1	4	as *Breme*		
Erdfurd	1	4		165	Ells
Ferrara	1	7	as *Ancona*		
Flanders	1	8	all the Province, as *Brabant*		
Flushing	1	8		104	Ells
Florence	1	7	for Woollen / Silks	116 / $122\frac{1}{2}$	Ells / Braces
France, the Kingdom, except the Places herein excepted. See *Paris.*					

Frankford

100 Ells of Antwerpe *compared with the Measure of other Places.*

The 100 Ells at *Antwerp* make at

Frankford	1	4	as *Ausborough*		
Gaunt	1	8	as *Antwerp* for Silks		
Gelderland	1	8		$104\frac{1}{2}$	Ells
Genes	1	7		122	Braces
			Silks (104, Palmes for 34^{2} Ells)	288	Palmes
			Woollen Cloth at 9 Palmes the Cane	32	Cannes
			Linnen Cloth at 13 Palmes the Cane	29	Cannes
Geneva	1	7		60	Stabb
Goes	1	8		97	Ells
Granada	1	14	as *Andaluzia*		
Gripswool	1	4	as *Breme*		
Halle	1	4		105	Ells
Hamborough	1	1	as *Breme*		
Harlem	1	8	in the Market for Linnen	$94\frac{1}{2}$	Ells
Henault	1	8	in the Market	$94\frac{1}{2}$	Ells
			in the Shops	$98\frac{1}{2}$	Ells
Hertogenbosh	1	8	See *Bolduc*		
Holland	1	8	in most places of the *Province*	$103\frac{1}{2}$	Ells
Honschotten	1	8	as at *Dunkerk*		
Hoye	1	8	as at *Bolduc*		
Ipre	1	8	as *Antwerp* for Silks		
Istrica, Istria	1	7	for Woollen Cloth	$101\frac{1}{3}$	Braces
			for Silk and Cloth of Gold	108	Braces
Lansan	1	7	as *Adler*		
Larta, *or* Laarta	1	5	as *Alexandria*		
Lavalona	1	5		111	Pichy
Lepanto	1	5		113	Pichy
Liege	1	8		114	Ells
Lipsich	1	4	for Silks and Linnen	105	Ells
			some say but $104\frac{1}{2}$		
			Cloth	120	Ells
Lisbon	1	10		62	Varras
			by some but 60		
			also	83	Varras
			and for Silks	100	Covados
Lisle	1	8	as *Cambray*		
London	1	2	for Linnen with the Palme and Thumb measured into it	60	Ells
			Woollen, the Thumb	75	Yards
			Frize at $1\frac{1}{2}$ for a Yard	50	Goads
			some say	59	Goads
			Roan Canvas, whereof the Centener is 120, being 10 Cords, of 12 Ells to a Cord	61	Ells cords
Loo, *or* Lowe, Louvaine	1	8	as *Bolduc*		
Lubeck	1	1	as *Collen*		
Luca	1	7		120	Braces
Lyons	1	3	for Linnen	60	Ells
			for Silks	$94\frac{1}{2}$	Ells
Maestricht	1	8	as *Acon* in the *Netherlands*		
Malaca, *or* Malaga	1	14	as *Adler*		
Mantua	1	7	as *Ancona*		
Maroco, *or* Moroco	3	2	as *Capo d' Algier*		
Marseilles	1	3	for Woollen Cloth	$33\frac{1}{2}$	Cannees
			for Silks	36	Cannees
Masiers	1	8	as *Cambray*		
Masilla	1	7		$34\frac{1}{2}$	Cannes
Meanen	1	8	as *Cambray*		
Melvyn	1	9	as *Dantsicke*		
Meydeborgh	1	4	as *Halle*		
Meysen	1	4	as *Lipsich*		

Middleburgh

100 Ells of Antwerpe *compared with the Measure of other Places.*

The 100 Ells of *Antwerp* make at

Middleburgh	1	8	in the Market for Linnen	$94\frac{1}{2}$	Ells
			otherwise	100	Ells
Millan	1	7	for Linnen	120	Braces
			for Silks	141	Braces
Mirandula, Modena	1	7	as *Ancona*		
Munster	1	4		65	Ells
Namen	1	8	as *Acon* in the *Netherlands*		
Nantes	1	3	as *Abbeville*		
Naples	1	7		116	Cannes
				$33\frac{1}{2}$	Cannes
Narva	1	15		125	Arsins
Negropont	1	7	as *Ancona*		
Nigropont	1	5	as *Lepanto*		
Norenborgh	1	4	as *Lipsich*		
Ockermonde	1	4		106	Ells
Offner	1	4		119	Ells
				130	Ells
Orsies, *or* Orchis	1	8	as *Cambray*		
Osenbridg, *or* Osenborgh	1	4		63	Ells
Overyssel	1	8	as *Gelderland*		
Padua	1	7	for Cloth	$101\frac{2}{3}$	Braces
			for Silks	$83\frac{1}{2}$	Braces
Palermo	1	7	as *Masilla*	$34\frac{1}{2}$	Canes
			One Cane, 4 Pichy		
Paris			*France*, and most part of all that Kingdom	59	Ells
			According to others	57	Aulnes
Parma	1	7		91	Braces
				$109\frac{1}{2}$	Braces
Perato	1	7	as *Ancona*		
Pesaro	1	7	for Cloth	107	Braces
			for Silks	103	Braces
Picardy	1	3	as *Abbevile*		
Piran	1	7	as *Istrica*		
Prague	1	4	as *Bautson*		
Provence	1	3		36	Cannes
Puglia	1	7	for Cloth	31	Cannes
			for Silks	33	Cannes
Raguza	1	13	as *Luca*		
Rama	2	8		115	Pichy
Ravenna	1	7	as *Corfu*. Some say	113	Braces
Rechanati	1	7	as *Bergamo*		
Regenburgh	1	4		$78\frac{1}{2}$	Ells
Revel, *or* Rivaile	1	15	as *Coningsbergh*		
Rhode	2	1	as *Adler*		
Riga	1	9	as *Coningsbergh*		
Rochel	1	3	as *Paris*		
Rome	1	7		33	Cannes
			for Woollen Cloth	$105\frac{1}{2}$	Cannes
Romerswal	1	8		99	Ells
Rostock	1	4		119	Ells
Rouen	1	3	The Centener of Ells being 112, that is 28 to a quarter	58	Ells
			According to some	52	Aulnes
Salonici	1	5		109	Pichy
Sapi	1	5	as *Archipelago*		
Saragossa	1	14		33	Cannes
Scio, *or* Sio	2	1	as *Corfu*		
Scotland, most part of that Kingdom, where they reckon 120 to the 100				72	Ells
Sebenico	1	13		112	Braces
Sevil, *or* Sivil	1	14	as *Andaluzia*		

100 Ells of Antwerp *compared with the Measure of other Places.*

The 100 Ells of *Antwerp* make at

Sicilia	1	7	as *Palermo*		
Sluys	1	8	as *Antwerp* for Silks		
Stetin	1	4	as *Ockermonde*		
Stockholme	1	15	as *Conninsbergh*		
Toledo	1	14	as *Castile*. *Hunt* saith	88	Varras
Tournay	1	8		108	Ells
Treviso	1	7	as *Bergamo*		
Tripoli	3	2	as *Alexandria*		
			1 Cane, 4 Pichy		
Tripoli	2	11		112	Pichy
Valentia	1	14		73	Cannes
Venice	1	7	as *Istrica*		
Vere	1	8		94 $\frac{1}{2}$	Ells
Verona	1	7	Long Measure	86	Braces
			Short Measure	104 $\frac{1}{2}$	Braces
			For Cloth of Gold	108	Braces
Vincenza	1	7	for Woollen Cloth	98 $\frac{3}{4}$	Braces
			for Silks	80 $\frac{1}{2}$	Braces
Vienna	1	4	for Linnen	77 $\frac{1}{2}$	Ells
			for Cloth and Silks	85 $\frac{1}{2}$	Ells
Ulme	1	4		120	Ells
			Woollen Cloth	96	Ells
Urbin	1	7	as *Bergano*		
Winockxborough	1	8	as *Bolduc*		
Wismar	1	4		118	Ells
Ysenghem			as *Antwerp* for Silks		
Zara	1	13	as *Sebenico*		
Zurich	1	4		116 $\frac{1}{2}$	Ells

Concave Measures.

Forrain Concave Measures have had the same fate as Long Measures to differ in Names and Quantities with most Nations; and by the aforesaid Authour have like Contents and Correspondencies as follow.

Forreign Measures of Wine *and* Oyle.

Fother *of Wine the Content.*

In *Germany* they call the Carriage of the drawing of 2 Horses, a Fother of Wine, and accompt 2 $\frac{1}{2}$ Rods for a Fother.

Rod *of Wine how much.*

At *Dort* in *Holland*, they call a great Vessel 10 Feet Square, and one Foot deep: A Rod of Wine, every such Foot containing 7 $\frac{1}{2}$ Gallons *Antwerp*, every Gallon called there *Stoop*, weighing 6 ℔.

Ame *of* Dort *the Content.*

An Hogshead of Wine *Dort* Measure, called an *Ame*, contains 100 Gallons or *Stoopen*; and every Gallon 10 *Schreaves*.

Some German *Measures.*

Comenius in his *Janua Aurea*, mentions among the *German* Measures besides the Fother, Half *Eymer*, and *Eymer*, 3 sorts of *Maasz*, 1 of 24 *Kannes*, 1 of 12, and another of 3, together with the *Noesel*, Half *Noesel*, and *Drittentheil*, *&c.* but gives no accompt of the particular Contents thereof.

Malines affirms at *Meysen* in *Saxony* 20 ℥ Ponderal, make 24 Mensural. And at *Lipsich* 32 ℥ Mensural 26 $\frac{2}{3}$ Ponderal, but at the rate of 6 Mensural for 5 Ponderal, it should be 26 $\frac{2}{3}$.

Hogshead *the Weight and Content.*

Pag. 30. he writes that an Hogshead of Wine weighs 500 *l.* the Cask 50 *l.* Wine *Netto* 450 ℔. An Hogshead of Corn 400 ℔. Cask 50 ℔. Corn *Netto* 350 ℔. so shall the Ton of Wine *Netto* be 1800 ℔. with the Cask 2000 ℔. of Corn but 1600 ℔. with the Cask. Four Hogsheads going to a Ton, and 2 Tons to a Last.

Milliar *of Oil.* Ame *of* Antwerpe *how much.*

Pag. 31. A Millier of Oyle at *Antwerp* 1100 ℔. a Butt 152 Stoopes.

One Ame of *Antwerp* containes 300 Stoops, every Stoop weighing 6 ℔. called a Stone. And 6 of these Ames of Wine make in

6 Ames *of* Antwerpe *compared with the Measure of other Places.*

1	10	*Algarve*	34	Starre
		Ansoy, or *Bastard Spain*	2	Pipes 16 Stoopes
1	8	*Artois*	4 $\frac{3}{4}$	Hogsheads
1	3	*Auxere*	3	Puncheon
1	3	*Ay*, as *Artois*		
1	7	*Bolognia*	13	Corbes
1	3	*Bourdeaux*	4 $\frac{1}{2}$	Hogsheads

1 7 *Calabria*

1 7 *Calabria* —————— 8 Salmes

3 3 *Canaries* —————— 2 Pipes of 150 Stoope, or 1 ^{9}. Butt. Every Butt at *Antwerp* 158 Stoopes. They meaſure by the Roove of 30 ℔, which at *Antwerp* is 5 Stoopes. Every Butt containes 30 Rooves. And the Pipe 30 Rooves of 28 ℔ weight.

Ames *of* Antwerp *compared with the Meaſures of other Places.*

1 5 *Candia* —————— 80 Moſtaches

Canado, or *Condado Spain* —————— 2 Butts

1 3 *Coniac* —————— 2 Pipes or 4 Hogſheads

1 5 *Conſtantinople* —————— 180 Almes

96 $\frac{1}{2}$ Almes of Oyle there is at *Venice* a Milliar.

1 5 *Corfu* —————— 37 Zare, or Sare.

1 7 *Ferrara* —————— 12 Naſtelli, of 8 Seccheio

1 7 *Florence* —————— 16 $\frac{3}{4}$ Barrels of 20 Fiaſchi or 18 Stoopes *Antwerpe*, 3 Barrels is 1 Starre, and 1 Starre is 54 Stoops *Antwerp*.

1 7 Iſtria —————— 15 Venas

1 3 Liborne —————— 5 $\frac{1}{2}$ Hogſheads

1 10 Lisbon —————— 37 $\frac{1}{2}$ Almudas.

1 Almuda is 1 $\frac{1}{2}$ Roove of Sevil, accompting 8 Sevil Somers or Covados to 1 Roove, that is 12 Cavados to 1 Almuda. Every Covado 4 Quartils, or Quarts.

Oyl Meaſure is by Alqueri or Canter, 1 Alqueri is 6 Covados, 1 Cantar 4 *Antwerp* Stoopes.

1 2 *London* —————— 252 Gallons.

So is 1 Ame of *Antwerp* 42 Gallons at *London*.

3 3 *Madera* —————— 2 Pipes, lacking 16 Stoops.

1 3 *Orleans* —————— 4 Hogſheads, lacking 10 Stoops, or 60 ℔ of *Antwerp*.

1 7 *Padua* —————— 1 $\frac{1}{25}$ *Cara*.

Oyl is by the Milliar of 1185 ℔.

1 3 *Paris*, as at *Orleans*.

1 Hogſhead 36 Sextiers, 1 Sextier 4 Quarts, 1 Quart 2 Pints, 1 Pint 2 Choppins or Obles, 1 Choppin 24 Poulceons. *Cotgrave* calls that a Muid of Wine which *Malines* calls an Hogſhead, and ſaies 3 of them go to a Ton by the Cuſtoms of *Clermont*. And between the Muid and Sextier in quantity placeth a Barril to contain 9 Septiers or 72 Pints. The Septier he counts all one with the Sextier or Sextary, and when taken for a Wine Meaſure is 8 *Paris* Pints, and in all theſe Veſſels the Cask ſhall hold ſo much clear Wine beſides the Lees. The Pint is the 288 part of the Muid, almoſt as big as our Quart, weighing 27 Ounces. The Chopine in *Latine Ciopina*, or *Ciopinta*, is called *Oble quaſi Obolus* being half the *Paris* Pint, But in *St. Denis* and ſome other places 3 go to a Pint. The Poulceon, a ſmall meaſure of little uſe, ſave to try the Gage of the Small Sextier of 3 ℥, and Semiſextier of 1 $\frac{1}{2}$ meaſures uſed by the *Apothecaries*. The Poulceon 1 ʒ. *Alſted* mentions a *French* Meaſure called *Arroba*, or *Roove* to contain 2 Sextaries. But *Cotgrave* makes it contain as much as will weigh 25 ℔. Others write of a *Congie* containing about 4 $\frac{1}{2}$ Pints of *Paris*. Some an *Amphora* of about 36 Quarts, but both theſe ſeem *Roman* Meaſures, and perhaps may be in uſe in thoſe parts of *France* that border on *Italy*.

1 3 *Poictou* —————— 2 $\frac{1}{2}$ Pipes, or 5 Hogſheads.

1 7 *Piran* —————— 12 Urna.

1 7 *Puglia*, as *Calabria*.

Oyle alſo 8 Salmes, 1 Salme 10 Star, 1 Star 32 Pignatoli.

1 7 *Rome* —————— 7, Brenten.

1 Brent 96 Pockal, (wrote alſo Bocal), or 13 $\frac{1}{2}$ Rubes or Stones of 10 ℔, of 30 ℥, or 42 Stoops of *Antwerp*.

For Honey the Pound is 44 ℥. The *Spaniards* call Bocal, *Azumbre*. 1 Barrillis 32 Bocals. 1 Bocal 4 Foglietta's, that is 128 in the Barrillis.

6 Ames of Antwerp compared with the Measure of other Places.

1 14 *Seres* or *Sherry*, as *Canaries*.

1 14 *Sevill* ———————— $56\frac{1}{4}$ Rooves.

1 Roove, or Arroba, (in *Latine*) 8 Somers or Azumbres, 1 Somer 4 Quartiles or Sextaries. 1 Quartil of a Stoope of *Antwerp*. They deliver 27 and 28 Rooves in a Pipe, but Oyle by 40 and 41 Rooves in a Pipe.

Hunt saith that 32 *Spanish* Sextaries are equal to 24 *Roman*, and that the *Spanish* Sextaries contain $3\frac{1}{2}$ Parillas, which he makes of Oyle $3\frac{3}{4}$ ℥ of Water 4 ℥ *ferè*, of Syrrup 6 ℥ *ferè* accompting $17\frac{1}{4}$ ℥ of Water = $23\frac{1}{2}$ of Syrrup. And mentions the *Modius* or *Moyo* to contain 16 Arroba's or Amphoras. *Heylin p.* 1044 affirms the Arroba of *Spain* to contain 25 Bushels.

1 7 *Treviso* ———————— 11 Consi. the 10. one Cara.

3 2 *Tripoli* ———————— 45 Metares, of 42 Rotules.

3 2 *Tunis* ———————— 60 Matali of 32 Rotules.

1 7 *Venice* ———————— 80 Mostati.

38 make 1 Butt, and 76 an Amphora, $16\frac{4}{5}$ Quarti Besonts Measure. The 4 one Bigontz. Bigonts is a *French* Hogshead 1 Quart 18 Stoopes of *Antwerp*, $15\frac{3}{5}$ Quarti Measure Secchio or small Measure of 4 Tischaufer.

Amphora is 4 Bigonts or Bigontines, 16 Quarti Bigots Measure, $18\frac{1}{2}$ Quarti Secchio. Lagel is a Puncheon, Amphora is 2 Ames. Oyle and Honey some measure by Amphora, but most by the Milliar of 1210 ℔.

Hunt reports the Follieta at *Venice* equal to the *Spanish* Sextarie or Quartil, and to contain in Water or Wine 16 ℥.

1 Congitella 4 Bocals, 1 Bocal 2 Medios.

1 Medius 2 Follieta's, that is 16 in the Congitella.

1 7 *Verona* ———————— $1\frac{1}{2}$ Cara, or 14 Brents.

1 Brent, 16 Basses. Oyle by the Milliar of 1738 ℔, which is 8 Brenten and 11 Basses.

1 7 *Vincenza* ———————— $1\frac{1}{16}$ *Cara*.

Oyle by the Milliar of *Venice*.

1 5 *Zant*, as *Corfu*.

Beer Measures.

Foreign Beer-Measures.

English Barrel the Gallon whereof how many Stoopes. Holland Barrel how much. Lubeck Barrel how much. Dantsick Fat how much.

The Barrel of Beer in *England* 36 Gallons, is 48 Gallons Wine-Measure. Every Beer-Gallon 2 Stoopes in *Flanders*, and at *Amsterdam* $1\frac{3}{4}$ Stoope.

The Barrel of Beer in *Holland* containeth 54 Stoopes, at *Amsterdam* $56\frac{1}{2}$ Stoopes, accompting 60 Stoops there for 64 *Flemish*.

The Barrel of Beer of *Lubeck* is just 50 Stoopes of *Antwerp*.

Foreign Corn-Measures.

Corn Measures. The Last. The Muid.

The Last is differently reckoned, but with the *English* just 2 Tons or 4000 ℔ in dead Weight, reckoning Barley 5 Score to the Hundred.

In *France* several parts of the *Netherlands* and other places, they use a Measure called a Muid, Mudde, Moyo, *&c.* differently according to the Language of the Countrey where used; derived as conceived from *Modius*. This Measure in *France* as used for Land and Wine is spoken to before. The more proper use thereof is for Corn, Coals, Salt, and dry Commodities.

A Table of Corn Measure.

Ordinary French *Corn-Measure according to* Cotgrave.

	Septiers.	Mines.	Minots.	Boisseaux.	Quarts.	Pints
Muid.	12	24	48	144	576	1152
Septier.		2	4	12	48	96
Mine.			2	6	24	48
Minot.				3	12	24
Boisseau.					4	8
Quart.						2

A Muid

A Muid of Coales is 16 Mines.

Muid of Coales.

Hunt, on whose Authority I cannot say, counts the Common Measure of *France* to be 1 Muid, 24 Boisseaux, and that the same is about 18 Bushels of our Water Measure, and 32 of these Muids go to the Hundred: But by another sort of Measure he calls *Oldron*, 1 Hundred is 20 Tons, or 36 Muids.

A Septier of Coales and Oates 21 Boisseaux, though of Wheat but 12, as above, and is said to weigh 220 ℔ by *Nicot*. Neverthelesse *Vigenere* will have the Septier in the Table above to be Rye Measure, and that of Wheat to weigh 240 ℔. The Septier of *Moulin*'s is 16 Boisseaux. The Small Septier is spoken of before as a proper Liquid Measure.

Septier *of Corn.* *The Sorts.*

The Mine above is ordinary. The Mine of *Clermont* double, *viz.* as much as the Septier in the Table.

Mine *of Corn.*

The Boisseaux is about 3 of our Gallons weighing 20 ℔ by *Cotgrave*, and our 3 Gallons 21 ℔ *Avoirdupois*.

Boisseaux *how much.*

Some report a Bichot to be a Corn Measure used in *Burgundy*, and to be 2 Mettres, and 1 Mettre to contain 2 ½ of that Countrey Boisseau.

Bichot *and* Mettre.

The *Russia* Chetfird is about 3 *English* Bushels, as *Fletcher* affirmes.

Chetfrid *of* Russia.

The Laste, or Last of *Amsterdam* is 27 Moyes or Mudden. 1 Mudde, 4 Scheppels. Or 1 Last is 29 Sacks, 1 Sack 3 Achtelings, or Archtelings.

Laste of Amsterdam *the content how much in several other places.*

This Last of *Amsterdam* maketh in the following places, as at

1	8	*Antwerp*——	37½	Vertules.
1	3	*Bourdeaux*——	38	Boisseaux, whereof 33 to the Last.
1	8	*Bridges*——	17½	Hoot.
1	8	*Brussels*——	10½	Mudden, by *Malines* the Mudde or Vertule there is One.
1	3	*Calais*——	18	Rasiers, agree with *England*.
1	9	*Conningsbergh*——	$\frac{6}{7}$	of a Last, 6 Last there being 7 at *Amsterdam*.
1	1	*Coppenhagen*——	23	Small Barrels, whereof 42 make a Last.
2	1	*Cyprus*——	40	Medimnos, 1 Medimnus 2 Cipros.
1	9	*Dantsick*——	56	Scheppels, whereof 60 make a Last.
			There 4	Scheppels are 1 Mudde, which is the Skippound of 340 ℔.
1	8	*Delfe*——	87	Achtelings.
1	8	*Dort*——	28	Sacks.
1	8	*Dunkirk*——	18	Rasiers, Water-Measure
1	1	*Ebbeltorff*	23	*Danic* Barrels of 36 to the Last.
				Some allow 42 to a Last.
1	4	*Embden*——	55	Werps, whereof 61 make a Last, or 15' of 4 Werps,
				Malines 15½, but then should the Last be 62.
1	8	*Enckhuysen*——	42	Sacks.
1	1	*Fan.eren*——	78	Scheppels, whereof 96 to the Last.
1	8	*Gaunt*——	4	Muddes and 7 Halsters, or 55 Halsters.
				One Mudde there is 12 Halsters.
1	7	*Genoa*——	23½	Mina.
2	6	*Goa*——		Their Bharo for Pepper is 3½ Quintals of *Portugal* Weight every Quintal 100 ℔. For Wheat, Rice, and other dry things 1 Candil is 20 Mao's or Hands, 24 Medida's 1 Mao, and 1 Medida 9 ℥.
1	8	*Groeninghen*——	33	Muddes.
1	1	*Hamborough*——	83	Scheppels, 90 whereof make a Last.
1	1	*Heyleger-haven*-	80	Scheppels, of 96 to the Last.
1	1	*Horne*, as *Enckhuysen*.		
1	10	*Lisbon*——	225	Alquiers, of which 240 to a Last, or 4 Moyo's of 60 Alquiers to the Moyo, and so in the *Portugal* Islands.
1	2	*London*——	10¼	Quarters, or 82 Bushels.
1	1	*Lubeck*—— }	85	Scheppels, whereof 96 to the Last.
1	4	*Mechelborough* }		
1	8	*Medenbiick*, as *Enckhuysen*.		
1	9	*Melvyn*——	[illegible]	of a Last.
1	8	*Middleburgh*——	40	Sacks 41 ½ to the Last in all *Zeland*.
1	1	*Nelleboghe*——	23	Barrels, whereof 42 to the Last.
1	7	*Puglia*——	32	Cara, of 36 Timani.
1	9	*Riga*——	42	Loops.
1	3	*Rochell*——	128	Bushels, 4 to every Sestier.
1	4	*Rostock*, as *Lubeck*		

Laſt of Amſterdam how much in ſeveral other Places.

1	8	*Rotterdam*, as *Delſe*.		
1	3	*Rouen*——	20	until 30 Mines. 1 Mine 4 Boiſſeaux.
1	8	*Schonehaven*——	88	Achtelings.
1	14	*Sevill*——	54	Hanegas. A Laſt there is 4 Cahis. 1 Cahi is 12 Hanegas.
1	7	*Sicilia*——	38	Medimno's, of 6 Moyo's.
1	4	*Stetin*, as *Coningsbergh*.		
1	15	*Sweden*——	23	Barrells.
1	8	*Texel*——	58	Loops.
1	7	*Venice*——	32	Starr.

Salt Meaſures.

Foreign Salt-Meaſures.

Sack of Armuyden, how much in ſeveral other Places.

At *Armuyden* in *Zeland* they reckon $8\frac{2}{3}$ weighes for a Hundred, 1 Weigh $11\frac{1}{2}$ Sacks, 1 Sack 4 Meaſures. And 15 Weighes of *Bruwage* Salt make the great Hundred.

The Sack of Salt of *Armuyden*, being 122 ſmall Barrells for the 100, make in the other places, as at

1	8	*Amſterdam*——	102	Scheppels
1	8	*Antwerp*——	144	Vertels of 24 to the Laſt, and 6 Laſt to the 100. But White Salt is meaſured by a Leſs Meaſure of 12 on the 100.
1	8	*Axells*——	102	Meaſures.
1	8	*Bruges*——	104	Meaſures.
1	3	*Bruwage*——	$\frac{4}{7}$	parts of 100 of 28 Moyo's, 1 Moyo, 12 Sacks, and by the Load, 10 Load in the 100, and 48 Moyos or Muys to the laſt, or 21 Barrels.
1	14	*Cadiz*——	22	Cays, or Caies.
1	3	*Calais*——	130	Barrels 19 to the Laſt, but 20 by Freighting.
1	8	*Damme*, as *Axells*.		
1	1	*Denmark*——	$6\frac{2}{3}$	Laſt.
1	8	*Deventer*, as *Amſterdam*.		
1	8	*Dunkirk*——	92	Water-Meaſures, but 104 Land-Meaſures.
1	4	*Embden*——	100	Barrels of 14 to our Laſt.
1	8	*Gaunt*——	108	Sacks, or Barrels.
1	1	*Hamburgh*——	7	Laſt, whereof 80 Barrels make the 100.
1	8	*Ipre*——	144	Meaſures.
1	10	*Lisbon*——	25	Moyo's.
1	2	*London*——	$7\frac{1}{2}$	Laſt of 18 Herring Barrels, but by Weys, $11\frac{1}{2}$.
1	1	*Lubeck*——	7	Laſt of 18 Barrels.
		Mary Port——	28	Moyo's.
1	8	*Oſtend*——	98	Meaſures.
1	7	*Piram*——	70	Moſe.
1	8	*Rotterdam*——	100	Meaſures, whereof 6 make 1 Mudde of 18 to the 100.
1	3	*Rouen*——	$16\frac{1}{2}$	Muys, and ſo almoſt all *France*.
1	14	*St. Lucar*——	21	} Cays, or Caies.
1	10	*St. Tubal*——	20	}
1	15	*Sweden*——	112	Tunnes, or Barrels of 16 to the Laſt.
1	7	*Venice*, as *Piran*.		
1	8	*Utrecht*, as *Amſterdam*.		

Seacoale Meaſures.

Foreign Sea-Coal-Meaſures.

Laſt of Newcaſtle how much at ſeveral other Places.

The Laſt of *Newcaſtle* Sea-coales is $7\frac{1}{2}$ Chalders, which *Malines* ſaith at *London* and *Yarmouth* make 10 Chalders. Some ſay more, beſides the allowance at *London* of 21 for 20.

The ſame Laſt is at

1	8	*Alſt*——	200	Muddes.
1	8	*Amſterdam*——	$13\frac{1}{2}$	Hoot of 38 Meaſures to an Hoot, or Hoet.
1	8	*Antwerp*——	175	Vertels.
1	8	*Bruges*——	100	Meaſures for Oates.
1	3	*Condet*——	44	Muys, 80 Muys make 1 Cherke
1	8	*Dort*——	12	Hoot, alſo by Weighs of 144 ℔. one Weigh 24 Stone, 1 Stone 6 ℔.

1 8 *Gaunt*

1	8 *Gaunt*————	144	Sacks, or 14 Muys.
1	8 *Middlebargh*, by		Weighs of 180 ℔ to a Weigh.
1	8 *Ostend*, as *Bruges*.		
1	3 *Rouen* ————	100	Barrels, allowing 104 for the 100.
1	8 *Zeland*————	68	Herring Barrels.

Laste *of* New-castle *how much at several other Places.*

Foreign Weights.

Generally 3 sorts of *Weights* are used for Merchandise.

Foreign Weights of 3 sorts.

1. Weights of great Content, as Hundreds, Kintalls, Centeners, Talents, Thousands, Weighs, Skippounds, Charges, Lispounds, Rooves, *&c.*

2. Weights of lesser Content, as Pounds, Mina s, Manehs, Rotuli, *&c.*

3. Small Weights, as Ounces of 12, 14, 16, 18, 20, 30, *&c.* to the Pound, and the Subdivisions of the Ounce.

Talents, of the *Hebrews*, *Greeks*, and *Latines* are seen before. Talents,

Cantars, Centeners, or Kintals sometime wrote Quintals, accompted by Merchants as Hundreds; are of 100, 112, 120, 125, 128, 132, and 140 Pounds. Cantars, &c.

Weighs, or Weys, are commonly 165 ℔, or 180 ℔, or 200 ½ ℔ for a Charge. Weighs, &c. Skippounds, used in many places *quasi Shippound*, or *Shippond*, for as in *Italy* and other Countreys the *Carga*, *Cargo*, or *Charge* is the Loading of an Horse of 300, or 400 ℔. So is the Skippound taken for the Divident of a Last of Corn Laden in a Ship. Skippounds are of 300, 320, 340, and 400 ℔ to the Skippound. *Cargo* is often taken for the whole Lading or Burden of a Ship. Skippounds. Carga's.

Lispounds, of 15, 16, and sometimes 20 ℔ to the Lispound. Lispounds.

Rooves, or Arrobas of 10, 20, 25, 30, and 40 ℔ to the Roove. Rooves.

Stones, of 6, 8, 10, 14, 16, 20, 21, 24, 32, and 40 ℔ to some Stones. Stones.

Poade, of *Russia* by *Heylin* 1, 40 ℔. Poade.

Mixias, is commonly understood to be 10000 Drams, of 8 to 1 ℥, and 12 ℥ to 1 ℔. Mixias.

Sestertio's of *Cleopatra* in *Ægypt* and other places in *Africa*, were 2 ½ ℔, for 50 Sestertio s made 125 ℔, but in *Thracia* it was but 2 ⅙ ℔. Sestertio of Africa, &c.

Pound is divided into more or less Ounces. Pounds.

Mark Weight, commonly 8 ℥. Markes.

Mark Pound 16 ℥, that is 2 Markes. Mark Pound.

Mina Ptolomaica, 1 ½ *Rotuli*, or 18 Ounces, or 144 Drams, and in lesser Divisions thus.

A Table of the Mina Ptolomaica.

	Rotuli.	Ounces.	Drams.	Scruples.	Oboli.	Lupines.	Siliquas. *or* Carrats.	Aereoli.
Mina.	1½	18	144	432	864	1296	2592	6912
	Rotulus.	12	96	288	576	864	1728	4608
		Ounce.	8	24	48	72	144	384
			Dram.	3	6	9	18	48
				Scruple.	2	3	6	16
					Obolus.	1½	3	8
						Lupine.	2	5⅓
							Siliqua, *or* Carrat.	2⅔

Mane, or Maneh, in *Arabia*, double 1 of 16 ℥, and 1 of 20 ℥.

That called *Alialica*, *Basaria*, *Alanthalica*, and *Aegyptia*.

This *Romana*, and is indeed of *Alexandria*, the Pound there being 20 Ounces.

Maneh *of* Arabia *the Sorts and Names.*

Rotulus, in *Arabia*, *Syria*, *Asia minor*, *Ægypt* and *Venice*, reckoned for a Pound is thus divided. Rotulus.

Rotulus,

A Table of the Arabian Rotulus.

	Sachosi, or Ounces.	Sextaries, or Cicles.	Deniers, or Aureos.	Darchiny, or Drams.	Scruples, or Garma.	Obolos, or Orloffs.	Danigs, or Lupines.	Kirats, or Siliquas.	Aereola's, or Kestuffs.
Rotulus, *or* Pound.	12	24	84	96	288	576	864	1728	3450
Sacros Sachos *or* Ounce.		2	7	8	24	48	72	144	288
Sextarie *or* Cicle			$3\frac{1}{2}$	4	12	24	36	72	144
Denier *or* Aureus Aunius Audanakus				$1\frac{1}{7}$	$3\frac{3}{7}$	$6\frac{6}{7}$	$10\frac{2}{7}$	$20\frac{4}{7}$	$41\frac{1}{7}$
Dram *or* Darchiny Alky Oliginat					3	6	9	18	36
Scruple Garme *or* Kenmer						2	3	6	12
Obolus Orloff *or* Onolaffat Onolum							$1\frac{1}{2}$	3	6
Daning *or* Danic Lupine								2	4
Carrat Kirat *or* Siliqua									2

Physick Pound at Venice. Lupines *there.* Kestuff, *how much.* *Pound of* Alexandria. Italian *Pound.*

Some mention the Physick Pound at *Veniee* to have but 7 Drams in the Ounce.

The Lupines at *Venice* called Sextula's, because 1 ℥ hath 72, which is 6 times 12.

Every Kestuff, or Aereolum (or Areolum) is the Weight of 2 Barley-Cornes, so is there in the Rotulus 6912 Graines.

The *Alexandrian* Pound 20 ℥, the Ounce 8 ʒ, *&c.*

The *Italian* Pound generally is divided into 12 ℥, 1 ℥ into 2 Staters, and 1 Stater into 4 Drams; so hath 1 ℔ 24 Staters, 96 Drams.

But in Physick there, and in other Places thus.

A Table of the Italian *Physick Pound.*

	Ounces.	Loots.	Sizaynes, or Siliqua's.	Drams.	Scruples.	Obolos.	Siliqua's.	Graines.
Pound.	12	24	48	96	288	576	1728	5760
Ounce.		2	4	8	24	48	144	480
Loot.			2	4	12	24	72	240
Sizayne *or* Siliqua				2	6	12	36	120
Dram					3	6	18	60
Scruple						2	6	20
Obolus							3	10
Siliqua								$3\frac{1}{2}$

Spaine,

Spanish Weights.

Spain, some say hath a *Mina Romana*, which contains 20 ℥. A Common Pound of 16 ℥. and a Physick Pound of 12 ℥. each Ounce divided into 8 Drams. The Ounce of the *Toletan* Physick Pound excepted, which hath 9 ʒ, as some affirm.

Pound Weights of Spain.

A Table of the Mina Romana *of* Spain.

	Libra.	℥.	Duels.	Quarterns.	Sixths.	ʒ.	Syrian Beans.	℈.	Obolos.	Carats.	Chalcos.	Graines
Mina Romana	$1\frac{2}{3}$	20	60	80	120	160	240	480	960	2880	5700	11520
	Libra	12	36	48	72	96	144	288	576	1728	3456	6912
		℥	3	4	6	8	12	24	48	144	288	576
			Duel	$1\frac{1}{3}$	2	$2\frac{2}{3}$	4	8	16	48	96	192
				Quartern	$1\frac{1}{2}$	2	3	6	12	36	72	144
					Sixth	$1\frac{1}{3}$	2	4	8	24	48	96
						ʒ	$1\frac{1}{2}$	3	6	18	36	72
							Syrian Beane	2	4	12	24	48
								℈	2	6	12	24
									Obolus	3	6	12
										Carat	2	4
											Chalcus	2

Tables of the Pounds of Spain.

The Common Pound of Spain.

	Marks.	Ounces.	Drams.	Adarmes *or* Adarames
Pound	2	16	128	256
	Mark	8	64	128
		Ounce	8	16
			Dram	2

The Physick Pound of Toledo.

	Ounces.	Drams.	Scruples.	Graines.
Pound	12	108	324	6480
	Ounce	9	27	540
		Dram	3	60
			Scruple	20

Pound Weights of France.

Pounds of France.

The Weight used by the Merchants for the most part is of 16 ℥, called *Liure d' Anvers*, though in some places but 14, others 18 ℥. *Cotgrave* writes the *Liure* or *Pound de Lyon* to be 15 ℥. that *de Spaigne* but 14 ℥. And divides the Pound of 16 ℥. into 32 Halfes, 64 Sezaines, 128 Treseaux, 256 Gross, 512 Demigross. And the Pound used by the *Farriers* consisting of 12 ℥ into 90 Drams, 270 Scruples, 540 Oboles. After *Malines* the Ordinary, or Pound commonly used for *Merchants* is parted thus.

Tables of the Pounds of Paris *and* Lyons.

The Pound Weight of Paris.

	Ounces.	Grosse.	Scruples.	Graines.
Pound	16	128	384	9216
	Ounce	8	24	576
		Grosse	3	72
			Scruple	24

The Physick Pound of Lyons.

	Ounces.	Drams.	Scruples.	Graines.
Pound	12	96	288	5760
	Ounce	8	24	480
		Dram	3	60
			Scruple	20

Cotgrave mentions a Weight called *Sextule* of 4 Scruples, or the Sixth part of 1 ℥. Sextule, *the Weight.*

Pounds of Germany.

Tables of the Pound of Vienna *and Physick Pound.*

Pound Weights of Germany.

The Pound Weight of Vienna *in* Austria.

	Ounces.	Loots.	Quints.	Pennings.	Grains.
Pound	16	32	128	512	12800
	Ounce	2	8	32	800
		Loot	4	16	400
			Quint	4	100
				Penning	25

The German *Physick Pound, by* Alsted.

	Ounces.	Drams.	Scruples.	Graines.
Pound	12	96	288	5760
	Ounce	8	24	480
		Dram	3	60
			Scruple	20

Pounds of the Low Countries.
Pounds of Bridges.

In the *Low Countries* they use Pounds of 12, 14, 15, *&c.* Ounces.

At *Bridges* in *Flanders* they have 1 ℔ of 14 ℥, and 1 ℔ of 16 ℥. The 100 ℔ of 16 ℥ make 108 ℔ of 14 ℥, but the Ounces of 14 to the ℔ are the heaviest for 100, of these are 105 $\frac{1}{3}$ Ounces of 16 to the Pound, This ℔ is thus divided.

Table of the Bridges *Pound.*

	Ounces	Loots	Sizaines	Drams.
Pound	16	32	64	128
	Ounce	2	4	8
		Loot	2	4
			Sizaine	2

Dram, *or* Quint.

Weights at Antwerp.

At *Antwerp* they use to weigh by the Hundred Pounds even Weight, called *Suttle*, for which commonly at the Weigh-House is allowed 101 ℔. A Stone is 8 ℔. The Skippound 300 ℔. The Weigh 165 ℔. The Carga or Charge 400 ℔. which is two Bales of 200 ℔ each for an Horse to carry. The Pound there is 16 ℥.

This 100 ℔ of *Antwerp* weigheth in the Places following,

The 100 ℔ *at* Antwerp *how much at divers other Places.*

1 3 *Abbevile* —— 94 $\frac{3}{4}$ ℔.
2 11 *Achri* —— 17 $\frac{3}{4}$ Rotuli. The 100 A Cantar Tambaran.
Adler —— { 138 ℔ Ordinary Weight. / 91 ℔ To weigh Steel, Tinne and Copper. }
1 8 *Ailst* —— 108 ℔.
3 1 *Alcario* —— { 164 ℔. / 78 Minas of 16 ℥ to the Mina. / 27 Rotuli of 6 ℔ to the Rotuli. }
1 Peso is 1 $\frac{1}{2}$ Metallicum, or a Dram.
50 Metallici 1 Mark. Our Mark 42 Metallici.
Musk and Amber sold by this Weight in *Ægypt*.
2 11 *Aleppo* —— 22 Rotuli of 100 to a Cantar.
1 Rotulus is 60 ℥, or 480 Metecalos or ʒ.
1 ℥ is 8 Metecalos or Dragmes.
1 ʒ or Metecalo is 1 $\frac{1}{2}$ Peso.
10 Peso's are 1 Onga, or Ongia, to weigh Civet.
2 1 *Alexandria* —— { 108 Rotuli, of 100 to a Cantar. / 78 Minas of 20 Ounces. }
America Malica - { 90 ℔, of 12 Ounces to the ℔. / 36 Minas Sestertias of 30 ℥. }
2 11 *Aman*, as *Aleppo*.
1 8 *Amsterdam* —— 94 $\frac{4}{5}$ ℔. And for Silkes they use the Weight of *Antwerp*.
1 7 *Aquila* —— 147 ℔.
1 3 *Aquismort* —— 102 ℔.
2 2 *Arabia* —— { 78 Rotuli. / 104 Maires, *or* Minas. / 148 Pounds. / 936 Ounces, or Sachosi 12 to 1 Rotulus. }

3 1 *Arcadia*

The 100℔. *at* Antwerp *now much at divers other Places.*

3	1	*Arcadia*	92℔, and 83 ℔ for Mavigetto.
1	5	*Archipelago*	120℔.
		Armaria bona	105℔. of 16 ℥ to the ℔.
			93℔. of 18 ℥ used for Silk and Copper.
			54℔. of 32 ℥ Flesh Weight.
2	3	*Armenia*	130℔.
1	14	*Arragon.*	106℔.
			96℔. Great Weight for Wooll.
1	8	*Arschot*	100℔. all one with *Antwerp*
1	8	*Audinarde*	110℔.
1	3	*Avignon*	111℔. A Centener is 2 Frailes of 56 ℔.
1	4	*Ausburgh*	95℔.
1	8	*Barrow Op Zome*	98℔.
1	14	*Barcelona*	95℔. Wooll Weight.
			106℔. Common Weight.
			131℔. Saffron Weight.
1	4	*Basil*	96℔. They use Centeners of 100℔, 120℔, and 132℔.
1	7	*Bergamo*	137℔. and 108℔. by the 2 Quintals.
1	1	*Bergen*	96℔. but uncertain weighing with a Sling.
1	4	*Bibrach*	92℔. of 16 ℥ to 1 ℔ as Constance.
2	11	*Barutti*	21 Rotuli.
1	7	*Bolognia*	53℔. of 30 ℥ to weigh Wax and Wooll by Rooves of 1℔.
1	3	*Borgoingne*, as *Abbevile*.	
1	4	*Botsen*	138℔. Ordinary Weight.
			91℔. To weigh Steel, Tinne, and Copper.
1	3	*Bourdeaux*, as *Abbevile*.	
1	4	*Breslau*	120℔. by the Centener of 24℔ to 1 Stone, and 5 Stone to 1 Centener. And 5½ Stone to the Centener of 132 ℔ there also used.
1	7	*Brescia*	184℔. and for *Venice* Gold 136 ℔.
1	8	*Bridges*	100℔.
			93℔. for Butter and Cheese, The Stone 6 ℔. and 20 Stone 1 Weigh, but Wooll Weight is 108 ℔. weighed by Stones of 6 ℔, called Nailes or Neiles. 18 Neiles to the Hundred, 45 Neiles to the Weigh, 2 Weighs to 1 Pocket of Wooll. *Hunt* saies 18 Neiles is 144 ℔ of our Wooll Weight.
1	8	*Brussels*, as *Arschot*.	
		Bucca	44 Ocha's.
1	14	*Burgos*	93 Rotuli.
2	1	*Bursa*	88 Rotuli.
3	4	*Cabo verde*	107½℔, or Rotuli, A Quintal is 128 ℔ of 4 Rooves of 32℔.
1	7	*Calabria*	147 ℔.
1	3	*Calais*	111 ℔. Ordinary Weight.
			92 ℔. Merchants Weight.
			114 ℔. *English* Wooll Weight.
2	6	*Calicut*	80 Aracoles. *Malines p.* 18. mentioning the Baccar or Bahar at *Calicut* to be at *Lisbon* 4 great Quintals of 112 ℔ to the Quintal, and that the 4 Quintals are 480 Aracoles, that is 120 Aracoles for 1 Quintal. And again that the Bahar is 20 Faracoles, which is 5 Quintals at *Lisbon* of 32 ℔ *per* Roove, is not well to be understood Seeing the great Quintal at *Lisbon* is 128 ℔ or 4 Rooves of 32℔ *per* Roove; whereas 4 Quintals of 112 ℔ is but 448 ℔, and 5 Quintals of 128 ℔ is 640 ℔, unless there be 2 sorts of Bahars at *Calicut*, 1 of 48 Aracoles, and another of 20 Faracoles. Or that the Bahar be 5 great Quintals at 129 ℔ the Quintal, that is 645 ℔ for so many Pounds or *Portuguise* Rotuli are in 480 Aracoles for 100 ℔ of *Antwerp*, which answer to 107½ ℔ of *Portugal* Weight by his own Concession in the same Page a little before.
3	3	*Canary-Islands*	107 ℔. as Sevill.
1	5	*Candia*	138 ℔. for Gold Thread.
			89 Rotuli, whereof 100 is a Cantar or Quintal.

1 7 *Carpi.*

The 100 ℔ at Antwerp how much at divers other Places.

1 7 *Carpi*, as *Aquila*.
1 14 *Castile*——————102 ℔.
Cataio—————— 87 Rotuli 100 to a Cantar.
1 7 *Cesena*, as *Bergamo*.
1 4 *Collen*——.———— $93\frac{1}{3}$ ℔.
1 7 *Como*, as *Aquila*.
1 9 *Coningsbergh*——125 ℔, which is a Centener. A Last of Wheat there 5200 ℔. a Stone 40 ℔, a Skippound 10 Stone, that is 400 ℔.
1 4 *Constance*—————— 92 ℔. of 16 ℥ or 32 Loot. Some by the Centener of 100 ℔, and some of 120 ℔.
1 5 *Constantinople*.——{ $87\frac{1}{2}$ Rotuli, 100 to a Cantar. / 39 Ochaa, *Hunt* writes it *Cohaa*.
$2\frac{1}{2}$ Metallici, which is their Dram, make 3 of ours.
1 1 *Coppenhagen*——— 96 ℔. There the Centener is 112 ℔. A Stone is 10 ℔. A Skippound 32 Stone, or 20 Lispound of 16 Mark Pound which is a Skippound, or 320 ℔.
1 5 *Corfu* ————{ 97 ℔. Great Weight. / 115 ℔. Small Weight.
1 8 *Cortrycke*, as *Audinarde*.
1 9 *Cracon*——————124 ℔. The Centener there is 136 ℔.
1 7 *Crema*, as *Aquila*.
1 7 *Cremona*————{ 143 ℔. of 12 ℥ most used. / 132 ℔ of 12 ℥, being 13 ℥ of the other. / 60 ℔. of 28 ℥ to the ℔. used for Flesh.
2 1 *Cyprus* —————— $20\frac{1}{4}$ Rotuli, 100 to the Cantar.
2 11 *Damascus*————— 26 Rotuli. There 1 Cantar is 5 Zurli, or Stone, and 1 Stone 20 Rotuli, 1 Rivola is 225 ℔. *Antwerp*.
1 9 *Dantsick*——————120 ℔. There 1 Last of Wheat is 4528 ℔. The Last of *Rye* 4245 ℔. 1 Skippound 340 ℔. of 10 great Stone. 1 Skippound 320 ℔. of 20 Lispound. 1 Centener 125 ℔. 1 Stone for Spices 24 ℔. 1 Great Stone for Gross Wares 34 ℔. 1 Lispound 16 Mark Pound.
1 3 *Diepe*, as *Abbevile*.
1 8 *Dixmude*, as *Ailst*.
1 8 *Doway*, as *Audinarde*.
1 6 *Dublin*, and in *Ireland* generally.—{ $91\frac{1}{2}$ ℔. by the Great Hundred. / 104 ℔. Subtle Weight.
1 12 *Edenburgh* and all *Scotland*.————} 96 ℔. and $103\frac{1}{2}$ ℔. for 112 ℔.
1 4 *Erdford*———— 85 ℔. as at *Vienna*.
1 7 *Faenza*——————132 ℔.
3 2 *Fez*, or *Fesse*——— 96 ℔. by *Hunt* wrote *Feas*, and noted as in *Portugal*.
1 7 *Ferrara*, as *Bergamo*.
Fio—————— 96 $\frac{3}{4}$ Rotuli, *or* Scrutarij.
1 7 *Fiume*, as *Venice*.
1 8 *Flanders*——————110 ℔. for the most part, the places herein excepted.
1 7 *Florence*——————125 ℔. of 12 ℥ to the ℔.
3 1 *Forfori*————— 65 Rotuli.
1 7 *Forli*, as *Aquila*.
1 3 *France* generally—111 ℔. except herein excepted.
1 4 *Frankford* } as *Basil*.
1 4 *Friburg*— }
1 8 *Gaunt* as *Ailst*.
1 8 *Gelderland*———— 99 ℔. The Places herein excepted.
1 7 *Genes*, by Rooves to a Quintal of 4 Rooves, and 4 ℔. over.
110 ℔. a Quintal of Pepper.
114 ℔. a Quintal of Ginger.
1 7——————*Geneva*{ 102 ℔. Weight for Spices. A Carga is 270 ℔ small Weight. / 85 ℔. Great Weight.
1 4 *Germany*, A Centener of the small Weights is 100 ℔, of the great 120 ℔. and 132 ℔. The Centener of 120 ℔. is 5 Stone, of 24 ℔. *per* Stone.

2 6 *Goa*,

2 6 *Goa*, as *Portugal* by Quintals, Arrobes or Rooves, *&c.* They have also another Weight called *Mao*, which signifieth the Hand, and weigheth 12 ℔. used for Butter, Honey, Sugar, *&c.* in the *Portugal* Dominions. *The 100 ℔. at* Antwerp *how much at divers other Places.*

1 14 *Granada*, as *Armaria bona*.

3 4 *Guynea*, as *Cabo verde*.

1 1 *Hamburgh* —— 96 ℔. The Centener 120 ℔. of 12 Stone, 1 Stone 10 ℔. A Lispound 15 ℔, and 20 Lispound 1 Skippound.

1 4 *Heidelbergh*, as *Basil*.

1 8 *Hertogenbosh*, as *Arschot*.

1 8 *Holland*, as *Gelderland*.

1 8 *Hulst*, as *Ailst*.

1 8 *Ipre*, as *Ailst*.

1 7 *Istria*, as *Venice*.

1 5 *Laarta* —— 87 Rotuli, 100 to a Cantar.

1 5 *Laconia* —— { 138 ℔.; $78\frac{1}{2}$ Rotuli. }

1 7 *Lansan*, as *Bergamo*

1 5 *Lavalona* —— 131 ℔.

1 14 *Leon* —— 109 ℔.

1 5 *Lepanto* —— { 156 ℔.; 26 Rotuli, 1 Rotulus 6 ℔. }

1 4 *Lipsich*, as *Basil*.

1 10 *Lisbone*, See *Calicut*.

1 8 *Lisle*, as *Audinarde*.

1 2 *London*, and all *England*. —— { $91\frac{1}{2}$ ℔ Gross Weight of the Kintal Weight 112 ℔.; 104 ℔. Subtle Weight; $189\frac{1}{2}$ Markes of 8 ℥ Troy. }

1 8 *Louvaine*, as *Arschot*.

1 1 *Lubeck*, as *Coppenhagen*.

1 7 *Luca*, as *Aquila*.

1 3 *Lyons* —— { 111 ℔. ordinary Weight. A Centener is 112 ℔.; 102 ℔. Almerick, or Weight of *Geneva* for Spices, abating 8 ℔. *per Cent.*; $94\frac{1}{4}$ ℔ by the Kings Weight to pay Custom by.; A Quintal is 100 ℔. A Charge 300 ℔. A Somme 400 ℔. }

3 3 *Madera*, as *Cabo Verde*.

1 8 *Malines*, as *Arschot*.

1 7 *Mantua*, as *Aquila*.

1 3 *Marseilles* —— 111 ℔

3 2 *Maroco*, or *Moroco*, as *Capo Verde*.

1 14 *Medina del Camporas Castile*.

1 9 *Melvin* —— 124 ℔. The Last of Wheat 5200 ℔. The Skippound, and Stone as *Coningsbergh*.

1 4 *Meysen* —— { 100 ℔. of 16 ℥ to the ℔. which is the Princes Weight, called *Zigostatica*.; 96 ℔. Merchants Weight.; 148 ℔. of 12 ℥ to the ℔. }

1 7 *Millan*, as *Cremona*.

1 3 *Mirabel*, as *Aquismort*.

1 7 *Mirandula*, as *Aquila*.

1 7 *Modena*, as *Faenza*.

2 7 *Molucco* —— 88 Rotuli, 112, a Cantar.

1 3 *Montpelier*, as *Avignon*.

1 4 *Munchen*, as *Ausburgh*.

1 7 *Naples* —— 120 ℔. and for *Venice* Gold 134 ℔.

1 15 *Nareca* —— 120 ℔. A Lispound or Stone is 20 ℔. and 20 Lispound a Skippound, that is 400 ℔. used for Rye, but for Wheat but 350 ℔. to a Skippound.

1 7 *Nicosia*, or *Nichosia*, as *Archipelago*.

1 5 *Nigropont* —— 119 ℔.

1 4 *Norenburgh*, as *Constance*.

1 4 *Norlingen*, as *Ausburgh*.

1 4 *Offen*, as *Basil*.

The 100 ℔. *at* Antwerp, *how much at divers other Places.*

3 2 *Oran* — 94 Rotuli, 1 Cantar 5 Rooves, 1 Roove 20 Rotuli.
138 ℔. for Spices. 1 Cantar 4 Rooves.
50 Rotuli for Corne, 1 Cantar 6 Rotuli.
61 Rotuli, for Cotton Wooll, 1 Cantar 15 Rotuli.
1 7 *Otranto*, 1 7 *Padua*, as *Bergamo*.
1 3 *Paris* — 93 ℔. accompting 4 Quarters of 25 ℔ to the Hundred.
1 7 *Parma*, as *Aquila*.
1 4 *Passau* — 87 ℔.
1 7 *Pavia*, as *Cremona*.
1 7 *Piran*, as *Venice*.
1 7 *Piedmont*, 1 7 *Plaissance*, as *Aquila*.
1 4 *Poosen*, as *Breslaw*.
1 8 *Popering*, as *Ailst*.
1 10 *Portugal* — 107½ Rotuli or Araters. The great Quintal is 128 ℔. of 4 Rooves, 1 Roove 32 ℔. The Small Quintal is 112 ℔. of 4 Rooves, 1 Roove 28 ℔. The Quintal of Wax 168 ℔, which is 1½ Quintal of 112 ℔. of 4 Rooves of 42 ℔. the Roove.
1 4 *Prague*, as *Passau*.
1 7 *Puglia*, as *Calabria*.
1 7 *Raguza*, *Raviano*, *Ravenna*, as *Faenza*.
1 7 *Rechanati* — 137 ℔. but to Gold Thread but 112 ℔.
1 4 *Regensbourgh*, as *Passau*.
1 15 *Revell* — 120 ℔ which is a Centener. The Skippound there is 400 ℔.
2 1 *Rhodes* — 19½ Rotuli, A Cantar is 100.
1 9 *Riga* — 120 ℔. A Lispound is 20 ℔. and 20 Lispound a Skippound.
1 7 *Rimano*, as *Faenza*.
1 3 *Rochel* — 111 ℔. and 119 ℔. by the small Weight.
1 7 *Romagna*, as *Naples*.
1 7 *Rome* — 132 ℔.
1 3 *Rouen* — 91 ℔. by the *Viconte*, accounting as at *Paris*.
94¾ ℔. by the ordinary weight, and 4 ℔. *per Cent* over.
1 4 *Saltsburgh* — 111 ℔. Small Weight.
83 ℔. great Weight.
1 3 *St. Antoine* — 127 ℔.
1 8 *St. Omar*, as *Audinarde*.
3 3 *St. Thomas*, as *Cabo Verde*.
1 14 *Saragossa* — 112 ℔. And the small Quintal 131 ℔.
1 7 *Savoy* — 137 ℔.
195 ℔. Small Weight.
1 4 *Saxony*, as *Meysen*.
3 1 *Sciba*, as *Antwerp*, 320 ℔. is there a Skippound.
2 1 *Scio*, as *Fio*.
1 13 *Sequia*, as *Venice*.
1 14 *Sevil* — 107 ℔. The great Quintal is 144 ℔ of 4 Rooves of 36 ℔.
The lesser Quintal is 120 ℔. of 4 Rooves of 30 ℔.
The small Quintal is 112 ℔. of 4 Rooves of 28 ℔.
1 7 *Sicilia* — 152 ℔. of 12 ℥ *per* ℔.
61 Rotuli of 30 ℥ is a Cantar of 24 Sestertios.
54 Rotuli for Flesh by Talents of 12 Sestertio's is 30 Rotuli.
1 4 *Silesia*, as *Breslaw*.
1 13 *Spelato*, as *Venice*.
1 4 *Spiers*, as *Bibrach*.
1 4 *Stetin* — 96 ℔. The small Stone 10 ℔. The great Stone 21 ℔.
The Centener 112 ℔.
1 15 *Stockholme* — 120 ℔. The Skippound 320 ℔. and also 340 ℔.
The Centener 120 ℔. The Stone 10 ℔.
1 4 *Straelsont* — 92 ℔. The Stone 10 ℔. and the Lispound 16 ℔.
3 2 *Suus*, or *Sus*, or *Fez*.

The 100 lb. at Antwerp how much at divers other Places.

2 11 *Syria*———————156 Minos, 1 Mina 100 Drams.
1 8 *Tergos*———————107 lb.
1 3 *Tholouse*, as *Avignon.*
3 2 *Thunes*, or *Tunis*— 63 Rotuli.
1 9 *Thoren*———————120 lb. The Stone is 24 lb.
1 8 *Tournay*, as *Ailst.*
1 7 *Treviso*, as *Bergamo.*
1 7 *Trieste*, as *Venice.*
3 2 *Tripoli*, as *Thunes.*
2 11 *Tripoli*——————— $26\frac{1}{2}$ lb.
1 14 *Valentia*——————— { 106 lb. by Quintals of 4 Rooves of 30 lb for Spices. 134 lb. by Quintals of 4 Rooves of 36 lb. }
The small Carga is 360 lb. that is, 3 Quintals of 120 lb.
The great Carga is 432 lb. that is 3 Quintals of 144 lb.
1 7 *Venice*——————— { $98\frac{1}{3}$ lb. Great Weight, called *Ala Grossa*, used for Flesh, Butter, [illegible], Leather, Dates, Yarne, Copper, Thread [illegible] Oile, Brimstone and Wooll. 156 lb. Small Weight of 12 ℥, called *Ala Sotile*, most used for all Merchandise. }
An Ounce is 6 Sassi, 1 Sassi 24 Carrats, 1 Carrat 4 Graines.
They also accompt by Thousands, *&c.* with allowance of 2 lb. *per Cent.* in the *Custom-House.*
1 Thousand 40 Mixti, 1 Mixti 25 lb.
1 Carga 400, lb 1 Starre 220 lb. The Starre is Mensural.
Starres for Corn 130 lb. Ginger 180 lb. Raisins 260 lb.
The Starre containes 54 Pottles of Wine at *Antwerp.*
1 7 *Veronna*——————— 90 lb. And for Gold Thread 143 lb.
1 4 *Vienna*——————— 85 lb. as at *Erdford*, where also a Summe of Quick-Silver is 275 lb.
1 14 *Villaco*, or *Vellica*— 80 lb.
1 4 *Vlme*, as *Basil.*
3 2 *Vna*——————— { 65 Rotuli for Cotton. 75 Rotuli for Spices. 94 Rotuli for Corn. }
1 7 *Vrbin*, as *Bergamo.*
Wallons Countrey, as *Ailst.*
1 8 *Walsland*, as *Gelderland.*
1 9 *Wilde*, as *Riga.*
1 4 *Wisel*, as *Ausburgh.*
3 1 *Zaidin*——— 77 Rotuli.
1 8 *Zeland*, as *Gelderland.*
3 1 *Zeroi*——————— 50 Rotuli.
1 8 *Zurich Sea*——— 110 lb.

Foreign Weights for Money.

Weights for Money

In *Florence* they use a Weight for Gold and Silver, and at *Geneva* for Silver called a Pound of 12 ℥. 1 ℥ is 24 Deniers, and 1 Denier is 24 Graines. So is there 6912 Graines in the Pound. *Pounds of* Florence *and* Geneva.

In *Naples* their Pound is likewise divided into 12 ℥, and every Ounce into 8 Octany, or Octavos. *Pound of* Naples.

The Mark Weight is used in many other Places, and at *Antwerp* containeth 8 ℥, and is heavier than their ordinary lb. by 5 upon the Hundred, as *Malines* saith. This Mark is divided in a double manner. Mark *of* Antwerp.

Tables of Antwerpe Mark.

	Ounces.	English.	Graines.
(1) Mark	8	160	5120
Ounce		20	640
English			32

	Ounces.	Peny-weights.	Graines.
(2) Mark	8	192	4608
Ounce		24	576
Peny-weight			24

The

A Table of the French Mark.

The Mark Weights of some other Places subdivided.

France.

	Ounce.	Grosses.	Deniers.	Graines.	Primes, *or* Garobs.	Seconds.	Tercies, *or* Mallooquen.
Mark	8	64	192	4608	110592	2654208	63700992
	Ounce	8	24	576	13824	331776	7962624
		Gross	3	72	1728	41472	995328
			Denier	24	576	13824	331776
				Graine	24	576	13824
					Garob, *or* Prime	24	576
						Second	24

How many Carrats *and* Graines *in their Ounce.*

In *France* the Ounce is also divided into 2 Carrats, and every Carrat into 12 Graines.

Tables of Mark *Weights of* Dantsick *and* Geneva.

Dantsick *in* Poland.

	Ounces.	Pence.	Hellers.
Mark	8	256	512
	Ounce	32	64
		Peny	2

Geneva *for* Gold.

	Ounces.	Deniers.	Graines.
Mark	8	192	4608
	Ounce	24	576
		Denier	24

Meysen *and* Norenburgh.

Meysen *in* Saxony.

	Ounces.	Deniers.	Graines, *or* Momenta.
Mark	8	192	4608
	Ounce	24	576
		Denier, *or* Peny	24

Norenburgh *in* Germany.

	Ounces.	Loots.	Quints.	Primes.	Sestertios.
Mark	8	16	64	256	1024
	Ounce	2	8	32	128
		Loot	4	16	64
			Quint	4	16
				Prime, Peny, *or* Numulus.	4

Portugal *and* Venice.

Portugal.

	Ounces	Octavos, *or* Oitavos.	Great Grains.
Mark	8	64	288
	Ounce	8	36
		Oitavo *or* Octavo	$4\frac{1}{2}$

Venice.

	Ounces.	Silicos. *or* Quarts.	Siliquas. *or* Carrats.	Graines.
Mark	8	32	1152	4608
	Ounce	4	144	576
		Quart, *or* Silico,	36	144
			Carrat, *or* Siliqua	144
				4

Spain.

Spain.

Spaine.

Gold.

	Ounces.	Castellanos.	Tomines.	Graines.
Mark	8	50	400	4800
	Ounce	$6\frac{1}{4}$	50	600
		Castellano	8	96
			Tomine	12

Silver.

	Ounces	Drams *or* Octavos.	Graines.
Mark	8	64	4800
	Ounce	8	600
		Dram, *or* Octavo	75

Rome.

Rome.

	Ounces.	Drams.	Scruples.	Obolos.	Siliquas,	Primi, *or* Graines.
Mark	8	64	192	384	1152	4608
	Ounce	8	24	48	144	576
		Dram.	3	6	18	72
			Scruple	2	6	24
				Obolus	3	12
					Siliqua	4

Romana Libra, *by* Malines.

A Table of Romana Libra

Libra.	12.	84	168	336	840	3320	5040.
	Ounces, *or* Guilders.	Denarios.	Victoriatas.	Sestertio's.	Asses.	Quadrantes.	Sexantes.

The Ton of Gold in *Latine*, *Tina*, *seu Tonna*, by some called *Roman*, but by *Alsted*, *German* is thus divided.

Tonne of Gold how called.

A Table of the Tonne of Gold.

	Pounds.	Marks.	Ounces.	Loots.	Drams.
Tonne of Gold.	$781\frac{1}{4}$	$1562\frac{1}{2}$	12500	25000	100000
	Pound	2	16	32	128
		Mark	8	16	64
			Ounce	2	8
				Loot	4

Scotland, divides their Pound into 24 Deniers, 1 Denier 24 Primes, 1 Prime 24 Seconds, 1 Second 24 Thirds, 1 Third 24 Fourths, *&c.*

Pound of Scotland.

The Correspondency of 100 Markes of *Antwerp* to the places following.

The 100 Marks *of* Antwerpe, *how much at some other Places.*

		Adler	$76\frac{1}{2}$	lb.
3	1	*Aegypt*	94	Besses
3		*Africa*	87	Markes
1	7	*Ancona*	$103\frac{1}{4}$	Markes
1	7	*Aquila*	71	lb.
1	4	*Ausburgh*	$105\frac{2}{19}$	Markes
1	4	*Bambergh* } *Bavaria* }	$103\frac{1}{4}$	Markes
1	4	*Bohemia*	87	Markes
1	4	*Bresla*	$121\frac{1}{4}$	Markes

The 100 Marks *at* Antwerp, *how much at some other Places.*

1	14	*Burgas*	$116\frac{2}{3}$	Markes
1	7	*Calabria*	$76\frac{1}{2}$	℔.
1	14	*Catalonia*	100	Markes
1	4	*Collen*	$105\frac{2}{19}$	Markes
1	5	*Constantinople*	87	Markes
1	7	*Crema*	$103\frac{1}{4}$	Markes
1	9	*Dantsick* }	$105\frac{2}{19}$	Markes
1	4	*Erdford* }		
1	7	*Florence*	72	℔.
1	4	*Franconia*	$103\frac{1}{4}$	Markes
1	4	*Frankford*	$105\frac{2}{19}$	Markes
1	4	*Friburgh*	$103\frac{1}{4}$	Markes
1	7	*Genes* for { Gold	116	Markes
		Silver	77	Markes, *or* ℔.
1	7	*Geneva*, as *Paris* and *Lyons*.		
1	5	*Gracia*	$105\frac{2}{19}$	Markes
1	4	*Hungary*	87	Markes
1	4	*Lipsich*	$105\frac{2}{19}$	Markes
1	2	*London*	$89\frac{8}{9}$	℔.
1	3	*Lyons*	{ 112	Markes, Merchants Weight
			{ $102\frac{1}{2}$	Markes, Merchants Weights
				The Kings Weight
1	4	*Ments* }		
1	4	*Meysen* }	$105\frac{2}{19}$	Markes
1	7	*Millan* }		
1	7	*Naples*	$76\frac{1}{2}$	℔.
2	6	*Narsinga*	87	Markes
1	4	*Norenburg*	$103\frac{1}{4}$	Markes
4	2	*Nova Spagnia*	$87\frac{1}{2}$	Markes
1	3	*Paris*, as *Lyons*.		
2	9	*Persia*	87	Mina's
4	3	*Peru*	$87\frac{1}{2}$	Markes
1	7	*Piedmont*	99	Markes
1	7	*Puglia*	$76\frac{1}{2}$	℔.
1	7	*Rome*	$103\frac{1}{4}$	Markes
1	4	*Saxony*	$105\frac{2}{19}$	Markes
1	14	*Spain*	107	Markes
1	4	*Trevers*, or *Triers*	$105\frac{2}{19}$	Markes
1	7	*Treviso*	$103\frac{1}{4}$	Markes
1	7	*Turin*	99	Markes
2 } 3 }		*Turky*	87	Markes
1	7	*Venice* }	$103\frac{1}{4}$	Markes
1	7	*Verona* }		
1	7	*Vicenza*	$105\frac{2}{19}$	Markes
1	4	*Vienna*	87	Markes
1	4	*Vlme*	$105\frac{2}{19}$	Markes
1	4	*Wissilburgh*	$103\frac{1}{4}$	Markes

Foreign Monies.

To close up the Forain *Geodaticks*, Moneys take their turn, concerning which three things are to be observed.

1st. Their Divisions, or greater and lesser Denominations.

2ly. The Accompts thereof, and Exchanges.

3ly. The Weight and Worth of the several Coines.

Accompts and Exchange at several Places.

Malines, p. 240, 241, 257, 258, 259, and other Authours inform us concerning the former two, as followeth, *viz.* at—

2 11 *Aleppo*, The Exchange is made by *Sultanies* of 120 Aspers, or Dollers of 80 Aspers, every Asper 10 Macherines.

3 1 *Alexandria*, They Accompt by Ducats, either *Ducat de Pargo*, of 120 Maids, Ducat of *Venice* of 40 Maids, or *Italian* Ducat of 35 *Maids*.

1 7 *Ancona*, Exchange is made on the Ducat of 21 Gross, (which is in Specie 23 Gross) which Ducat is also 14 Carlini, and every Carlini 6 Bollidini, So is the Ducat 84 Bollidini.

1 14 *Arragon*,

1 4 *Arragon*, The Rial, or Ryal of Plate is 23 Dinero's (*Hunt* ſaith 13) and the Ducat is 12 Ryals, whereon they make Exchange. And they Accompt by Pounds of 20 *s.* and 12 *d.* And the Ducat of 12 Ryals. Every Ryal of 1 *s.* or 12 *d.*

Accompts and Exchange at ſeveral Places.

1 8 *Artois*, And in ſeveral other Places they Accompt and Exchange by Pounds or Liures Toirnois of 20 Stivers, or 40 Pence *Flemiſh*, whereof 6 called Guilders or Florius makes the Pound *Flemiſh* in all the 17 Provinces of the *Netherlands*. Which Pound is divided into 20 *s.* and every Shilling into 12 *d.* &c.

Some reckon by the Pound *Paraſis*, which is but 20 Pence, whereof 12 make 1 Pound *Flemiſh*, but their Accompts, as alſo the Finances of the Princes are kept by Pounds Tournois, and both Pounds divided into 20 *s*, and every Shilling into 12 Pence, admitting alſo the Subdiviſions of Obolo's, Maille, Heller, Hallinck, Corte, Mites, Point, Engevin, Poot, and ſuch like Copper Monies.

Alſtead mentions the *Florin* in *Germany* to be 15 Batz, every Batz 2 Albes, every Albe 8 Oboli, or Nummos. So ſhall the Florin be 30 Albes or 240 Oboli.

1 4 *Auguſta*, or *Ausburgh*, Accompts on the Dollar coined at 65 Creutzers, riſen ſince to 72. Exchange is made on the imaginary rate of 65 Creutzers.

A Creutzer, is ſometime called a Schreikenborger, and in *Latine*, *Crucigerus* and *Cruciatus*, being pieces ſtamped with a Croſs. Their Groſs make 12 Creutzers or 3 Batz, ſo is their Batz 4 Creutzers. Their Lyon Piece half a Creutzer. They have their Snubourgh, Blaphart, or Bohemico's of 3 and $3\frac{1}{2}$ Creutzers. The Rix or Rycks Dollar is 30 Albes of 8 *d.* every Albe, or 72 Creutzers. Every Dollar as before. See the following Table, and afterwards in *Germany*.

Creutzer, how called and ſtamped.

A Table of the German Dollar.

	Groſs.	Batz.	Albes.	Creutzers.	Lyon.	Pence.	Black penys.
Dollar	6	18	30	72	144	240	288
	Groſs	3	5	12	24	40	48
		Batz	$1\frac{2}{3}$	4	8	$13\frac{1}{3}$	16
			Albe	$2\frac{2}{5}$	$4\frac{4}{5}$	8	$9\frac{3}{5}$
				Creutzer	2	$3\frac{1}{3}$	4
					Lyons	$1\frac{2}{3}$	2
						Peny	$1\frac{1}{5}$

3 2 *Barbary*, Generally Accompts are kept, and Commodities ſold by Ducats of 10 ℥, each Ounce divided into 8 parts, which eight part is in Value about 12 *d.* Sterling.

1 14 *Barcelona*, As at *Arragon*.

1 4 *Bavaria*, Accompts and Exchanges both are by Guilders of 7 *s.* and 30 Pence to Shilling.

1 4 *Bohemia*, As in *Germany*, generally by the Dollar of 24 Bohemico's, called alſo White Groſs, each of 3 Creutzers, other Diviſions ſee in the Table following.

A Table of the Bohemian Scoc.

	Marke.	Dollars.	Angſter's.	Bohemico's.	Creutzers.	Pence.
Scoc.	$1\frac{1}{2}$	$2\frac{1}{2}$	30	60	180	600
	Marke	$1\frac{2}{3}$	20	40	120	400
		Dollar	12	24	72	240
			Angſter	2	6	20
				Bohemico	3	10
					Creutzer	$3\frac{1}{3}$

1 7 *Bolognia*,

1 7 *Bolognia*, They Accompt by Piastra, or Pounds (called also Piastri,) each containing 20 Bolognesi. And exchange on the Ducat of 4 Piastri.

1 8 *Brabant*, And in most places of the *Low Countries*, Monies, are accompted by the Pound *Flemish*, containing 20 *s. Flemish*. And every Shilling 12 *d.* or Deniers called Single Stivers, 2 of which make 1 double Stiver. See *Flanders*.

1 4 *Breslaw*, They reckon by Markes of 32 Gross, of 12 Heller to the Gross, And exchange by 30 Florens to have at *Norenbergh* 32 Florins, and at *Vienna* 34 Florins.

1 7 *Calabria*, Exchanges are made by the *Naples* Ducat, of 10 Carlini.

1 14 *Castile*, Exchanges are made on the Ducat of 375 *Marvedies*, which they call in the Bill of Exchange *Ducadas d' oro*, or *de Peso* to be payd out of the Bank is better by 6, or 8 *pro Milliar*. See *Spain*.

1 14 *Catalonia*, as at *Arragon*.

1 4 { *Cleves*, *Collen*, } Both Accompts and Exchanges are made by Dollars of 72 Creutzers.

A Table of the Guilder of Cleves *and* Collen.

	Markes.	Morkens.	White pennies.	Shillings, *or* Stivers.
Their Guilder is	4	12	24	48
	Marke	3	6	12
		Morken	2	4
			White Peny	2

1 5 *Constantinople*, as *Aleppo*.

1 9 *Dantsick*, They accompt by *Polish* Guilders, of 30 Gross, every Gross 18 *d.* They buy with the Great Mark of 60 Gross, or the Little Mark of 15 Gross. Also by the Scoc of 3 Great Marks. And exchange upon the *Florin Polish*, or the Pound *Flemish*. They have Dollars of 35 Gross of 3 Shillings. And new Dollars of 24, 26, or 30 Gross. Their Gilden is 80 Gross. So is

A Table of the Scoc *of* Dantsick.

	Gildens.	Great Markes.	Dollars.	Guilders.	Little Markes.	Gross.	Pence.
Scoc.	$2\frac{1}{4}$	3	$5\frac{1}{7}$	6	12	180	3240
	Gilden	$1\frac{1}{3}$	$2\frac{2}{7}$	$2\frac{2}{3}$	$5\frac{1}{3}$	80	1440
		Great Marke	$1\frac{5}{7}$	2	4	60	1080
			Dollar	$1\frac{1}{6}$	$2\frac{1}{3}$	35	630
				Guilder	2	30	540
					Little Mark	15	270
						Gross	18

1 1 *Denmark*, They Accompt by Markes of 20 Shillings. And Exchange upon the Dollar.

1 6 *Dublin*. See *Ireland*.

1 12 *Edenbourgh*. See *Scotland*.

1 4 *Embden*, They reckon by Guilders, and exchange on the Rycks Dollar, but from *London* hither and thither upon the Pound *Sterling*.

1 8 *Flanders*, As before in *Brabant*. See a more particular Division of the *Flemish* Money in the following Table.

Flemish

A Table of the Flemish Pound.

	Gilders.	Shillings.	Double-Stivers.	Single-Stivers.	Groats.	Ortgens.	Negen-Manneken.	Copper-Pence.	Mites.
Flemish Pound.	0	20	00	120	240	480	900	1920	2880
	Gilder	$3\frac{1}{3}$	10	20	40	80	160	320	480
		Shilling	3	6	12	24	48	96	144
			Double-Stiver	2	4	8	16	32	48
				Single-Silver	2	4	8	16	24
					Groat	2	4	8	12
						Ortgen, *or* Ortken	2	4	6
							Negen-Manneken	2	3
								Copper-Penny.	$1\frac{1}{2}$

Five Single Stivers are Currant in several Places of the *Low-Countries* for Six pence *Sterling*. Ortkens in some places are called Duyts. Mites in some places of Flanders are called Cortes, Engcuni, Points, Pites, Pootes.

1 7 *Florence*, They accompt by Crownes of 20 *s.* and 12 *d.* to the Shilling. And exchange by a Ducat called *Largo*, or *Scripto in Banco*. A Florin there is 24 Quatrini.

1 3 *France*, Generally they use Liures Solx and Deniers, and commonly accompt by them, as the *English* by Pounds, Shillings and Pence, but by an Edict made 1577, their Accompts are to be kept in *French* Crownes of 60 Sols to the Crown, or 3 Liures, that is Pounds Tournois. And exchange is made thereupon unless for some places in *Italy*, where they exchange for Number to have so many Ducats for so many Crownes of the Sum, not in Specie but imaginary, yet respecting the Value, or Par. See further in the Table and Notes thereupon.

A Table of the French Crown.

	Liures.	Sols.	Liarts.	Doubles.	Deniers.
French Crown A	3	60	240	360	720
	Liure B	20	80	120	240
		Sols C	4	6	12
			Liart D	$1\frac{1}{2}$	3
				Double E	2

There are also Petit Deniers, and Mailles, but not considerable.

A. This Crown here to make Exchange by is equivalent to the Silver Coines of *Lewis* 13th and 14th, called *Lewisses* and imaginary, and not to be accompted for the *French* Gold Crowne, which is a real Coine, and of greater Value now, being worth 8 *s. Sterling*, or thereabouts, but when currant, at 6 *s. Sterling*. The Accompt and Exchange agreed in reality, 10 Sols then and yet commonly reckoned for an *English* Shilling. Of this Gold Crown was the Cardecue a quarter, and so valued in *Sterling* Money at 18 *d.* and should be wrote *Quartid'escue*, *Escue*, being *French* for a Crown.

French Crown *of Gold how much.*

Cardecue *what and how to be wrought.*

B, and *C*. The Liures (or Pounds sometime called Franks) and Sols (wrote sometime Soulx, sometime Solx, derived from the *Latine*, *Solidus*, as *Liure* from *Libra* are different. Those commonly used are called Tournois, and valued with *Sterling* Money as above. Of the Sols *Barrois* 14 make 20 Sols *Tournois*. The Sols *Mausais*, is 2 Sols *Tournois*. The Sols *Paris* is $1\frac{1}{4}$ Sols *Tournois*. The Sols *Bourdelois* is half the Sols *Paris*. And so accordingly is the Liure to be accompted.

Liures *and* Sals *how called, whence the word.*
Sorts of Sols.

D, and *E*,

Liarts, *and* Doubles *not used in Accompts.*

D, and *E*, Neither the Liarts nor Doubles, though both Copper Coines are used in Common Accompts.

1 4 *Frankford*, Their Guilder or Florin by which they reckon is 60 Creutzers divided by 20 *s.* and every Shilling into 12 Hellers according to the Pound. But they exchange by the Dollar of 65 Creutzers payable in the two Yearly Fairs or Marts, one the Week before *Easter*, and the other all the Moneth of *September*.

1 7. *Genoa*, All Accompts, and Exchanges are made by Crowns of 60 *s.* divided into 20 *s.* and every Shilling into 12 Pence.

1 4 *Germany*, Every Batz by which generally they keep Accompts, is 4 Creutzers. They Exchange on the Dollar imaginary at 65 Creutzers, and so coined as was noted before at *Augusta*, though since risen to 72 in Value.

They have Pieces of 3, 6, and 12 Creutzers, and by them, and their Batz they value their own and *Exotick* Coines as the *Hungarian* Ducat is 27 Batz. The Gold Guilder is 18 Batz. The *Polish* Guilder or Dollar is 15 Batz, Teston 5 Batz, *&c.*

Guilder *of the* Romans.

A Guilder was the name the Antient *Romans* gave to an Ounce, and 8 ℥ made a Mark, and 12 Ounces or Guilders a Pound.

Nummi Dragmi.

And there were Coined Pieces called *Nummi Dragmi*, or *Groshen* the 8*th* part of a Dollar. *Anno* 1520. was the Gold Guilder Coined for a General Coine, and valued in *Holland* at 28 Stivers, but now in *Specie* at double the Price. Nevertheless Corn brought from *Poland* and the *East Countries*, is bought and sold by the same at the old value of 28 Stivers.

Angelicies, *what.* *Tribute Peny, abased, whence the Name of* Batz.

Angelicies was the Sixth part of a Dollar, making 3 Batz, or 12 Creutzers. These Angelici becoming Tribute Pennies were allayed, and so being made worse, did obtain the Name of Batz or Bates (sometime wrote Barses) *quasi* Base. And in *Thuringia* they are called Gulielmi, and in *Bohemia*, Bohemici, whereof they have also 12 Peeces dividedly, for 12 Pence; which Peny is 2 Hellers in Accompt all over *Germany*.

Pardauue-Xerafin *the Stamp and Value.*

2 6 *Goa*, Their Common Accompt is by their Ordinary Silver Coine a *Pardauue-Xerafin*, having the Image of St. *Sebastion* at the one side, and 3 or 4 Arrows bound together at the other, which is worth 3 Testons, or 300 Res of *Portugal*, but varieth as the Exchange riseth or falleth: And accordingly their other Coines, and Accompts of which some are imaginary, and some real. They have also some good and some bad Monies; for 4 good Tanga's, or 5 bad Tanga's are reckoned to value 1 Pardauue-Xerafin. And 1 Tanga is 75 Basarves. Of these Basarves 375 make 1 Pardauue-Xerafin. And 15 good Basarves are valued with 18 bad, which are made of Bad Tinne. By these other Countrey Coines are rated, as the *Larin* of *Persia* is worth 105, and 108 Basarves, as the Exchange goes. A Paradauue of *Larins* is 5 Larins. And the Crowns of *Venice* or *Turkey* are almost worth 2 Paradauue-Xerafins. They have also a *Pagode*, or Gold Crown, on which is the Figure of their *Idol*, worth about 8 Tanga's, And Gold Crowns of St. *Thomas* with his Image on them, esteemed at 7, or 8 Tangas.

Tangas *good and bad.*

Larin *of* Persia *the Value.*

Pagode *what.*

1 1 *Hamborough*, Their Dollar was first Coined at 31 Shillings Lups, and many Years currant for 33, is now inhaunced to 54 *s.* Lups, of 3 White Peny, and every Shilling is 12 *d.* and every Peny 2 Hellers. They Accompt by Markes of 16 *s.* Lubish, and 12 *d.* to the Shilling: But Exchange for *London* upon the Pound *Sterling*, and for other Places on the Rycks Dollar of 33 *s.* now by them inhaunced to 54 *s.* Lubish, or so many Stivers *Flemish*.

1 8 *Henault*, As *Artois*.

1 4 *Hungaria*, They accompt by Guilders of 10 *s.* of 30 *d.* to the Shilling, And by *Florins* of 20 *s.* and 12 *d.* to the Shilling: And exchange on the Ducat and Rycks Dollar worth 8 Shillings formerly, but 7 *s.* 7 *d.*

1 6 *Ireland*,

1 6 *Ireland*, They as the *English* Accompt by Pounds of 20 *s. Sterling* and Pence of 12 to the Shilling Only their Harper valued in *England* but 9 *d.* was with them counted 1 *s.* So as their Pound is but $\frac{3}{4}$ of ours, or 15 *s. Sterling*. And thereon Exchanges are made.

1 4 *Lipsich*, as *Bresla*.

1 10 *Lisbon*, See *Portugal*.

1 8 *Low Countries*, generally as before at *Brabant*.

1 2 *London*, Exchanges are generally made for *Germany* and the *Low Countries*, on the Pound *Sterling*. For *France* on the *French* Crown of 60 Sols *Tournois*. For *Italy*, *Spain*, and other Places on the Ducat Dollar or Florin according to the Custom of the Place

1 7 *Luca*, For divers Places in *Italy* and *Lyons* in *France* Exchanges are made on the Ducat.

1 3 *Lyons*, as before in *France*.

1 14 *Madrid*, See *Spain*.

1 7 *Millan*, Accompts are kept by Ducats Imperial, divided by 20 *s.* and 12 *d.* to the Shilling. And Exchanges made on the same, accompting 80 *s.* to the Ducat Imperial: But they buy by a Ducat currant of 120 *s.*

1 7 *Naples*, They Accompt by Ducats, Taries and Graines. The Ducat is 10 Carlini or 5 Taries, for the Tarie is 2 Carlini, or Royals. And hereupon Exchanges are made for most places of *Italy*; but for *Lyons*, they Exchange by Number, as 125 Ducats for 100 Crowns.

1 4 *Norenbourgh*, The Exchange is made on the Dollar of 65 Creutzers, and many times on the Guilder of *Florin*, of 60 Creutzers, which they also divide into 20 *s.* and every Shilling into 12 *d.* to keep Accompts by; and some say the Creutzer is 4 *d.* every Peny is 2 Hellers. And 5 *d.* is called a Fynfer, or 5 Pennick.

1 7 *Palermo*, The Ducat is 13 Taries, 1 Tarie 2 Carlini, 5 Ryals of *Spain* are 6 Taries. They accompt by Ounces of 30 Taries, to 20 Graines every Tarie, and every Graine 6 Piccolie. And their Exchanges are made upon *Florins* of 6 Taries, or Tarij.

1 3 *Paris*, as before in *France*.

1 9 *Poland*, They Accompt by Markes and Exchange on the Dollar, and also on the Florin of 48 *s.* The Marke is One Third part of it.

1 4 *Pomerania*, They divide their Money as in the next Table following, Accompt by Markes of 16 Snudens, and Exchange upon the Ryckx Dollar of 32 *s.* or 2 Markes Snudens, so called to distinguish them from Markes Lups, and Shillings Lups.

A Table of the Ryckx Dollar *of* Pomeren.

	Marks-Snudens.	Lups-Shillings.	Shillings-Snudens.	Pence.	Hellers.
Ryckx Dollar Marke Lups	2	16	32	384	768
	Marke-Snuden	8	16	192	384
		Shilling-Lups.	2	24	48
			Shilling-Snuden	12	24
				Peny	2

1 10 *Portugal*, They Accompt by *Milrais*, Ducats, or Crusado's, &c. as in the Table following, And Exchange by the same Ducat of 400 Raies.

A Table of the Mille Raies.

	Ducats, *or* Crusado.	Testons.	Rials.	Vintaines, *or* Half Rial.	Raies.
Mille Raies	$2\frac{1}{2}$	10	25	50	1000
	Ducat, *or* Crusado.	4	10	20	400
		Teston.	$2\frac{1}{2}$	5	100
			Rial.	2	40
				Vintaine, *or* Half Rial.	20

Of

Of these Ducats, Rials, (or Royals) and Raies (wrote also Reas, Reyse, and Res) are most in use for Accompt. They have Testons also of 4 Vintaines, 40 Raies are commonly accompted for Six Pence *Sterling*, and so accordingly were the other Coines valued, till the late advance whereby the Teston of 100 Raies were stamped, and made currant for 120 Raies, and so rated at 1 *s.* 6 *d. Sterling*, when before but 1 *s* 3 *d.*

1 7 ***Puglia***, as ***Calabria***.

1 9 ***Riga***, They buy by Dollars, or Florins *Polish* of 18 Farthings, whereof 11 make 10 Dollars, but they Exchange upon the Ryckx Dollar.

1 3 ***Roan***, as before in *France*.

1 7 ***Rome***, Accompts and Exchanges are performed by Ducats *di Camera* of 13 *July*, or *Guili*, every Ducat which they divide into 20 *s.* and every Shilling into 12 *d.*

1 11 }
2 10 } ***Russia***, They have small Coine of 11 ℥. 2 penny weights fine, called *Dengen*, whereof 320 Pieces weigh but a Mark of 8 ℥. They Exchange upon the Dollar of *Germany*; but for *London* upon their Rubble which is valued as a Double Ducat formerly, accompted equal to a Marke Sterling, or 13 *s.* 4 *d.*

1 14 ***Saragossa***, as ***Arragon***.

1 12 ***Scotland***, They Accompt by Pounds, Shillings, and Pence, as in *England*, but one *Pund Scotch* is but 20 *d English*. Their Marke is 13 $\frac{1}{3}$ *s. Scotch*, currant in *England* at 13 $\frac{1}{2}$ *d.* Their Noble, or half Marke with them 6$\frac{2}{3}$ *s.* with us 6$\frac{3}{4}$*d.* Their half Noble, and third part of their Noble proportionally. They have also Turnoners, Pence, and Half-Pence, and base Money of Bodles, Achisons, Babees, Placks, &c. accompting 6 Bodles to 1 *d. Sterling*, or 12 *d. Scotch*, 4 Bodles to 1 Achison, 3 to 1 Babee, and 2 to 1 Plack. But they Exchange upon their Marke

Base Money of Scotland.

1 14 ***Spaine***, as in ***Madrid***, ***Sevil***, and other Places their Accompts are all kept by *Malvedies*, or *Marveides* (wrote also *Merveides* and *Maravides*) whereof 375 are esteemed to make a Ducat of 11 Rials, though really every Ryal is but 34 Marveides, and so maketh but 374, as in the following Table, and so others keep Accompts accordingly. Exchange is made on this Imaginary Ducat of 375 Marveides to be payd in Bank with 5 on the 1000, which is the Salary of the Banker, or without the Bank to be payd without the same.

A Table of the Spanish Ducat.

	Pieces of Eight.	Rials.	Quartilios.	Marveides.	Carnado's.
Ducat	$1\frac{3}{8}$	11	44	374	2244
Piece of Eight		8	32	272	1632
Rial			4	34	204
Quartilio				$8\frac{1}{2}$	51
Marveide					6

A Rial is about 6 $\frac{1}{2}$ *d. Sterling*.

1 4 ***Stratsborough***, or ***Strausburgh***, They have Blapharts, Gross, Bohemico's, all currant for 3 Creutzers a piece, 1 Creutzer at 2 *d.* One Peny at 2 Hellers, and 1 Heller at 2 Orthings.

1 15 ***Sweden***, They reckon by Markes, whereof 8 make a Dollar, whereupon they Exchange. And 2 Markes make a Clipping of 9$\frac{1}{2}$ Stivers.

1 4 ***Tirol***, The Dollar is 72 Creutzers, and the Creutzer 5 Fynfers or Hellers.

2 11 ***Tripoli***, as ***Aleppo***.

1 14 ***Valentia***, as ***Arragon***.

1 7 ***Venice***, Thirty Batz make 1 Souldey, and 20 Souldeys 1 Liure of *Venice*. Their Gold Ducat is valued equal to 40 Maides of *Alexandria*. They have also Copper Money, 1 Sessini make 2 Quatrini, and 1 Quatrine 4 Bagatini, and so 3 Quatrini, or 12 Bagatini make an Half-penny *Sterling*, or thereabouts.

They

They accompt by Pounds *Flemish* of 10 Ducats or 20 *s.* and divide the Ducat into 24 Gross, and the Shilling into 12 Pence. And also by the Ducat 124 *s.* called *Ducato di Banco*, or Currant, and thereon Exchanges are made.

1 7 *Verona*, Their Accompts are kept by 20 *s.* and 12 *d.* to the Shilling. And they Exchange on the Ducat of 93 *s.*

1 4 *Vienna*, Both Accompts and Exchanges are kept and made by Guilders or Florins of 8 *s.* a piece, 30 *d.* to the Shilling, and 2 Hellers to the Peny.
They esteem the Ricks Dollar at 8 *s.* and the Ducat at 12 *s.*

1 4 *Ulme*, They reckon by Pounds of 20 *s.* and 12 Heller to the Shilling. And Exchange on the Dollar of 60 Creutzers.

That which remains to finish this long and tedious Chapter of *Geodæticals* is only to consider the weight and worth of the Coines of other Countreys as valued by the *Sterling* Standart, wherein because of the New Coines which may dayly be added by the Laws of the present or succeeding Governours, and those of different Fineness; to obviate the difficulties as well occasioned thereby, as by the rise and fall of Exchange, and so consequently of particular Coines, practised by Merchants: Every one that would arrive at satisfaction besides what can be here wrote, must add his diligent observance. *Foreign Coins.*

It may be remembred that the *English* Pound *Troy* is divided for Weight into 12 Ounces, every Ounce into 20 Peny-weights, and every Peny-weight into 24 Graines. And to try the Fineness of Silver the same divisions are kept, but for the Fineness of Gold, every Ounce is divided into 24 Carrats, and every Carrat into 4 Graines. And the old *Sterling* Standart for Silver is 11 Ounces 2 Peny-weights fine, and for Gold 22 Carrats fine. *Weight to try the fineness of Gold and Silver by in England.* Sterling *Standart.*

They beyond Sea for Weight and Fineness of Silver divide their Ounce into 20 *English*, and every *English* into 32 Azes? And for weight of their Gold the like; But for the Fineness thereof divide their Ounce into 24 Carrats, and every Carrat into 12 Graines. *Weight beyond Sea to try the Fineness by.*

In the following Coines, understand the Value according to the *English* Division, allowing for the Ounce of Silver 11 Ounces 2 Peny-weights fine, 5 *s.* but for the Ounce of Gold 22 Carrats fine 3 *l.* 10 *s.* that is 3 *s.* 6 *d.* for the Peny-weight, and so proportionally for Coines of greater or lesser fineness, which Valuation makes the particular Pieces to differ from that found in several Printed Books, as they one from another, according to the times they were Printrd or Wrote in. Some valuing the Ounce of Silver so fine as aforesaid, and others that of 11 Ounces fine, at 5 *s.* and the Ounce of Gold of the fineness aforesaid but at 55 *s.* others at 3 *l.* some at 3 *l.* 6 *s.* and generally not above 3 *l.* 6 *s.* 8 *d.* *How the following Coines are valued.*

Neither are all Coines though of one and the same fineness alwaies valued alike proportionally. For K. *James*, *May* 14. 1612. by Proclamation ordered the bringers in of Forein Coine might receive at the Mint as followeth. *All Coines not valued according to their Fineness.*

	l.	*s.*	*d.*
For the Ounce of *Spanish* Silver Money of *Sevil*	0	5	0
The Ounce of *Mexico* Money	0	4	10
Ingots of Silver, being 11 ℥. 2 pwts. fine	0	5	0
And so rateably for Silver of other fineness.			

		Car.	*Gr.*				
For 1 ℥. of	*Spanish* Pistolets being	21	3½	fine	3	6	0
	French Crowns being	22	0	fine	3	6	0
	Milreys, *Crusado* long and short Cross.				3	6	2
	Barbary Gold being	23	0½	fine	3	9	0
	Hungary Ducats, being	23	1	fine	3	9	0
	Spanish Ducats and Sultaines being	23 *Car.*	1 *Gr.*	fine	3	8	8
	Zechines, or Checkeene of *Venice* being	23	1	fine	3	10	0
And for the Ounce of all other Gold being		22	0	fine	3	6	0

And in like sort to this day by the Artifice of *Merchants*, *Goldsmiths*, *Bankers*, &c. Some Coines are valued and currant in Traffique at a value higher, than by a due proportion in respect of their fineness to the *Sterling* Standart, they ought to be: But in the following Tables, the New Value is equally apportionated, yet without allowance for Coinage, which I take to be about 2 *s.* for the Pound *Troy* of Silver, and for the Pound *Troy* of *Gold* about 15 *s.*

		Foreign Gold.	Fine Car.	Gr.	Pieces to the ℔ Troy.	Weight by *Malines.* pwts.	gr.	Weight by others. pwts.	gr.	Old Value. l.	s.	d.	New Value. l.	s.	d.
		Albertines, See Ducats.													
Angels the sorts.	Angels	with the three Lions	22	0	76	3	$3\frac{15}{19}$	3	$3\frac{3}{4}$	0	8	6	0	11	$0\frac{1}{2}$
	Angels	with *O.*	23	0	72	3	8	3	6	0	9	0	0	12	$2\frac{1}{4}$
	Angels of	*Batenborgh*	21	3	72	3	8	3	6	0	9	0	0	11	$6\frac{1}{4}$
	Angels of	*Flanders*, or best *Flemish* Angel	23	0	72	3	8	3	6	0	9	0	0	12	$2\frac{1}{4}$
	Angels of	*H. M.*	17	0	72	3	8	3	6	0	6	9	0	9	0
	Angels of	*Horne*	23	$1\frac{1}{2}$	72	3	8	3	6	0	9	6	0	12	$4\frac{3}{4}$
	Angels of	*Thoron*	22	$1\frac{1}{2}$	72	3	8	3	6	0	9	0	0	11	$10\frac{1}{4}$
	Angels of	*Vienna*	18	3	72	3	8	3	6	0	7	6	0	9	$11\frac{1}{4}$
		Chastilion	23	3	$79\frac{3}{4}$	3	$0\frac{72}{319}$	2	23	0	8	10	0	11	$4\frac{1}{4}$
		Cros's Daggers of *Scotland*	22	0	72	3	8	3	6	0	11	0	0	11	8
		The half thereof	22	0	144	1	16	1	15	0	5	6	0	5	10
Cro. the sorts.	Crowns	Flemish Crown	22	0	108	2	$5\frac{1}{3}$	2	5	0	6	0	0	7	$9\frac{1}{4}$
	Crowns	Floret Crown of *France*	23	3	$100\frac{1}{2}$	2	$9\frac{21}{67}$	2	9	0	7	0	0	9	$8\frac{1}{4}$
	Crowns	Charles French Crown	23	3	$100\frac{1}{2}$	2	$9\frac{21}{67}$	2	9	0	7	0	0	9	$0\frac{1}{4}$
	Crowns	Old French Crown	22	0	108	2	$5\frac{1}{3}$	2	5	0	6	0	0	7	$9\frac{1}{4}$
	Crowns	New French Crown — Some	22	0	$107\frac{1}{2}$	2	$5\frac{25}{43}$	2	$5\frac{1}{2}$	0	6	0	0	7	$9\frac{3}{4}$
	Crowns	New French Crown — Others	22	0	108	2	$5\frac{1}{3}$	2	5	0	6	0	0	7	$9\frac{1}{4}$
	Crowns	Half Imperial Crown	22	$1\frac{1}{2}$	$107\frac{1}{2}$	2	$5\frac{25}{43}$	2	$5\frac{1}{2}$	0	6	0	0	7	$11\frac{1}{4}$
	Crowns	Italian Crown	—	—	—	—	—	—	—	0	6	0	0	7	6
	Crowns	Four Crowns of *Portugal*	—	—	—	—	—	—	—	1	6	2	0	10	0
	Crowns	K. *Philip*'s Crown of *Spain*	22	$1\frac{1}{2}$	$107\frac{1}{2}$	2	$5\frac{25}{43}$	2	$5\frac{1}{2}$	0	6	0	0	7	$11\frac{1}{4}$
	Crowns	Scotch Crown	22	0	108	2	$5\frac{1}{3}$	2	5	0	6	0	0	7	$9\frac{1}{4}$
	Crowns	Thistle Crown	22	0	186	1	$6\frac{30}{31}$	1	$6\frac{3}{4}$	0	4	$4\frac{3}{4}$	0	4	6
Crusadoes the sorts.	Crusado's	or Ducat with the † of *Portugal*	22	1	105	2	$6\frac{6}{7}$	2	6	0	6	0	0	8	1
	Crusado's	or Ducat with the ☩ of *Portugal*	22	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	2	0	8	$3\frac{1}{4}$
	Crusado's	Great Crusado, or the *Portuguise* of *Emanuel* of *Portugal*	23	3	$10\frac{1}{2}$	22	$20\frac{4}{7}$	22	16	3	8	0	4	6	$4\frac{1}{4}$
	Crusado's	*Joannes* Great Crusado	22	3	$10\frac{1}{2}$	22	$20\frac{4}{7}$	22	16	3	5	0	4	2	$8\frac{1}{2}$
		Dublion of Spain	—	—	—	—	—	—	—	0	14	6	0	15	2
Ducats the sorts.	Ducats of	Albertus, *or* Albertines — Single	23	$3\frac{1}{2}$	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	8
	Ducats of	Albertus, *or* Albertines — Double	23	$3\frac{1}{2}$	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	13	0	0	17	4
	Ducats of	Albertus of *Austria* — Single	23	3	$78\frac{3}{4}$	3	$1\frac{1}{7}$	3	0	0	9	0	0	11	$5\frac{3}{4}$
	Ducats of	Albertus of *Austria* — Double	23	3	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	13	6	0	17	$3\frac{1}{4}$
	Ducats of	$\frac{2}{3}$ parts of the same double Duc.	23	3	$70\frac{1}{2}$	3	$9\frac{33}{47}$	3	9	0	10	0	0	12	$10\frac{1}{4}$
	Ducats of	$\frac{1}{3}$ part of the same double Duc.	23	3	126	1	$21\frac{5}{7}$	1	$21\frac{1}{2}$	0	5	7	0	7	2
	Ducats of	*Arragon*	23	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$7\frac{1}{2}$
	Ducats of	*Barbary* and elsewhere in *Turkey.* — Some	23	$0\frac{1}{2}$	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	4	0	8	$3\frac{1}{2}$
	Ducats of	*Barbary* and elsewhere in *Turkey.* — Others	23	3	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	6	0	8	$6\frac{1}{4}$
	Ducats of	*Batenborg* with the †	19	0	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	5	2	0	6	11
	Ducats of	Bishops Ducat	23	$0\frac{1}{2}$	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	3	0	8	$5\frac{1}{2}$
	Ducats of	*Castile*	23	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$7\frac{1}{2}$
	Ducats of	*Denmark*	20	0	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	5	4	0	7	2
	Ducats of	*Emannuel* of *Portugal*	23	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$7\frac{1}{2}$
	Ducats of	*Ferdinand* of *Batenborg*	19	$0\frac{1}{2}$	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	5	2	0	6	$10\frac{1}{4}$
	Ducats of	*Ferdinand* and *Carolus* of *Horne*	18	0	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	4	10	0	6	$5\frac{1}{4}$
	Ducats of	*Florence*	23	1	108	2	$5\frac{1}{3}$	2	5	0	6	4	0	8	$2\frac{1}{2}$
	Ducats of	*George Rechem*	21	3	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	1	0	7	$9\frac{1}{2}$
	Ducats of	*Guelders*	23	1	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	3	0	8	4
	Ducats of	*Gulielmus* of *Batenborg*	21	3	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	12	4	0	15	$9\frac{3}{4}$
	Ducats of	*Hamborough*	—	—	—	—	—	—	—	0	7	2	0	8	9
	Ducats of	*Holland*	23	2	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	5	0	8	$6\frac{1}{4}$
	Ducats of	*Hungary*, or Half Noble	23	$3\frac{1}{6}$	$113\frac{1}{2}$	2	$2\frac{120}{227}$	2	$2\frac{1}{2}$	0	6	4	0	8	0
	Ducats of	Other *Hungary* Ducats	23	1	$104\frac{1}{2}$	2	$7\frac{25}{209}$	2	7	0	6	4	0	8	$5\frac{1}{4}$
	Ducats of	*Italy* — Some as *Venice*													
	Ducats of	*Italy* — others	23	1	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	3	0	8	4
	Ducats of	*Majorca*	23	1	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	13	0	0	16	$10\frac{1}{4}$
	Ducats of	*Marie* of *Batenborgh*	20	$0\frac{1}{2}$	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	5	4	0	7	$2\frac{1}{2}$
	Ducats of	*Navarre*, and some others, as *Majorca*													
	Ducats of	*Nimmeghen* with *Stephen*	21	1	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	12	0	0	15	$5\frac{1}{4}$
	Ducats of	*Nimmeghen* of 1565	18	2	108	2	$5\frac{1}{3}$	2	5	0	4	10	0	6	$6\frac{1}{2}$

Oswald

Foreign Gold.	Fine Car.	Gr.	Pieces to the lb Trov.	Weight by *Malines.* pwts.	gr.	Weight by others. pwts.	gr.	Old Value. l.	s.	d.	New Value. l.	s.	d.	
Ducats of *Oswald* Ducat *Cusa*	19	$0\frac{1}{2}$	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	5	2	0	6	$10\frac{1}{4}$	
Pancratius Alleb. H. as *Oswald*														
Peter Rechem, as *Geo. Rechem*														
Portugal, see *Crusados*, *Milreys*														
Rome Single Some	23	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$7\frac{1}{2}$	
Rome Single Others	23	3	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	6	0	8	$6\frac{1}{4}$	
Rome Double	23	3	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	13	0	0	17	3	
St. Victor Rancratius, as *George Rechem*														
Spain Single Some	23	1	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$5\frac{1}{4}$	
Spain Single Others	23	2	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$6\frac{1}{2}$	
Spain Double	23	2	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	13	0	0	17	1	
Spain Great	22	0	24	10	0	10	0	1	10	0	1	15	0	Ducats.
States of the United Prov. with Letters	22	0	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	12	4	0	16	0	
The Half thereof	22	0	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	2	0	8	0	
Stephanus of *Batenborg*	19	$0\frac{1}{2}$	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	10	5	0	13	$10\frac{3}{4}$	
Suevia	23	1	$104\frac{1}{2}$	2	$7\frac{25}{209}$	2	7	0	6	4	0	8	$5\frac{3}{4}$	
Valence	23	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$7\frac{1}{2}$	
Venice	23	3	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	6	0	8	$6\frac{1}{4}$	
Victor Batenborgh, as *Geo. Rechem*														
Victor H.B. as *Marie* of *Batenborg*														
W. B. Margaret Toren	21	3	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	1	0	7	$9\frac{1}{2}$	
Water Ducats, as *Marie* of *Batenborg*														
Zeland Single	23	$0\frac{1}{2}$	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	3	0	8	$5\frac{1}{2}$	
Zeland Double	23	$0\frac{1}{2}$	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	12	6	0	16	11	
Ducats with the *Checquer*, as *Denmark*														
Floret of *France*	22	0	$100\frac{1}{2}$	2	$9\frac{21}{67}$	2	9	0	6	5	0	8	$4\frac{1}{4}$	
The *New Floret*, See *Gulden St. Andrew*														
Golden Fleece, or *Toyson d'or*	23	$3\frac{3}{4}$	$81\frac{1}{6}$	2	$22\frac{470}{487}$	2	$22\frac{3}{4}$	0	9	2	0	11	$3\frac{1}{4}$	
Gold Guilder, or *Guildren*	18	3	$112\frac{1}{2}$	2	$3\frac{1}{5}$	2	6	0	4	9	0	6	$4\frac{1}{4}$	Guilders
Guilders of *St. Andrew* *Old*	18	1	108	2	$5\frac{1}{3}$	2	$3\frac{1}{4}$	0	4	10	0	6	$5\frac{1}{4}$	*the*
St. Andrew *New*, or *Floret*	18	3	108	2	$5\frac{1}{3}$	2	3	0	5	0	0	6	$7\frac{1}{2}$	*sorts.*
Arnoldus	12	0	138	1	$17\frac{17}{23}$	1	$17\frac{1}{2}$	0	2	7	0	3	$3\frac{3}{4}$	
Carolus	14	0	126	1	$21\frac{5}{7}$	1	$21\frac{1}{8}$	0	3	6	0	4	$2\frac{1}{4}$	
Clemmer	13	0	114	2	$2\frac{10}{19}$	2	2	0	3	6	0	4	$4\frac{1}{4}$	
Collen	17	3	114	2	$2\frac{10}{19}$	2	$2\frac{3}{4}$	0	4	8	0	5	$11\frac{1}{4}$	
David of The *Harp*	15	0	114	2	$2\frac{10}{19}$	2	2	0	4	0	0	5	$0\frac{1}{4}$	
David of *Triers*	17	2	114	2	$2\frac{10}{19}$	2	2	0	4	8	0	5	$10\frac{1}{4}$	
David of *Utrecht*	16	0	114	2	$2\frac{10}{19}$	2	2	0	4	3	0	5	$4\frac{1}{4}$	
Frederick of *Bavaria*	14	0	117	2	$1\frac{3}{13}$	2	1	0	3	8	0	4	$6\frac{3}{4}$	
Gulielmus	18	1	108	2	$5\frac{1}{3}$	2	5	0	5	10	0	6	$5\frac{1}{4}$	
Horne	—	—	$158\frac{3}{4}$	1	$12\frac{36}{127}$	1	$12\frac{1}{4}$	0	4	11	0	6	4	
Joannes	16	0	$109\frac{1}{2}$	2	$4\frac{44}{73}$	2	$4\frac{1}{2}$	0	4	6	0	5	7	
Peter of *Louvaine*	17	$0\frac{1}{2}$	114	2	$2\frac{10}{19}$	2	2	0	4	5	0	5	$8\frac{1}{4}$	
Philip	15	3	111	2	$3\frac{33}{37}$	2	$3\frac{1}{4}$	0	4	2	0	5	5	
The Half thereof	15	3	222	1	$1\frac{35}{37}$	1	$1\frac{5}{8}$	0	2	1	0	2	$8\frac{1}{2}$	
Renish Guilder	22	0	$102\frac{3}{4}$	2	$8\frac{24}{411}$	2	8	0	6	9	0	8	2	
Saxon	17	3	113	2	$2\frac{110}{113}$	2	$2\frac{3}{4}$	0	4	8	0	5	$11\frac{3}{4}$	
States of the United Provinces	20	0	$120\frac{3}{4}$	1	$23\frac{113}{161}$	1	$23\frac{1}{2}$	0	4	8	0	6	$3\frac{3}{4}$	
Lions, Golden Lion of *Flanders*	23	3	$89\frac{1}{4}$	2	$16\frac{64}{119}$	2	$16\frac{1}{2}$	0	7	8	0	10	$1\frac{3}{4}$	
$\frac{2}{3}$ parts thereof	23	3	$133\frac{7}{8}$	1	$19\frac{3}{119}$	1	19	0	4	11	0	6	9	
$\frac{1}{3}$ part thereof	23	3	$267\frac{3}{4}$	0	$21\frac{61}{119}$	0	$21\frac{1}{2}$	0	2	5	0	3	$4\frac{1}{2}$	
Louisses, of *Louis* 13*th* and 14*th* of *France*	22	0	54	4	$10\frac{2}{3}$	4	8	0	15	0	0	15	$6\frac{1}{2}$	
The Half thereof	22	0	108	2	$5\frac{1}{3}$	2	4	0	7	6	0	7	$9\frac{1}{4}$	
Markes and Milreys: Marke of *Bohemia*	—	—	—	—	—	—	—	0	6	0	0	7	6	
20 Markes of *Scotland*	22	0	36	6	16	6	10	1	2	0	1	3	4	Marks
10 Markes of *Scotland*	22	0	72	3	8	3	5	0	11	0	0	11	8	*and*
5 Markes of *Scotland*	22	0	144	1	16	1	14	0	5	6	0	5	10	Milreys.
6 Markes of *Suevia*	—	—	—	—	—	—	—	0	4	9	0	6	0	
Milreys or Ducat of *Portugal*	22	1	48	5	0	4	20	0	13	4	0	17	$8\frac{1}{4}$	
Halfe Milreys	22	1	96	2	12	2	10	0	6	8	0	8	10	
Conterfeit Milreys	21	0	48	5	0	4	20	0	12	6	0	16	$8\frac{1}{2}$	

Nobles

Margin	Group	*Foreign Gold.*	Fine Car.	Fine Gr.	Pieces to the lb Troy	Weight by *Malines* pwts.	Weight by *Malines* gr.	Weight by others pwts.	Weight by others gr.	Old Value l.	Old Value s.	Old Value d.	New Value l.	New Value s.	New Value d.
Nobles the sorts.	Nobles of	*Bridges*	23	0	$88\frac{1}{2}$	2	$17\frac{5}{9}$	2	17	0	7	4	0	9	11
	Nobles of	*Flanders*, or *Flemish* Noble	23	0	54	4	$10\frac{2}{3}$	4	10	0	12	0	0	16	3
	Nobles of	Half *Flemish* Noble	23	0	108	2	$5\frac{1}{3}$	2	5	0	6	0	0	8	$1\frac{1}{2}$
	Nobles of	*Gaunt*	23	0	54	4	$10\frac{2}{3}$	4	10	0	12	0	0	16	3
	Nobles of	Half Noble, as the *Hungary* Ducat													
	Nobles of	Half Noble with the *Lyon*, as *Bridges*													
	Nobles of	*Henry* Noble of *France*	22	0	51	4	$16\frac{16}{17}$	4	16	0	13	4	0	16	$5\frac{1}{2}$
	Nobles of	The Half thereof	22	$2\frac{1}{2}$	108	2	$5\frac{1}{3}$	2	5	0	6	8	0	7	$7\frac{1}{4}$
	Nobles of	*Hollan*	23	3	48	5	0	4	20	0	14	3	0	18	$10\frac{1}{4}$
	Nobles of	*Overyssel* and *Utrecht* as *Holland*													
	Nobles of	*Zeland*, as *Gaunt*													
		Pezo of *Peru*, by *Heylin*, p. 1064								0	6	6	0	8	0
Pistolets the sorts.	Pistolets of	*De Lege & Legion*	18	0	108	2	$5\frac{1}{3}$	2	5	0	4	9	0	6	$4\frac{1}{4}$
	Pistolets of	*Italy* Some	21	$2\frac{1}{2}$	108	2	$5\frac{1}{3}$	2	5	0	5	9	0	6	$7\frac{1}{2}$
	Pistolets of	*Italy* Others	22	0	108	2	$5\frac{1}{3}$	2	5	0	5	10	0	7	$9\frac{1}{4}$
	Pistolets of	*Scotland*	19	$2\frac{1}{2}$	108	2	$5\frac{1}{3}$	2	5	0	5	2	0	6	$11\frac{1}{4}$
	Pistolets of	*Spaine* Single Some	21	$3\frac{1}{2}$	108	2	$5\frac{1}{3}$	2	5	0	5	10	0	7	$9\frac{1}{4}$
	Pistolets of	*Spaine* Single Others	22	0	108	2	$5\frac{1}{3}$	2	5	0	5	10	0	7	$9\frac{1}{4}$
	Pistolets of	*Spaine* Double	22	0	54	4	$10\frac{2}{3}$	4	8	0	11	8	0	15	$6\frac{1}{2}$
		Of 26 Ryals	22	0	45	5	8	5	6	0	14	0	0	18	8
		Portuguise, See Great Crusado of *Portugal*													
Postulats the sorts.	Postulats of	*Bourbon*	12	$0\frac{1}{2}$	$136\frac{1}{2}$	1	$18\frac{18}{91}$	1	18	0	2	7	0	3	$4\frac{3}{4}$
	Postulats of	*Cleves*	9	$0\frac{1}{2}$	156	1	$12\frac{12}{13}$	1	$12\frac{3}{4}$	0	1	9	0	2	$2\frac{3}{4}$
	Postulats of	Dog and Cat, as *Bourbon*													
	Postulats of	*Fran. Frier*	9	0	156	1	$12\frac{12}{13}$	1	$12\frac{3}{4}$	0	1	8	0	2	$2\frac{1}{2}$
	Postulats of	*Horne*	10	$0\frac{1}{2}$	156	1	$12\frac{12}{13}$	1	$12\frac{3}{4}$	0	1	11	0	2	$5\frac{1}{4}$
	Postulats of	*Juliers*, or *Guliers*	9	3	156	1	$12\frac{12}{13}$	1	$12\frac{3}{4}$	0	1	10	0	2	$4\frac{1}{2}$
	Pounds	6 Pound *Scotch*	22	0	80	3	0	3	0	0	10	0	0	10	6
	Pounds	12 Pound *Scotch*	22	0	40	6	0	6	0	1	0	0	1	1	0
Riders the sorts.	Ryders of	*Burgundy*	23	$0\frac{1}{2}$	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	3	0	8	$5\frac{1}{2}$
	Ryders of	*Campen* and *Swoll*	12	3	114	2	$2\frac{10}{19}$	2	2	0	3	2	0	4	$3\frac{1}{4}$
	Ryders of	*Deventer*, as *Campen* and *Swoll*													
	Ryders of	*Flanders*	23	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$7\frac{1}{2}$
	Ryders of	*Friesland* / *Guelders* of the Year 1583	21	0	108	2	$5\frac{1}{3}$	2	5	0	5	6	0	7	5
	Ryders of	*Guelders* Ryder	14	0	114	2	$2\frac{10}{19}$	2	2	0	3	6	0	4	$8\frac{1}{4}$
	Ryders of	*Guelders* new Ryder	23	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	9	0	8	$7\frac{1}{2}$
	Ryders of	*Philip Clincart*	14	0	114	2	$2\frac{10}{19}$	2	2	0	3	6	0	4	$8\frac{1}{4}$
	Ryders of	*Ryder* with the Loaves	10	$0\frac{1}{2}$	114	2	$6\frac{10}{19}$	2	2	0	2	7	0	3	4
	Ryders of	*Scotland* Some others	19	$2\frac{1}{2}$											
	Ryders of	*Scotland* Others	22	0											
	Ryders of	*States of the United Provinces*	22	0	36	6	16	6	12	1	0	0	1	3	4
	Ryders of	The Half thereof	22	0	72	3	8	3	6	0	10	0	0	11	8
	Ruble of	*Muscovie* (by *Heylin* 13 *s.* 4 *d.*)	—	—	—	—	—	—	—	0	10	0	0	13	4
	Ruble of	*Poland*	—	—	—	—	—	—	—	0	13	4	0	13	4
Royals the sorts.	Royals or Reals of	*Austria* Single	23	$3\frac{1}{2}$	$49\frac{1}{2}$	4	$20\frac{4}{11}$	4	20	0	14	4	0	18	5
	Royals or Reals of	*Austria* Double	23	$3\frac{1}{2}$	$24\frac{3}{4}$	9	$16\frac{8}{11}$	9	16	1	8	8	1	16	10
	Royals or Reals of	*Campen* and *Swoll*	23	0	$40\frac{1}{2}$	4	$20\frac{4}{11}$	4	20	0	14	2	0	17	9
	Royals or Reals of	The Half thereof	23	0	99	2	$10\frac{2}{11}$	2	10	0	7	1	0	8	$10\frac{1}{2}$
	Royals or Reals of	*Flanders*, or the Key	23	0	$69\frac{3}{4}$	3	$10\frac{18}{31}$	3	$10\frac{1}{2}$	0	10	0	0	12	$6\frac{3}{4}$
	Royals or Reals of	The Half thereof	23	0	$139\frac{1}{2}$	1	$17\frac{9}{31}$	1	$17\frac{1}{4}$	0	5	0	0	6	$3\frac{1}{4}$
	Royals or Reals of	*Imperial Loyal*	23	$3\frac{1}{2}$	69	3	$11\frac{11}{23}$	3	11	0	11	0	0	13	$2\frac{1}{2}$
	Royals or Reals of	The Half thereof	18	0	$105\frac{1}{3}$	2	$6\frac{54}{79}$	2	$6\frac{1}{2}$	0	4	11	0	6	$6\frac{1}{4}$
	Royals or Reals of	*Philip* with the *Spread Eagle*	18	2	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	5	0	0	6	$7\frac{1}{2}$
	Royals or Reals of	*Philip* of *Spain*	23	0	$69\frac{3}{4}$	3	$10\frac{18}{31}$	3	$10\frac{1}{2}$	0	10	0	0	12	$6\frac{3}{4}$
		Ship of *Flanders*, or *Schuytken*	22	1	$109\frac{1}{2}$	2	$4\frac{44}{73}$	2	$4\frac{1}{2}$	0	6	3	0	7	9
		Shock of *Bohemia*, some say 8 *s.* some	—	—	—	—	—	—	—	0	9	0	0	9	0
		Stiver Pieces, 9 Stiv. Pieces of *Batenborg* and *Frize*	7	1	176	1	$8\frac{8}{11}$	1	$8\frac{1}{2}$	0	1	3		6	$6\frac{3}{4}$
		Sultanies of 120 *Aspers*	23	1	90	2	15	2	16	0	7	0	0	9	$10\frac{1}{4}$

Take

Foreign Gold.	Fine Car. Gr.	Pieces to the lb *Troy.*	Weight by *Malines.* pwts. gr.	Weight by others. pwts. gr.	Old Value. l. s. d.	New Value. l. s. d.
Tabe of China					0 5 4½	0 7 2
Toman of *Persia*, by *Heylin*, p. 839. valued at 20 Crowns, seemeth not to be any Coine, but a denomination used in Accompt					5 0 0	5 0 0
Unicorn of *Scotland*	22 0	99¼	2 10$\frac{14}{397}$	2 10	0 6 0	0 8 5½
Xeriffe of *Goa* in *India*, by *Heylin*					0 6 0	0 7 6
by *Grimstone* p. 197. worth 300 *Res* of *Portugal*						
Zechines, or *Checkeenes* of *Venice*	23 1	90	2 16	2 15¾	0 7 6	0 9 10¼

Foreign Silver.	Fine ℥ pwts.	Pieces to the lb. *Troy.*	Weight of the Pieces ℥. pwts. gr.	*Sterling* Value. s. d.	Value by some Authors. s. d.	*Foreign Silver Coines.*
Albi of *Bamberg*	4 18	273	0 0 21$\frac{9}{19}$	0 1 +	0 1	
Albi of *Collen*, Some	5 10	345	0 0 16$\frac{16}{23}$	0 1 +	0 1	Albi, *or* Albes.
Albi of *Collen*, Others	5 10	342	0 0 16$\frac{16}{19}$	0 1 +	0 1	
Albi of *Collen*, Others	5 10	179	0 1 8$\frac{32}{179}$	0 2 —	0 2	
Albi of *Frankford*, or *Bambergh*						
Albi of *Mentz*, as *Collen*						
Albi of *Norenbergh* and *Palatine* of the *Rhine*, as *Bambergh.*						
Albi of *Trier*, as *Collen*						
Alteen, or *Atten* of *Muscovia*					0 4+	
Altine of *Poland*					0 4½	
Angel of *Scrikelborg*	10 7½	78¾	0 3 1$\frac{1}{7}$	0 8½+	0 8	
Asper of *Turkie*					0 1¼	
Babee of *Scotland*					0 0½	
Batz of 4 Creutzes. *Bavaria*, *Brandenburgh*, *Colmograve*, *Cost.* of 1530. *Friburgh*, *Ottinge*, *Raynsburgh*, *Roy*, *Scafhuysen*, *Taunte*	5 7	109½	0 2 4$\frac{44}{73}$	0 3 +	0 3	Batz *of several sorts.*
Kemptor half *Batz*	4 12½	192½	0 1 5$\frac{71}{77}$	0 1½+	0 1½	
Munchien half *Batz*	4 12½	186	0 1 6$\frac{30}{31}$	0 1½+	0 1½	
3 *Batz*, See *Snaphane*						
Bemesh, or *Bemish* of *Switz*					0 2½	
Bianco, or *Biamco* of *Italy*					0 8	
Blanks					0 0¾	
Half *Ruyters* Blank of *Holland*	3 0	144	0 1 16	0 1¼+	0 1¼	
Boligneo					0 0¾	
Carlini of *Italy*					0 6	
Carolus Guilder, as ⅔ of the *Philips* Dollar						
3 Carolus of *Carolus* and *Salsburgh*, *Campidona*, *Ernestus*, *Frankford*, *Ottingus*, *Patavia*, *Reynsborgh*	9 0	78¾	0 3 1$\frac{1}{7}$	0 7¼+	0 7	*Pieces of* 3. Carolus.
Causleto, or *Cavelcto* of *Italy*					0 3¼	

	Foreign Silver.	Fine ℥.	pwts.	Pieces to the ℔. Troy.	Weight of the Pieces ℥.	pwts.	gr.	*Sterling* Value, s.	d.	Value by others. s.	d.
Creutz. *of divers sorts.*	*Creuciat* of *John* of *Cleve*	8	7	$39\frac{3}{4}$	0	6	$0\frac{48}{53}$	1	$1\frac{1}{2}$+	1	1
Creuciats, Creuzers, or Crosses.	of *Ausburge* and *Ulme*	5	5	384	0	0	15	0	$0\frac{3}{4}$+	0	$0\frac{3}{4}$
	of *Poland*									0	$0\frac{3}{4}$
	12 *Creutzers* of *Bavaria*, *Vienna*	8	$7\frac{1}{2}$	57	0	4	$5\frac{1}{19}$	0	$9\frac{1}{2}$+	0	9
	other 12 *Creutzer* Pieces	8	$7\frac{1}{2}$	$61\frac{1}{2}$	0	3	$21\frac{27}{41}$	0	$8\frac{3}{4}$+	0	9
	And some	10	10	same		same		0	11 +	0	11
	10 *Creutzers* of *Frise*, *Ravenburg*, *Salsburg*, *Saxony*	8	7	$64\frac{1}{2}$	0	3	$17\frac{13}{43}$	0	$8\frac{1}{4}$+	0	8
	6 *Creutzers* of *Insburgh*	10	10	$124\frac{1}{2}$	0	1	$22\frac{22}{83}$	0	$5\frac{1}{2}$—	0	5
	6 *Creutzers* of *Vienna*	8	$7\frac{1}{2}$	114	0	2	$2\frac{10}{19}$	0	$4\frac{3}{4}$+	0	5
	other 6 *Creutzer* Pieces	10	10	123	0	1	$22\frac{34}{41}$	0	$5\frac{1}{2}$+	0	5
	3 *Creutzers* of *Bavaria*	4	8	375	0	0	$15\frac{9}{17}$	0	$0\frac{3}{4}$+	0	$0\frac{3}{4}$
	3 *Creutzers* of *Vienna*	4	$8\frac{1}{3}$	129	0	1	$20\frac{28}{43}$	0	$2\frac{1}{4}$—	0	2
	other 3 *Creutzer* Pieces	5	10	$136\frac{1}{2}$	0	1	$18\frac{18}{91}$	0	$2\frac{1}{2}$+	0	2
	2 *Crosses* and *Harpe*	4	0	180	0	1	8	0	$1\frac{1}{4}$+	0	$1\frac{1}{4}$
	Crowns of *France*, See *Louis*										
	Crowns of *Italy*									5	0
	Crowns of *Turky*									6	0
	Cupstoke									1	0
	Deghen, or *Denghen* of *Muscovia & de Narde*, *Russia*	11	13	$545\frac{1}{3}$	0	0	$10\frac{230}{409}$	0	$1\frac{1}{4}$+	0	$1\frac{1}{4}$
	Denier, *Petit Denier* of *Paris*	1	10	270	0	0	$21\frac{1}{3}$	0	$0\frac{1}{4}$+	not currant in *England.*	
	Denier, *Petit Denier* of *Tor*	1	10	$337\frac{1}{2}$	0	0	$17\frac{1}{15}$	0	$0\frac{1}{4}$+		
	Dicken of a *Wing*									1	4
Dollars *of divers sorts.* Dollars or Dollers of	*Albania*, or the *Cross Dollar*									4	8
	Basil of 6 *Creutzers*	10	$13\frac{1}{2}$	15	0	16	0	3	10+	3	10
	Batenburgh, *Bohemia. Ne.Op.*, *Bommel*	7	15	$13\frac{1}{2}$	0	17	$18\frac{2}{3}$	3	$1\frac{1}{4}$	3	1
	Brisgau	10	15	15	0	16	0	3	$10\frac{1}{2}$—	3	10
	Cambray, The Sixteenth part	6	10	123	0	1	$22\frac{34}{41}$	0	$3\frac{1}{4}$+	0	3
	Christopher, 45.	10	10	$12\frac{3}{4}$	0	18	$19\frac{13}{17}$	4	$5\frac{1}{4}$+	4	5
	Frisland of 1601.	9	0	$13\frac{1}{2}$	0	17	$18\frac{2}{3}$	3	$7\frac{1}{4}$—	3	7
	See *Guelders*										
	Germany or *Ryckx Dollar Ouncia*	11	5	$12\frac{1}{2}$	0	19	$4\frac{4}{5}$	4	$10\frac{1}{4}$+	5	0
	Ryckx Dollar of 1567	10	12	$12\frac{1}{2}$	0	19	$4\frac{4}{5}$	4	7 +	4	7
	Others	10	13	$12\frac{1}{2}$	0	19	$4\frac{4}{5}$	4	$7\frac{1}{4}$+	4	7
	Others	10	14	$12\frac{1}{2}$	0	19	$4\frac{4}{5}$	4	$7\frac{1}{2}$+	4	7
	Others	11	0	$12\frac{1}{2}$	0	19	$4\frac{4}{5}$	4	2 +	4	9
	Others	11	3	$12\frac{1}{2}$	0	19	$4\frac{4}{5}$	4	$9\frac{3}{4}$+	4	10
	Guelders and *Frisland* Some	10	4	$14\frac{1}{2}$	0	16	$20\frac{4}{19}$	3	$10\frac{1}{4}$+	3	10
	Guelders and *Frisland* Others	9	0	$12\frac{3}{64}$	0	19	$22\frac{34}{257}$	4	$0\frac{1}{4}$+	4	0
	Guelders and *Utrecht* Some	9	0	15	0	16	0	3	$2\frac{3}{4}$+	3	3
	Guelders and *Utrecht* Others	10	10	13	0	18	$11\frac{1}{13}$	4	$4\frac{1}{4}$+	4	4
	Guelders and *Utrecht* Others	10	12	13	0	11	$11\frac{1}{13}$	4	$4\frac{3}{4}$+	4	4
	See *Zutphen*										
	Gulielmus of *Sweden*	10	10	$12\frac{3}{4}$	0	18	$19\frac{13}{17}$	4	$5\frac{1}{4}$+	4	5
	Gustavus of *Liege* the 38*th.*	10	4	$12\frac{3}{4}$	0	18	$19\frac{13}{17}$	4	$3\frac{3}{4}$+	4	4
	Holland	9	0	$13\frac{1}{3}$	0	18	0	3	$7\frac{3}{4}$	3	8
	Holland with the *Crown*	8	0	$13\frac{1}{3}$	0	18	0	3	$2\frac{3}{4}$	3	3
	Ijmensen, as *Basil*										
	Luneburgh	10	$16\frac{1}{2}$	15	0	16	0	3	$10\frac{3}{4}$	3	10
	Phillip	10	0	$10\frac{5}{7}$	1	2	$9\frac{3}{5}$	5	$0\frac{1}{2}$+	5	0

Dollar

	Foreign Silver.	Fine ℥	pwts.	Pieces to the ℔ Troy.	Weight of the Pieces ℥.	pwts.	gr.	Sterling Value, s.	d.	Value by others. s.	d.	
Dollar, or Dollers of	Half of the *Phillips Dollar*	10	0	$21\frac{3}{7}$	0	11	$4\frac{4}{5}$	2	$6\frac{1}{4}$ +	2	6	Dollars *of several sorts.*
	Fourth Part	10	0	$42\frac{6}{7}$	0	5	$14\frac{2}{5}$	1	3 +	1	3	
	Fifth Part	10	0	$53\frac{4}{7}$	0	4	$11\frac{13}{25}$	1	0 +	1	0	
	Tenth Part	10	0	$107\frac{1}{7}$	0	2	$5\frac{19}{25}$	0	6 +	0	6	
	Twentieth Part	5	0	$107\frac{1}{4}$	0	2	$5\frac{101}{145}$	0	3 +	0	3	
	Fortieth Part	5	0	$214\frac{1}{2}$	0	1	$2\frac{122}{143}$	0	$1\frac{1}{2}$ +	0	$1\frac{1}{2}$	
	Two third Parts of the same *Dollar*	10	0	$16\frac{1}{4}$	0	14	$22\frac{2}{5}$	3	4 +	3	4	
	Poland { Some as *Batenborgh*											
	Poland { Others of 60 *Creutzers*	11	$3\frac{1}{2}$	15	0	16	0	4	$0\frac{1}{4}$ +	4	0	
	Prince of Orange or *Lyon Dollar*	9	0	13	0	18	$11\frac{1}{13}$	3	$8\frac{3}{4}$ +	4	0	
	Reynsburgh, as *Basil*											
	Riga	10	$2\frac{1}{2}$	$13\frac{1}{2}$	0	17	$18\frac{2}{3}$	4	$0\frac{1}{2}$ +	4	0	
	Scotland with the *Cross Daggers*	11	2	$11\frac{3}{4}$	1	0	$10\frac{10}{47}$	5	$1\frac{1}{4}$ +	5	0	
	States General of the United Provinces	9	0	$12\frac{3}{64}$	0	19	$22\frac{34}{257}$	4	$0\frac{1}{4}$ +	4	0	
	Succia, or { Merchants *Dollar*									3	2	
	Succia, or { *Ryckx*, or *Imperial Dollar*									5	2	
	Tremone, as *Brisgau*											
	Utrecht, See *Guilders*											
	Zeland, with the *Eagles*	9	0	$13\frac{1}{2}$	0	17	$18\frac{2}{3}$	3	$7\frac{1}{4}$ −	3	7	
	Zutphen and *Guilders* of 1586	10	4	$13\frac{2}{3}$	0	17	$10\frac{19}{41}$	4	$0\frac{1}{4}$ +	4	0	
	Drier									0	$0\frac{3}{4}$	
	Duplus	2	0	324	0	0	$17\frac{7}{9}$	0	$0\frac{1}{4}$ +	} not currant		
	Dupli Simple	5	10	882	0	0	$6\frac{26}{49}$	0	$0\frac{1}{4}$ +	} in *England.*		
	Dupli Mavi $\frac{1}{9}$ of *Guliel.* of *Turing*	2	15	440	9	6	$13\frac{1}{11}$	0	$0\frac{1}{4}$ +	nor this.		
Duyts	9 *Duyts Peny* of *Charles* and *Philip*	4	14	129	0	1	$20\frac{28}{43}$	0	$2\frac{1}{4}$ +	0	$2\frac{1}{4}$	Duyts *of several sorts.*
	11 *Duyts* of { *Charles-Limburg*	4	15	120	0	2	0	0	$2\frac{1}{2}$ +	0	$2\frac{1}{2}$	
	11 *Duyts* of { *Holland*	6	0	144	0	1	16	0	$2\frac{1}{2}$ +	0	$2\frac{1}{2}$	
	11 *Duyts* of { *Philip* and *Marie*	11	$3\frac{1}{2}$	270	0	0	$21\frac{1}{3}$	0	$2\frac{1}{2}$ +	0	$2\frac{1}{2}$	
	17 *Duyts*	10	10	147	0	1	$15\frac{2}{49}$	0	$4\frac{1}{2}$ +	0	$4\frac{1}{2}$	
	17 *Duyts* of { *Charles*, *Guilders*, *Liege*, *Limburgh*, *Lodowick*, *Philip*, *Philip* of *Flanders*, Two *Standing Lions* }	9	10	145	0	1	$15\frac{21}{29}$	0	$4\frac{1}{4}$ −	0	$4\frac{1}{4}$	
	17 *Duyts* of *Sluce*	9	5	148	0	1	$14\frac{34}{47}$	0	4 +	0	4	
	Flabes in the *Low Countries*									1	4	
	Finferkin									0	$0\frac{1}{2}$	
	Fleece, See *Stivers.*											
	Florius, by *Heylin*									3	0	
	Franks of *Turky*									2	0	
	Franks of *France*, 3 to a Crown	10	0	$26\frac{1}{4}$	0	9	$3\frac{3}{7}$	2	$0\frac{1}{2}$ +	2	0	
	Gagatta of *Italy*									0	1	
	Gnibij of *Rome*									0	6	
	Grot, or *Groot*									0	$1\frac{1}{4}$	
Groots	3 *Groots* or *Deniers*	5	10	$117\frac{3}{4}$	0	2	$0\frac{144}{157}$	0	3 +	0	3	Groots *of divers sorts.*
	5 *Groots* of { *Flanders*	10	$6\frac{1}{2}$	$146\frac{1}{2}$	0	1	$15\frac{93}{39}$	0	$4\frac{1}{2}$ +	0	$4\frac{1}{2}$	
	5 *Groots* of { *Gaunt*	5	13	145	0	1	$15\frac{21}{29}$	0	$2\frac{1}{2}$ +	0	$2\frac{1}{2}$	
	5 *Groots* of { *Philip* of *Flanders*	11	3	135	0	1	$18\frac{2}{3}$	0	$5\frac{1}{4}$ +	0	$5\frac{1}{4}$	
	5 *Groots* of { Others	10	14	135	0	1	$18\frac{2}{3}$	0	5 +	0	5	
	$5\frac{1}{2}$ *Groots* of 1520 *Ma. Flanders*	9	14	120	0	2	0	0	$5\frac{1}{4}$ −	0	$5\frac{1}{4}$	

Grosh

	Foreign Silver.	Fine ℥	pwts.	Pieces to the ℔ Troy.	Weight of the Pieces ℥.	pwts.	gr.	Sterling Value, s.	d.	Value by others. s.	d.
Gross *of divers sorts.* Grosh, or Gross of	*Ambass*	4	$12\frac{1}{2}$	$94\frac{1}{2}$	0	2	$2\frac{20}{21}$	0	3 +	0	3
	Ausburgh Some	5	7	108	0	2	$5\frac{1}{3}$	0	3 −	0	3
	Ausburgh Others	6	$4\frac{1}{2}$	155	0	1	$13\frac{5}{31}$	0	$2\frac{1}{2}$ +	0	$2\frac{1}{2}$
	Ausburgh Others of 3 *Batz*									0	9
	Basil	9	0	$118\frac{1}{2}$	0	2	$0\frac{48}{79}$	0	$3\frac{1}{4}$ +	0	$3\frac{1}{4}$
	Bassaw	5	7	$116\frac{1}{2}$	0	2	$6\frac{6}{71}$	0	$3\frac{1}{4}$ +	0	$3\frac{1}{4}$
	Bohemia, $1\frac{1}{2}$ Silver Grosh	3	$7\frac{1}{2}$	87	0	2	$18\frac{6}{29}$	0	$2\frac{1}{2}$ +	0	$2\frac{1}{2}$
	Brisau, as *Bassaw*										
	Brisgrave, *Campido*, *Corinthia*, as *Basil*										
	Coningstein	5	7	108	0	2	$5\frac{1}{3}$	0	3 +	0	3
	Curiensis, as *Bassaw*										
	Duodena, or the 12. par. of the Sil. Grosh	3	$3\frac{2}{3}$	$874\frac{1}{2}$	0	0	$6\frac{342}{583}$	0	$0\frac{1}{4}$ −	not currant in *England.*	
	Ferdinando of *Dantsicke*	5	0	180	0	1	8	0	$1\frac{3}{4}$ +	0	2
	George and *Wormeser*, as *Ambass*										
	Kempton, as *Bassaw*										
	Markegrave, as *Ambass*										
	Mary									0	$1\frac{1}{4}$
	Melvin 3 *Grosh* of 1340	10	4	138	0	1	$17\frac{17}{23}$	0	$4\frac{3}{4}$ +	0	5
	Melvin 3 *Grosh* Others	10	10	138	0	1	$17\frac{17}{23}$	0	$4\frac{3}{4}$ +	0	5
	Meysen									0	$2\frac{3}{4}$ or 3d
	Noiling, as *Ambass*										
	Poland									0	$1\frac{1}{2}$
	Poland Six *Grosh*	6	0	$13\frac{7}{8}$	0	17	$7\frac{5}{37}$	2	4 +	2	4
	Prague	9	$12\frac{1}{2}$	180	0	1	8	0	$3\frac{1}{4}$ +	0	$3\frac{1}{4}$
	Prussia, 3 *Grosh alb.*	10	$10\frac{1}{2}$	138	0	1	$17\frac{17}{23}$	0	$4\frac{3}{4}$ +	0	5
	Reynsburgh	6	$4\frac{1}{2}$	155	0	1	$13\frac{5}{31}$	0	$2\frac{1}{2}$ +	0	$2\frac{1}{2}$
	Salsburgh Some	6	$2\frac{2}{5}$	$118\frac{1}{2}$	0	2	$0\frac{48}{79}$	0	$3\frac{1}{4}$ +	0	$3\frac{1}{4}$
	Salsburgh Others	4	$12\frac{1}{2}$	39	0	6	$3\frac{9}{13}$	0	$7\frac{1}{2}$ +	0	$7\frac{1}{2}$
	Saxony, as *Coningstein*										
	Scafhuysen, as *Basil*										
	Sigismund of 1532, and 1535	10	4	69	0	3	$11\frac{11}{23}$	0	$9\frac{1}{2}$ +	0	$9\frac{1}{2}$
	Sigismund of *Prussia* 1534	10	11	69	0	3	$11\frac{11}{23}$	0	$9\frac{3}{4}$ +	0	10
	Others with the Armes of *Dantsick*	10	$0\frac{1}{2}$	69	0	3	$11\frac{11}{23}$	0	$9\frac{1}{4}$ +	0	$9\frac{1}{4}$
	Silver *Grosh* Common									0	2
	Taven, as *Basil*										
	Vienna	6	4	132	0	1	$19\frac{7}{11}$	0	3 +	0	3
	$1\frac{1}{2}$ Silver *Grosh*	3	$7\frac{1}{2}$	87	0	2	$18\frac{6}{29}$	0	$2\frac{1}{2}$ +	0	$2\frac{1}{2}$
	4 *Grosh* Peny	8	0	81	0	2	$23\frac{1}{9}$	0	$6\frac{1}{4}$ +	0	6
Guilders *the sorts.* Guilders, or Guldens	*Albertus* Double	10	15	$14\frac{3}{5}$	0	16	$10\frac{38}{73}$	4	0 −	4	0
	Albertus Single	10	15	$29\frac{1}{5}$	0	8	$5\frac{19}{73}$	2	0 −	2	0
	Albertus Half	10	15	$58\frac{2}{5}$	0	4	$2\frac{46}{73}$	1	0 −	1	0
	Albertus Quarter	10	15	$116\frac{4}{5}$	0	2	$1\frac{23}{73}$	0	6 −	0	6
	Carolus, as $\frac{2}{3}$ of the *Philips Doller*										
	Flanders Silver *Guilder*									2	0
	Gulielmus of *Turing*	6	15	129	0	1	$20\frac{28}{43}$	0	$3\frac{1}{4}$ +	0	$3\frac{1}{4}$
	Harp of *Ireland*, or Silver *Harp*	11	0	82	0	2	$22\frac{10}{41}$	0	$8\frac{1}{2}$ +	0	9
	Half *Harp*	11	0	164	0	1	$11\frac{5}{41}$	0	$4\frac{1}{4}$ +	0	$4\frac{1}{2}$
	Base *Irish Harp*	3	0	82	0	2	$22\frac{10}{41}$	0	$2\frac{1}{4}$ +	0	$2\frac{1}{4}$
	Old *Harp*	9	6	102	0	2	$8\frac{8}{17}$	0	$5\frac{3}{4}$ +	0	6
	Junctine, or *Justine* of *Italy*									1	6

Lion

Foreign Silver.	Fine ℥.	*pwts.*	Pieces to the lb. *Troy.*	Weight of the Pieces ℥.	*pwts.*	*gr.*	*Sterling* Value, *s.*	*d.*	Value by others. *s.*	*d.*	
Lion of *Gulielmi* — Some	2	5	150	0	1	$14\frac{2}{5}$	0	$0\frac{3}{4}$ +	0	1	
Lion of *Gulielmi* — Others	2	5	179	0	1	$8\frac{32}{179}$	0	$0\frac{3}{4}$ +	0	1	
Lieure of *France*, See Quar.Cro by *Heylin*			—	—	—	—	—	—	2	0	
Louis of *France*	11	2	—	—	—	—	—	—	4	6	
Half, Quart. and Eight part accordingly									not currant in *England.*		
Lyarts of *France*, *H*	3	0	—	—	—	—	—	—			
Lyre of *Geneva*	—	—	—	—	—	—	—	—	1	4	
Lyre of *Venice*	—	—	—	—	—	—	—	—	0	9	
Maille, Old *Petit Maille*	1	0	450	0	0	$12\frac{4}{5}$	0	$0\frac{1}{8}$ +	not cur. in *En.*		
Magenburgh, 3 *Armes*	5	$8\frac{1}{3}$	27	0	8	$21\frac{1}{3}$	1	1 +	1	1	
Other Piece	11	$3\frac{1}{2}$	51	0	4	$16\frac{16}{17}$	1	2 +	1	2	
Mark of *Denmark*	—	—	—	—	—	—	—	—	2	2	
Mark of *Scotland*	11	2	54	0	4	$10\frac{2}{3}$	1	$1\frac{1}{4}$ +	1	$1\frac{1}{2}$	
Half and Quarter accordingly											
Markesticke of Lady *Mary*, *Lubeck*	10	$16\frac{2}{3}$	27	0	8	$21\frac{1}{3}$	2	2 +	2	2	
Medine of *Cairo*	—	—	—	—	—	—	—	—	0	$2\frac{1}{4}$	
Mursenigo	—	—	—	—	—	—	—	—	0	11	
Nummi Dragme — Some	6	0	140	0	1	$17\frac{1}{7}$	0	$2\frac{3}{4}$ +	0	3	
Nummi Dragme — Others	6	$2\frac{1}{2}$	$118\frac{1}{2}$	0	2	$0\frac{48}{79}$	0	$3\frac{1}{4}$ +	0	3	
Peny of *Bohemia* — White	5	7	924	0	0	$6\frac{18}{77}$	0	$0\frac{1}{4}$ +	not currant in *England.*		
Peny of *Bohemia* — Black	2	$13\frac{1}{2}$	990	0	0	$5\frac{9}{11}$	0	$0\frac{2}{55}$ −			
Peny of *Holland*	0	19	518	0	0	$11\frac{31}{259}$	0	$0\frac{1}{9}$ —			
Peny, called the Brass Peny	4	10	120	0	2	0	0	$2\frac{1}{4}$ +	0	2	
Half thereof	4	10	240	0	1	0	0	1 +	0	1	
Half *Ruyters* Black Peny	4	14	256	0	0	$22\frac{1}{2}$	0	1 +	0	1	
Psound, or *Psound*	—	—	—	—	—	—	—	—	0	$4\frac{3}{4}$	
Plappot	—	—	—	—	—	—	—	—	0	$2\frac{1}{2}$ or $2\frac{3}{4}$	
Poali of *Italy*	—	—	—	—	—	—	—	—	0	6	
Pound, 3 Pound of *Scotland*	11	2	—	—	—	—	—	—	5	0	
Polpate, or *Baldpate* of *Scotland*	11	2	—	—	—	—	—	—	0	$10\frac{1}{2}$	
Half thereof	11	2	—	—	—	—	—	—	0	$5\frac{1}{4}$	
Quart.Crow. or Quartid'escue of *France*	10	$6\frac{2}{3}$	39	0	6	$3\frac{9}{13}$	1	5 +	1	6	Cardecues the sorts.
Quart.Crow. or Quartid'escue of *Lorrain*	9	$8\frac{1}{2}$	39	0	6	$3\frac{9}{13}$	1	$3\frac{1}{2}$ +	1	4	
Quart.Crow. or Quartid'escue of *Philip*, *Savoy*	10	$16\frac{1}{2}$	39	0	6	$3\frac{9}{13}$	1	6 +	1	6	
Rappen Muntz	—	—	—	—	—	—	—	—	0	$2\frac{1}{2}$	
Roustick	—	—	—	—	—	—	—	—	0	$1\frac{1}{2}$	
Ryals of *Albertus* of *Austria*	10	15	120	0	2	0	0	$5\frac{3}{4}$ +	0	6	Ryals of several sorts.
Half and Quarter accordingly											
Pieces of his of 3 *Ryals*	10	15	40	0	6	0	0	$5\frac{1}{4}$ +	1	6	
Italy — Some	9	17	108	0	2	$5\frac{1}{3}$	0	$5\frac{3}{4}$ +	0	6	
Italy — Others	9	14	108	0	2	$5\frac{1}{3}$	0	$5\frac{3}{4}$ +	0	6	
Italy — Others	9	11	108	0	2	$5\frac{1}{3}$	0	$5\frac{3}{4}$ —	0	6	
Mexico, 8 *Ryals*	11	0	$13\frac{2}{3}$	0	17	$13\frac{12}{41}$	4	4 +	4	4	
Rome, Course *Ryals*	7	0	108	0	2	$5\frac{1}{3}$	0	4 +	0	4	
Spain	11	$3\frac{1}{2}$	108	0	2	$5\frac{1}{3}$	0	$6\frac{1}{2}$ +	0	6	
Spanish 8 *Ryals* called Pieces of 8	11	4	$13\frac{1}{2}$	0	17	$18\frac{2}{3}$	4	$5\frac{3}{4}$ +	4	4	
States General of the United Province.	10	0	$10\frac{5}{7}$	1	2	$9\frac{3}{5}$	5	$0\frac{1}{2}$ +	5	0	
The 20*th.* part of the same, with the Arrows accordingly											
Venice	11	10	96	0	2	12	0	$7\frac{3}{4}$ +	0	8	
Ryder of *Guelders* and *Frisland*	9	0	$12\frac{3}{64}$ +	0	19	$22\frac{34}{257}$	4	$0\frac{1}{4}$ +	4	0	
Salvator of *Venice*	11	10	96	0	2	12	0	$7\frac{3}{4}$ +	0	8	

	Foreign Silver.	Fine ℥	*pwts.*	Pieces to the ℔ *Troy.*	Weight of the Pieces ℥.	*pwts.*	*gr.*	*Sterling* Value, *s.*	*d.*	Value by others. *s.*	*d.*
	Saſſenars double	10	$6\frac{1}{2}$	$146\frac{1}{2}$	0	1	$15\frac{93}{293}$	0	$4\frac{1}{2}$ +	0	$4\frac{1}{2}$
	Scaby of *Turky*									0	6
	Schaneberger									0	$1\frac{3}{4}$
	Scya of *Turky*									0	$6\frac{1}{4}$
	Senube, or *S'nube* of *Bohemia*	5	7	129	0	1	$20\frac{28}{43}$	0	$2\frac{1}{2}$ +	0	$2\frac{1}{4}$
	Half thereof	5	7	258	0	0	$22\frac{14}{13}$	0	$1\frac{1}{4}$ +	0	$1\frac{1}{4}$
	Seſtling									0	$0\frac{3}{4}$
Shillings of ſeveral ſorts. — Shillings of	*Bridges* of 1582	5	0	57	0	4	$5\frac{1}{19}$	0	$5\frac{1}{2}$ +	0	5
	Dantſicke									0	$0\frac{3}{4}$
	8 Shilling of *Dantſicke* of 1541	10	12	156	0	1	$12\frac{12}{13}$	0	$4\frac{1}{4}$ +	0	4
	Flanders									0	$7\frac{1}{2}$
	Friſland of 1586	6	0	57	0	4	$5\frac{1}{19}$	0	$6\frac{3}{4}$ +	0	6
	Gaunt of 1583	7	7	54	0	4	$10\frac{2}{3}$	0	$8\frac{3}{4}$ +	0	9
	Germany									0	$5\frac{1}{4}$
	Guelders, as *Friſland.*										
	Hamborough									0	$9\frac{3}{4}$
	Lubeck									0	$1\frac{3}{4}$
	M. E. and *Philip* of *Flanders*	11	3	135	0	1	$18\frac{2}{3}$	0	$5\frac{1}{4}$ +	0	5
	Scotland									0	1
	Switz, or *Helvetia*									0	$1\frac{1}{4}$
	Utrecht / *Zeland* } as *Friſland*										
	Sicherling									0	$1\frac{1}{4}$
	Snaphanen, Coined for 3 *Batz*	7	$7\frac{1}{2}$	$39\frac{3}{4}$	0	6	$0\frac{48}{53}$	1	0 +	1	0
	Snaphanen of { *Cleve* / *Deventer* / *Nimmeghen* }	7	11	48	0	5	0	0	10 +	0	10
	Soldi of *Genoa*									0	$0\frac{3}{4}$
	Soli of *Wersburgh*, *Dantſick* and *Pruſſia*	5	$6\frac{1}{4}$	$157\frac{1}{2}$	0	1	$12\frac{4}{7}$	0	2 +	0	2
	Soulx, or *Solx* of *France*									0	1
	Soulx ſtamped, called *Soulx Marque*									0	$1\frac{1}{4}$
French Sols of divers ſorts. — Soulx	The Old *Soulx* with ✠.	4	5	175	0	1	$8\frac{32}{35}$	0	$1\frac{1}{2}$ +	0	$2\frac{1}{4}$
	Ordinary *French Soulx*	3	10	147	0	1	$15\frac{9}{49}$	0	$1\frac{1}{2}$ +	0	$1\frac{1}{4}$
	Late *French Soulx*	3	$6\frac{1}{2}$	147	0	1	$15\frac{9}{49}$	0	$1\frac{1}{4}$ +	0	1
	Double Hand of one *Soulx*	3	15	132	0	1	$19\frac{7}{11}$	0	$1\frac{3}{4}$ +	0	$1\frac{1}{4}$
	Two *Soulx* Pieces, or Doubles	6	$6\frac{2}{3}$	117	0	2	$1\frac{3}{13}$	0	$3\frac{1}{2}$ —	0	3
	Four *Soulx* Pieces accordingly.										
Styvers of divers ſorts. — Styvers of	*Cambray*	3	5	135	0	1	$18\frac{2}{3}$	0	$1\frac{1}{2}$ +	0	$1\frac{1}{4}$
	Embden									0	$1\frac{1}{4}$
	Gaunt of 1583	3	0	$175\frac{1}{2}$	0	1	$8\frac{32}{39}$	0	1	0	1
	Groeninghen / *Liege* } as *Cambray*										
	States General of the United Provinces	4	0	168	0	1	$10\frac{2}{7}$	0	$1\frac{1}{2}$ +	0	$1\frac{1}{4}$
	Utrecht	3	0	167	0	1	$10\frac{82}{167}$	0	1 +	0	1
	Old Styver	3	$14\frac{1}{6}$	120	0	2	0	0	2 +	0	2
	New Styver	3	$13\frac{1}{3}$	120	0	2	0	0	2 —	0	2
	Half Styver	3	10	201	0	1	$4\frac{44}{67}$	0	1 +	0	1
	Quarter Styver *Oort*	1	$17\frac{1}{2}$	158	0	1	$12\frac{36}{79}$	0	$0\frac{3}{4}$ +	not currant in *England.*	
	Eight part Styver *Duyt*	1	14	474	0	0	$12\frac{12}{79}$	0	$0\frac{1}{4}$		
	Old Double Styver	7	$7\frac{1}{2}$	120	0	2	0	0	4 —	0	4
	Old Three Styvers	11	$3\frac{3}{4}$	120	0	2	0	0	6 +	0	6
	Old Four Styvers { with the *Eagle* / *Charles* and / *Philip* }	7	$7\frac{1}{2}$	60	0	4	0	0	8 —	0	8

Three

Foreign Silver.	Fine ℥	pwts.	Pieces to the ℔. *Troy.*	Weight of the Pieces ℥.	*pwts.*	*gr.*	*Sterling* Value. *s.*	*d.*		Value by by others. *s:*	*d.*
Styvers											
Three Styvers, or Fleece	10	10	108	0	2	$5\frac{1}{3}$	0	$6\frac{1}{4}$	+	0	6
Flemish Six Styvers	10	0	54	0	4	$10\frac{2}{3}$	1	0	+	1	0
The *Bre* 1499 / The *Key* and *Joane* — 3 Styvers	10	4	156	0	1	$12\frac{12}{13}$	0	4	+	0	4
Five Styvers of *Cambray* Some	6	$6\frac{1}{2}$	48	0	5	0	0	$8\frac{1}{2}$	+	0	8
Five Styvers of *Cambray* Others	6	$6\frac{1}{2}$	51	0	4	$16\frac{16}{17}$	0	8	+	0	8
Five Styvers of *Guelders*	8	$1\frac{1}{2}$	48	0	5	0	0	$10\frac{3}{4}$	+	0	10
Five Styvers of *Horne*, as *Cambray*											
Five Styvers of *Liege* Some	7	11	48	0	5	0	0	10	+	0	10
Five Styvers of *Liege* Others	6	$6\frac{1}{2}$	48	0	5	0	0	$8\frac{1}{2}$	+	0	8
Five Styvers of *Liege* Others	6	$6\frac{1}{2}$	51	0	4	$16\frac{16}{17}$	0	8	+	0	8
Teastons, or Testons of											
Baden, Chrysostome	10	$10\frac{1}{2}$	39	0	6	$3\frac{9}{13}$	1	$5\frac{1}{2}$	+	1	5
Berne — *Ottoman* / *Vincent*	11	$5\frac{1}{6}$	45	0	5	8	1	4	+	1	4
Castile, as *Berne.*											
Ferrara, Hercules, and *Alphonsus*	11	$5\frac{1}{6}$	45	0	5	8	1	4	+	1	4
France, Franciscus	10	7	42	0	5	$17\frac{1}{7}$	1	4	—	1	4
Friburg, Nicolas, as *Berne*											
Geneva	10	$4\frac{1}{2}$	42	0	5	$17\frac{1}{7}$	1	$3\frac{3}{4}$	+	1	4
Lorrain of 1524, and 1529	10	7	42	0	5	$17\frac{1}{7}$	1	4	—	1	4
Lucerne, Episcopus / *Mantua, Francis* — as *Berne*											
Millan — *Galleacius*, and *Lodovicus*	11	$5\frac{1}{6}$	45	0	5	8	1	4	+	1	4
Montserat, George and *Guill.*	10	$4\frac{1}{2}$	42	0	5	$17\frac{1}{7}$	1	$3\frac{3}{4}$	+	1	4
Navarre *Henricus* / *Anna* — as *Baden*											
Portugal, Io. V. L	10	7	42	0	5	$17\frac{1}{7}$	1	4	—	1	4
Savoy, Carolus Some	11	$5\frac{1}{6}$	45	0	5	8	1	4	+	1	4
Savoy, Carolus Others	10	$10\frac{1}{2}$	39	0	6	$3\frac{9}{13}$	1	$5\frac{1}{2}$	+	1	5
Sedun, Nicol, dan, Adrian	11	$5\frac{1}{6}$	45	0	5	8	1	4	+	1	4
Solod, Ursus, as *Berne*											
Turones of *France*	10	18	$26\frac{1}{4}$	0	9	$3\frac{3}{7}$	2	$2\frac{3}{4}$	+	2	2
Vierysers, Double	4	10	138	0	1	$17\frac{17}{23}$	0	2	+	0	2
Single accordingly											

Styvers of several sorts. *Testons of divers sorts.*

Base Coines inconsiderable.

The Coines that are all Brass used in Foreign Countries are many, and admit of several Subdivisions: But (as the *Lawyers* say, *de minimus non currat Lex*) they being so small and inconsiderable, and few of them being currant in any other place than respectively where Coined, are not worth the remembrance here.

Geodæticals *how to be placed, or expressed.*

Nothing is further needful to finish this Chapter, then to shew how to set down or express any *Geodætical* Number, which to do, place the highest Denominate Number to the left hand, and all the rest in a straight line in order to the right hand, with a little line, or prick or two, or some such note of distinction between them; and over the head of every Number, or near the same, set the Character, Symbole, or Note, whereby it may be known of what nature or kind of *Geodætical* the same is, as to express Four Pounds Twelve Shillings and Three Pence, set them as at *A.* Ten Ounces Three Drams, Two Scruples, and Fifteen Graines, as at *B.*

Examples.

	l. s. d.	*l. s. d.*	*l. s. d.*
A.	4 : 12 : 3	or thus 4—12—3	or thus 4 + 12 + 3
	℥ ʒ ℈ *gr.*	℥ ʒ ℈ *gr.*	℥ ʒ ℈ *gr.*
B.	10 . 3 . 2 . 15	or thus 10— 3—2—15	or thus 10 + 3 + 2 + 15

Example in Fracted Geodæticals.

And if the *Geodæticals* be fracted in like manner after the Number is set down, place the Denomination at or near the head thereof, as $\frac{1}{4}$*d.* $\frac{2}{3}$*s.* $\frac{4}{5}$*l.* $3\frac{1}{20}$ Ton, *&c.* which Numbers, though Without those Denominations should have been as Abstract Fractions; yet now are restrained by those Denominations. The first to be one Quarter or Fourth part of a Peny, which is a Farthing. The Second to be Two Thirds of a Shilling, which is 8 Pence; for one Shilling broken into Three parts or Groats, two

of

of them will amount to so much. The third Fraction likewise now is Four Fifths of a Pound, which is in Value 16 *s.* for if one Pound or 20 *s.* be divided into 5 parts, and 4 of them parts be put together, it maketh 16 *s.* The Fourth Number is 3 Tons, and 17 Twentieth parts of one Ton, that is 17 Hundred, whence also is to be observed, ☞ That when an Integer and a Fraction is mixt, the Fraction is alwaies set to the right hand of the Integer, and is a part of parts of one of those Integers, let the *Geodætical* Denomination be what it will. So $4\frac{1}{4}$ *l.* the $\frac{1}{4}$ shall understand a quarter of 1 *l.* not a quarter of 4 *l.* and the like of all others.

English Geodæticals *where to be understood.* Further also may be observed, to save the often repeating the words *Sterling*, or *English* in the following Examples; let the same be understood to those *Geodæticals* of Coin Weight, *&c.* though not expressed, where the Denominations or Notes do not express them to be Foreign *Geodæticals.*

CHAP. II.

Reduction of Geodæticals.

Simple Elements of Geodæticals. *How agree and differ to Integers and Faactions.*

THE Nature of *Geodæticals* with their Notes and Denominations declared in the præcedent Chapter, the rest of their Simple Elements are next to be spoken to.

Geodæticals, as they partake of the Nature of Abstract and Contract Numbers, and arise from others; so their Numeration is both Original and Ortive; that in *Addition*, *Substraction*, *Multiplication* and *Division*; and this in *Reduction.* And (as Fractions) have properly their Ortive Numeration, though but accidental, and for conveniency, fall first under consideration, before the more Essential, and Original Numeration. Yet different herein, that *Reduction* of *Fractions* declareth the proportion of one Number to another, or of broken parts to broken parts; but *Reduction* of *Geodæticals*, the Denominations of one Number, lesser or greater contained in another.

Reduction *of what use and how called.* *Reduction* of *Geodæticals* bringeth Numbers of one Denomination to another, and so sheweth how to express one and the same Number in Value under different Names or Denominations. As 8 *s.* or $\frac{2}{5}$ of a Pound, which is alike valuable. For which reason *Reduction* is sometime called *Æquation.* And sometime is useful to avoid Fractions, sometime to facilitate those Operations which without *Reduction* are tedious. The Number given to be reduced is called The *Reducend.* The several Denominations, are *Reducers.* And the Number obtained by *Reduction* is the *Result.*

Reducend Reducers Result *what*

The Sorts of Reduction.

Reduction of *Geodæticals* is either General, or Special.

General Reduction of *Geodæticals* is { Proper { Synthetical. or Analytical. } or Proportional { Analytical. and Synthetical. } }

Proper of the First Sort included under 3 Cases.

Proper Synthetical Reduction serveth to reduce Subtiller or Smaller Denominations into Grosser or Greater, as Pence into Shillings or Pounds, *&c.* and is performed by *Division* under one of these 3 Cases.

1. Data *Single.* Quæsita *Single Rule.*

1. *Case.* When the Number to be reduced is Single-Integral, and the Denomination desired is single.

Then divide the Number to be reduced if it may be, by so many as one of the Greater do contain of the Lesser Denomination; If it cannot be divided, abbreviate it as a Fraction.

1. *Example.*

1. *Example.* To know how many Pounds *Sterling* are in 81600 Pence. I divide 81600 by 240 the Pence in one Pound, and the Quotient 340 *l.* is the Result.

	9	
Reducend	8~~1~~600(340 *l.*	Result.
Reducer	~~24~~4 0	
	z	

2. *Example*, To know what parts of a Pound 8 *s.* or 8 *d.* are, because 8 will not be divided by 20 the Shillings in a Pound, nor yet by 240 the Pence in a Pound; they are both abbreviated as Fractions, and 8 *s.* at *A.* is seen to be $\frac{2}{5}$ *l.* and 8 *d.* at *B.* $\frac{1}{30}$ *l.* 2. *Example.*

A. $\frac{8}{20} \Big| \frac{4}{10} \Big| \frac{2\,l.}{5}$ *B.* $\frac{8}{240} \Big| \frac{4}{120} \Big| \frac{2}{6} \Big| \frac{1\,l.}{30}$

2. *Case*, When the *Reducend* is Integral and single, and the Denominations desired are plural. 2. Data, *single*, Quæsita *Plural*

Then divide the *Reducend*, by so many of the Denomination given, as make one of the next Greater desired, and then successively continue the Division of the Quotients resulting by so many as make one of the next Greater Denomination desired. *Rule.*

1. *Example*, To know how many Shillings and Pounds there are in the former Sum of 81600 *d.* First I divide 81600 by 12 the Pence in a Shilling, and the resulting Quotient is 6800 *s.* which divided by 20, the Shillings in 1 *l.* the Result is 340 *l.* as before. 1. *Example.*

```
9
81600 (6800 s. (340 l.
122      2 0
1
```

2. *Example*, To know how far 285120 Barley-Cornes laid end to end will reach, dividing by 3 the Barley-Cornes that make 1 Inch, and by 12 the Inches in 1 Foot, and by 5 the Feet in 1 Pace, and by 1056 the Paces in 1 *English* Mile: The Result is $1\frac{1}{2}$ Mile. And the several intermediate Quotients declare the respective Inches, Feet, and Paces therein. 2. *Example.*

```
1       112     24     (528
285120 (95040 (7920 (1584 (1 528/1056 or 1 1/2 Mile.
3333    1222   5555   1056
        11
```

3. *Case*, When the *Reducend* hath some Fraction annexed thereto. 3. Data *mixt.*

Then reduce the *Reducend* into an Improper Fraction, and divide the same after the manner of a Fraction, by so many of the given Denomination, as make one of the desired Denomination, Or else divide the Whole Number first, and add the Fraction to the Quotient. *Rule.*

Example. To know how many Shillings are in $196\frac{1}{2}$ *d.* First I reduce $196\frac{1}{2}$ into the Improper Fraction $\frac{393}{2}$, and then divide by 12, the Result is $16\frac{3}{8}$ *s.* or dividing 196 by 12, the Result is 16 *s.* 4 *d.* to which the half-peny added makes it 16 *s.* $4\frac{1}{2}$ *d.* all one with $16\frac{3}{8}$ *s.* *Example.*

Thus $\frac{393}{196\frac{1}{2}}$ *d.* $\frac{\overset{4}{12}}{1}\Big)\frac{\overset{131}{393}}{2}\Big(\frac{131}{8} = 16\frac{3}{8}$ *s.* or thus

```
7(4
196(16 . 4 1/2
122
1    16 . 4 1/2
```

Proper Analytical Reduction, serveth to reduce Grosser or Greater Denominations into Subtiller, or Smaller, as Pounds into Shillings, or Pence, &c. and is performed by *Multiplication* under one of these 3 Cases. Proper Reduction of the second sort included under 3 *Cases.*

1. *Case*, When the Nnmber given to be reduced is of one Denomination and Integral, and the desired Denomination single. 1. Data, *Single* Quæsita *Single.*

Then multiply the *Reducend* by so many as one of the Greater do contain of the Lesser Denomination. *Rule.*

Example, To know how many Pence are in 340 *l.* *Sterling*, I multiply 340 by 240, because so many Pence are contained in one Pound, and the Product 81600 is the Result. *Example.*

```
Operation      340 l.                              34 0 Reducend
               240 d.                              24 0 Reducer
              ------           6                  ------
               13600         6 X 7     or thus     136
               680             6                   68
              ------                              ------
In 340 l. Sterl. 81600 pence                       81600 Result.
```

2. Data *Plural.* Quæsita *Plural* 2. *Case*, When the *Reducend* is Integral and of divers Denominations, or it is desired to know how many of the several intermediate Denominations (if any such be) are between the Denomination given, and that into which it is to be reduced.

Rule. Then multiply the Number to be reduced by so many as one of the next Lesser Denominations to the *Reducend* containeth, and successively the Product resulting by so many as one of the next Lesser Denomination to the Denomination of the Product doth contain. And if any odd Numbers belong to the respective Denominations add them to the respective Products after the manner of Integers.

1. Example. 1. *Example*, In the former Sum of 340 *l.* if it were desired to know how many Shillings as well as Pence were contained therein. I multiply 340 *l.* by 20, the Shilling in 1 Pound, and the Product 6800 are the Shillings therein, which multiplyed by 12, the Pence in 1 Shilling, produce 81600 *d.* as before. See *C.*

2. Example. 2. *Example*, If it be desired to know how many Shillings, Pence, and Farthings there are in 355 *l.* 15 *s.* 4 *d.* 3 *q.* *Sterling.* After *Multiplication* by 20, the 15 *s.* are added, and after multiplication by 12, the 4 *d.* are added, and after multiplication by 4 the 3 *q.* are added as at *D*, and the Total Result is 341539 *q.*

```
   340 l.
    20
  ------
  6800 s.
    12
  ------
 13600
 6800
  ------
C. 81600 d.
  ------
```

```
   355 l. 15 s. 4 d. 3 q.
    20
  ------
  7100
    15 added
  ------
  7115 s.
    12
  ------
 14230
 71154 added
  ------
 85384 d.
     4
  ------
341536
     3 added
  ------
D. 341539 q.
  ------
```

Examples in several Denominations. In like manner any *Geodætical* of like Nature *English*, or Foreign may be reduced to a Lesser Denomination, observing to multiply by the Number of Lesser Denominations contained in the Greater, and adding the odd Numbers if any be.

Examples in Long Measure.

Long Measure. How many Barley Cornes being laid end to end will reach from *Rye* to *London* being 60 Miles?

```
Thus   60 Miles
        8
  ---------
      480 Furlongs
       40
  ---------
    19200 Perches.
      16½
  ---------
   115200
   19200
     9600
  ---------
   316800 Feet
       12
  ---------
   633600
  316800
  ---------
  3801600 Inches.
        3
  ---------
 11404800 Barly Corns
  ---------
```

```
or Thus,   60 Miles
         1056
  ---------
          360
         300
        600
  ---------
        63360 Paces
            5
  ---------
       316800 Feet.
           12
  ---------
       633600
      316800
  ---------
      3801600 Inches.
            3
  ---------
     11404800 Barly Corns.
  ---------
```

Example

Example in Square Measure.

In 1423 Acres, 2 Roods, 30 Perches; How many Perches? *Square Measure*

Thus 1423 Acres.
4
5692
2 added
5694 Roods
40
227760
30 added
227790 Perches

or thus, 1423 Acres.
160 Perches in 1 Acre.
85380
1423
110 added 110 Perches.
227790 Perches resulting.

Example in Weight.

C. qr. ℔.
In 19—2—20 *Avoirdupois Weight.* How many Grains? *Avoirdupois Weight.*

Thus 19 C.
4
76
2 added
78 qr.
28
624
156
20 added
2204 ℔.
16
13224
2204
35264 ℥
8
282112 ʒ
3
846336 ℈
20
16926720 gr.

or Thus 19 C.
112 Pounds in 1 C.
38
196 added 76 ℔.
197
2204 ℔.
7680 Graines in 1 ℔.
176320
13224
15428
16926720 Graines resulting.

Example in Time.

In 40 Years, 12 Dayes, 10 Hours, How many Minutes? *Time.*

40 Years.
365
200
240
12022
14622 Dayes
24
58488
29244
10
350938 Hours
60
21056280 Minutes.

Besides the 12 Daies in the *Reducend*, are added 10 more, (which is 22 in all) because every 4*th*. year being Leap-Year, and that year having 366 Daies in 40 Years is 10 Daies.

3. *Case*

3. Data mixt. *Rule.*

3. *Case*, When the *Reducend* hath some Fraction annexed thereto.

Then Reduce the *Reducend* into an Improper Fraction, aud multiply the same after the manner of Fractions, by so many of the given Denominations as make one of that desired.

Example.

Example, To know how many Pence there are in $16\frac{3}{8}$ *s* I reduce them into the Improper Fraction $\frac{131}{8}$, and the multiply by 12, and the Result is $196\frac{1}{2}$ *d.*

$$\frac{131}{16\frac{3}{8}\,s.} \qquad \frac{131}{8} \times \frac{12}{1} = \frac{393}{2} = 196\tfrac{1}{2}\,d. \qquad \frac{393}{2}\Big(196$$

Proof of Proper Reduction.

Proper Reduction of both sorts, Case by Case is alternately proved by the other; as by the former Examples is evident without further instance.

Proportional Reduction *belongs to* Proportions.

Proportional Reduction, is both *Analytical* and *Synthetical*, using both *Multiplication* and *Division*, and helpeth to reduce Denominations of Measure, Weight, Coine, &c. of one Kind or Countrey to another, *English* to *Foreign*, and *Foreign* to *English*, and is called *Proportional*, because it properly belongeth to the Doctrine of *Proportions* and *Rule of Three* handled in the *4th. Book*, where more fully thereof may be seen. But

If an Unit, one of the Data *fall under* 3. Cases.

because often in that Rule an Unit falls to be one of the Three Numbers, which neither multiplying nor dividing becomes useless, and so such kind of Reductive Questions become transient when the given *Geodæticals* contain in them some Part, Denomination or Value common to both, and may be translated hither under 3 Cases.

1. Data, *Single. Rule.*

1. *Case*, When the *Geodætical* given is Integral and Single.

Multiply the given Number by such common Parts or Value, as make one of the Denominate Integers, and divide the Product by the common Parts or Value in the other.

1. *Example.*

1. *Example*, In 300 Ells *English*, How many Yards?

Here because Ells contain 45 such Inches as Yards 36, or 5 Quarters, as Yards 4: I multiply the 300 by one, and divide the Product by the other, and find 375 Yards.

Thus	300	278 yards	or thus	300	
	45	13500(375		5	$\frac{1500}{4}$(375
	1500	3666		1500	
	1200	33			
	13500				

1. *Example.*

2. *Example*, In 108 Pieces (or Ryals) of Eight, at 4 *s.* 4 *d. per* piece: How many Crowns *English* at 5 *s. per* Piece?

Here Pence being the Denomination common to both, the Product of 108 multiplyed by 4 *s.* 4 *d.* or 52 *d.* and divided by 5 *s.* or 60 *d.* giveth $93\frac{3}{5}$ Crowns, the Resolution, or 93 Crowns and 3 *s. Sterling.*

Thus,	108		or thus	108	
	$4\frac{1}{3}$	$\frac{468}{5}$(93		52	$\frac{5616}{60}$(93
	432			216	
	36			540	
	468			5616	

2. Data *Plural. Rule.*

2. *Case.* When the given *Geodætical* is Integral and Plural.

Reduce all the Denominations into the lowest, and then divide or abbreviate as before.

1. *Example*,

1. *Example*, To know what parts of a Pound 7 *s.* 6 *d.* or 12 *s.* 3 *d.* 2 *q.* are; These Numbers reduced, the first into Pence, and the second into Farthings, and abbreviated because they will not be divided: Declare the former $\frac{3}{8}$ *l.* at *E.* and the other $\frac{59}{96}$ *l.* at *F.* 1. *Example.*

E. 7 *s.* 6 *d.*

	12
	14
	76
	90

$\frac{90}{240} \mid \frac{9}{24} \mid \frac{3}{8}\,l.$

F. 12 *s.* 3 *d.* 2 *q.*

	12
	24
	123
	147
	4
	590

$\frac{590}{960} \mid \frac{59}{96}\,l.$

2. *Example.* Suppose a *Ropemaker* marry his Daughter to a *Sope-maker*, and for her Portion give her Twenty Ropes, and on every Rope 20 Knots, and on every Knot 20 Purses, and in every Purse 20 Three Half-pence: How much Money had she for her Portion? 2. *Example.*

To resolve this Question, the number of Three Half Pence is first known by *Multiplication*, and then dividing that Product by 8 the Three Half Pence in one Shilling, and after by 20 the Shillings in one Pound; or else by 160 the Three Half-Pence in one Pound: The Portion is found to be 1000 *l.*

20	Ropes
20	
400	Knots
20	
8000	Purses.
20	
160000	3 Half-Pence

$\frac{160000}{8}\left(\frac{2000|0}{1000|\,l.}\right.$ $\quad \frac{160000}{16\ \ 0}\left(1000\,l.\right.$

3. *Case*, When the *Geodætical* Reducend is a mixt Number, or a Fraction. 3. Data *mixt, or* Fractions.

If a mixt Number reduce it into an Improper Fraction, and let the Numerator of the Fraction Proper or Improper be multiplyed by the Parts that make one of the Denominate Integers, and divide that Product by the Denominator with the common parts desired if any be. And thus the value of any Fraction or Remainer upon a Division may be known.

1. *Example*, To know the value of $\frac{7}{10}$ *l.* The Numerator 7 being multiplyed by the parts of a Pound, which are either 20 Shillings, 240 Pence, 960 Farthings, *&c.* according to that part multiplying, dividing the Product by 10, the Denominator; shall the Quotient of that Division be denominate. 1. *Example.*

$\frac{7\,l.}{10}$ 20 *s.* × 7 = 140; $\frac{140}{1\ 0}\left(14\,s.\right.$ ‖ 240 *d.* × 7 = 1680; $\frac{1680}{1\ 0}\left(168\,d.\right.$ $\frac{168}{12}\left(14\,s.\right.$ ‖ 960 *q.* × 7 = 6720; $\frac{6720}{1\ 0}\left(672\,q\right.$ $\frac{672}{48}\left(14\,s.\right.$

2. *Example*, To know how many *Liures Tournois* at 20 *d.* per *Liure*, are in 100$\frac{1}{2}$ *l.* *Sterling*. The 100$\frac{1}{2}$ *l.* reduced is $\frac{201}{2}$ which multiplyed by 240 *d.* the parts of one Integer produce 48240, this divided by 2 the Denominator multiplyed into 20 the common parts of the *Liure* desired, resolve the Question into 1206 *Liures*. 2. *Example.*

Thus $\frac{201}{100\frac{1}{2}\,l.}$ 201 × 240: 8040, 402, = 48240 $\quad \frac{48240}{2} \times \frac{20}{1}$ = 40 $\quad \frac{48240}{4\ 0}\left(1206\right.$

Proof of Proportional Reduction.

Proportional Reduction of each sort is proved, by reversing the Question and Work, by making the *Divisors* and Multipliers in the one Question, the contrary in the other. As in the former Instance, If 300 Ells contain 375 yards, and they were given to know how many Ells were therein, Then multiply 375 yards by 4 (the former Divisor) and that Product 1500 divided by (the former Multiplier) 5, will return 300 Ells, and shew the Works right.

Special Reduction what and how done.

Special Reduction consists in some Select Rules more brief and commodious than the common way. As——

1. Pence brought into Pounds, &c. at once.

1. To bring Pence into Pounds, Shillings, and Pence at one Work, divide the Sum to be reduced by 24, and from the Quotient cut off the right hand Figure, which is Primes, every Unite in Value 2 *s.* and the Remainer of the Division is Pence.

Example.

Example, If 85390 Pence be divided by 24, I take 12 *d.* for 1 *s.* out of the 22 Pence remaining on the Division, and add to the 14 *s.* or 7 Primes cut off from the Quotient, and with the 10 *d.* left of the 22, I obtain the Total 355 *l.* 15 *s.* 10 *d.*

```
x pence
x3x(22 l. primes
85390(3557
24444
222  355:14
     -------
        1:10
   355:15:10
```

Common Way.

```
(1
117
85390(0 d. (71(15 s. (355 l.
22222      2220
111
```

2. Farthings brought into Pounds, &c. at once. Example.

2. To bring Farthings into Pounds, Shillings and Pence, at one work, divide the Sum by 96, and from the Quotient cut off the right hand Figure as before, and for every 48 remaining add a Shilling, the rest are Farthings.

Example, If 417231 Farthings be divided by 96, the Quotient right hand Figure 6 is 12 *s.* the 15 left on the Division is Farthings, or 3 *d.* 3 *q.* added make the Total 434 *l.* 12 *s.* 3 *d.* 3 *q.*

```
45(1 q.
3349(5 l. primes
417231(4346
96666
999 334:12:
           3¾
   ----------
   434:12:3¾
```

Common Way.

```
           1
1  (3 q. 812(3d.  1 s.   l.
417231( 104307( 8609(2 (434
444444  12222   2220
          111
```

3. Pounds brought into Shillings. Example.

3. To reduce Pounds into Shillings, double the Number of Pounds, and to the right hand adjoyne a Cypher.

Example, If 30 *l.* be brought into Shillings, 30 doubled is 60, and a Cypher adjoyned makes it 600 *s.*

The Common way.

```
 30 l.        30 l.
  2           20
-----        -----
600 s.       600 s.
```

4. Shillings brought into Pounds. Example.

4. To reduce Shillings into Pounds, cut off the right hand figure, and take half the residue, as in the 13*th.* *Section* of *Division of Integers* was taught to divide by 20.

Example, If 8692 *s.* be brought into Pounds, 2 cut off the half of 869, the rest is 434 *l.* and 1 remaining makes 2 cut off to be 12 *s.*

```
(1 s.
869'2
-----
434 l.
```

Common way.

```
(1
869(2 (434 l.
222
```

5. To

5. To reduce Shillings into Pence, double the Number given, and placing it one place nearer to the right hand, add it to the given Number. 5. *Shillings brought into Pence.*

Example, To bring 16 *s.* into Pence, the double of 16 is 32, placed and added accordingly, make 192 Pence. *Example.*

16 *s.*	Common way.
32	16 *s.*
192	12
	32
	16
	192 *d.*

6. To reduce Pounds into Primes or 2 *s.* adjoyn a Cypher to the right hand of the given Number. 6. *Pounds into Primes.*

Example, 45 *l.* brought to Primes is 450 *Primes*. *Example.*

450 *Primes*.	Common way.
	45
	10
	450 *Primes*.

7. To reduce Shillings into Primes of Pounds, Take half the given Number. 7. *Shillings into Primes.*

Example, 346 Shillings reduced make 173 Primes. *Example.*

$\frac{1}{2}$ 346
173 *Primes*

Common way.

346 (173 *Primes*
222

8. To reduce any Number to another Denomination, that hath more Common Parts than one, any of those Common Parts may be taken and those are to be chosen, that make the work shortest. 8. Reduction *by any of the Common Parts.*

Example, If one Dollar be worth 4 *s.* 8 *d.* How many Dollars are in 348 *l.*? Here because Groats as well as Pence are common parts to 4 *s.* 8 *d.* and also to Pounds; I accept Groats before Pence, and working thereby find 1491 Dollars and 6 Groats over. *Example.*

Common way.

4 *s.* 8 *d.*	348 *l.*	1	4 *s.* 8 *d.*	348 *l.*	5 (2 Pence
3 Groats	60	622(6 Groats.	12	20	2718(4
		20880(1491 Dollars.			83520(1491 Dollars
12	20880	14444	48	6906	56666
2		111	8	12	555
14 Groats.			56	13920	
				6960	
				83520	

Special Reduction may be proved by the General, as in all the last 8 Examples is apparent, the Result by the Common Way is equal to the other. *Proof of* Special Reduction

Johnson adviseth to prove the Fractionary Operations in *Geodæticals* by finding the value of each Fraction, and compare them with their value after *Reduction*, which will be alike if the work be right. Johnson's *Proof of* Reduction *of* Fractions.

1. *Example*, If $\frac{1}{2}$ *l.* and $\frac{1}{3}$ *l.* be reduced to one Denominator as *Vulgar Abstract Fractions*, they make $\frac{3}{6}$ and $\frac{2}{6}$, and because $\frac{1}{2}$ *l.* is 10 *s.* and so is $\frac{3}{6}$ *l.* and $\frac{1}{3}$ *l.* is 6 *s.* 8 *d.* and so is $\frac{2}{6}$ *l.* their Reduction appears right.

$\frac{1}{2}$ *l.* $\frac{20}{2}$(10 *s.* $\frac{3}{6}$ *l.* $\frac{3}{20}$ 60 $\frac{60}{6}$(10 *s.* || $\frac{1}{3}$ *l.* $\frac{20}{3}$(2 (6$\frac{2}{3}$ *s.* $\frac{2}{6}$ *l.* $\frac{2}{20}$ 40 $\frac{40}{6}$ 2(4 (6$\frac{2}{3}$ *s.*

2. *Example*,

2. *Example*, If $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$ of an old Harper, or Nine-pence be reduced, it will make $\frac{1}{3}$ or Three-pence; and so by finding the Value of the Right hand Fraction $\frac{7}{9}$ it is 7 Pence, of which $\frac{6}{7}$ taken is 6 *d.* and $\frac{2}{3}$ of that is 4 *d.* of which $\frac{3}{4}$ is 3 *d.* also.

CHAP. III.

Addition of Geodæticals.

Geodæticals *added.*

ADdition of *Geodæticals* may be considered according to the different nature of the *Geodæticals* proposed to be added.

1. Integers *of one Denomination.*

1. *Case.* If the Numbers to be added be *Integral Geodæticals* alone, and of the same Denomination,

Rule.

Then add the Numbers as *Abstract Integers* in *Book* 1. *Part* 1. *Chap.* 5. and to the Total adjoyne or understand the Denomination given with the Numbers to be added.

1. *Example.*

1. *Example*, Two Flocks of Sheep, one of 910, and the other of 563 are to be added, the Total shall be 1473 Sheep.

2. *Example.*

2. *Example*, Three Men are indebted to another, one 340 *l.* the second 520 *l.* and the third 600 *l.* The Total of those Debts shall be 1460 *l.*

First Example.

$$\left.\begin{matrix}910\\563\end{matrix}\right\}\text{Sheep.}$$

$$\text{Total } \quad 1473 \quad \text{Sheep.}$$

Second Example.

$$\left.\begin{matrix}340\\520\\600\end{matrix}\right\}l.$$

$$\text{Total } \quad 1460\ l.$$

2. Integers *of several Denominations.* *To place the* Data.

2. *Case.* If the Addends be *Integral Geodæticals* greater and smaller of different Denominations,

Then place the greater Denomination alwaies to the Left hand, and in order to the Right hand the next Denominations, and also Units under Units, and Tens under Tens, &c. of every Denomination respectively, and Numbers of like Quality or Denomination under Numbers of the some Quality or Denomination. And when there are wanting some Denominations to fellow with others, some supply the places of such wanting Denominations with Cyphers.

To add them.

Then begin with the right hand file, and smallest Denomination first, and adding all those smaller Numbers together, mark how many of the next Greater Denomination, you can take out of the Total of those smaller Numbers so reckoned up, and so many Units reserve in your mind, and the Overplus, if there be any, subscribe under the line and file where you are reckoning. And as in *Integers* and the former sort of Greater *Geodæticals* you reckoned the Article before, so reckon them you carry away now into the next Left hand Denomination. And this do if there be 2. 3. or more Denominations in the Numbers given to be added.

Example.

Example, If 4 Men were indebted to me, *viz. A.* 22 *l.* 13 *s.* *B.* 15 *l.* 10 *s.* 8 *d.* *C.* 10 *l.* 5 *s.* 2 *d.* and *D.* 3 *l.* 4 *s.* 7 *d.* or any other Sums, and I would know the Total that is owing by them all. Then I set them as at *E.* and beginning with the Pence, I say 7 and 2 is 9, and 8 is 17, which is 1 Shilling and 5 Pence over, wherefore I set down the remaining 5 Pence, and the one Shilling I carry to the Denomination of Shillings, and the Work will stand as at *F.* Then the one Shilling I carry and 4 is 5, and 5 is 10, and 3 is 13, and 10 is 23, and 10 is 33 Shillings, out of which I can take but one of the next greater Denomination, which is Pounds, and there resteth 13 *s.* which I set down as at G. Now the one Pound reserved I add to the next Denomination thus, saying 1 and 3 is 4, and 5 is 9, and 2 is 11, where this being the Greater *Geodætical* and highest Denomination of that kind, I reserve the Articles to the next file, and subscribe the digits as before in *Integers* and going forward find the Total to be 51 *l.* 13 *s.* 4 *d.*

Debts

Debts of	E. l. s. d.	F. l. s. d.	G. l. s. d.	H. l. s. d.
A.	22—13—00	22—13—00	22—13—00	22—13—00
B.	15—10—08	15—10—08	15—10—08	15—10—08
C.	10—05—02	10—05—02	10—05—02	10—05—02
D.	3—04—07	3—04—07	3—04—07	3—04—07
		—05	13—05	51—13—05 Total.

Some for the aid of their memory prick that Figure of the least Denomination, which being joyned to the other below it, gives more than one of the next Denomination, and carry the Overplus to the next Figure, where when they exceed one of the next Denomination, they set another prick, and so still carry the Remaines, and the last Remain or Overplus, if any be, subscribe under that File. And then carry so many Units as there be pricks unto the Numbers of the next Left hand Denomination, and there proceed in like sort, and so in the following Examples the Numbers will be thus pricked.

Pricks used to help the Memory.

Example.

l.	*s.*	*d.*	*l.*	*s.*	*d.*	*l.*	*s.*	*d.*	*q.*
398	10	10·	5469	19·	11·	34619	10	06·	2
964	15·	06	2492	08	10·	32418	18·	08	1
813	13·	04·	1983	14·	02	24818	10·	11·	3·
421	11	11	3112	13	03	12498	11	00	1
2598	11	07	13058	16	02	104355	11	02	3

In Addition of Weights, Measures, Time, Motion, and all other *Geodæticals English* and *Forreign* in this Case: The like Order may be observed as in the *Examples* ensuing.

Examples of

Addition of

Troy and *Avoirdupois-Weight.*

Weight.

℔.	*ounc.*	*pwts.*	*gr.*	*mites.*
30	11	19·	23·	19·
27	10·	16·	20·	16·
24	08	12	21	18
83	07	09	18	13

Ton.	*C.*	*gr.*	℔.	℥	ʒ	℈	*gr.*
120	19·	3	27·	15·	7·	2	19·
89	10·	2·	19·	12·	6·	1·	10·
30	13	0	10	11	4	0	18
241	03	3	02	08	2	2	07

Square and *Concave Measures.*

Measure.

Acres	*Roods*	*Daies-work*	*Perches*	*Feet.*
141	3	9·	3·	16·
141	2·	8·	2·	12·
191	0	4	1	10
474	3	3	0	05

Quarters	*Bushels*	*Gallons*	*Pints.*
1944	7·	7·	7·
1036	6·	5·	4
987	1	3	2
3969	0	0	5

Time and *Motion.*

Time and Motion.

Years	*Dayes*	*Hours*	*Min.*	*Seconds.*
234	230·	23·	59·	59·
120	142	22·	30·	45
110	30	10	40	10
465	173	09	10	54

Signes	*Degr.*	*Min.*	*Sec.*	*Thirds.*
10	29·	59·	59·	59·
0	10	20	15	11
0	10	13	12	00
11	20	33	27	10

3 *Case.* If the Addends be Fracted *Geodæticals*, whether greater, or lesser, or Integral when Fracted,

3. Fractions or Mixt. Rule.

Then proceed in the Addition with the Fractions as Fractions, and the Integers as Integers, according to their respective Rules.

Examples. *Example*, To add $\frac{1}{5}$ *l.* with $\frac{3}{5}$ *l.* the Total as at *I*, is $\frac{4}{5}$ *l.* by *Book* 1. *Part* 2. *Chap.* 3. *Case* 1. And to add $\frac{1}{2}$ *l.* and $\frac{1}{3}$ *l.* together, the Total as at *K.* is $\frac{5}{6}$ *l.* by *Case* 2. there. And to add 40 $\frac{3}{4}$ *l.* with 18 $\frac{1}{2}$ *l.* the Total as at *L.* is 59 $\frac{1}{4}$ *l.* by carrying an Unite from the Fractions to the Integers, and adding them after the manner of Integers in *Book* 1. *Part* 1. *Chap.* 5.

I.

$$\overset{l.}{\frac{1}{5}} \times \frac{3}{5} = \overset{l.}{\frac{4}{5}}$$

K.

$$\begin{array}{c} 5 \\ \hline \end{array} \quad \underbrace{\overset{3}{\frac{1}{2}} \times \overset{2}{\frac{1}{3}}}_{6} = \overset{l.}{\frac{5}{6}}$$

L.

$$\begin{array}{l} l. \\ 40\frac{3}{4} \\ 18\frac{1}{2} \\ \hline 59\frac{1}{4} \\ \hline \end{array}$$

Proof of Geodætical Addition. *Addition of Geodæticals* falling under the first Case, admits of like Proof with Addition of Integers. And those under the Third Case are to be proved, as the Addition of Integers and Fractions, according to the respective Operations made use of in the Addition: For if Integers be added, the Work is proved as Addition of Integers, if Fractions, as Addition of Fractions. *Addition of Geodæticals* under the second Case is to be proved by *Substraction*, as is shewed in the next Chapter.

Proof by Nines. Addition of *English* Pounds, Shillings and Pence, in the Second Case in this Chapter above shewed: Some Schoolmasters teach to prove thus. Cast away all the Nines from the Pounds to be added, and what remains double, and bring to the Shillings, and cast away 9 also thence, and what remains treble, and bring to the Pence, and all the Nines being cast away there, note the last Remain. Then reject Nines in like manner from the Total, and if the last Remain here be like the former, they approve the Work.

As in the former Example where the Total was 51 *l.* 13 *s.* 5 *d.* casting away 9 from the Pounds to be added, there remains 5, which doubled is 10, and 9 cast therefrom, 1 remains, which reckoned to the Shillings, and 9 cast away as oft as can be there remains 6, this trebled is 18, from which twice Nine cast, there rests 0 to be brought to the Pence, and from thence all the Nines rejected there remains at least 8. then in the Total the Pounds make 6 Units, which doubled are 12, and 9 rejected leaves 3 to be reckoned to 4 in the Shillings, which 7 tripled make 21, from which 18, which is twice 9 rejected, there rests 3 to be added to 5 in the Pence, which also make 8 parallel to the former Remain.

The Proof uncertain. *Best Proof.* But for the Reason above remembred in Addition of Integers, all the Proofs by casting away 9 is uncertain, and the true Proof of *Addition* is by *Substraction*, as before taught in *Substraction of Integers*, and needs no further repetition here.

Proof of Addition of Fractions by the Value. *Addition of Geodætical Fractions* besides the Work above-mentioned may be proved by finding the Value of each Fraction before Addition, and adding them as *Geodætical* Integers, and comparing the Total with the Value of the Total of the added Fractions.

Example, If $\frac{1}{2}$ *l.* be added to $\frac{1}{3}$ *l.* the Total will be $\frac{5}{6}$ *l.* The Value of $\frac{1}{2}$ *l.* is 10 *s.* and $\frac{1}{3}$ *l.* is 6 *s.* 8 *d.* which added together make the Total to be 16 *s.* 8 *d.* and so much is $\frac{5}{6}$ *l.*

$$\frac{1}{2}l.\ \frac{20}{2}(10s. \;\Big|\Big|\; \frac{1}{3}\ \frac{240}{3}(\frac{80}{12}(6s.\ 8d. \quad \begin{array}{rr} s. & d. \\ 10 & 0 \\ 6 & 8 \\ \hline 16 & 8 \\ \hline \end{array} \;\Big|\Big|\; \frac{5}{6}l. \quad \begin{array}{r} 240 \\ 5 \\ \hline 1200 \\ \hline \end{array} \quad \frac{1200}{6}(\frac{200}{12}(16s.\ 8d.$$

CHAP. IV.

Substraction of Geodæticals.

Geodæticals Substracted. IN *Substraction of Geodæticals*, the different Nature of the proposed Numbers is to be considered.

1. Case.

1. *Case.* If the Numbers given be Integral *Geodæticals* only, and of the same kind or denomination, *1. Integers of one Denomination. Rule.*

Then substract the Lesser Number from the Greater, as was taught in Abstract Integers, *Book* 1. *Part* 1. *Chap.* 6. and understand the remain to be of the same Denomination with the given Numbers.

1. *Example*, If there were delivered to one 546 *French* Crowns to buy some Commodities with, and he did disburse but 354 of them; then will remain in his hands 192. *1. Example.*

2. *Example*, If one deliver me to keep for him 1006 *Liures Tournois*, and receive again at several times 821, then will 185 rest in my hands. *2. Example.*

1. *Ex. Fr.* △.		2. *Ex. l. Tour.*	
Delivered	546	Delivered	1006
Disbursed	354	Received	821
Remaineth	192	Remaineth	185

2. *Case.* If the given Number be Integral *Geodæticals* of different Denominations, Greater or Smaller, English or Foreign, *2. Integers of several Denominations.*

Then place the Greater or highest Denomination of the Number from which Substraction is to be made alwaies to the left hand, and the other Numbers in order to the right hand, and under the same the Subtrahend, so as Pounds may stand under Pounds, Shillings under Shillings, *&c.* likewise the Arithmetical Places of Units under Units, Tens under Tens, *&c.* are to be kept. And where any *Geodætical* Denomination is wanting, the same may be supplyed with Cyphers to keep place. *To place the Data.*

Then begin at the right hand and deduct the Lower Numbers or Figures of the Subtrahend and least *Geodætical*, out of the upper standing over them particularly, subscribing the Remain respectively under the same Files. *To Substract them.*

If the neather Figure happen to be greater than the upper, then in imagination borrow one of the Denomination next to the left hand, and add to the upper Number, which is too little, and make Substraction from both, and subscribe the Remain as before; and for that borrowed accompt one back in the next File, reckoning the next Figure to be substracted 1 more than it is, or the next Figure to be substracted from 1 less than it is. *When borrowing needful.*

Example, If *A.* lend to *B.* 344*l.* 10*s.* 6*d.* and *B.* paid *A.* 124*l.* 6*s.* 9*d.* How much is yet owing? I place the Numbers as at *C.* and because 9*d.* is greater than 6*d.* I borrow 1 Shilling, which is 12*d.* and put to 6*d.* and from the Total 18*d.* I take 9*d.* and there resteth 9*d.* then coming to the Shillings, I reckon that 1 that I borrowed and 16 is 17, which because I cannot take from 10, the Number over them I borrow 1 Pound, which is 20*s.* and put to the 10 makes 30*s.* from which 17 taken there remaineth 13*s.* to be subscribed, and that 1*l.* borrowed reckoned to the 4*l.* next maketh 5, which I proceed with as in *Substraction of Integers*, and finish the rest of the work accordingly, and by the last Remain find yet due to *A.* 219*l.* 13*s.* 9*d.* as at *D.* *Example.*

	l. — *s.* — *d.*			*l.* — *s.* — *d.*	
Lent by *A.*	344—10—06	*C.*	Lent by *A.*	344—10—06	
Paid by *B.*	124—16—09		Paid by *B.*	124—16—09	*D.*
			Rest due to *A.*	219—13—09	

When many Numbers are given from which Substraction is to be made, or many Numbers to be substracted from one, or many: Let all the Plurals of each sort be severally added into one, as is taught in *Substraction of Integers*, and then proceed in Substraction as above. *When the Data are many.*

In Substraction of Weights, Measures, Time, Motion, and other *Geodæticals*, *English* and *Foreign* in this Case: Let the like order be observed, as in the ensuing Examples. *Examples of Weight, Measure, Time, and Motion.*

Example,

Weight.	Measure.	Time.	Motion.
Ton. C. qrs. ℔. ℥.	*Acres Roods Rods.*	*Years Daies. Ho.*	*Sig. Deg. Min. Sec.*
40—18—3—24—12	340—2—39	1648—330—19	11—20—40—31
29—19—3—12—14	121—3—36	345—360—20	3—19—50—33
18—19—0—11—14	218—3—03	1302—334—23	8—00—49—58

3. Fractions or mixt.

3. *Case.* If the given Numbers be Fracted *Geodæticals*, Greater or Lesser, or Integral mixed with Fracted,

Then proceed in the Substraction with the Fractions as Fractions, and Integers as Integers, according to their respective Rules.

Examples.

Examples, To substract $\frac{1}{15}$ *l.* from $\frac{7}{15}$ *l.* the Remain is $\frac{6}{15}$ *l.* as at *E.* by *Book* 1. *Part* 2 *Chap.* 4. *Case* 1. And to Substract $\frac{1}{4}$ *l.* from $\frac{1}{3}$ *l.* the Remain will be $\frac{1}{12}$ *l.* as at *F.* by *Case* 2. there. And if $3\frac{1}{3}$ *l.* be taken from 13 *l.* there will be left $9\frac{2}{3}$ *l.* by borrowing an Unite which is 3 Thirds, to take the $\frac{1}{3}$ from, and for the same paying 1 to the 3, which is 4, and substracting it from 13, after the manner of Integers above-mentioned, as at *G.*

E.

$$\overset{l.}{\frac{7}{15}} - \overset{l.}{\frac{1}{15}} = \frac{6}{15} \text{ or } \overset{l.}{\frac{2}{5}}$$

F.

$$\overset{\overline{\;\;1\;\;}}{\underbrace{\overset{4}{\overline{\;\;}}\frac{1}{3}l. - \overset{3}{\overline{\;\;}}\frac{1}{4}l.}_{12}} = \frac{1}{12}l.$$

G.

Lent—	13 *l.*
Paid—	$3\frac{1}{3}$
Rest—	$9\frac{2}{3}$

Proof of Geodætical Substraction *of the First and Third Cases.*

Substraction of Geodæticals falling under the First Case is proved like *Substraction of Integers* and *Fractions*, And the Substractions under the third Case, as the *Substraction of Integers*, according to the respective Operations used in the Substraction: For if Integers be taken from Integers, the Work is proved as *Substraction of Integers*, if Fractions from Fractions, as *Substraction of Fractions*, and generally *Addition* by *Substraction*, and *Substraction* by *Addition.*

Of the Second Case and reciprocally Addition *thereby.*

The *Substraction of Geodæticals* falling under the Second Case, admits of *Geodætical Addition* for Proof. For if the remain be added to the Number substracted, the Total will be parallel to the Number from which Substraction is made when the Work is right. And consequently when from the Total of any *Geodætical Addition* of this sort, one or more of the Numbers added, be substracted; the Remain of that Substraction will be equal to the Residue of those Numbers added into that Total; and thereby prove that Addition right.

Addition of this sort may also be proved by beginning at the left hand to substract the respective Files from the Figure of the Total under them, and if any thing remain to underwrite, or remember to make allowance of the same, in the next right hand Figure of the Total, as was shewed before in the Proof of *Substraction of Integers*, *Book* 1. *Part* 1. *Chap.* 6. For further Evidence view the Examples of *Addition* at *H.* and *Substraction* at *I.*

Addition — *Proof by Substraction.*

	l. s. d.	Thus	or	Thus
	22—13—00	22—13—00	22—13—00	51—13—05
H.	15—10—08	15—10—08		29—00—05
	10—05—02	10—05—02	15—10—08	22—13—00
	03—04—07	03—04—07	10—05—02	
			03—04—07	
	51—13—05	51—13—05	29—00—05	
		xx—x—0		

Substraction

Substraction

	l. — *s.* — *d.*	
	51—13—05	Lent by *H.* to *I.* at several times.
I.	15—10—08 10—05—02 03—04—07	Paid at times to *H.*
	29—00—05	Total of all the Payments to be substracted.
	22—13—00	Remaineth due to *H.*
	51—13—05	Proof of the *Substraction.*

Substraction of Geodætical Fractions; besides the Proof above mentioned may be proved by finding the Value of each Fraction before Substraction, and substract them as Integers, and compare the Remain with the Value of the Remain of the Substracted Fraction. *Proof of Substraction of Fractions by the Value.*

Example, To take $\frac{1}{4}$ *l.* from $\frac{1}{3}$ *l.* leaves $\frac{1}{12}$ *l.* the Value of $\frac{1}{4}$ *l.* is 5 *s.* and $\frac{1}{3}$ *l.* is 6 *s.* 8 *d.* from which 5 *s.* substracted leaves 1 *s.* 8 *d.* and so much is $\frac{1}{12}$ *l.*

l. $\frac{1}{4}\ \frac{20}{4}($ 5 *s.* || *l.* $\frac{1}{3}\ \frac{240}{3}(\frac{80}{12}($ 6 *s.* 8 *d.*

s.	*d.*
6	8
5	0
1	8

|| $\frac{1}{12}$ *l.* $\frac{240}{12}(\frac{20}{12}($ 1 *s.* 8 *d.*

CHAP. V.

Multiplication of Geodæticals.

TO multiply *Geodæticals*, consider what Case the proposed Numbers fall under and accordingly proceed in their *Multiplication*. *Geodæticals multiplyed.*

1. *Case.* If the Numbers given to be multiplyed be Single *Geodæticals* both Integral, both Fracted, mixt Numbers, or the one an Integer, and the other a Fraction. *1. Single. Rule.*

Then multiply the Integers as Integers, and the Fractions as Fractions, and the Mixt Numbers as mixt Numbers; of the first sort is to be seen, *Book* 1. *Part* 1. *Chap.* 7.

1. *Example*, If 30 Men stood in a straight line, every one 3 Feet before the other; and it were desired to know how many Feet there were between the Front and the Rear. The Answer is the Product at *A.* 90 Feet. *Example* 1.

2. *Example*, If 40 Planks each of $10\frac{1}{2}$ Feet long were laid end to end; and the Question were, how far they would reach, The Answer is the Product at *B*, 420 Feet. *Example* 2.

3. *Example*, If a piece of Land contain in length $10\frac{1}{4}$ Yards, and in breadth $5\frac{1}{3}$ Yards: And the Question were to know the Area, or Content of the same, or how many Square Yards the whole did contain. The Answer is the Product at *C.* $54\frac{2}{3}$ Square Yards. *Example* 3.

4. *Example*, If $\frac{1}{2}$ *l.* be multiplyed by $\frac{1}{6}$ *l.* The Product is $\frac{1}{12}$ *l* as at *D.* *Example* 4.

A.
30 Men.
3 Feet.
90

B.
40 Planks.
$10\frac{1}{2}$ Feet.
400
20
420 Feet.

C.
656 Yards.
$$\underbrace{\frac{41}{10\frac{1}{4}} \times \frac{16}{5\frac{1}{3}}}_{12} = \frac{656}{12} \text{ or } 54\tfrac{2}{3}$$
Square.

D.
$$\frac{1}{\dfrac{1}{2}\,l. \times \dfrac{1}{6}\,l. = \dfrac{1}{12}\,l.}{12}$$

2. *Case.*

2. *One of the Data Single, and the other Plural.*

2. *Case.* If one of the given Numbers be a Plural *Geodætical*, and the other a Single. Then reduce the Plural Number into the lowest Denomination, and multiply the Result by the other, or if the Single *Geodætical* be a Digit or other small Number; you may easily multiply every of the Denominate Numbers severally by that Number, and carry in imagination or otherwise to the next greater Denomination, so many as the respective Products will afford, subscribing the Remaines.

Example.

Example, If a Merchant pay to 20 Porters 1 *l.* 6 *s.* 8 *d.* a piece, and it were desired to know how much Money in all was paid. First 1 *l.* 6 *s.* 8 *d.* reduced into Pence is 320, which multiplyed by 20, produceth 6400, and reduced back into Shillings and Pounds giveth 26 *l.* 13 *s.* 4 *d.* as at *E*, or otherwise 20 Eight-pences which is 40 Groats, or 13 *s.* 4 *d.* and the 13 *s.* carried to the place of Shillings leaveth the 4 *d.* to be subscribed, then 20 times 6 *s.* is 6 times 20 *s.* or 6 *l.* to be carried to the Denomination of Pounds, and the 13 *s.* left to be subscribed. So at last the 6 *l.* added to the 20 times 1 *l.* or 20 *l.* makes 26 *l.* 13 *s.* 4 *d.* as at *F*.

```
1 l. 6 s. 8.        E.          |  Thus,        or Thus,        or Thus,
 20                             |  l.  s.  d.   l.          F.  l.  s.  d.
---                             |  1—6—8        1 1/3           1—6—8
 20            4(4 d.           |       20      20                  20
  6          6400(              |  ---------    ---             ---------
---           xz  (53|3 s.      |  26—13—4      20                 13—4   Product of 20 by 8d.
 26                -------      |  ---------    6 2/3            6—0     Product of 20 by 6s.
 12   320 pence    26:13:4      |               —— l.          20        Product of 20 by 1l.
---   20 Porters   -------      |               26 2/3          ---------
 52   ----                      |               ——             26--13—4
268   6400                      |
---   ----                      |
320
```

3. *Data Plural, and the Product Simple. Rule. Example.*

3. *Case.* If both the given Numbers be Plural *Geodæticals*, and the Product required Simple, and of the lowest Denomination,

Then reduce the Numbers into their lowest Denominations, and multiply the Results as Integers.

Example, If 20 *l.* 10 *s.* 3 *d.* be multiplyed by 5 *l.* 4 *s.* 2 *d.* both being reduced make 4923 *d.* and 1250 *d.* which multiplyed produce 6153750 *d.* and they if occasion be may be reduced into Shillings and Pounds, as at *G*.

```
 l.  s.  d.      l.  s.  d.
 20—10—3         5—4—2          4923
 20              20             1250
----            ----           --------
400             100            246150                             G.
 10               4            9846         39x3(6                 s.
----            ----           4923         6x53750(51281 | 2
410             104            -------      xzzzzzz-------------
 12              12            6153750      xxxxx    2564c—12—6
----            ----           -------
820             208
4103            1042
----            ----
4923            1250
```

4. *Data Plural and Product Compound. Variety 1.*

4. *Case*, If both the Given Numbers be Plural *Geodæticals*, and the Product required Compound,

Then first after Reduction of the Numbers and Multiplication of the Results as before, divide the Product by the Product of every Lesser Denomination contained in one Greater multiplyed together.

Example.

Example, In the Numbers last Exemplary, because 240 Pence are contained in 1 Pound, and Pence are the Lesser Denominations in both the given Numbers, the Product of 240 multiplyed by 240, which is 57600 dividing the 6153750 the Product above obtained, the Quotient will be 106 *l.* and if the Value of the Fraction remaining be gotten, and added thereto; the whole *Geodætical* will be 106 *l.* 16 *s.* 8 $\frac{1}{4}$ *d.* as at *H*.

H. 240 × 240 = 9600, 480 — 57600

(4 / 28(21 / 61547(5 0(106 *l.* / 57666 0|0 / 577 / 5

4815 × 20 = 96300

(41 / 387|4 / 96300|0(16 *s.* / 5766|0 / 57

414 × 12 = 828, 414 — 4968

(360 / 4968(8 *d.* / 576

$\frac{360}{576} \mid \frac{180}{288} \mid \frac{90}{144} \mid \frac{5}{8}$

Or Secondly, Under the Reduced Numbers, Place the Number of Lesser Denominations contained in one Greater; and multiply as in Fractions. *Variety* 2.

Example, Under the before-reduced Numbers 4923 and 1250, placing 240, and multiplying them as Fractions, the alike Product will arise as followeth at *I*. *Example.*

I. 205125 / 1641 125 / 4923 1250 — 240 240 / 24 80 — 1920

|1 / 13|60 / 205125|5(106*l.* / 19222 0 / 199 / 1

1605 × 20 = 32100

(13 / 1198 / 32100|0(16 *s.* / 19220|0 / 19

138 × 12 = 276, 138 — 1656

(120 / 1656(8 *d.* / 192

$\frac{120}{192} \mid \frac{60}{96} \mid \frac{30}{48} \mid \frac{15}{24} \mid \frac{5}{8}$

Or Thirdly, If the *Geodæticals* be of one Nature, turn all the Lesser Denominations into Fractionary Parts of the Greater, and multiply as in the 5th. Case of *Multiplication of Fractions*, *Book* 1. *Part* 2. *Chap.* 5. or multiplying Number by Number carry out of each Multiplication so many of the next Greater Denomination as are therein, and Subscribe the Remain. *Variety* 3.

Example, Because the *Geodæticals* aforesaid are both one Nature, *viz.* Sterling Money, the 10 *s.* 3 *d.* and 4 *s.* 2 *d.* turned into Fractions, and after Multiplication of the Integers 20 by 5, proceeding to add the parts of the Multiplicand thereto: The like Product appears at *K*. *Example.*

20 *l.* 10 *s.* 3 *d.* or 20 $\frac{1}{2}$ and $\frac{1}{80}$ × 5 *l.* 4 *s.* 2 *d.* or 5 $\frac{1}{5}$ and $\frac{1}{120}$ produce 106 *l.* 16 *s.* 8 *d.* $\frac{5}{8}$.

	l. *s.* *d.*	
Thus	20—10—03	
	5— 4— 2	
	100	Product of 5 by 20.
	2—10	Five Angels.
	1— 3	Five Three pences.
K.	4—00—00	Fifth part of 20 *l.*
	2	Fifth part of 10 *s.*
	00$\frac{3}{5}$	Fifth part of 3 *d.*
	3—04	The 120 part of 20 *l.*
	01	The 120 part of 10 *s*
	00$\frac{1}{40}$	The 120 part of 3 *d.*
	106—16—08$\frac{5}{8}$	The Total Product.

	l.				*l.* *s.* *d.*
or Thus	20 $\frac{1}{2}$ and $\frac{1}{80}$				
	5 $\frac{1}{5}$ and $\frac{1}{120}$				
	100	The Product of	5	by 20 is 100	100—00—00
	2$\frac{1}{2}$		5	by $\frac{1}{2}$ or half pounds	2—10—00
	0$\frac{1}{16}$		5	by $\frac{1}{80}$ abbreviated is	0—01—03
	4		$\frac{1}{5}$	of 20 *l.* which is	4—00—00
	0$\frac{1}{10}$		$\frac{1}{5}$	of $\frac{1}{2}$ *l.* is $\frac{1}{10}$ *l.* or	0—02—00
	0$\frac{1}{400}$		$\frac{1}{5}$	of $\frac{1}{80}$ *l.* or reduced, is	0—00—0$\frac{3}{5}$
	0$\frac{1}{6}$		$\frac{1}{120}$	of 20 *l.* abbreviated is	0—03—04
	0$\frac{1}{240}$		$\frac{1}{120}$	of $\frac{1}{2}$ *l.* is $\frac{1}{240}$ *l.* or	0—00—01
	0$\frac{1}{9600}$		$\frac{1}{120}$	of $\frac{1}{80}$ *l.* reduced is	0—00—0$\frac{1}{40}$
	l. 106$\frac{2663}{3200}$				106—16—8$\frac{5}{8}$

Or, Fourthly, Multiply Number by Number, beginning at the left hand, and to find the Denomination of the several Products observe the first line shall be denominate, as the Multiplicand, the first left hand Figures of the other Lines of Production shall be denominate, as the Figures of the Multiplicand under which they stand, and all the other Figures in each line respectively are parts of one of that denomination, and are Numerators, under which for Denominator you may place or imagine placed as followeth, that is to say, under the second placed Figures to the right hand, the number of the next lesser Denomination to the greater of the Multiplicand, and under the next right hand Figures the number of the next lesser Denomination, and so accordingly under all the Numbers And to add these several Lines of Production into one Total, first begin at the left hand, and subscribe the left hand Figures under a line in the place of the greater Denominations as they stand, and then collect all the next right hand Collumne Integers together, and subscribe them as in *Addition of Integers*, and so do with the *Variety* 4.

Integers

2. *One of the Data Single, and the other Plural.*

2. *Case.* If one of the given Numbers be a Plural *Geodætical*, and the other a Single. Then reduce the Plural Number into the lowest Denomination, and multiply the Result by the other, or if the Single *Geodætical* be a Digit or other small Number; you may easily multiply every of the Denominate Numbers severally by that Number, and carry in imagination or otherwise to the next greater Denomination, so many as the respective Products will afford, subscribing the Remaines.

Example.

Example, If a Merchant pay to 20 Porters 1 *l.* 6 *s.* 8 *d.* a piece, and it were desired to know how much Money in all was paid. First 1 *l.* 6 *s.* 8 *d.* reduced into Pence is 320, which multiplyed by 20, produceth 6400, and reduced back into Shillings and Pounds giveth 26 *l.* 13 *s.* 4 *d.* as at *E*, or otherwise 20 Eight-pences which is 40 Groats, or 13 *s.* 4 *d.* and the 13 *s.* carried to the place of Shillings leaveth the 4 *d.* to be subscribed, then 20 times 6 *s.* is 6 times 20 *s.* or 6 *l.* to be carried to the Denomination of Pounds, and the 13 *s.* left to be subscribed. So at last the 6 *l.* added to the 20 times 1 *l.* or 20 *l.* makes 26 *l.* 13 *s.* 4 *d.* as at *F.*

```
1 l. 6 s. 8.        E.            |  Thus,        or Thus,      or Thus,
 20                               |  l.  s.  d.   l.       F.   l.  s.  d.
 ---                 4(4 d.       |  1—6—8        1 1/3         1—6—8
 20                 6400(         |       20      20                20
  6                  xx  (53|3 s. |  ---------    ---           ---------
 ---                ---------     |  26—13—4      20               13—4   Product of 20 by 8d.
 26                 26:13:4       |  ---------    6 2/3          6—0     Product of 20 by 6s.
 12  320 pence      -------       |               — l.          20       Product of 20 by 1 l.
 --- 20 Porters                   |               26 2/3        ---------
 52  -----                        |               ---           26—13—4
268  6400                         |                             -------
 --- ----
320
---
```

3. *Data Plural, and the Product Simple. Rule. Example.*

3. *Case.* If both the given Numbers be Plural *Geodæticals*, and the Product required Simple, and of the lowest Denomination,

Then reduce the Numbers into their lowest Denominations, and multiply the Results as Integers.

Example, If 20 *l.* 10 *s.* 3 *d.* be multiplyed by 5 *l.* 4 *s.* 2 *d.* both being reduced make 4923 *d.* and 1250 *d.* which multiplyed produce 6153750 *d.* and they if occasion be may be reduced into Shillings and Pounds, as at *G.*

```
 l.   s.  d.      l.  s.  d.
 20—10—3          5—4—2         4923
 20               20            1250
 ----             ----          ---------
 400              100           246150                            G.
  10                4           9846         39x3(6               s.
 ----             ----          4923         6x53750(51281 | 2
 410              104           ---------    xxxxxxx-------.-----
  12               12           6153750      xxxxx   25640—12—6
 ----             ----          ---------            ------------
 820              208
4103             1042
 ----             ----
4923             1250
 ----             ----
```

4. *Data Plural and Product Compound. Variety 1.*

4. *Case*, If both the Given Numbers be Plural *Geodæticals*, and the Product required Compound,

Then first after Reduction of the Numbers and Multiplication of the Results as before, divide the Product by the Product of every Lesser Denomination contained in one Greater multiplyed together.

Example.

Example, In the Numbers last Exemplary, because 240 Pence are contained in 1 Pound, and Pence are the Lesser Denominations in both the given Numbers, the Product of 240 multiplyed by 240, which is 57600 dividing the 6153750 the Product above obtained, the Quotient will be 106 *l.* and if the Value of the Fraction remaining be gotten, and added thereto; the whole *Geodætical* will be 106 *l.* 16 *s.* 8 $\frac{1}{4}$ *d.* as at *H.*

```
H.  240
    240    (4                      4815  (41           414
    ----   26(81                     20  387|4          12
    9600   61547(5 0(106 l.        ------ 9830|0(16 s. ----   (360       360 | 180 | 90  | 5
    480    57666 0|0               96300 5766|0        828    4968(8 d.  --- | --- | --- | -
    ----   577                     ------ 57           414    576        576 | 288 | 144 | 8
    57600  5                                           ----
                                                       4968
```

Or Secondly, Under the Reduced Numbers, Place the Number of Lesser Denominations contained in one Greater; and multiply as in Fractions. *Variety* 2.

Example, Under the before-reduced Numbers 4923 and 1250, placing 240, and multiplying them as Fractions, the alike Product will arise as followeth at *I*. *Example.*

```
      205125        |1          (13          (120
      1641   125    13|60       119|8        1656(8 d.
      4923   1250   20512|5(106 l. 3210|0(16 s. 192
I.    ----   ----   19222 0     1922|0
      240    240    199         19           120 | 60 | 30 | 15 | 5
      24     80     1                138     --- | -- | -- | -- | -
      -----------        1605         12     192 | 96 | 48 | 24 | 8
         1920              20        ----
                         -----        276
                         32100       138
                                     ----
                                     1656
```

Or Thirdly, If the *Geodæticals* be of one Nature, turn all the Lesser Denominations into Fractionary Parts of the Greater, and multiply as in the 5th. Case of *Multiplication of Fractions*, *Book* 1. *Part* 2. *Chap*. 5. or multiplying Number by Number carry out of each Multiplication so many of the next Greater Denomination as are therein, and Subscribe the Remain. *Variety* 3.

Example, Because the *Geodæticals* aforesaid are both one Nature, *viz*. Sterling Money, the 10 *s*. 3 *d*. and 4 *s*. 2 *d*. turned into Fractions, and after Multiplication of the Integers 20 by 5, proceeding to add the parts of the Multiplicand thereto: The like Product appears at *K*. *Example.*

20 *l*. 10 *s*. 3 *d*. or 20 $\frac{1}{2}$ and $\frac{1}{80}$ × 5 *l*. 4 *s*. 2 *d*. or 5 $\frac{1}{5}$ and $\frac{1}{120}$ produce 106 *l*. 16 *s*. 8 *d*. $\frac{5}{8}$.

	l. *s*. *d*.	
Thus	20—10—03	
	5— 4— 2	
	100	Product of 5 by 20.
	2—10	Five Angels.
	1— 3	Five Three pences.
K.	4—00—00	Fifth part of 20 *l*.
	2	Fifth part of 10 *s*.
	00$\frac{3}{5}$	Fifth part of 3 *d*.
	3—04	The 120 part of 20 *l*.
	01	The 120 part of 10 *s*
	00$\frac{1}{40}$	The 120 part of 3 *d*.
	106—16—08$\frac{5}{8}$	The Total Product.

	l.				*l*. *s*. *d*.
or Thus	20 $\frac{1}{2}$ and $\frac{1}{80}$				
	5 $\frac{1}{5}$ and $\frac{1}{120}$				
	100	The Product of	5	by 20 is 100	100—00—00
	$2\frac{1}{2}$		5	by $\frac{1}{2}$ or half pounds	2—10—00
	$0\frac{1}{16}$		5	by $\frac{1}{80}$ abbreviated is	0—01—03
	4		$\frac{1}{5}$	of 20 *l*. which is	4—00—00
	$0\frac{1}{10}$		$\frac{1}{5}$	of $\frac{1}{2}$ *l*. is $\frac{1}{10}$ *l*. or	0—02—00
	$0\frac{1}{400}$		$\frac{1}{5}$	of $\frac{1}{80}$ *l*. or reduced, is	0—00—0$\frac{3}{5}$
	$0\frac{1}{6}$		$\frac{1}{120}$	of 20 *l*. abbreviated is	0—03—04
	$0\frac{1}{240}$		$\frac{1}{120}$	of $\frac{1}{2}$ *l*. is $\frac{1}{240}$ *l*. or	0—00—01
	$0\frac{1}{9600}$		$\frac{1}{120}$	of $\frac{1}{80}$ *l*. reduced is	0—00—0$\frac{1}{40}$
	l. $106\frac{2663}{3200}$				106—16—8$\frac{5}{8}$

Or, Fourthly, Multiply Number by Number, beginning at the left hand, and to find the Denomination of the several Products observe the first line shall be denominate, as the Multiplicand, the first left hand Figures of the other Lines of Production shall be denominate, as the Figures of the Multiplicand under which they stand, and all the other Figures in each line respectively are parts of one of that denomination, and are Numerators, under which for Denominator you may place or imagine placed as followeth, that is to say, under the second placed Figures to the right hand, the number of the next lesser Denomination to the greater of the Multiplicand, and under the next right hand Figures the number of the next lesser Denomination, and so accordingly under all the Numbers And to add these several Lines of Production into one Total, first begin at the left hand, and subscribe the left hand Figures under a line in the place of the greater Denominations as they stand, and then collect all the next right hand Collumne Integers together, and subscribe them as in *Addition of Integers*, and so do with the Integers *Variety* 4.

Integers in the other Columnes, and then all the Fractions belonging to each line of Production severally, and add them together; and what of the other Denominations are contained in them, place in their Order, and if any thing remain, reduce it to a part of the lowest Denomination of the Total. And lastly add all these subscribed Numbers into one Total.

Example.

Example, The former Numbers thus multiplyed produce for the First Line 100. 50. 15. for the Second 80. 40. 12. and for the Third 40. 20. 6. as at *L. M. N.* then for 40 in the second Line, because Shillings is the next lesser Denomination to Pounds, 20 shall be the Denominator, and the Denominator to 12 shall be 240, the Pence in 1 *l.* and so shall they be $\frac{40}{20}$ *s.* and $\frac{12}{240}$ *s.* And the like in the third line $\frac{20}{20}$ *d.* and and $\frac{6}{240}$ *d.* Then collecting them as before directed for the $\frac{40}{20}$ *s.* is set 2 *s.* and $\frac{12}{240}$ is reduced to $\frac{3}{5}$ of a Peny by Abbreviation, and multiplying the Numerator by 12, and so proceeding, the other Fractions make 1 $\frac{1}{40}$ *d.* and the Numbers stand as at *O.* And for plainer Demonstration another Example is added at *P*, where 2 *l.* 10 *s.* 3 *d.* 2 *q.* is multiplyed by 2 *l.* 5 *s.* 2 *d.* 2 *q.*

		l.	s.	d.		
Denominations		l.	s.	d.		
Multiplicand		20	10	3	O	
Multiplier		5	4	2		
Lines of Production	L.	100	50	15		
	M.		80	$\frac{40}{20}$	$\frac{12}{240}$	
	N.			40	$\frac{20}{20}$	$\frac{6}{240}$
Collection of		100				
50 *s.* & 80 *s.*		6	10			
15 *d.* & 40 *d.*			4	7		
$\frac{40}{20}$ *s.* & $\frac{12}{240}$ *s.*			2	$0\frac{3}{5}$		
$\frac{20}{20}$ *d.* & $\frac{6}{240}$ *d.*				$1\frac{1}{40}$		
Total Product		106	16	$8\frac{1}{8}$		

l.	s.	d.	q.			
2	10	3	2			P.
2	5	2	2			
4	20	6	4			
	10	$\frac{50}{20}$	$\frac{15}{240}$	$\frac{10}{960}$		
		4	$\frac{20}{20}$	$\frac{6}{240}$	$\frac{4}{960}$	
			4	$\frac{20}{20}$	$\frac{6}{240}$	$\frac{4}{960}$
4						
1	10					
	10					
	2					
2	6	$3\frac{1}{2}$				
	1	$0\frac{7}{60}$				
		$1\frac{7}{240}$				
5	13	8	$0\frac{3}{4}\frac{1}{8}$			

Proof of Geodætical Multiplication.

Multiplication of Geodæticals is proved, if the Numbers be Integral, as *Multiplication of Integers*, if Fracted, as *Multiplication of Fractions*, that is in both by *Division*, as is further noted in the next Chapter.

Proof of Multiplying Fractions by the Value.

Moreover the *Multiplications of Geodætical Fractions* may be examined, by finding the Value of the Fractions to be multiplyed, and the Product thereof, and comparing the same with the Product of their Values, multiplyed as Integers; only observing to divide this Product by the Number of one of the Lesser Denominations contained in the Greater.

Example, If $\frac{2}{3}$ *l.* be multiplyed by $\frac{3}{4}$ *l.* the Product will be $\frac{1}{2}$ *l.* And $\frac{2}{3}$ *l.* being 13 *s.* 4 *d.* or 160 Pence, and $\frac{3}{4}$ *l.* being 15 *s.* or 180 Pence, these multiplyed produce 28800, which divided by 240, the Pence or Lesser Denomination contained in one Pound the Greater Denomination, the Quotient will be 120 Pence, or $\frac{1}{2}$ *l.* as before.

$$\frac{1}{2}\qquad \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} = \frac{1}{2}\ l. \text{ or } 10\ s.$$

$$\frac{2}{3}\ l. = 13\ s.\ 4\ d. \text{ or } 160\ d.$$

$$\frac{3}{4}\ l. = 15 : 0 \quad \text{or } 180\ d.$$

$$\begin{array}{r} 12800 \\ 160 \\ \hline 28800 \end{array}$$

$$\begin{array}{l} \quad 4 \\ \cancel{2}8800(\cancel{1}\cancel{2}0(10\ s. \\ \cancel{2}44\ 0\ \cancel{1}\cancel{2} \\ \ \cancel{2} \end{array}$$

Proof of Multiplication *in the Third and Fourth Cases.*

And so also the Operations of multiplying *Geodæticals* in the Third and Fourth Cases of this Chapter may be compared the one with the other, the Lesser Denominations being in that Fourth Case considered as Fractions, and dividing for their Value accordingly.

ly. Because though in multiplying Integers the Multiplicand is increased in Figures and Value so many times as the Multiplier containeth Units; yet in Fractions, and that sort of Multiplying *Geodæticals*, though the Figures may be increased so many times, yet in Value the New Fraction or Product is made so much less. For a Fraction being properly less than one, and making another Number so many times less also; Must needs produce a Number so much less, as the multiplying Fraction containeth parts in it, and the Product is but the Value of a Fraction of a Fraction.

As in the last Instance, If $\frac{3}{4}$ *l.* be taken for the Multiplier, then $\frac{3}{4}$ of 13 *s.* 4 *d.* or $\frac{2}{3}$ *l.* shall be 10 *s.* And so is $\frac{2}{3}$ of 15 *s.* which is $\frac{3}{4}$ *l.* If $\frac{2}{3}$ be taken for the Multiplier, the Product $\frac{1}{2}$ *l.* or 10 *s.* is less than $\frac{2}{3}$ *l.* or 13 *s.* 4 *d.* by $\frac{1}{4}$ thereof, which is 3 *s.* 4 *d* and less than $\frac{3}{4}$ *l.* or 15 *s.* by $\frac{1}{3}$ thereof, which is 5 *s.* But in Integers if they be multiplyed under the Denomination of Shillings, the Product is 20 times 10 *s.* or 200 *s.* if in Pence 28800, as before, which is 240 times as much. And this was the Reason for that Division before to equalize their Value. Nevertheless this is to be understood of Proper Fractions and Lesser *Geodæticals*, for Improper Fractions are redundant, and increase by their Units they contain after the manner of Integers.

CHAP. VI.

Division of Geodæticals.

Geodæticals divided. TO Divide *Geodæticals*, observe the Case the Numbers proposed fall under, and proceed accordingly in their Division.

1. Single. 1. *Case*, The Numbers given to be divided being Single *Geodæticals*, both Integral, both Fracted, or Mixt Numbers, or one an Integer or Mixt Number, and the other a Fraction.

Rule. Then divide the Integers as Integers, and the Fractions as Fractions, and the Mixt Numbers as Mixt Numbers, Of the first sort, See *Book* 1. *Part* 1. *Chap.* 8. and of the other, *Book* 1. *Part* 2. *Chap.* 6.

Example 1. 1. *Example*, If 41984 *l.* be divided equally among 164 Men, and it be desired to know how much each should have for his Part, The Answer is 256 *l.* the Quotient at *A*.

Example 2. 2. *Example*, If a Lane be 11880 Feet long, and I would know, how many Rods that is, I divide them by $16\frac{1}{2}$ the Feet in a Rod. And the Answer is 720 Rods, the Quotient at *B*.

Example 3. 3. *Example*, If a Field be $54\frac{2}{3}$ Square Yards, and 1 side thereof be $10\frac{1}{4}$ Yards, and the other side be demanded, The Quotient answereth at *C*. $5\frac{1}{3}$ Yards.

Example 4. 4. *Example*, If $\frac{1}{4}$ be divided by $\frac{1}{2}$ *l.* The Quotient is $\frac{1}{2}$ *l.* as at *D.*

A.

9
9x
41984(256 *l.*
x6444
x66
x

B.

$16\frac{1}{2} = \frac{33}{2}\Big)\frac{11880}{1}\Big(720$ Rods (cancelled to 1 and 360)

C.

$10\frac{1}{4} = \frac{41}{4}\Big)54\frac{2}{3} = \frac{164}{3}\Big(\frac{16}{3} = 5\frac{1}{3}$ Yards. (cancelled to 1 and 4)

D.

$\frac{1}{2}\Big)\frac{1}{4}\Big(\frac{1}{2}$ *l.* (cancelled to 1 and 2)

2. One of the Data Single, and the other Plural. 2. *Case*, One of the given Numbers being a Plural *Geodætical*, and the other a Single,

Rule. Then reduce the Plural into the Lowest Denomination, and divide the Result by the other. Or if the Single *Geodætical* be a Digit, or other small Number you may easie enough divide every one of the Denominate Numbers severally, carrying in imagination from the Greater Denomination to the next Lesser, the Number remaining on Division of the Greater, if any be, reduced and added to the Lesser, and make the Division from the Total.

Example. *Example*, If 15 *l.* 13 *s.* 1 *d.* be divided equally among 13 Men, to know how much each Man had. First, 15 *l.* 13 *s.* 1 *d.* reduced make 3757, and divided by 13, and after by 12 and 20, gives 1 *l.* 4 *s.* 1 *d.* to every Man, as at *E.* or otherwise dividing 15 *l.* by 13, gives 1 *l.* in the Quotient, and 2 *l.* left, which 2 *l.* or 40 *s.* being reduced brought and added imaginary to the 13 *s.* makes 53 *s.* out of which 13 may

be had 4 times to be set in the Quotient, and 1 *s.* will be left, which being 12 *d.* and added to the 1 *d.* makes 13, out of which the Divisor may be had once, and the Quotient as at *F.*

```
 l.  s.  d.        E.                          F.
15—13—1
20
———                   d.
300                                         2    1
 13          111    4(1   s. l.            ——————————
———          3757(289 (2(4(1          ) l.  s.   d.( l. s. d.
313          1333 122  20           13)15—13— 1 ( 1—4—1
 12           11   1                ————————————
626                                   13   13   13
3131
————
3757
```

3. Data *Plural*, and the *Quotient Simple.* *Rule.* *Example.*

3. *Case.* Both the given Numbers being Plural *Geodæticals*, and the Quotient desired Simple, and of the lowest Denomination,

Then reduce the Numbers into their Lowest Denominations, and divide the Results as Integers.

Example, If 25640 *l.* 12 *s.* 6 *d.* be divided by 5 *l.* 4 *s.* 2 *d.* both being reduced into Pence make 6153750, and 1250, which divided yield in the Quotient 4923 *d.* which if occasion be, may be reduced into Shillings and Pounds, as at *G.*

```
    l.     s.  d.      l. s. d.
  25640—12—6          5—4—2
     20                20
 ————————            ————
  512800              100                G.
      12                4          3
 ————————            ————       1 28     1  d.     s.
  512812              104       6153750(4923 (3 (41|0
      12               12       1255550 122
 ————————            ————        1222    1     20:10:3
 1025624              208         11
 5128126             1042
 ————————            ————
 6153750             1250
```

4. Data *Plural*, and the *Quotient Compound.* *Rule by* Reduction. *Example.*

4. *Case*, Both the given Numbers being Plural *Geodæticals*, and the Quotient required Compound,

Then after Reduction of the Numbers as above, if the Dividend be the Reduction of the Simple Product, Multiply the Divisor thus reduced by the Number of the Lesser Denomination contained in one Greater, and by this Product divide the Dividend.

Example, If 25640 *l.* 12 *s.* 6 *d.* were to be divided by 5 *l.* 4 *s.* 2 *d.* their Reductions are as before 6153750, and 1250, and 1250 the Divisor multiplyed by 240, the Pence in one Pound, produce for the New Divisor 300000, by which Division being made, the Quotient is 20 *l.* and if the value of the remaining Fraction be gotten and added thereto, the whole *Geodætical* will be as at *H.* 20 *l.* 10 *s.* 3 *d.* as before.

```
  1250     H.  6|153750(20 l.     30|75 | 00(10 s.    900( 3 d.
   240          |                   |—  | —           300
 ——————        3  00000           3  00 | 00
  50000
 2500          15375               75
 ——————           20               12
 300000       ———————             ————
 ——————        307500              150
                                    75
                                  ————
                                   900
```

Rule without Reduction.

But Secondly, If the Product given to be divided were compound without Reduction.

Then first reduce the given Numbers into their least Termes, and divide them as Integers or Fractions, as the Case happens.

Example.

If 106 *l.* 16 *s.* $8\frac{5}{8}$ *d.* be divided by 5 *l.* 4 *s.* 2 *d.* both reduced make $\frac{205125}{8}$ and $\frac{1250}{1}$ and after Division the Quotient is $\frac{1641}{80}$, or 20 *l.* 10 *s.* 3 *d.* is found at *I.* as before.

106 *l.*

```
  l.  s.  d.         l. s. d.                            l.          s.     d.
 106—16—8 5/8        5—4—2             I.           16 41 (20   8|20 (10  24 (3
  20                 20                              8  0       8  0       8
 ————                ———                             41         2
 2120                100                             20        12
   16                  4            10   1641        ———       ———
 ————                ———           ....  ......      820        24
 2136                104           1250 )205125 ( 1641 ———     ———
   12                 12             1 )   8   (  80
 ————                ———
 4272                208
 21368               1042
 ————                ———
 25640               1250
    8                ———
 ————
 205125
```

Or Secondly, Without *Reduction*, divide the Greater Denomination of the Dividend by the Greater Denomination of the Divisor, and an apt Quotient Figure or Figures found thereby, Multiply the whole Divisor by this Quotient, and placing the Product under the Dividend, substract it therefrom, and set the Remain at top, then by the Greater Denomination of the Divisor inquire out of the Remainer for another Quotient Figure, and thereby multiply the whole Divisor, and substract the Product as before, and so proceed to the end of the Work. *Variety.*

Example, In the Numbers last Exemplary, If 5 *l.* divide 106 *l.* there may be taken for the Quotient 20 *l.* by which 20, if the whole Divisor be multiplied, the Product will be 104 *l.* 3 *s.* 4 *d.* which substracted, the Work will stand as at *K*. Then out of the 2 *l.* 13 *s.* or 53 *s.* left, inquiring by 5 *l.* for another part of the Quotient 10 *s.* may be had, which multiplying the whole Divisor as before, and substracting the Product, the Work will stand as at *L.* And likewise proceeding for the remaining Numbers, the Work will stand as at *M.* *Example.*

K.

```
                 2 —13— 4
              ——————————————  l.
 l.  s.  d. )  l.   s.   d. ( 20
 5— 4— 2   ) 106— 16— 8 5/8 (
 20
 ——————————————————————————————
 100—80—40   104— 3— 4
```

L.

```
                     1—3
                   2—13—4
                 ———————————
 l.  s.  d. )  l.   s.   d.  ( l.  s.
 5— 4—2     ) 106— 16— 8 5/8 ( 20—10
 20—10
 ——————————————————————————————
 100—80—40      104— 3—4
    50—40— 20    2—12—1
       20  240
```

M.

```
                      1—3
                    2—13—4
                  ——————————
 l.  s.  d. )  l.   s.   d.  ( l.  s.  d.
 5— 4— 2    ) 106— 16— 8 5/8 ( 20—10—3
 20—10— 3
 ——————————————————————————————
 100—80—40     104— 3—4
    50—40— 20   2—12—1
       20  240     1—3 5/8
    15— 12 — 6
        20  240
```

Division of Geodæticals in general is proved by *Multiplication*, and *Multiplication* by *Division* alternately: *viz. Integers* as *Integers*, *Fractions* as *Fractions*, *Simple* as *Simple*, and *Compound* as *Compound*. For by making the Product of any *Multiplication* the Dividend, and the Multiplicand or Multiplier, the Divisor; the one of them will be returned in the Quotient, if the *Multiplication* be right. So on the contrary, if the Quotient of any Division be made the Multiplicand or Multiplier, and the Divisor be the other; the Product will return the Dividend, if the *Division* were right. Observing only to add to the Product the Remain of the *Division*, if any be. *Proof of* Geodæticals. Division *and reciprocally* Multiplication *thereby.*

Example,

Example whereof may be seen, *Book* 1. *Part* 1. *Chap.* 8. *Page* 36. and by comparing the *Multiplications* in the former Chapter with the Divisions in this will be sufficiently evident without further Example.

Last sort of Division *proved by* Addition.

In particular the last sort of *Division* at *M.* may be proved by *Addition*, for by adding all the Multiplees substracted in the *Division*, the Total with the Remain, when any is, will return the Dividend.

	l.	*s.*	*d.*	
Products	104	03	04	Multiplees
	2	12	01	
		1	$03\frac{5}{8}$	
Total	106	16	$08\frac{5}{8}$	Dividend

Proof of Division of Fractions *by the Value.*

Furthermore *Division of Geodætical Fractions* may be tryed by finding the Value of the *Fractions* to be divided, and the Quotient, and comparing the same with the Quotient of their Values divided as *Integers*, the Dividend being first multiplyed by the Number of one of the Lesser Denominations contained in the Greater.

Example, If $\frac{1}{2}$ *l.* be divided by $\frac{3}{4}$ *l.* the Quotient shall be $\frac{2}{3}$ *l.* and $\frac{1}{2}$ *l.* the Dividend being 10 *s.* or 120 *d.* multiplyed by 240 the Pence, or Lesser Denomination contained in 1 *l.* the Greater Denomination, make 28800, which divided by 180 *d.* the Value of $\frac{3}{4}$ *l.* gives in the Quotient 160 *d.* which is 13 *s.* 4 *d.* or $\frac{2}{3}$ *l.*

$$\frac{3}{4}\,l. = 15\,s. \text{ or } 180\,d.$$

$$\frac{3}{4}\Big)\frac{1}{2}\Big(\frac{2}{3} \text{ or } 13\,s.\ 4\,d. \qquad \frac{1}{2}\,l. = 10\,s. \text{ or } 120\,d.$$

$$\begin{array}{r} 240 \\ \hline 4800 \\ 240 \\ \hline 28800 \\ \hline \end{array}$$

$$\frac{\begin{array}{c}4\\ \cdot\cdot\\ 2\end{array}}{} \qquad \frac{\begin{array}{c}2\\ \cdot\cdot\\ 1\end{array}}{}$$

$$\begin{array}{llll} & & & d. \\ \cancel{1}\cancel{0} & & 4(4 & s. \\ \cancel{2}8800(& \cancel{1}\cancel{6}\cancel{0}(13 & & \\ \cancel{1}88\ 0 & \cancel{1}\cancel{2}\cancel{2} & & \\ \cancel{1} & \cancel{1} & & \end{array}$$

In Fractions *a Greater Number may divide a Lesser.*

Hereby it is evident that in *Fractions* a Greater Number may divide a Lesser, though in *Integers* it cannot, and that when *Division* is said to make a Sum less in Numeration, though the Quotient may be of greater Denomination than the Dividend was: It is to be understood of *Integers*, and not of *Fractions* that are Proper *Fractions*, Seeing in this Example $\frac{3}{4}$ of a Pound the Greater Number both in Figures and Value, can divide $\frac{1}{2}$ of a Pound the Lesser Number both in Figures and Value, and bring forth a Quotient bigger than the Dividend. For 13 *s.* 4 *d.* is greater than 10 *s.* by $\frac{1}{3}$ thereof, and yet divided by $\frac{3}{4}$ or 15 *s.* which is also greater than 10 *s.* by $\frac{1}{2}$ thereof.

But in *Integers* and *Improper Fractions* the Case is otherwise; for in the former alwayes the Numbers will be lessened, and in the latter sometime lessened, and sometime increased, according as the Greater *Improper Fraction* is the Divisor or the Dividend, which without Example is obvious enough in every Operation.

Partis primæ Libri Secundi

FINIS.

THE

THE SECOND PART OF THE SECOND BOOK.

CHAP. I.

Of FIGURALS.

Next sort of Numbers generally contrary, are Figurals, why so called.

THE next kind of Numbers to be viewed are called *Figural Numbers*, because they either do or may represent some Geometrical Figure, and are ever considered in relation to those Formes, and from thence borrowing their particular Names or Denominations, are rightly placed among Numbers generally Contract.

What of them belongs to Geometry, and what to Arithmetick.

The knowledge of *Figures*, their various Formes, and how to make and measure them, belongs to *Geometry*, and is there to be sought, but the Numbers contained in the Figures, and how to Order, Increase, and Diminish them, belong to *Arithmetick*, and those of special use therein, to be found here.

Figural Numbers what.

These Numbers are those understood by the Name of Figural Numbers, and are as various, as the Figures in *Geometry*, from whence they take their Names.

Geometrical Figures of Chief Note and most conveniently fitted to serve our turn may be thus aspected.

A Table of some Geometrical Figures.

- Figures
  - Linear — Sides. / Roots.
  - Superficial
    - Orbicular
      - Perfect ...... Circles.
      - Imperfect .... Ovals. &c.
    - Cornered
      - Sided
        - Uniforme
          - Triangles.
          - Quadrangles. — Oblongs. / Trapezia's &c.
          - Multangles.
        - Omniforme .... Like Flats.
      - Rooted ..... Squares.
  - Solid
    - Globular
      - Perfect ..... Sphæres.
      - Imperfect — Cilinders. / Cones
    - Angled
      - Sided — Pyramides / Prismes.
      - Rooted — Cubes. / Long-Cubes. / Square Cubes, &c.

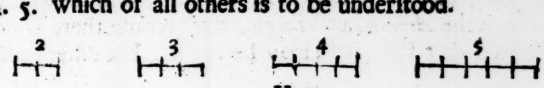
Figurals of 3 sorts.

Accordingly *Figural Numbers* are of Three sorts, *viz. Linear*, *Superficial*, and *Solid.*

1. Linear *what.*

Lineary, have comparison and relation to length only, and are therefore so called, because Unite is imagined to stand by Unit along in a Line, as in the following Figures of 2. 3. 4. 5. which of all others is to be understood.

Demonstrated.



Y y   This

This Name, although it be properly referred to such Numbers as will make no other form duely; yet it may also be applyed to any Number Abstract, because all such Numbers may be taken as the sides of other Figural Numbers.

Side of a Number, what and how called. Root *of a Number what, how called, and why.*

The Side (in Latine, *Latus*) is the Length of the Line containing the Side of the Figure: And if the Figure be equal-sided, or æquilateral, then is the side called a *Root* or *Radix* (after the Latine) by a Metaphor, because from thence other Figural Numbers arise as Branches from the Root of a Vegetable: Yet are not all equal Sides Roots, unless by *Multiplication* they can produce their Figures.

2. *Superficial, what and how called.*

Superficial Numbers, called also *Flat*, or *Plain Numbers*, and sometime *Surfaces* are considered according to such Form as they make in *Multiplication* or *Progression*, and have both Length and Breadth.

Sorts of Superficial Numbers. Oval, *whence the Name.*

Of *Superficies*, Some are Orbicular, and of them one sort only perfectly round, as Circles, and others imperfectly round as Ovals, so called from *Ovum*, Latine for an *Egg*, because they bear the resemblance thereof. Others are partly round, and partly of other Formes. The Figural Numbers of all which Figures I shall say nothing to, the Circle excepted, and of that but sparingly.

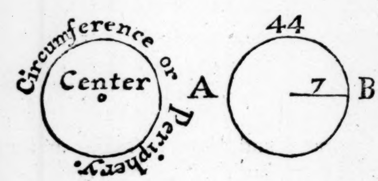
Circle *what*, Circumference, Periphery *what*, Center *what*. Semidiameter *what*. Diameter *what*.

A Circle is a plain Figure, determined with one bowed Line called the Circumference or Periphery, in whose midst is a Point named the Center, From which all right Lines drawn to the Circumference are equal, the Circumference being alwaies æquidistant from the Center, as appeareth at *A*. These Lines are called *Semidiameters*. And if the Line be drawn through the Center, and have both its Extremes ending in the Circumference, this is a *Diameter*, and the Longest Line a Circle is capable to contain.

Circular Number *what*. Area *of a* Circle.

A *Circular Number* is the Superficial Content, or after the *Latine* the *Area* of a Circle found by multiplying the *Semidiameter* into the *Semicircumference*, as at *B*.

Demonstration.



Semicircumference	22
Semidiameter	7
Circular Number.	154 or *Area.*

Numbers called Circular *on another accompt.*

Some Numbers sometime are called Circular, because as a Circle turns to the Point whence it began; so they being multiplyed by themselves, end in themselves; as 5 and 6, for 5 times 5 is 25, and 6 times 6 is 36, but here Circular Numbers are not taken in that Sence, but to be understood as before described.

Other Sorts of Superficies.

Other *Superficies* are Cornered or Angled, of which some are æquilateral, others inæquilateral; but none Rooted save the Square. The rest not Rooted of divers sorts, if considered *per se* simply are of one Form, if *inter se*, comparatively are of divers Formes, and therefore called *Like Flats*, or *Flats* that are alike.

Uniforme.

The *Uniforme Superficies* not Rooted are distinguished according to the Number of *Angles* therein: If 3 then called a *Triangle*, if 4 a *Quadrangle*, if more a *Multangle* or *Polygone*.

Triangle, *what*

A *Triangle* is a Figure comprehended of 3 Lines, and containeth as many *Angles*, or *Corners*.

Sorts of Triangles.

Triangles are Plain or Sphærical, according as the Lines are straight or bowed, whereof they are made; and are named both from their *Sides*, and from their *Angles*.

Æquilateral. If the 3 Sides be equal, the *Triangle* is called *Æquilateral*.

Isoscheles. If but 2 of the Sides be equal, it is named an *Isoscheles*.

Schalenum. If all 3 Sides be unequal, it is called a *Schalenum*.

Right Angled. If it have one *Right Angle*, that is containing 90 Degrees of a *Circle*, then it is called a *Right Angled Triangle*, or *Rectangled Triangle*, and after the *Greek* an *Orthogon*.

Broad Angled. If it have an *Angle* greater than a *Right Angle*, it is called an *Obtuse* or *Broad Angled Triangle*, or an *Ambligonium*.

Sharp Angled. If it have all the *Angles* less than a *Right Angle*, it is called an *Acute*, or *Sharp Angled Triangle*, or an *Oxigonium*.

And accordingly by the mixture of such *Sides* and *Angles* are the *Triangles* known asunder.

Leggs of a Triangle *how called.*

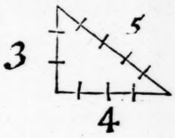
Every of the 3 Lines or Sides sometimes called the Leggs, and in *Latine*, *Crura*, have their distinct Names, as the *Perpendicular*, the *Base*, and the *Hypotenusa*.

Triangular Number *what*.

A *Triangular Number* is the *Area* of a *Triangle*, but because there is no certain Number from which, as from a *Root* such *Area* may be found, but differs according to the

Sides

Sides and *Angles* of every *Triangle*; it will not be meet to digress further than only to view the Plain Right Angled Triangle at *C*, and the rather, because some of the Ensuing Discourse may depend upon the knowledge thereof.

C 

3 Perpendicular. *Demonstration.*
4 Base.

12 Product.

6 *Area*, or the *Triangular Number* of this *Triangle*.

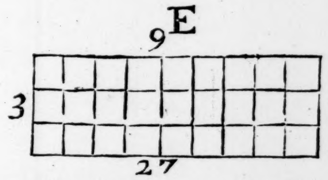
The Genus of *Quadrangles* include Squares as well as other Four-corned Figures, but because they are Rooted Figures, they take their Place by themselves hereafter. The other Species of *Quadrangled Figures* are an *Oblong*, a *Trapezium*, a *Rhombus*, and a *Rhomboide*. Quadrangles *what, and their kinds.*

An *Oblong*, called also a *Long Square*, and sometime a *Rectangle Figure* is an *Inæquilateral Paralellogram* consisting of 4 *Right Angles*, but only the opposite sides equal, as the Figure at *D*. Oblong *what.*

An *Oblong Number* is the *Area* of such Figure, which divided by the Lesser side bringeth the Greater in the Quotient, and if by Greater bringeth forth the Lesser, as 27 coming of 9 and 3, if divided by 3, giveth 9; if by 9 yieldeth 3 in the Quotient represented at *E*. or *F*. Oblong Number *what.*

D

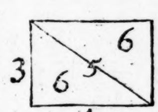
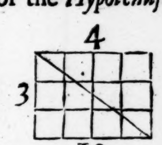
Oblongum.

E  *Demonstration.*

9 F. 27/9 (3
IIIIIIIII
IIIIIIIII
IIIIIIIII 27/3 (9
27

If a Right Line be drawn through the Center of this Figure from one Opposite Angle to the other; this Figure will be divided into Two Right Angled Triangles, as the Figure at *G*. sheweth. This *Diagonal Line* is called by some a *Diameter*, and from thence the Name of *Diametral Numbers* came. Diagonal, *by some called* Diameter, *and thence the Name of* Diametral Numbers.

A *Diametral Number* serveth to find out the length of this *Diagonal Line*, and hath two parts of that Nature, that if they be multiplyed together, will make the said *Diametral Number*, and the Squares of these two parts added together will make a *Square*, whose *Root* is the Length of the *Diametral* or *Diogonal Line* to that *Diametral Number*, as 12 is called a *Diametral Number*, because it hath Two parts, *viz.* 3, and 4, which produce it by *Multiplication*, and the Square of 3 is 9, and of 4 is 16, which 9 and 16 make together 25, whose *Square Root* is 5, the length of the *Diagonal Line* to the *Platform* 12, and of the *Hypotenusa* of each *Triangle* as at *G*. or *H*. Diametral Numbers *the use.*

G.  H  *Demonstration.*

3	3	4	9	5
4	3	4	16	5
12	9	16	25	25

A *Trapezium* hath all Four Sides unequal. Trapezium, *what.*

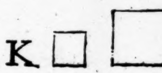
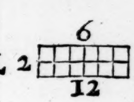
A *Rhombus* hath all four sides equal, but never a *Right Angle*. Rhombus *what.*

A *Rhomboide* hath the Opposite sides equal, but never a *Right Angle*. Rhomboide, *what.*

These and all *Multangles* being uncertain in the Measure of their Sides, and so consequently in their Figural Numbers are set aside here.

Those *Superficial Figures* called *Like Flats*, whether *Triangular*, *Quadrangular*, or otherwise, are such Plain *Homogeneal Superficies*, as bear a certain Proportion in their Sides unto each other, when compared together, as the Figures at *I*. and *K*. declare. Omniforme Superficies *how called.*

And Numbers called *Like Flats*, have the Sides of one bear like proportion to the Sides of another Platform of the same kind. As the Long Squares *L*. and *M*. If the Sides be 2 and 6 of the one, and 3 and 9 of the other, the Figures are *Like Flats*. And so the Numbers that express their Quantities, which are 12 and 27, are called *Like Flats*, because 6 is Triple to 2, as 9 is to 3. Numbers Like Flats, *what.*

I.  K  L  M  *Demonstration.*

6/2 (3

9 (3

A *Square*

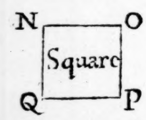
Square Figures, *the Names.*
Rooted.

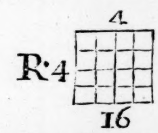
A *Square Figure*, called also a *Quadrate*, from the *Latine*, *Quadratus*, and by the *Arabians Zensus*, is a *Regular Superficies*, or an *Æquilateral Paralellogram* made by 4 *Equal Right Lines*, and as many *Right Angles*; every one of which Line is called the *Side* or *Root* of the *Square*, as by the Figure *N. O. P. Q.* is demonstrated, whose *Sides* are *N. O.* and *O. P.* and *P. Q.* and *Q. N.*

Square Numbers *what.*

A *Square Number* answering to the Similitude of this Figure is the *Content* or *Area* thereof, which is produced by multiplying any Number into it Self, and divided by his *Side* or *Root*, bringeth his Root (or *Divisor*) again in the *Quotient*, as 16, the *Product* of 4 by 4, divided by 4, returneth the *Root* represented, as at *R.* or *S.*

Demonstration.





S. 4

4
1111
1111
1111
1111
16

$\frac{16}{4}($ 4

3.
Solid *what, and how called.*

Solid Numbers, called sometime Sound and Corporeal, or Bodily Numbers, are the Third sort of Figural Numbers, and imploy both Length, Breadth, and Depth or Thickness; otherwise called Height, according to the Number of *Multiplications* whereof they are produced.

Sorts *of* Solid Numbers.
Axis *what.*

Of *Solid Numbers*, the only perfect *Globular* is the *Sphære*, or *Globe* represented in *Plano* as the Circle, and the *Axis* thereof as the Diameter of the other. This Term being often used for that, though the Diameter of a *Circle* is never properly called an *Axis.*

Cilinders *and* Cones, *their Forms.*
Pyramis, *whence the Name.*
Prismes. *no* Rooted Solids.

Cilinders, representing the Form of a Round Pillar; and Cones that are round at the Base, and pointed at Top; with such Sided Solids not Rooted, as are Angled, as a *Pyramis* so called from πυς, *Greek* for Fire, the Flame whereof being great at the bottom, ascendeth into a smaller form till it end in a Point) of *Triangular*, *Quadrangular*, or *Multangled* Form at the Base, equally ascending to a Point; And *Prismes* looking like a Writing-Desk, &c. for the same Reason before noted in some of the *Superficies* not Rooted; find no other Room here than to be mentioned.

Rooted Solids.

Among the Regular Rooted Solids, a *Cube* hath the præcedency.

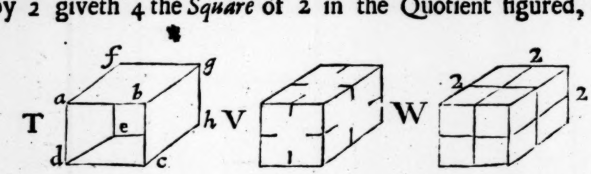
Cube, *what.*

A *Cube*, in *Latine*, *Cubus*, is a Right Angled *Paralelipipedon* æquiangled, and æquilateral, having 6 equal Surfaces, 8 Solid Angles, and 12 Sides, every one of which is the *Root*, represented by a Figure like a *Dye*, at *T*, whose Surfaces are *a. b. c. d.* and *a. b. f. g.* and *a. d. e. f.* and *b. c. h. g.* and *c. d. e. h.* and *e. f. g. h.* The *Angles a. b. c. d. e. f. g. h.* And the *Sides a. b.* and *a. f.* and *b. g.* and *b. c.* and *c. d.* and *c. h.* and *d. a.* and *d. e.* and *e. f.* and *e. h.* and *f. g.* and *g. h.*

Cube Number *what.*

A *Cube* or *Cubick Number* is the *Solid Content* of such Figure which is the Product of the *Square* multiplyed by the *Root*, and if divided by the *Root* or *Side*, bringeth the *Square* of the *Divisor* in the *Quotient*, as 8 the *Cube* of the *Root* 2, divided by 2 giveth 4 the *Square* of 2 in the Quotient figured, as at *V.* or *W.*

Demonstration.



2 Root
2
—
4
2
—
8

$\frac{8}{2}($ 4 Square.

Long Cube, *or* Squared-Square *what.*

A *Long Cube* (the next Regular Rooted Solid to a *Cube* is a Body of greater Length than Breadth or Depth, yet is the Length in proportion to them; for every Unit in his Depth or Breadth, begets a *Cube* equal to the first *Cube* in his Length, as the Figure at *X.* or *Y.* arising from the *Root* 2. make evident.

The Number thereof how called.

A *Long Cube Number*, called also a *Square of Squares*, sometime a *Biquadrate*, and commonly a *Zenzizenzike* Number, is the Number expressing the Solidity of that Figure; produced by multiplying the *Square* by it self, or the *Cube* by the *Root*, and being accordingly divided, returneth them in the *Quotient*, as 16 in Relation to the Root, 2 is a *Zenzizenzike* Number divided by 4 gives 4 the *Squares*, or by 2 yields 8 the *Cube* of 2 in the *Quotient*.

Demonstration.

2 Root
2
—
4
2
—
8
2
—
16

$\frac{16}{4}($ 4 Square

$\frac{16}{2}($ 8 Cube.

A *Squared*

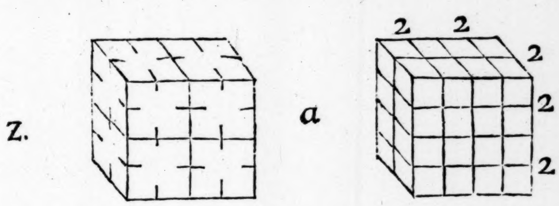
Squared Cube or Sursolide what.

A *Squared Cube* (the next in order) hath Length and Depth alike; but wants equal Breadth, yet so as every *Unite* of his Breadth begets a *Cube* both in Length and Depth, as the Figures at *Z*, or *a* from the aforesaid *Root* 2 make evident.

The Number thereof why called Sursolide.

A *Squared Cube Number*, in *Arithmetical Termes* is called a *Surdesolide*, or *Sursolide*, (perhaps as *Solid* upon *Solid*, *Sur* in *English* implying as much as *Super* in *Latine*) and contains the Solid Quantity of the Figure, being begotten by multiplying the *Square* into the *Cube*, or the *Squared Square* by the *Root*. And by *Reciprocal Division* returns them in the *Quotient*, as 32 is the *Sursolide* to the *Root* 2. If divided by 4 gives 8 the *Cube*, and by 2 brings 16 the *Squared Square* of 2 in the *Quotient*.

Demonstration.



2 Root.
2
4
2
8 $\frac{32}{4}$(8 *Cube.*
2
16 $\frac{32}{2}$(16 *Squared Square.*
2
32

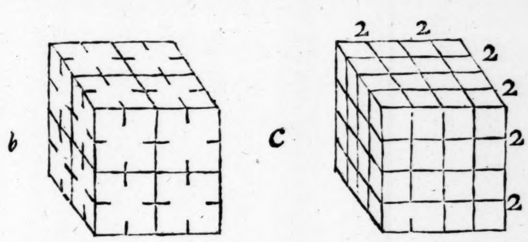
Cubed Cube or Squared Cube what.

A *Cubed Cube* is the Second Regular Body. *Arithmetick* takes notice of having *Square Solidity*, and is every way increased according to the *Units* originally in the *Root* thereof, as the Figures at *b*. or *c*. arising from the *Root* 2 demonstrate.

The Number and Names thereof.

A *Cubick Cube Number* is so called in relation to his Form only, otherwise the Number which declareth the Solidity of such Figure is called a *Zenzicube*, or a *Squared Cube Number*, made by multiplying the *Sursolide* into the *Root*, or the *Squared Square* by the *Square*, or the *Cube* by it self, and divided accordingly, brings forth the respective Numbers in the Quotient, as plain in 64 the *Zenzicube* of 2.

Demonstration.



2 Root.
2
4 $\frac{64}{16}$(4 Square.
2
8 $\frac{64}{8}$(8 Cube.
2
16 $\frac{64}{4}$(16 Squared Square.
2
32 $\frac{64}{2}$(32 Sursolide.
2
64

Names Geometrically and Arithmetically.

Further Instances may be spared, since you may proceed infinitely on *Rooted Solides*. But it is to be noted that every *Figure* receiveth its name *Geometrically*, according to his Form. And every Number containing the solid quantity thereof, being divided by his *Root*, giveth in the Quotient, the Number of the next lesser quantity. And these Numbers so containing the solidity in *Arithmetick*, have names not so much respecting the *Geometrical* Forms of the Figures, as the forming or producing of the Numbers themselves: For in the last *Example* 64. according to his Form *Geometrical* is a *Cubed Cube*, but in *Arithmetick* goes by the name of a *Squared Cube*, because the number 64, is formed by squaring 8, the *Cube* of 2, the *Root*. And so before 16, the *Squared Square* was so called, not because of his form, which was a *Long Cube*; but because it was the square of 4, which was the square of the *Root* 2.

Reason of the Arithmetical Names.

How to name the Higher Powers Geometrically.

And briefly to know how to name *Geometrically* every greater Form or higher Power (as some call them) that in order are increased by their *Roots*; observe that the fifth Body is named like the second, only doubling the *Cubes*, as *Long Cubick Cubes*. And the sixth Body like the third, as *Square Cubick Cubes*. And so the seventh Body like the fourth, tripling the *Cubes*. And so keeping the words *Long*, *Square*, and *Cube*, to every *Ternary*, the word *Cube* is added, as in the *Table* of *Rooted Numbers* in the next *Chapter* is sufficiently clear.

How to name them Arithmetically.

But the Terms *Arithmetical* retained to these *Rooted Powers*, take their rise from the original *Rooted Numbers*, viz. the *Square* and the *Cube*, except the *Sursolids*, and keep this order in accompt. The first *Rooted Number* is called a *Square*, the second a *Cube*, the third a *Squared Square*, the fourth a *Sursolide*, the fifth a *Squared Cube*, the sixth a

Second Surſolide, the ſeventh a *Square of Squared Squares*, the eighth a *Cube of Cubes*, the ninth a *Square of Surſolides*, the tenth a *Third Surſolide*, the eleventh a *Square of Squared Cubes*, the twelfth a *Fourth Surſolide*, &c. And in *Coſſical Numbers* in the next *Book* ſhall be ſhewn, how to proceed infinitely to name ſuch Numbers by their *Indices*.

Names of all Figurals *by their Quantities.*
Moſt regular to call the Root *the firſt Quantity.*

Yet ſome content themſelves to call theſe *Figural Numbers* neither after their *Geometrical* Forms, nor yet their antient *Arithmetical* Terms; but according to their Content or Quantity. And ſo they call a *Square* the firſt quantity, a *Cube* the ſecond quantity, a *Squared Square* the third quantity, *&c.* And others more regularly call the *Root* a Number of the firſt quantity, a *Square* the ſecond quantity, a *Cube* the third quantity, *&c.*

Obſervations.

All further needful to this *Chapter* may be conſidered in the following Obſervations.

1. Which have ſides.

1. All Angled Superficial and Sound Numbers have their ſides.

2. Unequal ſides many, Equal not more than 2, unleſs all.

2. One and the ſame Plain Number may have many ſides unequal, but ſeldom more than two equal ſides, except all be equal; as 36 hath 3, and 12 alſo 4, and 9, and 2, and 18, for the unequal ſides; but hath but only 6 and 6 for the equal ſides.

3. Which the Root.

3. The one ſide which is equal to the other in *Squares*, is the *Root*, and no *Flat Number*, ſave only a *Square*, hath a *Root*.

4. Which of the Solides *Rooted.*

4. Among *ſolid Numbers* they only have *Roots*, which be made of many Multiplications of ſome one Number by it ſelf, or by that which ariſeth thereof.

5. Which a Surde Number.

5. That Number whoſe Sides cannot be expreſſed by a *Whole Number* is called a *Surde Number*, and is no exact *Square* nor *Cube Number*, and ſuch are all *Prime Numbers*, and (*Squares* only excepted) the moſt part of all *Compound Numbers*. For if any *Whole Number* have a *Root*, that *Root* ſhall be a *Whole Number*.

6. What the Root *is, and how differs from the* Side.

6. The *Root* of a Number is a Number alſo, and is the ſide of the *Figural Number*: But every *Side* is not a *Root*, only the *Equal Side*, as aforeſaid, yet ſometime *Root* and *Side* are uſed *Synonimically*.

7. Roots *are infinite, and are differenced by* Adjectives.

7. *Roots* are as infinite as *Figures*; for any *Number* may be a *Root*, and the *Root* is always denominate according to his *Number*: For the *Root* of a *Square* ſhall be called a *Square Root*; the *Root* of a *Cube Number* is called a *Cubick Root*; ſo the *Root* of a *Squared Square* is a *Squared Square Root*; and the *Root* of a *Surſolide*, a *Surſolide Root*, &c.

8. One Number diverſly called as he ſtands related.

8. One and the ſame *Solide Number* may be diverſly named, according to the *Root* he ſtands related to; as 16, if it relate to the *Root* 4, is a *Square* or *Zenzike Number*, but if to the *Root* 2, is a *Zenzizenzike*. So 64, if related to the *Root* 8, is a *Zenzike Number*, and if to the *Root* 4, is a *Cubick Number*, but if to the *Root* 2, it is a *Zenzicube*, or a *Squared Cube Number*.

CHAP.

CHAP. II.

Production of Figurals.

Production of Figurals.

IN *Figural Numbers* is further to be learned the *Genesis* and *Analysis*. The first of these I call *Production*, and teach in this *Chapter* the manner thereof. The other is to be found in the next. And because *Figural Numbers* principally converse with *Integers*; *Figurate Fractions* are deferred to the *Fourth Chapter*.

To lay by those *Figural Numbers* of uncertain Product, and proceed orderly in the production of the rest most usual: Observe the Method used in the following *Sections*.

The Figurals whose Production is taught in this Chapter.

- *Production* of *Figural*
  - *Numbers* not *Rooted*
    - *Circulars* ——— §. 1.
    - *Oblongs* ——— §. 2.
    - *Diametrals* ——— §. 3.
    - *Like Flats* ——— §. 4.
  - *Rooted Numbers*
    - in General ——— §. 5.
    - in Particular
      - *Squares* ——— §. 6.
      - *Cubes* ——— §. 7.
      - *Higher Powers* ——— §. 8.

Circular Numbers produced.

§. 1. The multitude of *Figural Numbers* both *Superficial* and *Solide* wanting *Roots*, whose Formes in *Geometry* deserve inspection: As they are of less concern in *Arithmetick*, so they occupy the less room here, but only 4 being touched, and that briefly.

Circular Numbers (as in the next precedent *Chapter*) before noted in the sense here used, are but the *Areas* of the *Circles* found out as aforesaid, by multiplying the *Semidiameter* into the *Semiperiphery*, which are both usually given to produce them. If but one of them be given, the other is found by the proportion of the one to the other.

Proportion of the Diameter to the Periphery, by Archimedes and Van Ceulen.

Archimedes found the Proportion of the *Diameter* of a *Circle* to the *Circumference*, to be a very small deal greater than of 7 to 22. And of late *Ludoph van Ceulen* insisting in the same steps more precisely, found it to be of 1 to 3, 14159265358979 3. for which 3, 1416. may be taken; but most keep the former Numbers of 7 to 22, which make the *Circumference* 3 times as big as the *Diameter*, and $\frac{1}{7}$ part more.

Oblongs produced.

§. 2. *Oblongs*, or *Long Squares*, are produced by Multiplying one Side by the other adjoyning. As a *Field* or other *Superficies* being 3 Rods broad, and 8 long, the Form of that *Superficies* is a *Long Square*, and the Content thereof 24 Rods; obtained by Multiplying 3 into 8.

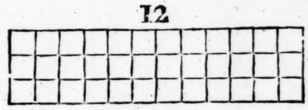
To find a Side unknown.

If one Side be unknown, having the Content and the other Side, Divide the Content by the known Side, and the Quotient will shew the Side unknown. As 24 divided by 3, gives 8; or by 8, gives 3, in the Quotient.

Which may be turned into Squares.

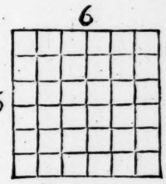
Those *Long Squares* whose Area is a Square Number, may be reduced from an Oblong Form to a perfect Square Figure. As a *Long Square*, whose Sides are 3 and 12, or 4 and 9, &c. and consequently the Area thereof 36, which because it is a Square Number of the Root 6, if each Side of the Platform be reduced to 6, the Figure will be a Regular Square, as here appeareth.

12

3 

Area 36.

6

6 

Area 36.

12 Side	6 Root
3	6
36	36

Diametral Numbers produced.

§. 3. *Diametral Numbers* were described before, and are produced as *Oblongs*, by Multiplying their proper parts together; or one Side of the Rectangle Figure by the other. As 60, produced by the Sides or Parts thereof, 5 and 12. Several others with their respective Sides may be seen in the *Table* following.

The

A Table of Diametral Numbers.

The Table of Diametral Numbers *unto the Lesser Side* 40.

Lesser Side.	Greater Side.	Diameter or Diagone.	Diametral Number.
3	4	5	12
5	12	13	60
6	8	10	48
7	24	25	168
8	15	17	120
9	12	15	108
	40	41	360
10	24	26	240
11	60	61	660
12	16	20	192
	35	37	420
13	84	85	1092
14	48	50	672
15	20	25	300
	36	39	540
	112	113	1680
16	30	34	480
	63	65	1008
17	144	145	2448
18	24	30	432
	80	82	1440
19	180	181	3420
20	48	52	960
	99	101	1980
21	28	35	588
	72	75	1512
	220	221	4620
22	120	122	2640
23	264	265	6072
24	32	40	768
	45	51	1080
	70	74	1680
	143	145	3432
25	60	65	1500
	312	313	7800
26	168	170	4368
27	36	45	972
	120	123	3240
	364	365	9828
28	96	100	2688
	195	197	5460
29	420	421	12180
30	40	50	1200
	72	78	2160
31	480	481	14880
32	60	68	1920
	126	130	4032
	255	257	8160
33	44	55	1452
	180	183	5940
	544	545	17952
34	288	290	9792
35	84	91	2940
	120	125	4200
	612	613	21420
36	48	60	1728
	105	111	3780
	160	164	5760
	323	325	11628
37	684	685	25308
38	360	362	13680
39	52	65	2028
	252	255	9828
	760	761	29640
40	75	85	3000
	96	104	3840
	198	202	7[illegible]20
	399	401	15960

Touching *Diametral Numbers*, 3 things are further considerable.

What is considerable in Diametral Numbers.

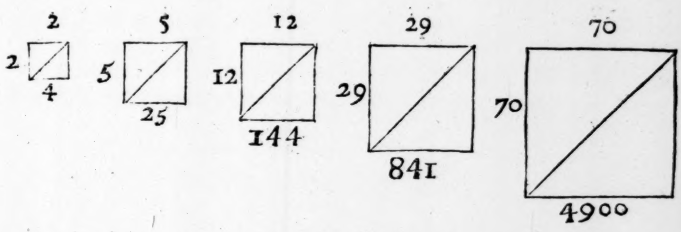
1. Somewhat concerning the knowledge of them in general.
2. If the Lesser Side of a *Diametral Number* be given, to find out the other.
3. When a Number is propounded, to discover if it be a *Diametral Number*, and consequently to find the Sides

For the first of these, let be minded.

Of the knowledge of them.

1. Diametral Numbers *set forth a Plain Rightangled Triangle.*

1. All *Diametral Numbers* do set forth a Plain Rectangled Triangle, having all 3 Sides known; which as it is rare, and of great use in many Geometrical Conclusions, so is it to be found in no other Numbers than only in *Diametral Numbers*: For though in Geometrical Figures you may ever infallibly find a Line, that will make a Square equal to the two Squares of any other two Lines; yet the certain measure of those Sides are not known in whole Numbers. And though other Numbers may go very nigh, yet it can never be done exactly but with *Diametral Numbers*, as in the following Examples of some *Square Numbers*.



Whose Doubles I take for the Squares of the Sides unknown, and they make 8, 50, 288, 1682, and 9800. All which differ only by an Unite from being Square Numbers; for 9 is a Square, and so are 49, 289, 1681, and 9801. and their Roots 3, 7, 17, 41, and 99. But those Doubles being no Square Numbers, cannot render their Sides in whole Numbers

2. Diametral Numbers *may have more parts than be apt for the Sides of the Rectangle.*

2. A *Diametral Number* may have more parts, then be apt for the Sides of the Platform or Rectangle Figure it represents: For it is not every two parts of the *Diametral Number*, that by Multiplication will produce the Number, that be meet Sides. As 60 hath these parts, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30. And beginning with the two extreams, *viz.* 2 and 30, they will, being multiplyed together, produce 60. And so will likewise any two of those Numbers or Parts equally distant from those extreams; as 3 and 20, also 4 and 15, likewise 5 and 12, and 6 and 10, but none of them save 5 and 12 are apt Sides to find the Diameter by. For if the other Sides be multiplyed squarely, and the Squares added together, there will arise no Square Number.

3. *If the least Side be odd, so shall the Diameter.*

3. As there are 3 Numbers, to wit, the 2 Sides and the Diameter; so alwayes if the first or least Side be odd, then shall 2 of the 3 Numbers be odd also; and the Diameter shall be the second and great odd Number.

4. Diametrals *even Numbers.*

4. All *Diametral Numbers* are even, and no odd Number can be a *Diametral Number*.

5. *Odd Numbers above 1, may be the least Side.*

5. All odd Numbers save 1, may be the Lesser Side to *Diametral Numbers*, but even Numbers do not serve so generally. For if the Lesser Side be an even Number, he must exceed 4; and if the Greater Side be an even Number, he must be such an one as may be divided by 4.

6. *Lesser Side odd what the Square shall be.*

6. If the Lesser Side be an odd Number, then ordinarily the Square of it is just equal with the summe which amounteth by Addition of the Diameter to the Greater Side. As in the former Platform of 60, where 5 being the Lesser Side, his Square is 25, and so much do the Diameter 13, and the Greater Side 12 make added together.

7. *One Side may serve divers* Diametrals.

7. One Side may serve to divers *Diametral Numbers*, as 9 to the *Diametral Numbers* 108 and 360, begotten of the Sides 12 and 40, *&c.*

8. Diametrals *divided by 12.*

8. There is no *Diametral Number*, but it may be evenly divided by 12, wherefore they be all even Numbers, evenly and oddly.

9. *How they end.*

9. There is no *Diametral Number*, but it endeth in 0, in 2, or in 8.

10. *They cannot have more than one Diameter,* &c.

10. There is no *Diametral Number* can have any more Diameters than one, and yet may one Number be the Diameter to divers other. As 25 is the Diameter to 168 and 300, *&c.*

11. Squares *no* Diametrals.

11. No Square Number can be a *Diametral Number*.

12. *They may be the* Area *of an* Oblong, *&c.*

12. Though every *Diametral Number* be the Area of a Long Square, yet the Area of every Long Square is not a *Diametral Number*.

Of finding the Side.

Secondly, When the Lesser Side or Part of a *Diametral Number* is given to find the other thereby. This Lesser Side will be a—

Number { Odd or Even. } { Uncompound or Compound.

When the Datum is Odd and Uncompound.

Example.

If the propounded Number be an odd Uncompound Number, multiply that Number by it self, and the Product (being an odd Square Number) as near the half as may be part into two. The Number less than the half by an Unite will be even, and shall be the second or Greater Side of the *Diametral Number*. And the other part greater than the half by an Unite shall be odd, and the Length of the Diameter. As if 3 be propounded, the Square thereof is 9, the Lesser Part 4 shall be the Greater Side, and the Greater Part 5 the Diameter.

When the Datum is Odd and Compound.

Example.

If the given Number be an odd Compound Number, then hath he more Greater Sides than one; for he hath the benefit not only of the former Rule, but also he followeth the form of that Number of which he is Compound. As 9, whose Square is 81, hath not only 40, the Lesser Part for the Greater Side, and 41, for the Diameter; but being Compound of 3 followeth that form also; and therefore as 3 hath 4 for the Greater Side, so 9 being thrice 3, shall have 12 which is thrice 4, for a match Side with him, and 15, which is thrice 5, for his Diameter. Likewise 15, being compounded of 5 and 3, shall have both Forms in making of the *Diametral Numbers*. For as 3 hath 4, so 15, being 5 times 3, shall have 20, which is 5 times 4 for the second Side, and 25 for his Diameter, which is 5 times 5. Again, as 5 hath 12, so shall 15, being 3 times 5, have 36, which is 3 times 12 for his second Side, and 39, which is 3 times 13, for his Diameter.

Proportion between the Diameters *and* Diametrals.

Here by the way may be noted, that though both the *Diameters* and *Diametral Numbers* (as of necessity they must) vary from the former Numbers; yet is there a marvellous proportion between them. For the proportion of both the Sides in one Figure, to both the Sides in the other, being added together, will be like the proportion between the two *Diametral Numbers*. As if 3 and 4 be the Sides of a *Diametral Number* they make 12, and 9 and 12 being Sides, make 108, that is, 9 times 12. Now 9 to 3 is triple, and so is 12 to 4, and both triples added together (Addition of *Ratio's* being as Multiplication of *Fractions*) make the proportion or amounting *Ratio Noncuple*, or ninefold, and so are the two *Diametral Numbers*, 12 and 108, in proportion each to other.

When the Datum *is Even.*

Example.

If the Lesser Side propounded be an even Number, then square the Number as before, and of that Square take two Quarters; from one Quarter take an Unit, and put to the other; so have you two odd Numbers, the Lesser of which shall be the Greater Side of the *Diametral Number*, and the other the *Diameter*. As 8 squared is 64, the Quarter 16, from whence 1 taken leaves 15 for the Greater Side of the Platform, and adding 1 to 16, the total 17 shall be the *Diameter*.

Proportion of the Greater Sides and Diameters.

Such even Numbers as have more Greater Sides than one, yet have they the like Numbers in proportion for their Greater Sides and Diameters, as the Numbers have of which they be Compound. As 20, compound of 4 and 5, shall have the Greater Side and Diameter belonging to 5 fourfold, and so the Greater Side of 5 is 12, and of 20 is 48, which is 4 times 12, and the Diameter 52, which is 4 times 13, the Diameter to 5.

Of discovering Diametrals.

Thirdly, To discover if a Number propounded be *Diametral* or not, and consequently to find the Sides; take these 9 Directions.

1. *By the ending.*

1. If it end with any other Figure than 0, 2, or 8, it can be no *Diametral Number*.

2. *Evenly divided by 12.*

2. If it may not be evenly divided by 12, although it end as abovesaid, it is no *Diametral Number*.

3. *By the Parts of the Number.*

3. If the Number propounded have those two properties, then set out all the Parts thereof, so as the Lesser Part stand over the Greater Part, which being multiplied together will make the whole Number, and then examine those Parts according to the former Doctrine.

4. *Take the Parts most apt.*

Example.

4. Observe which of the Parts that stand for the Sides of the Platform be most apt to constitute a *Diametral Number*, and make tryal of them, for some Parts at first sight appear unapt. For if among the Parts, the Lesser Number be odd, the Square thereof must contain double to that Greater Number that is coupled with it, and 1 more. As in the *Diametral Number* 12, where the Sides are 3 and 4, there 9, the

Square

Square of 3, is double to 4, and 1 over. And if the Lesser Number be even, then must the Square of it contain the Greater Number that stands by it, 4 times and 4 more. As in the *Diametral Number* 48, is 6 coupled with 8, which 6 times 6 is 36, that is 4 more than 4 times 8; and this holds in all Numbers not Compound of other *Diametral Numbers.*

5. When the given Number hath many Parts, to save work, guess at one which seems probable, and making Proof thereby, if he be found too small, assay with the rest of the Parts greater; and if he be too big, refuse all the Parts above, and examine only the smaller Parts till a just Part be found. But if thus examining you still find the Part either too great or too little, then is the Number given no *Diametral Number.* As if 120 be the Number propounded, because it ends in 0, and may be evenly divided by 12, it s probable to be a *Diametral Number.* I therefore set out the Parts which are these.

5. If a Part too great or too little be taken.

Example.

Parts of 120	2.	3.	4.	5.	6.	8.	10.	12.	15.	20.	24.	30.	40.	60.	Parts.
coupled.	60.	40.	30.	24.	20.	15.	12.	10.	8.	6.	5.	4.	3.	2.	Numbers.

Here though I see many Parts, yet I need examine but few; because several have no likelyhood of producing a *Diametral Number.* For all even Numbers under 6, cannot be the Lesser Side of any such Number, therefore the second and fourth Parts are rejected. Also all Numbers above the tenth Part are refused, because the Numbers under them are too little to answer proportionably for the Greater Side to the Parts standing over, which should be the Lesser Side, and these are greater than they. Again the third Part is set aside, as having under him too great a Number; for under 3 ought to stand no other Number than 4, to make a *Diametral Number.* Moreover, under 5 I find 24, but if I square 5, it is 25, which is but 1 more than the Number under 5, when it should be 1 more than the double. Therefore I either pitch upon the sixth, eighth or tenth Parts for the Sides of the *Diametral Number*, or else 120 cannot be *Diametral*; then examining 6, his Square is 36, but this is not 4 times 20, and 4 more; so the sixth Part is laid by also as too little. Then I square 10 its 100, but then 4 times 12 and 4 more is but 52, and therefore the tenth Part is too big. So that 8 and 15 must be the Sides, or else the Number is no *Diametral.* And squaring 8 it is 64, which is 4 times 15, and 4 more, whereby 120 is seen to be a *Diametral Number*, and hath 8 and 15 for the Sides of the Platform.

On the contrary, proving 72 by his Parts, though he end in 2, and may be divided by 12, yet doth it appear to be no *Diametral Number.*

6. By observing the proportion between the *Diametral Sides* it is easie to discern, whether the Parts be apt to constitute a *Diametral Number* or not: For the two Sides of all *Diametral Numbers* keep a constant proportion either to other in the order following, and continue in the same accordingly.

6. Observe the Proportion between the Sides.

The Orders for placing the Sides.

The First Order of Odd Numbers for the Least Sides.

3	5	7	9	11	13	15	17	19	21	23	25	27	&c.
4	12	24	40	60	84	112	144	180	220	264	312	364	

The Second Order of Even Numbers for the Least Sides.

8	12	16	20	24	28	32	36	40	44	48	&c.
15	35	63	99	143	195	255	323	399	483	575	

In both these Orders the Lesser Sides stand at top, and the Greater Sides beneath, as Antecedent and Consequent.

Stifelius sets the Greater Side at top, and the Lesser Side below, and reduceth the Antecedent into Units, like Integers and Fractions.

The Order of Stifelius.

The First Order after Stifelius.

$$1\frac{1}{3} \quad 2\frac{2}{5} \quad 3\frac{3}{7} \quad 4\frac{4}{9} \quad 5\frac{5}{11} \quad 6\frac{6}{13} \quad 7\frac{7}{15} \quad 8\frac{8}{17} \quad \&c.$$

The Second Order after Stifelius.

$$1\frac{7}{8} \quad 2\frac{11}{12} \quad 3\frac{15}{16} \quad 4\frac{19}{20} \quad 5\frac{23}{24} \quad 6\frac{27}{28} \quad 7\frac{31}{32} \quad \&c.$$

And this for observance is best approved, because in the first Order you see both in the whole Numbers and Numerators of the Fractions the Natural Order of Numbers, as 1, 2, 3, 4, *&c.* And in the Denominators, the Natural Progression of odd Numbers, as 3, 5, 7, 9, *&c.*

But

But in the second Order the whole Numbers go in their Natural Order, and the Numerators and Denominators keep an *Arithmetical Progression* by equal distance of 4. So that in the Numerators all the Numbers be odd, in the Denominators they be all even.

7. *Parts of the Number may be abbreviated.* 7. The great Parts of any Number to be examined may be abbreviated (like a Fraction) into its least Terms. For if the proportion hold in the least Terms it will hold in the greatest. As if 540 be proposed to find whether it be *Diametral*, I set down so many of the Parts as are necessary; thus,

Example.

2	3	4	5	6	10	12	15	18	20	&c.
270	180	135	108	90	54	45	36	30	27	

Where I may abbreviate many of the Parts, as $\frac{6}{90}$, $\frac{10}{54}$, *&c.* but comparing their Proportions with the former, cannot find the Numbers alike proportional; but 15 and 36 are in like proportion, and so they continue if abbreviated to 5 and 12, therefore is 540 *Diametral*, and 15 and 36 the Sides thereof.

8. *What Cyphers may be cut off.* 8. From such given Numbers as end in Cyphers, cut off even Cyphers as often as you can, as 2, 4, 6, *&c.* and if the rest be a *Diametral Number*, so was the given Number: For if 540, or 432, *&c* be *Diametrals*, then 54000 and 43200 be the like.

9. *By a Square Divisor.* 9. If any Number being divided by a Square Number, make the Quotient a *Diametral Number*; then is the Number divided a *Diametral Number* also. As 48 divided by 4, (a Square Number) yieldeth 12 in the Quotient, (a *Diametral Number*) therefore is 48 a *Diametral Number* likewise.

Like Flats *produced.* §. 4. *Like Flats*, because of their proportion in their Sides, are thence so termed, and therefore no one Number without relation to another can be termed a *Like Flat*. Some call them *Square-like Figures*, because they have some properties with *Square Numbers.*

To produce these Numbers, Multiply any two Square Numbers by one other Number, and the Products shall be *Like Flats*. As 4 and 9, if Multiplyed by 3, give 12 and 27, which be *Like Flats.*

Also if 2 Square Numbers will admit of 1 Divisor, then divide them thereby, and the Quotients shall be *Like Flats*. As 36 and 9, divided by 3, give 12 and 3, which are *Like Flats*. And so are 4 and 9, being the Quotients of 16 and 36, divided by 4.

Properties of Like Flats. The 4 following Properties of *Like Flats* are collected out of *Euclid. lib.* 8. *prop.* 18. 20. *&* 26. and *lib.* 9. *prop.* 1. *&* 2.

1\. 1. Every two Numbers *Like Flats*, have one mean Number between them in proportion to the *Lesser Flat*, as the *Greater* is to him. As 4 and 9 have 6 for a mean between them. For as 6 is $1\frac{1}{2}$ to 4, so is 9 to 6.

2\. 2. One *Flat Number* beareth unto the other double that proportion their Sides do. As 4 and 9, whose Sides 3 and 2, or $\frac{3}{2}$ are in proportion *Sesquialter*, and the *Flats* themselves 9 to 4, are in double *Sesquiquarta* proportion, and so will the *Sesquialter Ratio* make doubled.

3\. 3. Numbers that be *Like Flats* have such proportion together, as one of the Square Numbers used in their Composition beareth to the other. For in the former Examples, 12 to 3 is as 16 to 4, or 36 to 9, and if one of them be divided by the other, a Square is brought forth in the Quotient.

4\. 4. Any two Numbers being *Like Flats* multiplyed together, will produce a Square Number. As 4 and 9 make 36, so 12 and 27 make 324, the Square of 18.

Rooted Numbers *produced Generally.* §. 5. *Figural Numbers Rooted*, in General are produced by *Multiplication* thus. To Multiply any Number by it self makes a Square Number. Again, that Square Number multiplyed by the Root produceth a Cubick Number. To multiply by the Root that Cubick Number, giveth a Squared Square Number. And to multiply again by the Root yieldeth a Sursolide Number. And so multiplying the last Product by the first Root, bringeth forth a Number of the next greater Quantity, and so successively. So that to produce a Square, one Multiplication will serve (one Number being accounted for the Length, and the other for the Breadth). To make a Cubick Number, two Multiplications are required, by the second whereof the Number taketh Depth, as by the first Length and Breadth. Thus to make a Squared Square, 3 Multiplications are requisite, and then there is made a Line of Cubes. A Sursolide Number produced this way must have 4 Multiplications, which make a Square, wherein every Unite is a Cube. So the fifth Multiplication maketh a Cube of Cubes, accounting every Lesser Cube for an Unite. And then the sixth Multiplication returneth the multiplyed Numbers to the nature of Lineary Cubes. And the seventh to the nature of Squared Cubes. And the eighth to the nature of Cubick Cubes, and so forth infinitely as was before expressed. These 3 Names of *Long*, *Square*, and *Cubick Cube*, may be reiterated, but a fourth Form can never be devised. For further discovery inspection may be made into the *Table* of *Rooted Numbers* procreated after the common way thus.

The

A Table of Rooted Numbers.

The TABLE *of* Rooted Numbers, *consisting of* Fifteen Figural Formes, *and* Twelve Roots.

	Names Arithmetical.												*Names Geometrical.*
1	Roots.	1	2	3	4	5	6	7	8	9	10	11	12	Roots.
2	Squares.	1	4	9	16	25	36	49	64	81	100	121	144	Squares.
3	Cubes.	1	8	27	64	125	216	343	512	729	1000	1331	1728	Cubes.
4	Squar'd Squar.	1	16	81	256	625	1296	2401	4096	6561	10000	14641	20736	Long Cubes.
5	Surfolides.	1	32	243	1024	3125	7776	16807	32768	59049	100000	161051	248832	Squar'd Cubes.
6	Squar'd Cubes.	1	64	729	4096	15625	46656	117649	262144	531441	1000000	1771561	2985984	Cubick Cubes.
7	Second Surfolides.	1	128	2187	16384	78125	279936	823543	2097152	4782969	10000000	19487171	35831808	Long Cubick Cubes.
8	Squares *of* Squar'd Squar.	1	256	6561	65536	390625	1679616	5764801	16777216	43046721	100000000	214358881	429981696	Square Cubick Cubes.
9	Cubes *of* Cubes.	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489	1000000000	2357947691	5159780352	Cubes *of* Cubick Cubes.
10	Squares *of* Surfolides.	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401	10000000000	25937424601	61917364224	Long Cubes *of* Cubick Cubes.
11	Third Surfolides.	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609	100000000000	285311670611	743008370688	Square Cubes *of* Cubick Cubes.
12	Squares *of* Squared Cubes.	1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481	1000000000000	3138428376721	8916100448256	Cubick Cubes *of* Cubick Cubes.
13	Fourth Surfolides.	1	8192	1594323	67108864	1220703125	13060694016	96889010407	549755813888	2541865828329	10000000000000	34522712143931	106993205379072	Long Cubick Cubes *of* Cubick Cubes.
14	Squares *of* Second Surfolides	1	16384	4782969	268435456	6103515625	78364164096	678223072849	4398046511104	22876792454961	100000000000000	379749833583241	1283918464548864	Square Cubick Cubes *of* Cubick Cubes.
15	Cubes *of* Surfolides.	1	32768	14348907	1073741824	30517578125	470184984576	4747561509943	35184372088832	205891132094649	1000000000000000	4177248169415651	15407021574586368	Cubed Cubick Cubes *of* Cubick Cubes.
	&c.													*&c.*

The Table *explained.*

The *&c.* at the foot of the *Table* denotes it may be increased; but this is large enough to view the orderly production of *Rooted Numbers*, to the 15th *Quantity* and 12th *Root*, where is to be seen,

Indices *of* Figurals, *what and why so called.*

1. The Numbers of *Quantity* in the first Left Hand Column, called *Indices* in *Latin*, because the Power or Quantity of the *Figural Number*, and how far he is removed from the *Root*, is shewed thereby.

2. The *Roots* from which each *Figural Number* ariseth, in the head of the *Table*.

3. The *Arithmetical Names* proper to each Number, at the left side.

4. The *Names* according to the Forms of each Number, at the right side.

And 5. The Numbers themselves in the body of the *Table*, which are to be accepted for *Figural Numbers* of that Denomination, which you find written against them at the sides, and have that Number for their *Root*, that standeth over them in the head of the *Table*.

Rooted Numbers *produced particularly. The* Square.

§. 6. *Rooted Numbers*, admit of some variety in production, which in particular is set forth in this, and the two following *Sections*.

The first *Rooted Number* a *Square*, besides multiplying the *Root* into it self as aforesaid, may be obtained by breaking the *Root* into Parts, called also Segments; and Squaring the Parts severally, and orderly adding them with the double of the Product produced by Multiplication of the Segments one into another.

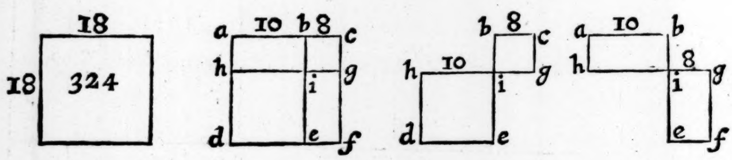
Theoreme *of* Euclide.

This Device is grounded on that *Geometrical Theoreme* in *Euclid. lib.* 2. *prop.* 4. That if a Right Line be cut into two Segments, the Square of the whole Line shall be equal to the Squares of the Segments, and to the two Right-Angled Figures made of the Segments.

Example and Demonstration where the Root *hath but* 2 *Figures.*

As if the Right Line, whose Square I would produce, be 18, I cut the Line into two Segments, as suppose 10 and 8, then shall the Squares of 10 and 8, added with the double of 10, multiplyed into 8, be the Square of 18.

At the Figure following (when the Line or Side of the Square is cut into 2 Segments) there appears 4 plain Figures; of which 2 are Squares, and 2 Rectangle Figures or Long Squares, all which added together, make the whole Square.

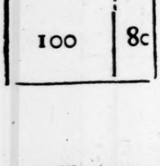


And of these 4 plain Figures is the whole Square obtained by the 4 Numbers of their Area; thus,

1. The Greater Square *h. i. e. d.* is known by multiplying the Line *h. i.* (equal to *a. b.*) into it self, and so 10 squared is 100.

2. The Lesser Square *b. c. g. i.* is known by multiplying the Line *b. c.* (equal to *i. g.*) into it self, and so 8 multiplyed by 8, gives 64

3. The Long Squares are known by multiplying the 2 Segments one into another, and then doubling the Product. As the Segment *a. b.* 10, into the Segment *b. c.* 8, which gives 80, for the Area of each Rectangle Figure, and added to the Squares are their Complement to the Square of the whole Line 18, as by the ordinary way may be proved.



18 . Radix
10 : 8 . Segments
100 . Square of the Greater Segment
80 }
80 } . Rectangle Figures
64 . Square of the Lesser Segment
324 Zensus

Proof.
18 Radix
18
144
18
324 Zensus

Where the Root *hath more than* 2 *Figures.*

In like manner, if the *Root* consist of many Figures, or the Line be cut into different Segments, yet is the work alike. As if 140 were the side of a Square, whose Square Number were desired, I may cut 140, into 100 and 40, or 90 and 50, or 95 and 45, or any other parts.

Example.

140	Radix
100 : 40	Segments
10000	Great Square
4000 / 4000	Rectangle Figures
1600	Lesser Square
19600	Zensus

140	Radix
95 : 45	Segments
9025	Great Square
4275 / 4275	Rectangle Figures
2025	Lesser Square
19600	Zensus

Proof.

```
  140 Radix
  140
 -----
 5600
 140
 -----
 19600 Zensus
```

Variety of working with 2 Figures.

Some instead of cutting the Line of the Square in but two Segments, as before, do after a sort cut the same into several Segments, even as many as the Root hath Figures; and to shorten the Multiplying work, accompt every Figure in what place so ever, or though never so great an Article, but as a Digit, and to supply the Cyphers wanting, place the Numbers gotten as aforesaid, each one place nearer to the Right Hand than the other, and some add the two Rectangle Figures together, ere they set them down under the Root. As for instance, To get the Square of 46, I accompt the Segments not 40 and 6, but 4 and 6, then place under 4 his Square 16, and multiplying the double of 4, which is 8, by 6, or 4 by 6, and double the Product (for it's all as one) the Product 48 is the summe of both the Rectangle Figures, which I place under 16, one place nearer to the Right Hand, then under the Segment 6, I set his Square 36, and the Total added together, the Square of 46 is found to be 2116, and by the former wayes may be proved true. *Example.*

Proofes.

```
  4 6   Root         46            46 Root
                    40 : 6         46
 |16|  |            ------        -----
 | 4|8 |            1600           276
 |  |36|             240          184
 ------              240          -----
 2116   Square        36          2116 Square
                    ------
                    2116
```

With many Figures.

And if the Root propounded consist of many Figures, then after the manner last mentioned, when you have gotten the Square of the first two Figures to the Left Hand in the same sort proceed to seek the Power or Quantity of the rest. As in seeking the Square of 46808, thus;

Example.

	4	6	8	0	8	Root
	16					
	4	8				
		36				
Square of	21	16				46
		73	6			
			64			
Square of	21	90	24			468
				0		
				0		
Square of	21	90	24	00		4680
			74	88	0	
					64	
Compleat	21	90	98	88	64	Square

Proof.

```
     46808 Root
     46808
   ----------
     374464
    3744640
   280848
  187232
  ----------
  2190988864 Square
```

Touching Square Numbers, and their production, further observe. *Observations.*

1. A Square Number doth never end in 2, 3, 7, 8, or a single 0; but in order terminate thus, 1, 4, 9, 6, 5, 6, 9, 4, 1, 0, 1, 4, 9, 6, &c. beginning again as in the *Table* of *Rooted Numbers* is apparent. *1. Of their Terminations.*

2. The

2. *How made of* Diametrals.

2. The Squares of the Parts of a *Diametral Number* added together, make a *Square Number*, as was seen before in *Diametral Numbers*.

3. *How made of* Like Flats.

3. Numbers called *Like Flats*, multiplyed together make *Square Numbers*. As 2 and 18 make 36 the Square of 6; so 3 and 48 make 144, the Square of 12.

4. *Odd Square half lacking, 1 added makes a Square.*

4. If 1 be taken from any Square Number which is odd, the Square of half the remainer being added to the first Square will make a Square Number. As 9 the Square of 3, from which 1 be taken there resteth 8, the half 4 squared is 16, to which if 9 be added the total is 25, the Square of 5.

5. *How made of an Even Number.*

5. The Square of half any even Number, if the even Number be added to it, and 1 more, will make a Square Number. As 10 whose half is 5, the Square whereof is 25, to which 10 added, and 1 more, make the Total 36, the Square of 6.

6. *How otherwise.*

6. If to the Square of half any even Number 1 be added, and the even Number then substracted, there will remain a Square Number. As if to 25, the Square of 5, the half of 10, there be 1 added, it will be 26, from which if 10 be substracted, 16 a Square Number remaineth whose Root is 4.

7. *How by Addition of Odd Numbers.*

7. Odd Numbers continually added from an Unit successively to the antecedent Squares, make the Totals, Square Numbers. As 1 and 3 is 4, so 4 and 5 is 9, and 9 and 7 is 16, *&c.*

8. *How by Cubes.*

8. Cubick Numbers added successively from the Unit, produce Square Numbers. As 1 and 8 is 9, so 9 and 27 is 36, *&c.*

Cube *produced particularly.*

§. 7. The particular construction of the *Cube* (the first and least Rooted Body) is next to be seen.

A *Cube* by the fifth *Section* of this *Chapter* is produced the common way, by Multiplying the Square Number by the Root. As 2 by 2 makes 4, which 4 again multiplyed by 2 makes 8, the *Cube* of 2.

This is a kind of triple Multiplication, the Root being alwayes valued in himself once. For 2 times 2 twice maketh 8, and so 3 times 3 thrice yieldeth 27, the *Cube* of 3. And 4 times 4 four times giveth 64, the *Cube* of 4, *&c.*

Device of Ramus.

The production of these Numbers vary from the ordinary way: If the Root be broken into Parts, and the *Cubes* of the Parts or Segments be added to the two solid Figures, comprehended 3 times under the Square of one Segment multiplyed by the other. Which Device *Ramus*, *lib.* 24. *sect.* 10. imitating that in *Euclide* of the Square, delivereth. As if there be a *Cube*, whose Root is 18 Inches, and I would know the *Cube Number* thereof, which shall declare how many solid Inches there are in that Body;

Example and Demonstration where the Root hath 2 Figures.

I cut the Root into two Segments, as 10 and 8, and thereby doth the Body at *K.* appear (as much as can be aptly demonstrated in *Plano*, or by *Flats*) to be parted into 8 Bodies or Solidities, *viz.*

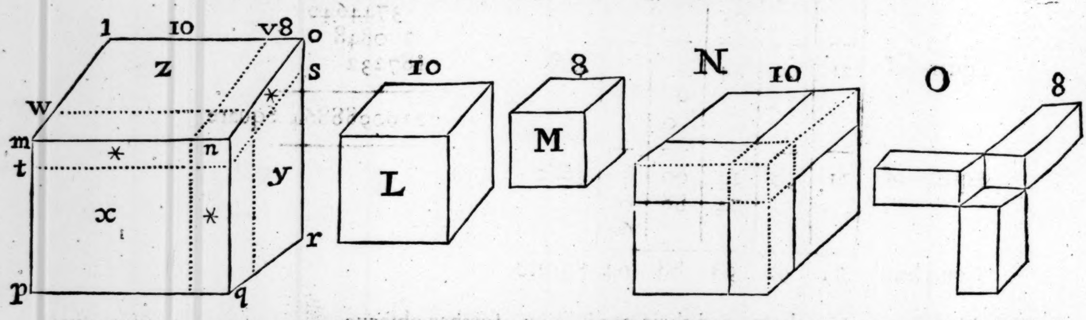
The first a *Great Cube* of the Greater Segment, which cannot well be made visible by a Paper description (*Cubes* pourtrayed like *Dice*, whose Form most fitly they represent, some parts will be hid from the prospect, and must be imagined) but is placed under the Greater Segment *Z.* and is that part of the *Cube K.* from the Line *t.* downward.

The second a *Lesser Cube* of the Lesser Segment, cut off in the corner from the *Cube K.* at *n*, joyning together the 3 Lesser Paralelipipedons to the 3 Greater.

The 3 Greater Paralelipipedons marked with *x. y. z.*

The 3 Lesser Paralelipipedons marked with 3 *Asterisques.*

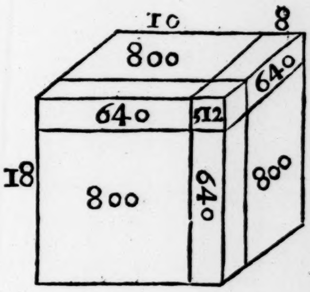
And as if the *Cube* were taken in pieces, be partly discerned at *L. M. N. O.*



And as all these solid Figures joyned together make the *Cube* at *K.* compleat, so will their several solid Numbers added together complement the *Cube Number*. For the *Cube* of the Greater Segment is 1000, obtained by multiplying the Greater Segment Cubically. And the *Cube* of the Lesser Segment also is gotten by multiplying the same *Cubically*,

Cubically, and so 8 *Cubed* is 512. Then the Greater Paralelipipedons are found by multiplying the Square of the Greater Segment by the Lesser Segment; that is 100, the Square of 10 by 8, maketh 800, for every one of the 3 Greater Paralelipipedons. Lastly, the Lesser Paralelipipedons are known by multiplying the Lesser Segment squared by the Greater Segment. As 64, the Square of the Lesser by 10 the Greater Segment, give 640 for every of the 3 Lesser Paralelipipedons, all which Numbers added together render the *Cube Number* 5832. Thus,

18	Radix
10 : 8	Segments
1000	Cube of the Greater Segment
800 800 800	3 Greater Paralelipipedons
640 640 640	3 Lesser Paralelipipedons
512	Cube of the Lesser Segment
5832	Cubus



Proof by the Common way.

18	Radix
18	
144	
18	
324	Zensus
18	
2592	
324	
5832	Cubus

Hereby is the Theoreme of *Ramus* also clear, That the *Cube* of the whole Line is equal to the *Cube* of the Segments, and the 2 solid Figures comprehended 3 times under the Square of his Segment, and the remaining Segment.

And it may further be noted, that as in the vulgar way of production you first get Plain Numbers, and then Multiply those Plains by the Roots, to bring forth the solid Numbers: So may you here proceed likewise; for if you get the Squares of the Segments, and the Rectangle Figures, as before in the precedent *Section* was taught, and then Multiply those 4 Plain Numbers by the Segments severally, you will produce 8 Solids, the Total whereof will be the *Cube Number*. As in the former Number the 4 Plain Numbers in the Square of 18, were 100, 80, 80, 64, which Multiplyed by 10, give the several Products of 1000, 800, 800, 640, and when Multiplyed by 8, give 800, 640, 640, 512, all which added together produce the *Cube*, as before, 5832.

100 . 80 . 80 . 64

Greater 10 Segment.

1000 . 800 . 800 . 640

100 . 80 . 80 . 64

Lesser 8 Segment.

800 . 640 . 640 . 512

If there be more Figures in the Root than two, yet this course is still kept. As in seeking the *Cube* of 468; thus, *Where the Root hath more than 2 Figures.*

Example.

468	Radix
400 : 68	Segments
64000000	Cube of the Greater Segment
10880000 10880000 10880000	3 Greater Paralelipipedons
1849600 1849600 1849600	3 Lesser Paralelipipedons
314432	Cube of the Lesser Segment
102503232	Cubus

Proof.

468	Root
468	
3744	
2808	
1872	
219024	Square
468	
1752192	
1314144	
876096	
102503232	Cube

Variety of working with 2 Figures.

Some shorten the work, accompting the Segments as Digits only, and to supply the place of Cyphers (as before in the Square) place the Numbers orderly to the Right Hand. As where the Root hath but 2 Figures, the first Right Hand Figure of the Cube of the Greater Segment under the first Left Hand Figure of the Root. And next that one place nearer to the Right Hand, the summe of the 3 Greater Paralelipipedons gotten by tripling the Square of the Left Hand Digit, and Multiplying that triple by the Right Hand Digit of the Root And under this one place nearer to the Right Hand is set the summe of the Lesser Paralelipipedons obtained by tripling the Square of the Right Hand Digit, and Multiplying that triple by the Left Hand Digit of the Root. And lastly, the Cube of the Right Hand Digit of the Root (or Lesser Segment) placed under him compleat the work As if the Cube of 56 be demanded, I place under 5 his Cube 125, and next 450, which is the Product of 6 into 75, the triple of 25, the Square of 5; and next 540, the Product of 5 into 108, the triple of 36, the Square of 6; and lastly, 216 the Cube of 6, in their orderly places before directed, and adding them, find the Cube sought to be 175616.

Example.

```
   5   6    Root
 125
  45 0
   5 40
     16
 175 616    Cube
```

```
   56
 50 · 6
 125000
  15000
  15000
  15000
   1800
   1800
   1800
    216
 175616
```

```
Proofes.   56 Root
           56
          336
         280
         3136 Square
           56
        18816
       15680
       175616 Cube
```

With many Figures.

If the Root consist of many Figures, then after in the former manner you have wrought for the Cube of the first 2 Figures to the Left Hand, go forward to seek the quantity of the residue. As in seeking the Cube of 46808, this is the work.

Example.

```
           4   6   8   0   8
           64
           288
            432
             216
Cube of    97336                46
            50784
              8832
                512
Cube of    102 503 232          468
                   0
                    0
                     0
Cube of    102 503 232 000      4680
                52 565 760 0
                     8 985 60
                          512
Compleat   102 555 806 746 112  Cube
```

```
Proof.
         46808 Root
         46808
        374464
       3744640
      280848
     187232
    2190988864 Square
         46808
    17527910912
   175279109120
  13145933184
  8763955456
 102555806746112 Cube
```

How Cubes are gotten by Addition of Odd Numbers.

Further in general may be noted, That Cubick Numbers are begotten by Addition of odd Numbers from an Unit successively. As 3 and 5 make 8, so 7, 9, and 11, the next odd Numbers are 27, the next Cube to 8; also 13, 15, 17, 19, make 64, and the like of others; and so many Units in the Root, so many odd Numbers in the Cube.

How Cube Numbers end.

One thing more is remarkable in the termination of the Cube, *viz.* that he may end in any Figure, but he maketh exchange in some. For if he have 0, 1, 4, 5, 6, or 9, in

in the first place of his Root, he will have the same in the first place of the Cube. But if 2 be first in the Root he will exchange for 8 in the first place of his Cube; and if 8 be the first of his Root, 2 shall be the first of the Cube. And in like sort do 7 and 3 make exchange in the Cubick Number.

§. 8. The varieties of production of *Higher Powers*, or *Rooted Solides*, greater than the *Cube* shall close up this *Chapter*. And because it would be tedious to recite, and consequently to remember many particulars, take this General Rule to know how many wayes every such *Higher Power* may be produced. Higher Powers *produced.*

Mark their *Indices*, or how many degrees the Number you would produce is removed from the Root, as whether it be the second, third, fourth, fifth, &c. Quantity (accompting the Root alwayes for the first, as before in the *Table* of *Rooted Numbers*), and couple each two Numbers together, the one next increasing from the Root, with the other next decreasing from the Quantity or Number sought to be produced, and so proceeding, at last there will be either an even Couple, or an odd Number. If they be all even Couples, then so many wayes may the Number desired be produced by Multiplying each two Numbers together that answer to their coupled *Indices*. And if among the coupled *Indices* there be an odd Number which hath none to match him, then may the Number you seek, by Multiplying the *Figural Number* answering to this odd *Index* into it self, be produced, as well as by the Coupled Numbers. As to know how many wayes the *Third Sursolide*, or 11*th Rooted Figural Number* (accompting the Root for the Prime or Original) may be produced, I couple their *Indices* as at *P*, and supposing the Root 2, place the correspondent Figural Numbers as at *Q* and find he may be produced 5 wayes, by Multiplying, 1. The Root into the Square Sursolide. 2. The Square into the Cubed Cube. 3. The Cube into the Zenzizenzizenzike. 4. The Squared Square into the Second Sursolide. And 5. The Sursolide into the Squared Cube. *By their* Indices. 1. *Example.*

Indices Coupled.

P.	10	9	8	7	6
	1	2	3	4	5
	11	11	11	11	11

Index of the Third Sursolide.

Numbers Placed.

Q.	1024	512	256	128	64
	2	4	8	16	32
	2048	2048	2048	768	128
				128	192
				2048	2048

The Third Sursolide of 2 produced.

Likewise 8 several varieties of producing 65536, the Zenzizenzizenzizenzike of 2, or the 16*th Figural Rooted Number*, by this way are found. For besides the 7 Couples of *Indices*, the 8th *Index* is odd, and his Number Multiplyed squarely will effect as much as the other. See the Operations at *R*, and *S*. 2. *Example.*

R.

Added								
	15	14	13	12	11	10	9	8
	1	2	3	4	5	6	7	
	16	16	16	16	16	16	16	16

S.

Doubled								
	32768	16384	8192	4096	2048	1024	512	256
	2	4	8	16	32	64	128	256
	65536	65536	65536	24576	4096	4096	4096	1536
				4096	6144	6144	1024	1280
							512	512
				65536	65536	65536		
							65536	65536

And further, because this Rule depends upon the proportions between the *Indices* and their Quantities; for as the one increase by Addition, so the other by Multiplication (as in process of this Treatise hereafter may be seen); therefore whatever *Indices* added together, will make the Total, the *Index* of the Figural Number sought, those Figural Numbers answering to the added *Indices*, Multiplyed one into another shall produce the Figural Number desired, as in the last Example, because 5, 5, and 6, make 16, and also 4, 4, and 8, and 11, 4, and 1, and several other Numbers do the like, the Figurals of those Quantities Multiplyed together, as at *T. V. W.* will produce the Figural Number of 16, and the Root being 2, will be 65536, as before. *Varieties thereof.*

T.

T.		*V.*		*W.*	
32	5	16	4	2048	11
32	5	16	4	16	4
64		96		12288	
96		16		2048	
1024		256		32768	
64	6	256	8	2	1
4096		1536		65536	16
6144		1280			
		512			
65536	16				
		65536	16		

Of the ending of Squared Squares.

Some have observed the Terminations of these *Higher Powers*, and find,

The Squared Square never endeth in 2, 3, 4, 7, 8, or 9, but either in 0, 1, 5, or 6, and thus according to the Natural Order of Numbers in their Roots, 6, 1, 6, 5, 6, 1, 6, 1, 0, 1, and then begin again.

Sursolides.

The Sursolides imitate their Roots. For if the Root be a Digit, then hath the Sursolide the same Digit in his first place: But if his Root be an Article, then as the Sursolide is the fifth Quantity, so hath he 5 times so many Cyphers together in the Right Hand places as the Root had, and the next signifying Figure after these Cyphers is the first Figure significative of his Root. And if the Root be a mixt Number, yet still is the first Figure of the Sursolide, the first of the Root.

Others higher.

The next Figural Number, as Sursolides follow the manner of their Roots, so do they the manner of the Squares. And the next to them the manner of the Cubes. And the next the manner of the Squared Squares. And then they begin again, and are like the Sursolides, *&c.* All which may fully be seen in the foregoing *Table* of *Rooted Numbers.*

Proof of Production of Rooted Numbers.

The Proof of Production of Rooted Numbers (besides the varieties proved one by another) must be deferred till Extraction of their Roots be learned, which shall be the Subject of the next *Chapter.*

CHAP. III.

Extraction of Roots.

Extraction of Roots.

THE *Genesis* of *Figural Numbers* now finished, I come to their *Analysis*, vulgarly called *Extraction of Roots.*

Of what made.

As Production of *Figural Numbers* was made up of Multiplication and Addition, so is Extraction of their Roots of Substraction and Division.

Divisor to seek.

But whereas in Division of Integers, the Divisor is known, here it is to seek, every remove requiring a new Divisor.

Sides of the Numbers not Rooted, *found by Division.*

The Sides of those *Figural Numbers* not Rooted, mentioned in the former *Chapter*, being sufficiently known by their Production to be the Factors of such Products by one single Multiplication, render their Invention easie by one single Division of those produced Area's by the Side known, and need no further remembrance here.

Roots *by Extraction.*

But the finding of the Root of a *Figural Number* made of several Multiplications, is much more difficult. For the clear understanding whereof, and all needful thereto, see,

Extraction of *Roots*
- in General — §. 1.
- in Particular of
  - *Squares* — §. 2.
  - *Cubes* — §. 3.
  - Compounds of both — §. 4.
  - All Higher Powers, by the *Table* — §. 5.
  - *Surdes*, and to denominate the Remains — §. 6.

§. 1. To

§. 1. To the Extraction of *Roots* in General, 2 things are necessary. *What necessary to Extraction of Roots.*

1. To have the *Figural Numbers* of every Digit perfectly in mind: For it were superfluous to seek Rules for them, since they may be easier remembred then Rules for their production, and readily found in the *Table* of *Rooted Numbers*: As for the Square and Cube, thus; 1. *To know the Figural Number for every Digit.*

Roots	1	2	3	4	5	6	7	8	9	
Squares	1	4	9	16	25	36	49	64	81	The like of others.
Cubes	1	8	27	64	125	216	343	512	729	

2. To prick the *Figural Numbers* given whose Root is to be extracted, according to their Quantities, beginning at the Right Hand. As the Square, because a Number of the second Quantity, prick every second Figure, and leave one unprickt. The Cube prick every third Figure, and leave two unprickt, and so accordingly upward. And let it be noted that so many Pricks as the Number will admit of, so many Figures must be in the Quotient for the Root to consist of. 2. *To prick the Number according to the Quantity.*

§. 2. In Particular, the Square Root of a Number is extracted commonly thus: *To Extract the Square Root.* *Rules.*

1. When the Number is placed with a Crooked or Rectangle Line to separate the Quotient or Root from the Square, and pricked as abovesaid, then seek the greatest Square Number contained in the Figure or Figures that belong to the last Prick to the Left Hand, which Square Number set thereunder, and substract therefrom, and the Root of that Square set in the Quotient, cancelling the Figure or Figures standing over this Square, and if there be Remainders set them in order at top, and so is the work for that Prick ended. 1.

2. Double this Root, and set him down for a Divisor, thus; if he be a Digit, place him under the next unprickt Figure inclining to the Right Hand; and if an Article or Mixt Number, then set the first Figure or Cypher of this doubled Root in the unprickt place as before, and the other Figure one place nearer to the Left Hand, and divise with this Number for a new Quotient Figure, which must be no greater than that the Square of the same Figure may be also substracted from the remaining Numbers left to the next Right Hand Prick after you have, as in common Division, substracted the Product of your Divisor multiplyed into this Quotient Figure, still cancelling the Figures from which any thing is taken, and setting down the Remainer if any be. 2.

3. Then set down the Square of this Quotient Figure, the first Figure thereof, if more than one, under the next Prick inclining to the Right Hand, and the other Figure one place nearer to the Left Hand, and substract it, cancelling the Figures after Substraction, and setting the Remainers, if any, at top as before. 3.

4. If your Number admit of more Pricks than 2, then for every Prick exceeding 2 must you repeat the work in the second and third Directions, to double the Quotient and divide thereby, and substract the Square of each Figure in their order. 4.

For further Explanation; suppose I would prove whether 46808 be the Square Root of 2190988864, then having placed and pricked the Number, I find 21 to belong to the last or Left Hand Prick, I therefore inquire the Greatest Square in 21, and it is 16, whose Root is 4, which placed in the Quotient, and substracting 16 from 21, there rests 5 at top, as at *A*. *Example.*

Then I double the Root 4, and it is 8, which placed under the unprickt Figure, and divising thereby, I can take but 6 in the Quotient, or else I shall not leave enough to the next to make Substraction of his Square; for should I take 7, I should leave but 3 to the 0, which would be but 30, and the Square of 7 is 49; therefore I first place 6 in the Quotient, and take 6 times 8, which is 48, out of 59, there rests 11, as at *B*.

Then do I square 6, and it is 36, which I set under 110, the Numbers belonging to the Prick, and substract it, as at *C*.

Then do I double the whole Quotient 46, and the summe 92 is the next Divisor to be placed under 749, and divising thereby find the Divisor may be taken 8 times from 749, and yet leave enough to take the Square of 8, from the remaining Figures, the next pricked Figure being joyned thereto, I therefore set 8 in the Quotient, and substracting the Divisor 8 times, there resteth 13; and the Square of 8, which is 64, substracted leaves 74, uncancelled as at *D*.

Then I again repeat this last work, and double the whole Root 468, and place the summe 936, in order beginning under the next unprickt Figure, and finding the Divisor bigger than the uppermost Number, I set a Cypher in the Quotient, and also under the next Prick to the Right Hand; for the Square of 0 is 0, and cancel the Divisor, as at *E*.

Lastly, Doubling the whole Quotient 4680, the next Divisor will be 9360, thereby 8, gotten for the Quotient, and the Divisor substracted 8 times, and afterward the Square of 8, as before, there will be nothing left remaining, as at *F.*; whereby 2190988864, is seen to be a Square Number, and hath 46808 for his Root. And thus is the production of the Square in the former *Chapter* proved true. And reciprocally the truth of Extraction thereby.

```
              5                                    1
                                                   51
   A.    2190988864 | 4              B.    2190988864 | 46
         16                                168

         17                                1717
         524                               52434
   C.    2190988864 | 46             D.    2190988864 | 468
         1686                              168624
          3                                 396

         1717                              1717
         52434                             52434
   E.    2190988864 | 4680           F.    2190988864 | 46808 Root
         168624600                         168624600004  46808
          3963                              3963366     ------
           9                                  99        374464
                                                       3744640
                                                      280848
                                                     187232
                                                    ----------
                                         Square     2190988864 Proof
```

How to work and prove it by Addition.

Example.

Gnomon *what.*

Extraction of the Square Root may be proved by Addition: If the substracted Numbers be orderly placed one under another, and then added, the Total will return the Square whose Root was extracted. As in the former Example, after the second Figure is placed in the Quotient, set him under the Divisor, and multiply the Divisor thereby, and the amounting Product joyned with the Square of the Quotient Figure, makes the second Number to be substracted, called a *Gnomon*; and so continue this work according to the Number of Pricks on the Great Square, and then add all the Gnomons together with the Left Hand Square first substracted.

5 74 74

Zensus	21	90	98	88	64	46808 Radix
	16	8				
		8				
	4	8				
		36				
Gnomon	5	16				
		9	2			Divisor.
			8			
		73	6			
			64			
Gnomon		74	24			
			93	6		Divisor.
				0		
			00	0		
				0		
Gnomon			00	00		
			9	36	0	Divisor.
					8	
			74	8	0	
					64	
Gnomon			74	88	64	

Proof.

```
16
 516
  7424
    0000
    748864
----------
2190988864
```

Some

Some omit to place the Divisors underneath, and others multiply the Root by 20, and take the Product for the Divisor; but set the Cypher under the next Right Hand Prick, and by the new Quotient Figure multiply this Divisor, to which the Square added shall make the Total the Gnomon to be substracted, and then multiply again by 20, this whole Quotient, to get a new Divisor, if there be more Pricks, reiterating this work till all be finished; as in the former instance. Again thus; *Variety of work.*

Example.

```
            57474
            . . . . .
            2190988864 | 46808        4    4    46    468    4680
           ____________               4   20    20     20      20
            16                       __   __   ___   ____   _____
             516                     16   80   920   9360   93600
              7424                   __    6     8      0       8
                 0000                     __   ___   ____   _____
                 748864                  480  7360   0000  748800
           ____________                   36    64      0      64
Collection 2190988864 Proof               __   ___   ____   _____
           ____________                  516  7424   0000  748864
                                          __   ___   ____   _____
```

§. 3. The *Cube Root* is extracted with some variety, but the most received way is thus. *To extract the Cube Root.*

Rules.

1. When the Number is placed and pricked as before directed, from the Figures belonging to the last Prick to the Left Hand, substract the Greatest Cube Number you can have out thereof, and cancelling the same Figures, if there be any Remains set them at top, and the Root of this Cube set in the Quotient, and so have you done with this Prick: For this work, as in the Square, is wrought but once. 1.

2. Triple the Root, and multiply this triple by the Root, the Number that ariseth shall be the Divisor, and set one place nearer to the Right Hand, by which inquire for a new Quotient Figure from the Figures standing there over, and when found out place in the Quotient; and as in Division of Integers, substracting the Product of your Divisor multiplyed by the new Quotient Figure, cancel the Figures from which any thing is substracted, and set the Remainders, if any be, at top. 2.

3. Square the last Quotient Figure, and multiply this Square by the triple of the former Quotient Figure, and place the Product one place nearer to the Right Hand, and substract it from the upper Figures, still cancelling and setting the Remains at top, as before. 3.

4. Under the next Right Hand Prick, set the Cube of the last Quotient Figure, and substract it likewise: For if this Cube, and the Number last above mentioned, cannot be substracted, the last Quotient Figure is too big, and a less must be taken. 4.

5. If there be more Pricks on the given Cube, reiterate the work in the second, third and fourth Directions. 5.

For *Example*: If I would know what is the Cube Root of 102503232, when I have placed and pricked the Number, I find 102 to belong to the Left Hand Prick, the Greatest Cube I can have from thence is 64, of which the *Cube Root* is 4, therefore I set 4 in the Quotient, and taking 64 from 102 leave 38 over the same, as at *G*. *Example.*

Then 4 tripled is 12, which multiplyed by 4 is 48 for a Divisor, which standing under 385, I see I may take him 6 times from thence, wherefore putting 6 in the Quotient, and substracting 48 the Divisor, 6 times, I cancel the Number 385, and set the Remainder over the same in order as at *H*.

Then I square 6, and it is 36, which multiplyed by 12, the triple of 4, makes 432, placed under 970 and substracted, leaves remaining 538, and then placing the Cube of 6, which is 216, under 5383, and substracting it, leave 5167, as at *I*.

Then because there is yet one Prick remaining, I square the Quotient 46, and it is 2116, which I triple, and it is 6348; and this I take for a new Divisor under 51672, and by it get 8 in the Quotient, and substracting this new Divisor 8 times, leave behind 888, as at *K*.

Then do I square 8, and it is 64, which multiplyed by the triple of 46, produceth 8832, this I substract from the next place to the Right Hand, as at *L*.

Lastly, the Cube of 8 substracted, cuts off all the Figures, whereby it appears, 468 is the *Cube Root* of 102503232, and the whole work stands as at *M*. And the truth of this Extraction is proved by the Production of the Cube according to the former *Chapter*, as this by Extraction reciprocally.

G

```
       38                               9
                                        387
       .  .  .                          .  .  .
G.   102503232 | 4              H.    102503232 | 46
     64                               648
                                      4

       51                              518
       936                             9368
       38787                           387878
       .  .  .                         .  .  .
I.   102503232 | 46             K.    102503232 | 468
     64826                            648268
       431                             4314
       42                              423
                                       6

       518                             518
       93685                           93685
      3878781                         3878781
       .  .  .                         .  .  .
L.   102503232 | 468            M     102503232 | 468
     6482682                          64826822    468 Root
      43143                            431431
      4238                             42385     3744
      68                               68       2808
                                               1872

                                               219024 Square
                                                  468

                                               1752192
                                              1314144
                                              876096

                                        Proof  102503232 Cube
```

How to work and prove it by Addition. Cubical Extraction may alſo be proved by Addition : If when you have gotten the ſecond Quotient Figure, you place him under the Diviſor, and multiply the Diviſor thereby, and under this Product, one place nearer to the Right Hand, the Product of the Multiplyer ſquared and multiplyed by the tripled Root, and under this one degree nearer to the Right Hand, the Cube of the Multiplyer, and ſubſtract all theſe Numbers added into one Total Gnomon from the given Cube, cancelling the Figures you make Subſtraction from, and if you pleaſe thoſe underneath, except the Gnomon. And ſo continue this work till all the Pricks be done with, then adding all the uncancelled Gnomons with the firſt ſubſtracted Cube orderly placed, and you will have the Cubical Number returned ; as appeareth by the former Example wrought and proved this way.

Example.

```
              5
             28 167
           ---------------
Cubus     | 102 | 505 | 232 |  468 Radix
          |  64 |     |     |
          |   4 | 8   |     |  Diviſor
          |     | 6   |     |
          |  28 | 8   |     |
          |   4 | 32  |     |
          |     | 216 |     |
Gnomon    |  33 | 336 |     |
          |     | 634 | 5   |  Diviſor
          |     |     | 3   |
          |   5 | 078 | 4   |
          |     | 8   | 42  |
          |     |     | 512 |
Gnomon    |   5 | 167 | 23  |

                    Proof.

                    64
                    33336
                     5167232
                    ---------
                    102503232
```

Some

Variety of work.

Some after pricking the Number, and substracting the Greatest Cube out of the last Prick, and getting the Divisor, and thereby a second Figure in the Quotient as before; by this Quotient Figure multiply the Divisor, and substract the Product, then triple the first Quotient Figure, and to the Right Hand of the triple set the last Quotient Figure, and the Product of this Number multiplyed by the Square of the last Quotient Figure, place under the given Number, so that the Right Hand Figure thereof may stand under the next Prick to the same Hand, and substract. And so reiterate this manner of work till all be finished.

Example.

As in the former Number, after 64, as before, is substracted, the Divisor 48 I multiply by 6, and the Product 288 substract from 385, then I triple 4 the first Quotient Figure, and by the 12 amounting, I set 6 the second Quotient Figure; this 126 I multiply by 36, the Square of 6, and the Product 4536 I withdraw from 9703, and leave at top 5167: Then getting the new Divisor, as before, 6348, and multiplying it by the new Quotient 8, there is produced 50784, after Substraction of which to the triple of 46, which is 138; I adjoyn 8, and increase this 1388 by 64, the Square of 8, and the Product 88832, abated from the given Cube, leaveth nothing as before.

```
              5
             9188
            387078                                          Proof.
            .  .  .
Cubus     102503232 | 468 Radix
           64  :  :                 126      1388          64
           48  :  :                  36        64          288
            6  :  :                ----      ----           4536
          ---  :  :                 756      5552           50784
Gnomon {   288 :  :                378       8328              88832
       {   4536   :                ----      ----          ---------
          ------- :                4536     88832          102503232
            6348  :                ----      ----          ---------
               8  :
          ------- :
Gnomon {   50784  :
       {    88832
          ---------
```

Other variety.

Others after the Greatest Cube out of the Left Hand Prick is substracted, and by the Divisor a second Figure placed in the Quotient, as before, triple the whole Quotient, and multiply that triple by the first Quotient Figure, and again the Product by the latter Quotient Figure, and to that subjoyn the Cube of the said latter Figure one place nearer to the Right Hand, and then deduct the Total out of the given Cube, repeating the like work for every Prick.

Example.

As in the former Example thus: After 6 is found for the second Figure of the Quotient, I triple 46, and it's 138, which multiplyed by 4, gives 552, that again by 6 produceth 3312, then 216, the Cube of 6, subjoyned makes 33336, for the Gnomon to be substracted from 38503, so rests 5167 to that Prick; this reiterated for the next leaves 0, as before.

```
            5                        46        468       Proof.
           38167                      3          3
           .  .  .                  ----      -----
Cubus    102503232 | 468 Radix       138       1404       64
          64  :  :                     4         46       33336
Divisor   48  :  :                  ----      -----        5167232
         ---- :  :                   552       8424      ---------
Gnomon   33336   :                     6      5616       102503232
         -----   :                  ----      -----      ---------
Divisor    6348  :                  3312      64584
         ------- :                   216          8
Gnomon   5167232                    ----     ------
         -------                    33336    516672
                                    -----       512
                                             -------
                                             5167232
                                             -------
```

Tap his way of work.

Tap in his *Seamans Kalendar* implyes 300 and 30 in the work; thus: After the Number is pricked, and the Cube of the Left Hand Prick substracted as before, the Square of the Root found multiplyed into 300, shall be the Divisor, to be placed so as the Right Hand Cypher thereof shall stand under the next Prick, and a new Quotient Figure gotten thereby, multiply the Divisor, and then multiply the first Quotient Figure by 30, and that Product by the Square of the second Quotient Figure, and add these, with the Cube of this last Quotient Figure, into one total Gnomon; and for every Prick do the like.

Example. As in the former Number, first 64, the Cube of 4, substracted from 102, I set down 300, and 30, and against 300, to the Left Hand the Square of 4, which is 16, and to the Left Hand of 30, the Root it self 4, then multiplying 300 by 16, the Product 4800 is the first Divisor, by which 6 gotten in the Quotient, I place it on the Right Hand of 300, and the Square of 6 on the same Hand of 30, and then multiplying all the Numbers in the upper row, I have 28800, and the Product of those in the lower row is 4320, which added with 216, the Cube of 6, make the Gnomon 33336; and so proceed to deal with the Numbers remaining to the other Prick, as here appeareth.

	5						
	38167						
	· · ·						
Cubus	102503232	468 Radix	16 . 300 : 6		2116 . 300 . 8		
	64 : :		4 . 30 . 36		46 . 30 . 64		
Divisor	4800 :	300	30		2116	46	
	:	16	4		300	30	
Gnomon	33336 :			28800			5078400
	:	1800	120	4320	634800	1380	88320
Divisor	634800	300	36	216	8	64	512
Gnomon	5167232	4800	720	33336	5078400	5520	5167232
		6	360			8280	
Proof	102503232						
		28800	4320			88320	

Adding the uncancelled Gnomons with the Cube first substracted, as they stand will return the Great Cube, and prove the work true in this, as the other varieties.

To extract the Root of Compounds. §. 4. Of the Prime or Original Figurate Rooted Numbers, a *Square* and a *Cube*: Several Higher Quantities are Compound, as their Names *Arithmetical* denote. And perhaps this may be the reason why they retain Names different from their *Geometrical* Forms, that it might be an easie memento to the speedy Extraction of their Roots: For according to the Composition so shall you draw the Root from thence, whether Zenzick or Cubick, and so often as the Name is found in the Composition, in their order beginning at the Left Hand.

Compounds of 3 sorts. These two Names *Square* and *Cube*, or *Zenzick* and *Cubick*, make three sorts of Nominal Compounds, *viz.*

Either Squares with Squares, as Zenzizenzikes, Zenzizenzizenzikes, *&c.*

Or 2. Cubes with Cubes, as Cubicubicks, Cubicubicubicks, *&c.*

Or 3. Squares with Cubes, as Zenzicubes, Zenzizenzicubes, Zenzicubicubes, *&c.* Examples of each sort follow.

Extraction of the first sort. A Zenzizenzike is the least and first Number of the first Composition, in which, because the Zenzick is twice repeated, I extract the Square Root of the given Number, and from that Root (which also will be a Square Number) I extract the Square Root again; and so is this last Root, the Zenzizenzike, or Squared Square Root of the first given Number.

1 Example. As if I would extract the Zenzizenzike Root of 796594176, I first extract the Square Root, which is 28224, and from this Square Number extract the Square Root again, and so have 168, which I accept for the Squared Square Root desired.

	11225	3	
	372335671	1666	
	· · · · ·	· · ·	
Zenzizenzike	796594176	28224	168 Radix
	444644446	12624	
	6556641	336	
	5		

2 Example. And so for the Zenzizenzizenzikes, or of greater Quantity, it is but to reiterate the Zenzick Extraction. As to know the Zenzizenzizenzike Root of 16983563041, I first extract the Square Root, which is 130321, and then the Square Root of that, which is 361, and also the Square Root thereof, and so take 19 for the Root desired.

```
                      260
                     54726        447     28
                    . . . . . .   . . .   . .
Zenzizenzizenzike  16983563041 | 130321 | 361 | 19 Radix
                   12960096441   966221   121
                     2266006       37      8
                       226
```

Numbers of the ſecond Compoſition have the Cubick Root extracted in like manner. As to know the Cubicubike Root of 10604499373, the firſt Extraction gives the Cubick Root 2197, which is alſo a Cube Number, and his Root extracted is 13, the Cubicubike Root of the given Number. *Extraction of the ſecond ſort.* *Example.*

```
            10132
          13520423
          244376024    122
          . . . .      . .
Cubicube 10604499373 | 2197 | 13 Radix
          8261339333   1377
          1132202894    22
            527813
             1432
               3
```

The like is to be done for Numbers of greater Quantity under this Compoſition; as Cubes of Cubick Cubes to extract the Cube Root 3 times, *&c.*

When the Number is of the third Compoſition, as in Zenzicubes, Zenzizenzicubes, Zenzicubicubes, *&c.* ſo often extract the Zenzick Root as that Name is in the Compoſition, and ſo likewiſe the Cubick Root, and in ſuch order as they ſtand compounded. *Extraction of the third ſort.*

For in a Zenzicube, firſt extract the Square Root, this Root ſhall be a Cube, whoſe Cube Root extracted ſhall be the Zenzicube Root of the given Number. As 729 is a Zenzicube, whoſe Square Root is 27, which is a Cube Number, and hath 3 for his Root. *1 Example.*

In a Zenzizenzicube, extract the Square Root twice, and the Cube once at laſt. As in 4096, whoſe Square Root firſt extracted is 64, the Square Root of which is 8, and 8 is the Cube of 2; ſo 2 is the Radix Zenzizenzicubick of 4096. *2 Example.*

But in a Zenzicubicubick extract the Square Root once at firſt, and then the Cube Root twice. As in 262144, the Square Root is 512, the Cube Root whereof is 8, which is alſo a Cube Number, and hath 2 for his Root. *3 Example.*

All theſe Extractions may be proved by common Production, and if the Numbers ſubſtracted in the Work be collected into Gnomons, by Addition, as aforeſaid. *Proof.*

§. 5. Becauſe the Surſolides are excluded out of the foregoing Compoſitions, whoſe Roots nevertheleſs it is requiſite to know how to extract, when occaſion ſhall require; and becauſe it would be tireſome to ſet down particular Rules for every Quantity of the Higher Powers, and troubleſome to the Memory to retain them, with the varieties of work, ſeeing as a Figural Number may be produced divers wayes, his Root may be many wayes extracted. And beſides the particular Rules for each increaſing according to their Quantities, and ſo in effect are as endleſs and various as the Numbers themſelves, it will be more commodious to perform the Extraction of the Roots of all the Higher Powers by ſome one General Direction, which by the help of the *Table* following is perfectly to be done. *To extract the* Roots *of the Higher Powers by the* Table *following.*

And indeed all the Particular Rules that are given for Extraction of Roots, even thoſe of the Square and Cube, have their ground in the *Table*, and come from thence; as might very eaſily be demonſtrated. And by comparing the Tabulary Numbers and Operation therewith, hereafter in this *Section* ſet forth, with the Particular Rules here ſet down for the Zenzizenzike, Surſolide, and Zenzicube, will be ſufficiently clear without further illuſtration. *Particular Rules grounded on the* Table.

Particular Rules for the Zenzizenzike.

Particular Rules for the Squared Square.

1. After the Number is pricked according to his Quantity from the Left Hand Prick, ſubſtract the greateſt Zenzizenzike Number, and ſet the Zenzizenzike Root thereof in the Quotient. 1.

2. Multiply

2. 2. Multiply this Root Cubically, and quadruple the Product for a Divisor to be placed one place nearer to the Right Hand, and a new Quotient Figure gotten thereby, Multiply the Divisor, and reserve this Product to be added with the 3 Numbers in the next Directions to make up the Gnomon.

3. 3. Square the first Quotient Figure, sexcuple the Square, and multiply the Product by the Square of the last Quotient Figure.

4. 4. Cube the last Quotient Figure, and Multiply the Cube by the quadruple of the first Quotient Figure.

5. 5. Take the Zenzizenzike Number of the last Quotient Figure, and with the 3 Numbers last above-mentioned, add them (duely placed one nearer than another to the Right Hand) into one total Gnomon, and substract the same from the given Number: And if the Number have more Pricks than 2, the work in the second, third, fourth, and fifth Directions is to be repeated.

Example. Thus the former Number 796594176 pricked, and 1 the greatest Squared Square Number in 7 substracted, I multiply the Cube of 1, which is 1 by 4, and 4 is Divisor; which though standing under 69, yet will afford but 6 for the Quotient. This 6 multiplyed into 4, gives 24 Then 1 squared is 1, and multiplyed by 6 is 6, and again by 36, the Square of the last Quotient Figure, makes 216. And then 216, the Cube of 6, by 4 the quadruple of 1, the Product is 864. Lastly, 1296, the Zenzizenzick of 6, added with 24, 216, 864, makes the Gnomon for the second Prick.

Then the Cube of 16 quadrupled is 16384, the next Divisor, whereby 8 is gotten for a new Quotient Figure, which multiplying the Divisor gives 131072 Then the Square of 16, sexcupled and multiplyed by the Square of 8, makes 98304. And the Cube of 8, by the quadruple of 16, produceth 32768. And the Zenzizenzike of 8 is 4096, added to the other Numbers make the next Gnomon, which substracted as the former leave 0 remaining.

	~~1~~	
	~~64123~~	
Zenzizenzike	~~796594176~~	168 Radix
	1	
Gnomons	55536	
	141234176	
Proof	796594176	

$$\left.\begin{array}{r} 1 \\ \hline 24 \\ 216 \\ 864 \\ 1296 \end{array}\right\} 55536$$

$$\left.\begin{array}{r} 131072 \\ 98304 \\ 32768 \\ 4096 \end{array}\right\} 141234176$$

$$\begin{array}{r} \hline 796594179 \end{array}$$

Particular Rules for the Sursolide.

Particular Rules for the Sursolide.

1. 1. The Number being pricked according to his Quantity, take from the Figures belonging to the Left Hand Prick, the greatest Sursolide Number therein, and place the Sursolide Root thereof in the Quotient.

2. 2. Multiply the Zenzizenzike Number of the Root by 5, (or quintuple it) and the Product shall be the Divisor to be set one place nearer to the Right Hand, by which get a new Quotient Figure, and Multiply the Divisor thereby, and reserve the Product for the first Number of the Gnomon.

3. 3. Cube the first Quotient Figure, decuple the Cube (or Multiply it by 10) and Multiply the Product by the Square of the last Quotient Figure.

4. 4. Square the first Quotient Figure, decuple the Square, and Multiply the Product by the Cube of the second Quotient Figure.

5. 5. Quintuple the first Quotient Figure, and Multiply the Product by the Zenzizenzike of the next Quotient Figure in the Root.

6. 6. Add these 4 last mentioned Numbers with the Sursolide of the last Quotient Figure, orderly placed every one nearer by one place to the Right Hand than the other, into one Gnomon, and substract the summe from the given Number. And for every other Prick on the given Number, go over again with the work in these 5 last Directions.

Example. As to extract the Sursolide Root of 28153056843, which marked as directed, it doth appear that 1 is the greatest Sursolide, and Root also of 2. Then the Zenzizenzike of 1 is 1, which quintupled is 5, this 5 is Divisor to 18, by which I get but 2 for the Quotient, this multiplyed into 2 makes 10. Then the Cube of 1 is but 1,

which decupled is 10, and multiplyed by 4 is 40. And the Square of 1 decupled is 10, and multiplyed by 8, the Cube of 2, gives 80. And the quintuple of 1 is 5, which multiplyed by 16, the Zenzizenzike of 2, yields 80. Lastly, 32, the Sursolide of 2, added with 10, 40, 80, and 80, in their order make up the Gnomon 148832, for the second Prick. And by reiterating the work for the next, get the Gnomon 3269856843.

	~~1~~~~3~~~~2~~~~6~~~~9~~8	
Sursolide	28153056843	168 Radix
	1 : :	
Gnomons	148832 :	
	3269856843	
Proof	28153056843	

1
10
40
80
80
32 } 148832

311040
155520
38880
4860
243 } 3269856843

28153056843

Particular Rules for the Zenzicube.

Particular Rules for the Squared *Cube.*

1. The Number pricked according to his Quantity, deduct the greatest Zenzicube Number from the Left Hand Prick, and put the Zenzicube Root thereof in the Quotient. 1.
2. Sexcuple the Sursolide Number of the Root, this Product shall be Divisor, by which get another Figure for the Quotient, and multiply the Divisor thereby, reserving this Number for the Gnomon. 2.
3. Multiply the Zenzizenzike Number of the first Figure of the Root by 15, and the Product again by the Square of the next Figure of the Root. 3.
4. Multiply the Cube of the first Quotient Figure by 20, and the Product again by the Cube of the next Quotient Figure. 4.
5. Multiply the Square of the first by 15, and that again by the Zenzizenzike of the second Quotient Figure 5.
6. Multiply the first Figure of the Root by 6, and the Product by the Sursolide of the second Figure of the Root. 6.
7. Let all these five last mentioned Numbers be added with the Zenzicube of the second Quotient Figure, orderly placed one nearer to the Right Hand than the other, into one Gnomon, and substract the same from the given Zenzicube; and repeat the work of these six last Directions for every other Prick. 7.

Example. As in extracting the Zenzicube Root of 244140625; first out of 244, I take the greatest Zenzicube I can, which is 64, whose Root is 2, the Sursolide of which 32 sexcupled is 192, for Divisor, by which 5 is gotten for the Quotient; this 192 multiplyed by 5 is 960. And 16, Zenzizenzike of 2 by 15 is 240, and again by 25, the Square of 5 is 6000. And 8, the Cube of 2, by 20 is 160, which by 125, the Cube of 5, is 20000. And 4, Zenzike of 2, by 15 is 60, which by 625, the Zenzizenzike of 5, is 37500. And 2, the Root by 6 is 12, that by 3125, the Sursolide of 5, is 37500. Lastly, 15625, the Zenzicube of 5, makes up the Gnomon.

	~~1~~8~~0~~	
Zenzicube	~~244140625~~	25 Radix
	64 :	
Gnomon	180140625	
Proof	244140625	

64
960
6000
20000
37500
37500
15625 } 180140625

244140625

A Table *for* Extraction of Roots.

The Table for Extraction of Roots.

Zenzizenzicube.	12	66	220	495	792	924	792	495	220	66	12
Third Surfolide.	11	55	165	330	462	462	330	165	55	11	
Square Surfolide.	10	45	120	210	252	210	120	45	10		
Cubicube.	9	36	84	126	126	84	36	9			
Zenzizenzizenzike.	8	28	56	70	56	28	8				
Second Surfolide.	7	21	35	35	21	7					
Zenzicube.	6	15	20	15	6						
Surfolide.	5	10	10	5							
Zenzizenzike.	4	6	4								
Cube.	3	3									
Squa.	2										
Root.	1										

The Table *explained.*

The Table Explained.

The little Iſſuants at Top denote the *Table* may be increaſed as occaſion requires, though this reaching to Zenzizenzicubes be large enough for Example.

On the exterior parts on either ſide aſcending from the Root, are Numbers declaring the Quantities of the Powers whoſe Names are placed diſtinctly on the Head of each Quantity.

The Numbers in the Body of the *Table* are thus gotten, after the Numbers of the Quantities are placed in the exterior parts, add the two Numbers belonging to the Cube together, which being 3 and 3, they make 6, to fill up the middle Square belonging to the Zenzizenzike, between the 4 and 4, ſignifying his Quantity which 6 and 4 added make 10, for the two middle Squares of the Surſolide ; and this 10 added to 5 is 15, for the Squares of the Zenzicube next the middle, and for that 10 to 10 gives 20 : And thus adding one Number with another alternately, the other Numbers are found ; and the *Table* may be enlarged *ad infinitum.*

Uſe of the Table.

The Uſe of the Table.

To extract the Root of any Number by help of the *Table* ; after the Number is pricked according to his Quantity as before taught, and the Greater Number of that Quantity whoſe Root you would extract, ſubſtracted out of the Figures belonging to the Left Hand Prick, and the Root thereof ſet in the Quotient, as before ; then ſet apart this Root with his Square, Cube, Zenzizenzike, *&c.* until you come to the Number before ſubſtracted. And in order under them, beginning at the Root, place the Numbers found in the *Table* belonging to the Quantity whoſe Root you are extracting. Then Multiply the Numbers ſtanding one over another, one into another, the Multiplication next the Right Hand ſhall be the Diviſor, which after you have gotten another Figure of the Root by to be ſet in the Quotient, ſet him down under the Diviſor, and his Square under the next Multiplication to the Left Hand, and his Cube under

under the next, and so proceed to the Quantity you are extracting, and so will every of the former multiplyed Numbers have another Number under them, and one Number over; then multiply the one by the other, and add all these Products with the Left Hand Quantity together, in such order that every Number be placed one place nearer to the Right Hand than the other; and the Total of these Products make up the Gnomon for the next Right Hand Prick to be substracted, as before: And so for every other Prick on the given Number, repeat this work.

1. *Example.* To extract the Square Root of 28224. *Example of the* Square.

Zenzike 28224 | 168
Gnomons { 156, 2624
Proof 28224

Rad.	Zen.		Rad.	Zen.	
1	1		16	&c.	
2			2		Tabulary Number
2		First Divisor.	32		Second Divisor.

Zen.	Rad.		Zen.	Rad.	
36	6		64	8	
36	12		64	256	

12
36
156 Gnomon.

256
64
2624 Gnomon

2. *Example.* To extract the Cube Root of 4741632. *Example of the* Cube.

Cubus 4741632 | 168
Gnomons { 3096, 645632
Proof 4741632

Rad.	Zen.	Cub.		Rad.	Zen.	Cub.	
1	1	1		16	256	&c.	
3	3			3	3		Tabulary Numbers.
3	3		First Divisor	48	768		Second Divisor.

Cub.	Zen.	Rad.		Cub.	Zen.	Rad.
216	36	6		512	64	8
216	108	18		512	3072	6144

108
216
3096 Gnomon

3072
512
645632 Gnomon

3. *Example.* To extract the Zenzizenzike Root of 796594176. *Example of the* Squared Square.

Zenzizenzike 796594176 | 168 Radix
Gnomons { 55536, 141234176
Proof 796594176

Rad.	Zen.	Cub.	Zenz.		Rad.	Zen.	Cub.	Zenz.	
1	1	1	1		16	256	4096	&c.	
4	6	4			4	6	4		Tabulary Numbers.
4	6	4		First Divisor.	64	1536	16384		Second Divisor

Zenz.	Cub.	Zen.	Rad.		Zenz.	Cub.	Zen.	Rad.
1296	216	36	6		4096	512	64	8
1299	864	216	24		4096	32768	98304	131072

216
864
1296
55536 Gnomon.

98304
32768
4096
141234176 Gnomon.

4. Example.

Example of the Surfolide.

4. *Example.* To extract the Surfolide Root of 28153056843.

Surfolide 28153056843 | 123 Radix

1

Gnomons { 148832
3269856843

Proof 28153056843

Rad.	Zen.	Cub.	Zenz.	Sur.	
1	1	1	1	1	
5	10	10	5		
5	10	10	5		First Divisor.

Rad.	Zen.	Cub.	Zenz.	Sur.
12	144	1728	20736	*&c.*
5	10	10	5	Tabulary Numbers
60	1440	17280	103680	Second Divisor

Sur.	Zenz.	Cub.	Zen.	Rad.
32	16	8	4	2
32	80	80	40	10

40
80
80
32

Gnomon 148832

Sur.	Zenz.	Cub.	Zen.	Rad.
243	81	27	9	3
243	4860	38880	155520	311040

155520
38880
4860
243

Gnomon 3269856843

Example of the Squared Cube.

5. *Example.* To extract the Zenzicube Root of 164170508913216.

Zenzicube 164170508913216 | 234 Radix

64

Gnomons { 84035889
16134619913216

Proof 164170508913216

Rad.	Zen.	Cub.	Zenz.	Sur.	Zenzic.
2	4	8	16	32	64
6	15	20	15	6	Tabulary Numbers
12	60	160	240	192	First Divisor

Zenzic.	Sur.	Zenz.	Cub.	Zen.	Rad.
729	243	81	27	9	3
729	2916	4860	4320	2160	576

2160
4320
4860
2916
729

84035889 Gnomon

Rad.	Zen.	Cub.	Zenz.	Sur.	Zenzic.
23	529	12167	279841	6436343	*&c.*
6	15	20	15	6	Tabulary Numbers
138	7935	243340	4197615	38618058	Second Divisor

Zenzic.	Sur.	Zenz.	Cub.	Zen.	Rad.
4096	1024	256	64	16	4
4096	141312	2031360	15573760	67161840	154472232

67161840
15573760
2031360
141312
4096

16134619913216 Gnomon

§. 6.

§. 6. A *Surde Number*, ſometime called *Irrational*, is as much as to ſay a Number from which it is not poſſible to take the Root, but there will remain ſomething, which declares that the Number given was not a perfect Figural Number, and the ſide thereof cannot therefore be expreſſed by an Integer. As the Square Root of 12 is 3, and there will remain 3; ſo the Cube Root of 28 is 3, and 1 will remain. *Surdes what.*

The Extraction of the Roots of *Surde Numbers* differs nothing from the wayes already ſet forth, and may be proved by the common way of Production, or Addition of the Gnomons, adding in the Remain to the Product or Total. But all the difficulty is to denominate the Remain, to know what part of the whole is ſignified thereby, ſeeing the Diviſors are alwayes uncertain. Some Authors for the Square double the Root; others add 1 to the double for the Denominator; and others double the Remainer for the Numerator, and to the quadruple of the Root add 1 for the Denominator. And for the Cube, triple the Square of the Root, and add thereto the triple of the Root, and 1 more for the Denominator; yet all theſe, and ſeveral other Rules, fail to find out a true Denominator exactly, but that which comes neareſt the truth is thus. *To get the greateſt Root therein.*

Adjoyn Cyphers to the Right Hand of the given Number according to his Quantity. As if a Square, as many times two Cyphers, as you pleaſe: If a Cube, as many times three Cyphers, as you pleaſe, *&c.* and continue the Extraction of the Root to the end of the Cyphers, and the more Cyphers are adjoyned the nearer the true Root you come. Then divide the Quotient by an Unit, and half the Numbers of Cyphers you added if it were a Square, and the third part if a Cube, the fourth part if a Squared Square, *&c.* *The neareſt Denominator to the Remain.*

1. *Example*, in a Square Surde. Suppoſe a Square Plot of Ground were 18 Perches, and I would know the Side thereof: If then I add 2 Cyphers, and extract the Root, the Quotient is 42, which divided by 10, the Root is $4\frac{1}{5}$. And if I add 4 Cyphers, and divide the Quotient 424 by 100, the Root is $4\frac{6}{25}$. But if I add 6 Cyphers, the Quotient will be brought nearer the truth, and be $4\frac{121}{500}$ beſides the ſmall Fractions ſtill left upon the Extractions, which denominate after the other wayes uſed in Authors, and added will ſomewhat increaſe the Number. *Example in a Square Plot of Ground.*

2 | 36	12		Proof of both.
Zen. 1800 | $\frac{4\|2}{10}$ Radix $4\frac{1}{5}$ Perches	2|36|24		42 424
168	1800 00 | $\frac{4\|24}{100}$ Radix $4\frac{6}{25}$		42 424
164	168:		84 1696
	154		168 848
	84	Remain	36 1696
	3376		224 added
			1800 180000

2. *Example*, in a Surde Cube. It is noted of the *Greeks*, that through their great Luxurioufneſs and Riot, they had brought Contagious Diſeaſes upon them, and conſulting their Oracles for redreſs thereof, received anſwer, *That when they would double their Altar* (which was of a Cubick Form) *they ſhould be delivered from thoſe Plagues*; meaning, the beſt Method to deliver Realms from ſuch contagion breeding Vices, was to abate of their Voluptuouſneſs, and apply themſelves to Literature. But now ſuppoſe the Altar was 4 Feet Square every way, and the Altar were doubled, what muſt the Side be? Here if I double the Cube of 4, which is 64, it is 128, from which becauſe I cannot extract the Cube Root without leaving a Remain, I aſſay to come near the Truth, and adjoyn 6 Cyphers, and extracting the Root yet 503 in the Quotient, and 736473 remaineth, then dividing 503 by 1, and 2 Cyphers, which are the third part of 6, I have 5 Feet, and $\frac{3}{100}$ of a Foot for the Root or Side, beſides the odd Remain, which denominated after the common way, will be $\frac{736473}{760537}$ of $\frac{1}{100}$ or $\frac{736473}{76053700}$. *Example in a doubled Cube.*

		3 | 736473	Radix		503
Root	4	128000000	503(5 $\frac{3}{100}$ Feet		503
	4	125 : :	100		
	——				1509
Square	16	75 : :			25150
	4	: :			
	——	0000 :			253009
Cube	64	:			503
	——	7500 :			
Double	128	:			759027
	——	2263527			12650450
				Remain	736473 added.
					128000000

CHAP. IV.

Figurate Fractions.

To Figurate Fractions, And Mixt Numbers.

WHen a Fraction is given to be multiplyed Figurally, Multiply the Numerator by himself into the Quantity desired, and the Denominator likewise.

If an Integer and a Fraction be given, reduce them into an Improper Fraction, and then Multiply Figurally the Numerator by himself, and also the Denominator into the Quantity desired.

Examples.

As { To Square $\frac{3}{4}$ is $\frac{9}{16}$, and $2\frac{3}{4}$ reduced is $\frac{11}{4}$ Squared is $\frac{121}{16}$.
To Cube $\frac{3}{4}$ is $\frac{27}{64}$, and $2\frac{3}{4}$ reduced is $\frac{11}{4}$ Cubed is $\frac{1331}{64}$.

To extract the Roots.

To extract the Root of a Fraction: First, extract the Root of the Numerator, and then the Root of the Denominator of the same Quantity.

Examples.

As to extract the { Square Root of $\frac{9}{16}$ renders $\frac{3}{4}$.
Cube Root of $\frac{27}{64}$ renders $\frac{3}{4}$.

No Roots *in* Heterogeneal Fractions.

But if the Numerator and Denominator be Heterogeneal, that is, not both of one Nature, though the one be a Square, and the other a Cube, as $\frac{16}{27}$, or the like; yet can the Fraction have no Root extracted, but must remain as a Surde broken Number; of which more hereafter in the next *Book*.

Partis Secundæ & Libri Secundi

FINIS.

ARITH-

ARITHMETICK.

The Third BOOK,

CONCERNING

Numbers ſpecially contract;

In Six PARTS.

WHEREIN

DECIMALS	are	Diſcovered.
ASTRONOMICALS	are	Anatomized.
LOGARITHMES	are	Overlooked.
COSSICKS	are	Characterized.
SURDES	are	Surveyed.
SPECIES	are	Inſpected.

AND THEIR

SIMPLE ELEMENTS.

CHAP. I.

Of DECIMALS.

IN the former *Books* enough hath been ſaid of *Abſtract* and *Generally Contract*, or *Vulgar denominate Numbers*, to make repetition here would be tautological. The next ſort of Numbers I ſhall therefore fall upon are *Numbers ſpecially Contract*, which are Numbers challenging to themſelves ſome ſpecial Operations, and reſtrained by ſome Denomination either implyed or expreſſed: Implyed if the Denominators be certain; expreſſed if otherwiſe. *Numbers ſpecially contract follow thoſe generally ſo.* *What Numbers ſpecially Contract are.*

Denominators are certain in *Decimals*, *Aſtronomicals* and *Logarithmes*, and therefore omitted: Denominations uncertain in *Coſſicks*, *Surdes* and *Species*, and therefore expreſſed. Of theſe in order, and firſt of *Decimals*. Denominators *where certain where uncertain.*

Decimals

Decimals their converse and practice.

Decimals converse very much with *Geodaeticals*, and practice to work both Integers and Fractions together in *Addition*, *Substraction*, *Multiplication* and *Division*, thereby facilitating many laborious and intricate works in Common Fractions.

Whence the Name, and what it signifies. Denominator of Decimals what, and how known.

They take their Name from the *Latin* word *Decimus*, signifying a Tenth, or Tenth part; because as the Integers above the Unit increase by Tens towards the Left Hand, so the Decimal Fractions below the Unit decrease by Tens towards the Right Hand; the Denominator of every Decimal Fraction being alwayes an Unit with Cyphers: As 10, 100, 1000, *&c.* that is to say, an Unit and so many Cyphers as there are Figures in the Numerator. And therefore because the Denominator doth alway consist of one place more than the Numerator, and is thereby certainly known, the Denominator is omitted, and the Numerator only used. For if the Numerator consist of 3 Figures or Places, the Denominator shall be 1000, which is 3 Cyphers and an Unit; and if the Numerator have 4 Figures, the Denominator shall be 10000, *&c.*

Several wayes of distinguishing the Decimal.

For distinguishing of the Decimal Fraction from the Integers, it may truly be said, *Quot Homines, tot Sententiae*; every one almost fancying severally. For some call the Tenth Parts, *Primes*; the Hundredth Parts, *Seconds*; the Thousandth Parts, *Thirds*, *&c.* and mark them with *Indices* equivalent over their heads. As to express 34 Integers and $\frac{1426}{10000}$ Parts of an Unit, they do it thus, $34.\overset{\prime}{1}.\overset{\prime\prime}{4}.\overset{\prime\prime\prime}{2}.\overset{\prime\prime\prime\prime}{6}$. Or thus, $34.\overset{(1)}{1}.\overset{(2)}{4}.\overset{(3)}{2}.\overset{(4)}{6}$. Others thus, $34,1426^{\prime\prime\prime\prime}$; or thus, $34,1426^{(4)}$. And some thus, 34. 1 . 4 . 2 . 6 . setting the Decimal Parts at little more than ordinary distance one from the other, and they read them thus, 34 Integers, 1 Prime, 4 Seconds, 2 Thirds, 6 Fourths: Or 34 Integers 1426 Fourths. Others distinguish the Integers from the Decimal Parts only by placing a Cōma before the Decimal Parts thus, 34,1426; a good way, and very useful. Others draw a Line under the Decimals thus, $34\,\underline{{}^{1426}}$, writing them in smaller Figures than the Integers. And others, though they use the Cōma in the work for the best way of distinguishing them, yet after the work is done they use a Rectangular Line after the place of the Unite, called *Seperatrix*, a separating Line, because it separates the Decimal Parts from the Integers, thus $34\lfloor\underline{1426}$. And sometimes the Cōma is inverted thus, 34‘1426, contrary to the true Cōma, and set at top. I sometime use the one, and sometime the other, as cometh to hand.

Cōma, a good way. Seperatrix what.

For further discovery of the orderly progression of *Decimals* behold the following *Tables*, one with *Figures* and *Letters*, and the other with *Letters*, *Figures* and *Cyphers*, sometime called *Numeration Tables*.

A Table for Numeration of Decimals.

Table with Letters *and* Figures.

			Integers.						Unit.				Decimals.					
9.	8.	7.	6.	5.	4.	3.	2.	1.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
M	M	M	M	M	M	M	C	X	V	X	C	M	M	M	M	M	M	M
M	M	M	M	C	X				The Units place.				X	C	M	M	M	M
M	C	X														X	C	M

Another.

Table with Letters, Figures *and* Cyphers.

			Integers.						Unit.				Decimals.					
I.	H.	G.	F.	E.	D.	C.	B.	A.	V.	a.	b.	c.	d.	e.	f.	g.	h.	i.
9.	8.	7.	6.	5.	4.	3.	2.	1.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
1000000000	100000000	10000000	1000000	100000	10000	1000	100	10	The Units place.	10	100	1000	10000	100000	1000000	10000000	100000000	1000000000

In

In these *Tables* (for both are as one) there are four Progressions, two *Arithmetical* in the Figures above, which are called *Indices*, proceeding both wayes from Unity by the difference 1, though contrarily; two *Geometrical*, in the lower continued Proportions from Unity both wayes, by the *Ratio* 10. So that as the Series of the Numbers from the Units place are continued in a decuple proportion from the Right Hand towards the Left, and increase in their value, as before was said; so their value decreaseth in a subdecuple proportion from Unity towards the Right Hand. *The Tables explained. Indices what.*

Example, by the latter *Table* suppose, 4. 3. 2. 1. 0. |1. 2. 3. 4. be a Number given, and stand with his Letters thus,

D. C. B. A. V. a. b. c. d.
4. 3. 2. 1. 0. |1. 2. 3. 4.

then shall *D.* be equal to 10 *C*, and *C.* equal to 10 *B*, and *B.* equal to 10 *A*, and *A.* equal to 10 *V*, and *V.* equal to 10 *a*, and *a.* equal to 10 *b*, *&c.*

Again, if *A.* be equal to 10 *V*, then shall *a.* be equal to $\frac{1}{10}$ of one *V*, and if *B.* be equal to 100 *V*, then shall *b.* be equal to $\frac{1}{100}$ of one *V*; and so if 1 Prime be $\frac{1}{10}$ of 1 Integer, every Second shall be $\frac{1}{100}$ Part, and every Third $\frac{1}{1000}$ Part, *&c.* of that Integer. And hence it is that *Decimals* are set down in a retrograde order to *Integers*: As if I were to set down Eight Hundred in *Integers*, it is thus 800, but in *Decimals* thus, |008.

The Unit, or one Integer, is alwayes understood to be divided into Parts, bearing denomination or name of the place of the last Figure to the Right Hand in the Decimal Fraction.

As 0,1 signifieth One Tenth Part, and 0,2 Two Tenth Parts, as if they were writ at large thus, $\frac{1}{10}$, $\frac{2}{10}$, the denomination of the place being Tenths, as the first place from the Unit towards the Right Hand, and noted as above with 10 in the *Table*. In like manner shall 0,12 signifie Twelve Hundred Parts, and 0,34 Thirty Four Hundred Parts, as if written at large thus, $\frac{12}{100}$, $\frac{34}{100}$, the denomination of the last place being noted with 100, under the last Figures, 2 and 4. The like is to be understood of others.

Note further, That Cyphers before Integers, or after Decimals, that is to the Left Hand of the one, and to the Right Hand of the other, signifie nothing at all; but after the Integers and before the Decimals, they are significant, for making up the places whereby the value of the other Figures are estimated. As 0001, signifieth but 1 in Integers; and ,1000, but $\frac{1}{10}$ in Decimals: Therefore in writing of Decimal Parts, mark well the Units place, by some one or other of the distinguishing Notes before mentioned, and let the void places (if any be) be filled up with Cyphers, and the place of the Unit set down, though there be no Integers. As to set down 3 Fourths, and 4 Fifths, thus 0,00034. *Cyphers before Integers or after Decimals signifie nothing. Units place to be well noted.*

To remove the *Seperatrix* one place nearer to the Right Hand increaseth the Number ten times in value, and toward the Left diminisheth it as much: For 0,125 shall be ten times less than 01,25. *Removing the Seperatrix what effect.*

The *Indices* are very considerable, because of singular use almost in all Contract Numbers, as well as Decimals, and serve here to find out the true value of the Products in Multiplication, and Quotients in Division, as hereafter may be seen. *Indices very considerable.*

The *Index* of the Unit is 0, as in the former *Tables*. And the *Index* of any other place may easily be known thus: *Indices how known.*

In Integers, abate one from the number of places from the Units place, and the Remain is the *Index*. As the *Index* of 8 in this Number 8921, is 3, of 9 is 2, of 2 is 1, and 0, the *Index* of 1, because he standeth in the Units place.

In Decimals, the just number of the places from Unity is the *Index*. As the *Index* of 8 in this Number 0,8921, is (1), and in this Number 0,1298 is (4), of 9 in the former is (2), in the latter (3), of 2 in the former (3), in the latter (2), of 1 in the former (4), in the latter (1).

These *Indices* are expressed by the Learned *Oughtred* in his *Clavis limata*, affirmatively and negatively; that is, of Integers affirmatively by the sign +, signifying more; and *How expressed by Mr. Oughtred.*

and of Decimals negatively, by the ſign —, ſignifying *leſs*. And where the ſign — is not ſet, the ſign + is underſtood, though not expreſſed. And ſo 34,1426 ſtands thus, $34,\overline{1426}$.

Signs of a Number either Negative or Affirmative.

When a Number is wholly negative or wanting, the ſign *leſs* ſhewing its defect, is ſet at the Left Hand, thus, —34,1426; but if affirmative, the Number is placed without the ſign *more*, as 34,0000, as well as with it thus, +34,0000. And though the Number be not wholly affirmative, but negative in the Decimals, yet it may be ſet without, thus, 34,1426, as well as +34,1426. And Decimals themſelves albeit wholly negative, yet not taken privatively may be marked or not, as 0,1426; or thus, +0,1426.

These Signs helpful.

How auxiliary the right underſtanding of theſe ſigns + and — is to the Simple Elements of Decimals and all Contract Numbers will be ſeen hereafter. It remains therefore, as neceſſary to this *Chapter*, that Decimals be counted Pure, when ſet without Integers. Mixt, when ſet with them. Simple, when affirmative or negative in their Signs. Compound, when their Signs are both affirmative and negative.

Decimals Pure or Mixt, Simple or Compound.

CHAP. II.

Reduction of Decimals.

Decimals reduced, how many wayes, and the uſe thereof.

REduction of *Decimals* ſerveth to reduce *Fractions* or *Geodæticals* into *Decimals*, or *Decimals* into them, that without impediment they may be wrought together, and Operation being ended their value may be found.

To turn Common Fractions into Decimals.

§. 1. To reduce *Common Fractions* into *Decimals*: Adjoyn to the Right Hand of the Numerator of the Fraction as many Cyphers as you pleaſe, and divide by the Denominator, the Quotient ſhall be the Decimal, and denominate according the Number of Cyphers adjoyned; that is to ſay, the Right Hand Figure or Cypher of the Quotient, as the Right Hand Cypher adjoyned to the Number before Diviſion, and ſo the other Figures or Cyphers in order, proceeding towards the Left Hand accordingly. So that if I adjoyn but one Cypher, then the *dexter* Figure of the Quotient ſhall be Primes, if 2 Cyphers, Seconds, if 3 Cyphers, Thirds, *&c.*

Example in a ſingle Fraction turned into a Perfect Decimal.

As to ſet $\frac{3}{4}$ in Decimals, I adjoyn to 3 the Numerator, 2 Cyphers, and divide by 4 the Denominator, and the Quotient is 75, which is 7 Primes, 5 Seconds, there being but 2 Cyphers adjoyned, therefore the Quotient can be but Seconds.

$$\frac{3}{4} \qquad \begin{array}{l} \quad\cancel{2} \quad (1)(2) \\ \cancel{3},\cancel{0}\cancel{0}\,(75 \\ \quad\cancel{4} \end{array}$$

When any Number to be reduced into a Decimal cannot be brought into a perfect Decimal, but there will be ſome Remain upon the Diviſion, then the more Cyphers are adjoyned the nearer the true Decimal you come, but ſeldom above 7 or 8 Cyphers need to be uſed; for that the Quotients of ſuch Diviſions come ſo near the truth that what is wanting is very inconſiderable.

Example in a ſingle Fraction turned into one Imperfect.

As to reduce $\frac{2}{3}$ to Decimals, if 5 Cyphers be adjoyned the Quotient is 66666, divided by 3; and if the Diviſion be continued (more Cyphers being adjoined) the ſame Quotienary Numbers will ſtill be increaſed, becauſe the Fraction cannot be reduced into a perfect Decimal.

$$\frac{2}{3} \qquad \begin{array}{l} \cancel{2}\cancel{2}\cancel{2}\cancel{2}\cancel{2}\,2 \quad (1)(2)(3)(4)(5) \\ \cancel{2},\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\,(66666 \\ \qquad 3 \end{array}$$

Reaſon of Reduction.

So any other proper ſingle Fraction whatſoever may be turned into a Decimal, the Reaſon being the ſame: For as the Denominator of any Fraction is to the Numerator, ſo ſhall any Denominator be to a Numerator which ſhall have the ſame value to the Denominator given, as the firſt Numerator had to his Denominator, as is evident in the

the foregoing Examples: For as 4 to 3, so is 100 to 75; and as 3 to 2, so is 100000 to $66666\frac{2}{3}$.

Fractions conjunct and divided reduced into Decimals.

If the Fractions given to be reduced into Decimals be conjunct or divided Fractions, then first by Reduction of Fractions reduce them into one Denomination, and abbreviate them into their least Terms, and if Conjunct add them together, and then turn them into Decimals, as above. Example of both.

Examples of both.

To reduce $\frac{3}{4}$ and $\frac{1}{5}$ into a Decimal: First by Reduction of Fractions they make being reduced $\frac{15}{20}$ and $\frac{4}{20}$, and added $\frac{19}{20}$, then adjoyning to 19, 2 Cyphers, and dividing by 20, the Quotient 9 Primes, 5 Seconds, is the Decimal.

To reduce $\frac{3}{4}$ of $\frac{1}{5}$ into a Decimal: First by Reduction of Fractions they are brought to $\frac{3}{20}$, and by adjoyning 00, to 3, and the summe divided by 20, gives 1 Prime, 5 Seconds, for the Decimal.

$$\frac{19}{\begin{matrix}15 & & 4\\ \frac{3}{4} & \times & \frac{1}{5}\\ & 20 & \end{matrix}} \qquad \begin{matrix}\cancel{1} & \\ \cancel{19},\cancel{00} & (\overset{(1)(2)}{95}\\ \cancel{2}\ \cancel{0} & \end{matrix} \qquad \frac{3}{\begin{matrix}\frac{3}{4} \text{ of } \frac{1}{5}\\ 20\end{matrix}} \qquad \begin{matrix}\cancel{1} & \\ \cancel{3},\cancel{00} & (\overset{(1)(2)}{15}\\ \cancel{2}\ \cancel{0} & \end{matrix}$$

To reduce Decimals into Common Fractions.

§. 2. To reduce *Decimals* into *Common Fractions*. If the Denominator be given to find a Numerator, then multiply the given Decimal by the given Denominator, and the Numbers exceeding the place of Primes shall be the Numerator to the given Denominator. And if the Numerator be given to find a Denominator, then by the given Numerator multiply the Denominator of the Decimal, and divide the Product by the given Decimal. Example of both.

Examples.

Suppose 0,75 be the given Decimal, and 4 the Denominator given, to which a Numerator is desired; then multiplying 0,75 by 4, there exceeds the *Seperatrix* 3, which shall be the Numerator, and so 0,75 reduced to a Common Fraction shall be $\frac{3}{4}$. For as 100 to 75, so is 4 to 3.

And if 3 the Numerator were given, and 0,75 the Decimal to find a Denominator, then I multiply 100 the Denominator of the Decimal by 3, and the Product 300 divide by 75 the Decimal, the Quotient is 4 the Denominator to 3. For as 75 to 100, so is 3 to 4.

$$\begin{array}{r}0,75\\ 4\\ \hline 3,00\end{array} \qquad \frac{3}{4} \qquad \begin{array}{r}1,00\\ 3\\ \hline 3,00\end{array} \qquad \begin{matrix}\cancel{3},\cancel{00} & (4\\ \cancel{75} & \end{matrix} \qquad \frac{3}{4}$$

To reduce Geodæticals into Decimals.

§. 3. To reduce *Geodæticals* into *Decimals*, observe whether the *Geodætical* be of one Denomination or more, single or plural.

1. Case. Single and next the Great Integer.

If the Geodætical be single, have but one Denomination, and that next the greatest Denomination of the Integer, then adjoyn to the Right Hand thereof so many Cyphers as you please, and divide the summe by the whole Number of Units of that Denomination contained in the Integer, and the Quotient is the Decimal, which shall be denominate according to the number of Cyphers adjoyned, as was shewed before.

Examples.

As to reduce 9 Shillings, which is the next denomination to Pounds, into Decimals; I adjoyn 2 Cyphers to 9, and this 900 I divide by 20, the whole number of Shillings contained in one Pound, or greatest Integer, and the Quotient is 45, of which 5 shall be Seconds, because 2 Cyphers were adjoyned to 9, and the 4 shall be Primes; so is $0,\overset{(1)(2)}{45}$ the Decimals for 9 *s*.

Also to reduce 9 Months into Decimals, 00 adjoyned to 9, and divided by 12, the Months in a Year, the Quotient $0,\overset{(1)(2)}{75}$ is the Decimal desired.

$$\begin{matrix}\cancel{1} & \\ \cancel{9}\cancel{00} & (\overset{(1)(2)}{45} \text{ Decimal for } 9\,s.\\ \cancel{2}\ \cancel{0} & \end{matrix} \qquad \begin{matrix}\cancel{6} & \\ \cancel{9}\cancel{00} & (\overset{(1)(2)}{75} \text{ Decimal for 9 Months.}\\ \cancel{12} & \end{matrix}$$

If

2. Case. Single, and not next the great Integer.

If the single Geodætical given be not next to the greatest Denomination of the Integer, then the Decimal may be had either working as above, accompting one of the next Denomination for the Integer, or by dividing the given Geodætical increased with his Cyphers by the parts of the greatest Integer.

Example.

As to reduce 9 *d.* into Decimals, because Pence is not the next Denomination to Pounds, the great Integer, I either divide 9 with Cyphers by 12, the parts of one Shilling, which is the Denomination next to Pence, and so have I $\overset{(1)(2)}{75}$, the Decimal of a Shilling; or by 240, the Pence in a Pound the greatest Integer, and thereby get $\overset{(2)(3)(4)}{375}$, which is the Decimal of a Pound for 9 *d.*

```
  6     (1)(2)            1
 9,00 (75                182    (2)(3)(4)
 12                     9,0000 (375
                        2 4440
                          22
```

3. Case. Plural.

If the given Geodætical to be reduced consist of more Denominations than one, reduce the given Numbers by Geodætical Reduction into their lowest Denomination, and then work as above.

Examples.

As to reduce 3 *s.* 3 *d.* into Decimals: 3 *s.* 3 *d.* reduced into Pence, make 39 Pence, to which 4 Cyphers adjoyned, and the summe divided by 240, the Pence in one Pound, the Quotient is 1625, which is $\overset{(1)(2)(3)(4)}{1\,6\,2\,5}$, because the Right Hand Cypher was of the same Denomination, there being 4 Cyphers adjoyned.

So 1 *s.* 4 *d.* 2 *q.* reduced into Decimals make $\overset{(2)(3)(4)(5)}{6\,8\,7\,5}$, being first reduced into Farthings make 66, and then divided with 00000 adjoyned by 960, the Farthings in one Pound.

```
 s. d.                               s. d. q.
 3—3                                 1—4—2
 12           1                      12            74
 ——         15 62                    ——          8 428
 36        39,0000 (1625 (4)         16        66,00000 (6875 (5)
  3        24 4440                    4        9 66660
 ——          2 22                    ——          999
 39                                  66
 ——                                  ——
```

4. Case. Many Plural.

If Plural Geodæticals, of one or more Denominations, be given to be reduced into Decimals; first add them as Geodæticals are to be added, then reduce them into their lowest Denomination, and turn them into Decimals, as before.

Example.

As to reduce into Decimals, 3 *s.* 2 *d.*, 2 *s.* 6 *d.* and 1 *s.* 1 *d.* being added the total is 6 *s.* 9 *d.* and in Pence 81, then by adjoyning 4 Cyphers, as above, and dividing, I get in the Quotient $\overset{(4)}{3375}$ for the Decimal of all the 3 given Numbers.

```
 s. d.    s. d.
 3—2      6—9
 2—6      12         11
 1—1      ——         9 82     (1)(2)(3)(4)
 ——       12        81,0000 (3375
 6—9      69        24 4440
 ——       ——          2 22
          81
          ——
```

The end of Tables.

The like may be understood of other Geodæticals of Weight, Measure, Time, &c. as well as those instanced of Coyn; but some to ease this continued work, prepare Tables for each Denomination, out of which the Decimals taken for each given Number, may soon, by common Addition, be brought into one total.

Tables easily made.

And these Tables are easily made and understood, considering (as was before shewed in the *First Chapter* of this *Book*) that every Integer is equal to 10 Primes, therefore half that Integer shall be equal to 5 Primes, and the quarter of that Integer equal to 2 Primes, 5 Seconds; and so the Decimals for the even aliquot parts of the Integer found, the rest of the intermediate

termediate places may be easily had by Addition or Substraction, *&c.* as the case requireth.

Example in the *Table* of *English* Coyn. If 1 Pound or Integer (containing 20 Shillings) in Decimals set thus, 1,00, be 10 Primes, then $\frac{1}{2}$ *l.* or 10 *s.* must be 5 Primes, and $\frac{1}{4}$ *l.* or 5 *s.* shall be 2 Primes, 5 Seconds; and the fifth part of 5 *s.* which is 1 *s.* shall give the fifth part of 2′, 5″, which is 5″; then if I substract 5″, the Decimal of 1 *s.* from 2′, 5″, the Remain, 0,2′, is the Decimal of 4 *s.* the half of which is 2 *s.* and the Decimal thereof 0,1′, being half of the Decimal 0,2′, and so also if I double 5″, the Decimal of 1 *s.* I have 0,1′, as before. And in like manner all the other intermediate places for the 19 *s.* may be filled with Decimals. *Example.*

Then taking 0,05″, the Decimal of 1 *s.* and breaking it in two parts, which is 0,025‴, you have the Decimal of 6 *d.*; and again 0,0125⁗, half the latter, the Decimal of 3 *d.*, and so of others, as the following *Tables* make conspicuous.

DECIMAL TABLES, FOR *Sterling*-Money.

Decimal Tables for Sterling *Money.*

Shillings

Geodæticals.		Decimals.
l.	*s.*	
1, or	20	1,00
	19	0,95
	18	0,90
	17	0,85
	16	0,80
	15	0,75
	14	0,70
	13	0,65
	12	0,60
	11	0,55
	10	0,50
	9	0,45
	8	0,40
	7	0,35
	6	0,30
	5	0,25
	4	0,20
	3	0,15
	2	0,10
	1	0,05

Pence and Farthings.

Geodæticals.		Decimals.
s.	*d.*	
1, or	12	0,050000
	11	0,045833
	10	0,041667
	9	0,037500
	8	0,033333
	7	0,029167
	6	0,025000
	5	0,020833
	4	0,016667
	3	0,012500
	2	0,008333
	1	0,004167
d.	*q*	
1, or	4	0,004167
	3	0,003125
	2	0,002083
	1	0,001041

Troy-Weight.

Troy-*Weight.*

Ounces.

Geodæticals.		Decimals.
lb.	*Ounces.*	
1, or	12	1,000000
	11	0,916667
	10	0,833333
	9	0,750000
	8	0,666667
	7	0,583333
	6	0,500000
	5	0,416667
	4	0,333333
	3	0,250000
	2	0,166667
	1	0,083333
Pen.w.	*Gra.*	
1, or	24	0,004166
	12	0,002083
	6	0,001041
	3	0,000520
	1	0,000173

Penny-Weights.

Geodæticals.		Decimals.
Ounce.	*Pennyw.*	
1, or	20	0,083333
	19	0,079167
	18	0,075000
	17	0,070833
	16	0,066667
	15	0,062500
	14	0,058333
	13	0,054166
	12	0,050000
	11	0,045833
	10	0,041667
	9	0,037500
	8	0,033333
	7	0,029166
	6	0,025000
	5	0,020833
	4	0,016667
	3	0,012500
	2	0,008333
	1	0,004166

The *Table* of *Troy* Weight for Ounces, may serve for Pence, if One Shilling be accompted the Integer: And also for Months, One Year being accompted the Integer.

The Decimal Table *for* Avoirdupois *Weight, accompting the Hundred Weight for the Integer.*

A Decimal Table *for* Avoirdupois *Weight.*

Pounds.

Geodæticals. C.	℔.	Decimals.
1, or	112	1,000000
¾, or	84	0,750000
½, or	56	0,500000
¼, or	28	0,250000
	27	0,241071
	26	0,232143
	25	0,223214
	24	0,214286
	23	0,205357
	22	0,196429
	21	0,187500
	20	0,178571
	19	0,169643
	18	0,160714
	17	0,151786
	16	0,142857
	15	0,133929
	14	0,125000
	13	0,116071
	12	0,107143
	11	0,098214
	10	0,089286
	9	0,080357
	8	0,071429
	7	0,062500
	6	0,053571
	5	0,044643
	4	0,035714
	3	0,026786
	2	0,017857
	1	0,008929

Ounces.

Geodæticals. ℔.	℥.	Decimals.
1, or	16	0,008929
	15	0,008371
	14	0,007812
	13	0,007254
	12	0,006696
	11	0,006138
	10	0,005580
	9	0,005022
	8	0,004464
	7	0,003906
	6	0,003348
	5	0,002790
	4	0,002232
	3	0,001674
	2	0,001116
	1	0,000558

Or thus, accompting 1 ℔. for the Integer.

℔.	℥.	
1, or	16	1,0000
	15	0,9375
	14	0,8750
	13	0,8125
	12	0,7500
	11	0,6875
	10	0,6250
	9	0,5625
	8	0,5000
	7	0,4375
	6	0,3750
	5	0,3125
	4	0,2500
	3	0,1875
	2	0,1250
	1	0,0625
	0¾	0,046875
	0½	0,031250
	0¼	0,015625

The Table *for* Ounces *serves for* Nails *in a* Yard.

The last *Table* for Ounces may serve for the Nails in one Yard, which are 16, as well as the Ounces in one Pound, and so the Decimals the same, accompting one Yard the Integer instead of one Pound.

The

The Table *for the Dayes of one Month, accompting the Integer One Year, or 365 Dayes.*

A Decimal Table for the Dayes in a Month.

Dayes.	Decimals.	Dayes.	Decimals.
1	0,002740	16	0,043836
2	0,005479	17	0,046575
3	0,008219	18	0,049315
4	0,010959	19	0,052055
5	0,013698	20	0,054795
6	0,016438	21	0,057534
7	0,019178	22	0,060274
8	0,021918	23	0,063014
9	0,024658	24	0,065753
10	0,027397	25	0,068493
11	0,030137	26	0,071233
12	0,032877	27	0,073973
13	0,035616	28	0,076712
14	0,038356	29	0,079452
15	0,041096	30	0,082192

To get the Decimals for Hours.

In like manner taking the Decimal for one Day, and breaking it into parts, I may have the Decimals for Hours; as for 12 Hours 0,00137, for 6 Hours 0,000685, *&c.* And in the same method Tables may be made for other Geodæticals, and by these Tables the Decimals may be speedily had. As if 16 *s.* 9 *d.* were to be turned into Decimals; in the *Table* of *Coyn* against 16 *s.* is found 8 Primes, and against 9 *d.* is 3 Seconds, 7 Thirds, and 5 Fourths, which added together make the total Decimal 0,8375. So if 17 *C.* 13 ℔. 5 ℥. be represented in Decimals, the *Table* of *Avoirdupois* Weight sheweth the Decimal of 13 ℔. to be 0,116071, and the Decimal of 5 ℥. 0,00279, both which together represent the Decimal Number thus, 17,118861.

Use of the Tables.

To reduce Decimals *into* Geodæticals.

§. 4. To reduce *Decimals* into *Geodæticals.* If the Decimal be found in the Table, then the correspondent Number, without further work, effects the desire: But if not, multiply the Decimal given by the denominate parts contained in the Integer, and what exceeds the Prime Line shall be the Number desired, and shall be denominate according to the denomination of the Multiplyer. One Example will make all plain. As if 0,9875 be given, and it is desired to know how much the same is in *English* Money: Then if I multiply by 20, the Shillings in one Pound, I find 19 exceed the Prime Line or *Seperatrix*, which shall be 19 Shillings, because 20 the Multiplyer was Shillings, and the remaining Decimal 0,75, multiplyed by 12 the parts of one Shilling, the Product is 9, which shall be Pence, because 12 Pence are the denominate parts of a Shilling that were multiplyed by. But if I multiply the Decimal first given by 240, the Pence in one Pound, I find 237 exceed the Prime Line, which 237 shall be Pence because the Multiplyer 240 was Pence.

Example.

```
      0,9875           0,9875              2
        |  20              | 240          22(9 d.
    ----|----        ------|----         237(19 s.
 s. 19|7500            39|5000            22
        |  12           197|50
    ----|----        ------|----
       1|5000        d.237|0000
       7|500
    ----|----
 d.   9|0000
```

What to be done with Imperfect Decimals.

Many times in imperfect Decimals, when there is left in the place of Primes near another Integer, and upon a Division almost the Divisor is left remaining, another Unit is commonly taken for the same. And so is done in many of the Tabellary Numbers before. And when it happeneth may be done in all the Cases of the Four *Sections* of this *Chapter*.

Proof of Reduction of Decimals.

The Reductions of the first and second *Sections* are alternate Proofs of each other; and so are they of the third and fourth *Sections*.

CHAP.

CHAP. III.

Indices *Added and Substracted.*

Addition and Substraction of Indices.

SO much in the *First Chapter* of this *Third Book* hath been said of *Indices*, as may be sufficient to give a right understanding of them, that they are Numbers which, as their Name implyeth, do shew the number or distance of places any Number or Species to which they are annexed is from the Unit, and how many places such Number or Species hath ; so as there needs no further explanation here.

What shewed by their Addition.

Addition of *Indices*, discovers the number of places any *Factum* or Product should consist of, and thereby consequently sheweth the true nature of the Product.

What by their Substraction.

Substraction of *Indices*, declareth the true nature of any Quotient after Division of Mixt Numbers, or Numbers of several distances from the Unit.

Indices to be added will be either alike, that is, both Integers, or both Decimals; or else unlike, that is, the one an Integer and the other a Decimal.

How added if alike.

If they be both alike, then add the *Indices* as Integers are added, and the summe shall be the *Index* of the same kind the added *Indices* were.

Examples.

Examples.	Integers.		Decimals.	
Addends	4	5	(4)	(5)
	3	3	(3)	(3)
Totals	7	8	(7)	(8)

How added if unlike.

If the *Indices* be unlike, then substract the Lesser Number out of the Greater, the difference shall be the *Index* of the same kind with that Number which was the greatest.

Examples.

Examples.	Integers and		Decimals.		
Addends	7	2	(2)	(7)	
	(2)	(7)	7	2	
Index	5	(5)	5	(5)	remaining

In both these Cases the Operation and Reason thereof is so plain they need no explanation.

Indices to be substracted are to be placed one over the other, as the Numbers to which they belong ought to be placed ; for it matters not though the uppermost be the least.

These *Indices* also will be either alike or unlike.

How substracted if alike.

If both the *Indices* be alike, take the difference for the *Index* desired, and his nature is thus discerned : If the Subtrahend or Number to be substracted be the least, the difference remaining shall be an Integer in Integers, and a Decimal in Decimals : But if the greatest the contrary, to wit, in Integers a Decimal, and in Decimals an Integer.

Examples.

Examples.	Integers.		Decimals.	
	5	4	(5)	(4)
Subtrahends	4	5	(4)	(5)
Remains	1	(1)	(1)	1

How substracted if unlike.

If the *Indices* be unlike, add them together, and take the summe, and as the upper Number from which Substraction is to be made, so shall this *Index* be, whether Integer or Decimal.

Examples.

Examples.	Integers and Decimals.								
	5	(5)	2	(2)	4	(4)	(3)	0	
Subtrahends	(2)	2	(5)	5	(4)	4	0	(3)	
Index	7	(7)	7	(7)	8	(8)	(3)	3	amounting.

In

In the Examples of Substraction, where the *Indices* are alike, the work is plain; for taking the difference between 4 and 5, the Remain 1 is the *Index*, which is either an Integer or a Decimal, according as the Number to be substracted was greater or lesser than the other.

And in the latter Examples the Addition of both the *Indices* make the remaining *Index* denominate alwayes as the higher *Index* or Number from which Substraction is made. And the Reason of both is evident, for that the more is substracted the less of necessity must remain; and the lesser Number either in Nature or Figure substracted, the greater will remain.

In these last Examples, c, or the place of the Unit, is seen to be accompted as the *Index* of an Integer.

CHAP. IV.

Addition of Decimals.

THE variety of adding Decimals is occasioned as the Numbers given to be added are Simple or Compound. *Decimals added.*

Simple whether Pure or Mixt, being orderly placed every one under his like Denomination, are added as Integers. *Simple.*

Examples.

Examples.	Pure.		Mixt.		
	0,990625	—0,5875	352,7250	—51,025	Addends.
Simple	0,725000	—0,1250	240,0375	—20,050	
	1,715625	—0,7125	592,7625	—71,075	Totals.

Compound Decimals are added by substracting, as in Integers, the Lesser Number out of the Greater, and to the Remain which shall be the Total, subscribe the sign of the Greater Number. *Compound.*

Examples.

Examples.	Pure.		Mixt.		
	+0,950	—0,950	+34,125	—34,125	Addends.
Compounds	—0,825	+0,825	— 3,950	+ 3,950	
	+0,825	—0,825	+30,175	—30,175	Totals.

If Compounds given to be added be many, let the Numbers of each sign be added with their fellows, as Simple Decimals, and then the Totals, as Compounds. As if 34,125 —0,95, were to be added with 1,45 —3,5, the + being added with + and — with —, the Totals are 35,575 —4,45, then taking the — from the +, the remaining Total is 31,125. *Compounds when many.* *Example.*

34,125	—0,95	+35,575
1,45	—3,5	— 4,45
35,575	—4,45	+31,125

Besides the Proof of these works by Substraction, noted in the next *Chapter*, the truth may be brought to the Test by reducing your Decimals into Geodæticals, and adding them in their order, the Totals of both Operations will agree when rightly done. *Proof of Decimal Addition.*

As in the last Example.

Decimals.		Geodæticals.				
		l. s. d.		l. s. d.		l. s. d.
34,125	—0,95	34—02—06	lacking	00—19—00	or	33—03—06
1,450	—3,50	1—09—00	lacking	03—10—00		— 2—01—00
35,575	—4,45	35—11—06	wanting	04—09—00		31—02—06
— 4,450		— 4—09—00				
+31,125	Correspondents	+31—02—06	Proof.			

CHAP. V.

Substraction of Decimals.

Decimals *Substracted. Simple.*

THE variety in substracting Decimals, as well as their Addition, is according to the Simplicity or Composition of the Numbers given to be substracted.

Simple Decimals Pure or Mixt, being orderly placed every Denomination under his like, are substracted by withdrawing the nether Number from the upper, as Integers borrowing 10, if need be, and subscribing the Remain, except the Subtrahend be the greatest Number, then take the Lesser Number from the Greater, and change the Sign to the difference.

Examples.

Examples.	Pure.	Mixt.	Pure.	Mixt.	
Simple	0,990625	352,7250	0,725	—1,725	
	0,725000	240,0375	0,950	—1,950	Subtrahends.
	0,265625	112,6875	—0,225	+0,225	Remains.

Compound.

Compound Decimals having contrary Signs, for their Substraction require Addition; so the Totals of the Numbers shall be the Remains; to which is to be affixed the upper Numbers sign, that is, the sign of the Number from which Substraction is to be made.

Examples.

Examples.		Pure.			
Compound	+0,950	—0,950	+0,125	—0,125	
	—0,125	+0,125	—0,950	+0,950	Subtrahends.
	+1,075	—1,075	+1,075	—1,075	Remains.
			Mixt.		
	+34,125	—34,125	+ 3,950	— 3,950	
	— 3,950	+ 3,950	—34,125	+34,125	Subtrahends.
	+38,075	—38,075	+38,075	—38,075	Remains.

Compounds when many.

If Compounds given to be substracted be many, let the Numbers of each sort be substracted as Simple Decimals, and then the Remains one from the other as Compounds. As if 3,5 —1,45 were to be substracted from 22,475 —0,8, the + taken from the + leaves +18,975, and the — from the — leaves +0,65, and this Remain added to the other makes the Remain at last +19,625. Otherwise if I take each — from the respective + to which they are joyned, the 2 Numbers will be +21,675 and +2,05, and then Substraction made as in Integers, the Remain will be as before.

Example.

Proof of Decimal Substraction.

The Proof of Addition is by Substraction, and of Substraction by Addition alternately, as before in Integers and Geodæticals hath at large been seen, with this difference only here, that Simple Decimals reserve their Proof by the Simple Operations, and the Compounds by the Compound Operations respectively: So as Addition of the Simple shall be proved by Simple Substraction, and Compound Addition by Compound Substraction, and accordingly Substraction Simple or Compound by the Simple or Compound Addition.

Besides this Proof, the truth of Decimal Substraction as well as Addition may be proved by turning the Decimals into Geodæticals, and substracting them in their order; so will the Remains of both Operations agree, if done aright. As in the last Example.

	Decimals.		Geodæticals. *l. s. d.*		*l. s. d.*		*l. s. d.*
	22,475 —0,80		22—09—06	wanting	0—16—0	or	21—13—6
	3,500 —1,45		3—10—00	wanting	1—09—0		2—01—0
Thus	18,975 +0,65		18—19—06	more	0—13—0		19—12—6
	0,65		+ 0—13—00				
	19,625	Correspondents	19—12—06				
Or	+22,475		+22—09—06				
	— 0,800		— 0—16—00				
	+21,675		+21—13—06				
Thus	+ 3,50		+ 3—10—00				
	— 1,45		— 1—09—00				
	+ 2,05		+ 2—01—00				
	+19,625	Correspondents	+19—12—06				

CHAP. VI.

Multiplication of Decimals.

THE varieties of multiplying Decimals are sorted under the like double head of Simple and Compound. *Decimals multiplyed.*

Multiplication of Simple Decimals is so ordered, that the Product may be procured compleat or contracted, as occasion shall require. *Simple and*

To procure the Product compleat, multiply Number by Number, as in Integers, and add the Decimal *Indices* of the Multiplicand and Multiplyer (or the *Indices* of the Right Hand Figures of them) together, and the summe shall be the *Index* of the Product, and the sign thereof alwayes +. *The Product compleat.*

Examples.

Examples.		Pure.		Mixt.			
0,9875	(4)	— 0,125	(3)	— 3,625	(3)	13,625	(3)
0,8875	(4)	— 0,025	(3)	— 2,5	(1)	12, 75	(2)
49375	(8)	625	(6)	18125	(4)	68125	(5)
69125		250		7250		95375	
79000						27250	
79000		+0,003125		+9,0625		13625	
0,87640625		Compleat Products.				173,71875	

Not only the Total *Index* of the Product by adding the *Indices* as abovesaid is found, but the true place in the Product of any two Numbers multiplyed may be had by adding the particular *Indices* of the particular *Factores*. As in the last Example; if 5 in the Multiplicand be multiplyed by 1 in the Multiplyer, to know of what place the *Factus* or Product will be, or how far distant from Unity, and whether a Decimal or Integer; the *Index* of 5 is (3), and the *Index* of 1 there is 1, added make the Total *Index* (2), and so shall the Product 5 fall in the place of Decimal Seconds. Also the Product of 2 in the Multiplicand, whose *Index* is (2), multiplyed into 2 in the Multiplyer, whose *Index* is 0, because he standeth in the Units place, makes the Product Decimal Seconds, shewed by Addition of their *Indices* as aforesaid. ☞

Sometime

Simple and the Product contracted.

Sometime it happens that all the Figures of the Product need not be expressed, but only some of those towards the Left Hand, because those towards the Right Hand are of small value, and in many Operations inconsiderable, and so the Product is contracted into a less number of places.

Contraction 3 wayes.

To contract the Product proceed in one of these three wayes.

1. *By* Nepaires Bones.

First, by *Nepaires Bones*, having set the Multiplicand on the *Bones*, and by the *Index* of the *Bones*, having the several Products answering the Figures of the Multiplyer, take off each *Bone* so many Figures as will be sufficient, having respect to the Tens that rise on the next Right Hand *Bone*, and placing them orderly one under the other, add them into one Total Product.

Example.

As if 321,125 were to be multiplyed by 43,25, where though there be 5 Decimals in both the *Factores*, yet needing but 3 in the Product, I Tabulate 321,125 on the *Bones*, and take out the Multiplees, and in that appertaining to the multiplying 5, omit two places, and one place in the Multiplee of 2, and add the Residue in order, as followeth.

1	3	2	1	1	2	5
2	6	4	2	2	4	1/0
3	9	6	3	3	6	1/5
4	1/2	8	4	4	8	2/0
5	1/5	1/0	5	5	1/0	2/5
6	1/8	1/2	6	6	1/2	3/0
7	2/1	1/4	7	7	1/4	3/5
8	2/4	1/6	8	8	1/6	4/0
9	2/7	1/8	9	9	1/8	4/5

```
   64225 .
  963375
 1284500
   16056 . .
```

```
     321,125
       43,25
    --------
    16056| . .
    64225| .
   963375|
  1284500|
  ----------
  13888,656
```

2. *By cutting off the Right Hand Figures.*

But Secondly, for that the whole Art of *Arithmetick* may be performed by the Pen, and many times the *Bones* are not at hand, multiply all as Integers, and cut off from the particular Multiplees or Total Product, so many of the Right Hand Figures as are useless, respecting the Tens that rise in the next Right Hand File. And so the work of the former Example stands.

Example.

```
Thus,  321,125              Or thus,  321,125
         43,25                          43,25
      ---------                      ---------
       16056|25                       16056|25
       64225|0                        64225|0
      963375|                        963375|
     1284500|                       1284500|
     ---------                      ---------
     13888,656                      13888,656|25
```

3. *By Mr.* Oughtred's *retrograde way.*

Thirdly, Mr. *William Oughtred* in his *Clavis* proceeds in a retrograde order thus; If the Product be desired Pure, that is, to be wholly Integral without any Decimal Fraction, then place the Unity of the Multiplyer, or the Lesser Number, under the Unity of the Multiplicand, or Greater Number, and so the Integers in order under the Decimals, and the Decimals under the Integers counterchanged. And if the Product be desired Mixt, then place the Unity of the least Number under that Decimal which the Product is desired to be. And if the Product be desired less then Pure, by so much remove the Units place of the Lesser Number towards the Left Hand. And the Numbers thus placed, begin to multiply by each Figure of the least Number or Multiplyer at that Figure of the Multiplicand, or greatest Number, which stands thereover, and the Products of the Left Hand Multiplyers let be set first, and so the rest in order by a Perpendicular Line.

Example.

As for further instance in the former Example thus, I place 3 in the Lesser Number being in the place of Unity, under the place of Thirds, which is 5 in the Greater Number,

Number, and the other Numbers in order alternately, and then beginning to multiply by 5 in the Lesser with 1 in the Greater Number which stands over him, adding in the Article that will arise, if he be multiplyed by 2, the next Right Hand Figure in the Greater Number, I subscribe 6 under the Units place in the Lesser Number, and so proceed towards the Left Hand, and the like is done with the other Multiplying Figures, save because the last 4 hath no Figure over him, the Product arising by him is set one place distant from the Unity towards the Left Hand: And the work stands thus;

```
  321,125
     52|34
 ---------
    16056|
    64225|
   963375|
 1284500.|
 ---------
13888,656|
```

And if the Product had been desired purely Integral, or less than Pure, the Products would have been found thus by,

The second way.		Mr. *Oughtreds* way.	
321,125	321,125	321,125	321,125
43,25	43,25	52|34	52|34
16|05625	1|605625	16|	1|
64|2250	6|42250	64|	6|
963|375	96|3375	963|	96|
1284|500	1284|500	12845|	1284|
13889|	1389|	13889|	1389|
Pure.	Less than Pure.	Pure.	Less than Pure.

In all these is added more than really found in the Right Hand File, an Unit in the pure Products, and two Units in the others, respect had to the Tens, or almost Tens arising from the Figures cut off, as is usual in such Cases. *An Unit or more added, and why.*

Compound Decimals convert into Simple, if by Addition or Substraction they may, and then multiply them as before: Otherwise multiply every Number of the Multiplicand by every Number of the Multiplyer, beginning at the Left Hand, as in *Geodæticals*, *Book* 2, *Part* 1, *Chap*. 5. *Case* 4. And for the Signs, when Numbers of like Signs, as + with +, or — with —, are multiplyed together, the Product shall be +; but unlike Signs, as + with —, or — with +, shall make the Product —. *Compounds multiplyed.*

As if 5,9 —2,8 were to be multiplyed by 3,4 —1,2, I either convert them into Simple by substracting the — out of the +, and then multiply the Remains 3,1 by 2,2; as at *A.*: Or otherwise multiply like Geodæticals, as at *B.*; and by adding + with +, and — with — in the Multiplees, and then substracting the — from the +, the Product of both Operations are alike 6,82. *Example.*

```
                 A.                                 B.
+5,9 —2,8          +3,4 —1,2          5,9  —2,8
—2,8               —1,2               3,4  —1,2
----               ----               ---------                 +20,06
+3,1               +2,2               236   112                 + 3,36
                                      177   84                  ------
          3,1  (1)                    ----------------          +23,42
          2,2  (1)                    20,06 —9,52               —16,60
          ---  ---                    ----------------          ------
          62    2                           118      56         + 6,82
          62                                59       28         ------
          ----                              -----   -----
          6,82                              —7,08   +3,36
          ----                        ----------------------
                                      20,06 —16,60  +3,36
```

Proof of Decimal Multiplication.

Multiplication of Decimals is proved by their Division, as is mentioned in the next *Chapter*. But besides that Proof, if the Decimals be turned into Geodæticals, and multiplyed, the Products of both will correspond when the work is without Error: As in the last Example at *A*.

Decimals.		Geodæticals.
		l. s. d. less *l. s. d.* or *l. s.*
5,9 —2,8 or 3,1		5—18—0 less 2—16—0 or 3—02
3,4 —1,2 or 2,2		3—08—0 less 1—04—0 or 2—04
62		6—04
62		12—$\frac{8}{20}$
6,82	Correspondents	6—16 $\frac{2}{5}$

Or if the Geodæticals be reduced into Shillings, and their results 62, and 44 multiplyed, and then returned again into Pounds by 20 times 20, as in Multiplication of Geodæticals in like Case before was taught, there will be obtained 6 *l.* 16 *s.* $\frac{2}{5}$ equivalent to 6,82, as before.

l. s. *s.*		(3
3—2 or 62	20	27|28(6 *l.*
2—4 or 44	20	400
248	400	2(1
248		65|60(16 *s.* $\frac{2}{5}$.
2728		400

CHAP. VII.

Division of Decimals.

Decimals *divided.*

Simple.

THE varieties of dividing Decimals are ordered according to the Simplicity or Composition of the given Numbers.

Division of Simple Decimals runs into a Bivity of dividing a Greater Number by a Lesser, or a Lesser by a Greater.

1. *A Greater by a Lesser.*

In the first Case, where a Greater Number is to be divided by a Lesser, divide Number by Number as in Integers, and to find the true *Index* of the Quotient, substract the *Indices* Decimal of the Divisor from the Dividend, or the *Indices* of the Right Hand Figures of them the one from the other, and the Remain shall be the *Index* of the Quotient, and the sign thereof +.

Examples in Pure Decimals.

Examples in Pure.

To divide 0,87640625 by 0,8875, the Quotient will be 9875, which shall be Decimal Fourths, because the *Index* of the Dividend is (8), from which (4) the *Index* of the Divisor substracted, the Remain is (4).

So —0,003125 divided by —0,125, shall make the Quotient Decimal Thirds; for (3) the *Index* of the Divisor substracted from (6) the *Index* of the Dividend leaves (3) remaining.

644	Units place supplyed by		Places supplyed by
76537	(4) a Cypher.	6	(3) Cyphers.
,87640625	(9875, or 0,9875	— 3125	(25, or 0,025+
,8875	(8) *Index* of the Dividend.	—,125	(6)
,8875	(4) *Index* of the Divisor.	,125	(3)
,8875			
,8875	(4) *Index* of the Quotient.		(3)

Examples

Examples in Mixt Decimals.

As 173,71875 divided by 13,625, the Quotient 1275, shall be 12,75; for by substracting (3) the Decimal *Index* of the Divisor from (5) the Decimal *Index* of the Dividend, the Remain is (2). *Examples in Mixt.*

Also if 758 Yards of Cloth cost 284 *l.* 5 *s.*, or in Decimals 284,250, and the one be divided by the other, the Quotient will be 375, which by substracting 0, the *Index* of 758, from (3) the *Index* of the Dividend, will be found Decimal Thirds, or 7 *s.* 6 *d.* for the price of one Yard.

Likewise if 77448,288 be divided by 0,2864, the Quotient shall be 27042, and by substracting the *Index* (4) of the Divisor from (3) the *Index* of the Dividend, the Remain 1 shews the *Index* of the Right Hand Quotient Figure to be removed one place distant from the Unit towards the Left Hand, and therefore Cyphers are placed to fill up the void places.

```
 68
 10 21                (2)      3 7         (3)      25                    1
 37 4612                       56 89                201 60 72
173,71875( 12,75              284,250( 375         77448,288( 270420,0
 13,625               (5)     758          (3)     ,2864
  13,625              (3)      758         (0)      ,2864               (3)
   13,625            ----       758        ---       ,2864              (4)
    13,625            (2)                  (3)        ,2864             ---
                                                       ,2864             1
```

The second Case, where a Lesser Number is to be divided by a Greater, trebly branches it self. As first by supplying the defect of the Dividend with Cyphers. Secondly, by contracting the Divisor. Thirdly, by both. So as in Multiplication a Product may be contracted, in Division a Quotient may be increased negatively, as far as shall be needful, and is sometime phrased to divide by an Irrational or Infinite Number. 2. *A Lesser by a Greater, 3 wayes.*

First Branch, adjoyn to the Right Hand of the Dividend as many Cyphers as shall be requisite, and divide as before (accompting the *Index* of the Dividend, as it now stands with the Cyphers adjoyned) and separate the Integers, if any, from the Decimals, or supply the void places with Cyphers, and the work is done. 1. *By adjoyning Cyphers.*

As if 21 *l.* were to be divided among 24 Men, I adjoyn 000 (which are as many as need here) to 21, and divide by 24, the Quotient is 875, and by Substraction of the *Indices* appears to be (3) for each Mans Share, which reduced into Godæticals is 17 *s.* 6 *d.* *Example.*

```
              1
              82     (3)
 (3)        21,000( 875, or 0,875.
  0           24
 ---           24
 (3)            24
```

Second Branch in dividing by the *Bones*, supposeth Cyphers to be adjoyned as before, but otherwise taketh as many of the divising Figures to the Left Hand as are sufficient for the first Divisor, and divideth the given Number thereby, then for every particular subsequent Division lessen the Divisor, cutting off from the Right Hand of the Divisor each time one Figure, having respect to the Tens arising in the next Right Hand place (as was taught before in Multiplication) and so proceed in the Division till a Quotient be gotten as large as is desired. 2. *By contracting the Divisor.*

As to divide 467023 by 357,09264, the first Divisor is but 357093, the next 35709, *&c.* as is plain by the work it self subscribed, and so the Quotient 130785 negative. *Example.*

```
                        317                       Multiplees.
                       2 803                      35709264
Divisor               109 930   Quotient          587031
357,09264+)467 023( 1307,85 ———             ——————————
 . . . . . .          357,093                     357093
                       357,09                     107127
                        357,0                      0000
                         357                       2500
                          35                       286
                           3                        17
```

Third

3. *By both. Example.* Third Branch both adjoyneth Cyphers, and contracteth the Divisor, as aforesaid.

As if 34 were to be divided by 4,326481, and 5 Decimals only required in the Quotient: After 5 Cyphers are adjoyned to 34, I cut off 1 from the Divisor, and divide first by 4,32648, and then by 4,3264, &c. The rest of the work needs no further Explanation.

	1		
	36		
	252		Multiplees.
	3723		
	25345		4,326481
Divisor	3 72463	Quotient	858587
4,326481)	34,00000	(7,85858	
. . . . . .	4,32648		3028537
	4,3264		346118
	4,326		21632
	4,32		3461
	4,3		216
	4		35

Some in the Divisions of this and the second Branch, use not to subscribe the Divisor as above, but in stead thereof the Multiplees, and so make Substraction, applying the Divisor in a Loose Paper, as was shewed in the *First Book*, among the varieties of Division of *Integers*.

Compounds divided. Compound Decimals convert into Simple, by Addition or Substraction, if they may, and then divide them as before. Otherwise divide the Numbers of the Dividend by the Numbers of the Divisor, like the Division of *Geodæticals*, in the *Third* and *Fourth Cases*, *Book* 2. *Part* 1. *Chap.* 6. as they happen. The *Index* of the Quotienary Numbers are got as above. And the signs that are like give +, and unlike —, as in the precedent *Chapter* of *Multiplication*.

Example. As if 20,06 —16,60 +3,36 were to be divided by 3,4 —1,2, being turned into Simple Decimals they are 6,82 and 2,2, and Division ended as at *A.* the Quotient is 3,1. If otherwise, divided as Geodæticals at *B.* after 34, gotten by the first setting down of the Divisor, and the Divisor multiplyed thereby, the Product is 20,06 —9,52, which substracted from the Dividend leaves —7,08 +3,36, which is the Product of —12 the next Quotient Figures multiplyed into the Divisor.

A.

+20,06   +3,4
3,36   —1,2
+23,42   +2,2
—16,60
+ 6,82

2
(2) 6,82 (3,1
(1) 2,2
2,2
(1)

B.

(1)
+5,9 —2,8) 20,06 —16,60 +3,36 (3,4 —1,2
—1,2 +3,4   20,06 — 9,52
— 7,08 +3,36

236   112
177   84
20,06 —9,52

118   56
59   28
—7,08 +3,36

— 7,08

Proof of Decimal Division. Division of Decimals is to be proved by Multiplication, as Multiplication by Division counterchanged; so as the Simple Multiplication of Decimals shall be proved by their Simple Division, and Compound by Compound accordingly, and on the contrary their Division by their Multiplication.

Also those Divisions that set down the Multiplees of the Divisors underneath to substract, may be proved by Addition of those Multiplees, which will with the Remains, if any, return the Dividend. As in the last Example at *B.* like *Geodæticals* of the fourth Case before noted, and the Divisions of the second and third Branches in this *Chapter*, thus to be placed.

Second

Second Branch.		Third Branch.
(0	Remain added.	(1
17		35
286		216
2500		3461
00000		21632
107127		346118
357093		3028537
467023	Dividend returned.	34,00000

And besides this let the Decimals be turned into Geodæticals, and accordingly divided, and the Quotients of both works, when done without Error, will clearly correspond. As in the last Example at *A*.

Decimals.		Geodæticals.
		l. *s.*
6,82	Dividend.	6—16 $\frac{2}{5}$
2,2	Divisor.	2—4
3,1	Quotient.	3—2

4
l. *s.*) *l.* *s.* (*l.* *s.*
2—4) 6—16 $\frac{2}{5}$ (3—2
3—2
6—12
6—12
4 $\frac{8}{20}$ or abbreviated $\frac{2}{5}$.
4—$\frac{8}{20}$

Or if the Geodæticals be all reduced into Shillings, then must 136$\frac{2}{5}$ *s.* be divided by 44 *s.* the Quotient of which Division will be 3 *l.* and the 4$\frac{2}{5}$ left if reduced into an Improper Fraction multiplyed by 20, and as a Fraction divided by 44, the Quotient will be 2 *s.* as before.

CHAP. VIII.

Figuration of Decimals.

TO produce *Figurate Decimals* hath no difficulty therein; for by multiplying any Decimal Simple or Compound into it self, the Square Decimal is produced. And the Square multiplyed by the Root, produceth the Cube, *&c.* as *Figural Numbers* before in the *Second Part* of the *Second Book* were declared to be commonly produced. And because the same is done by Multiplication, the *Indices* of such Figurated Decimals are found by adding together the *Indices* of both the Factors, and the signs of the Product known, as in Multiplication of Decimals before was shewed. Figurate Decimals *produced.*

Examples. Pure Simple } Decimals { Mixt. Compound. *Examples.*

	Examples.		Pure Simple	Mixt. Compound.			
(2)	0,15	Root	1,2	—0,5			Root
(2)	0,15		1,2	—0,5			
	075		24	10			
	015		12	5			
			1,44	—,60			
(4)	0,0225	Square		—,60	+,25		
(2)	,15		1,44	—1,20	+,25		Square
	01125		1,2	—0,5			
	00225		288	240	50		
			144	120	25		
(6)	0,003375	Cube	1728	—1440	+300		
				— 720	+600	— 125	
	&c.		1,728	—2,160	+,900	—,125	Cube
					&c.		

Compound may be turned into Simple.

If the Compound Decimal be turned into a Simple, and Figurated accordingly, and the Number produced compared with the Product of the Compound, they will agree in summe when the Operation is right. As if 1,2 —0,5 be turned into a Simple Decimal, by substracting the 5 which is lacking out of the 1,2 affirmative, the remaining Root is 7, whose Square is 49, and Cube 343. And so the summe of the Compound Square and Cube Decimal thereof appears by adding + to +, and — to —, and then deducting the — out of the + thus;

+ 1,2	+ 1,44	+ 1,728	—2,160
— ,5	+ ,25	+ ,900	— ,125
7 Root.	+ 1,69	+2,628	—2,285
49 Square.	—1,20		
,343 Cube.		+,343 Cube.	
	+ ,49 Square.		

Roots of Simple Decimals how extracted when the Index will be parted.

To extract the Root of a Decimal, if it be Simple, and the Decimal Denomination to the Right Hand be such as will be parted by 2 for the Square, 3 for the Cube, *&c.* then prick the Number, and extract the Root according to the Quantity desired, as before *Book* 2. *Part* 2. *Chap* 3. And to find the *Index* of the Root do thus: For the Square take half the *Index* of each pricked Figure, for the *Index* of the several Figures of the Root. For the Cube take $\frac{1}{3}$ part of the *Index* of the pricked Figures. For the Biquadrate take $\frac{1}{4}$ part, *&c.*

Example in the Square.

As in extracting the Square Root of 0,0225, the Root will be found 15, which is 0,15, because half the *Index* of the pricked 2 being Seconds in the Square shall be Primes in the Root, and half the *Index* of 5 in the Square being (4) in the Root, shall be Seconds

Example in the Cube.

So in extracting the Cube Root of 0,003375, the *Index* of the pricked 3 in the Cube being Thirds, shall be Primes in the Root. And the *Index* of 5 in the Cube being (6), shall make the *Index* of 5 in the Root (2), which is the third part of (6).

	2 (4) (2)		3 (6) (2)
Square	225 (15 Root.	Cube	3375 (15 Root.
	1 :		1 :
	10:		15 :
	25		75:
	225		125
			3375

When the Index will not be parted.

If the Decimal Denomination of the Simple Decimal, whose Root you would extract, will not be parted as above, then adjoyn a Cypher or more to the given Number to make the *Index* so divisible, and then extract the Root as before.

Example in the Square.

As if the Square Root were desired of 0,0015129, because (7), the *Index* of 9, cannot be halfed, I adjoyn a Cypher or 3,5, or more Cyphers proportionably, and extracting the Root with 1 Cypher the Root shall be (4), with 3 Cyphers 5, *&c.* according to the Cyphers adjoyned.

Example in the Cube.

Also if the Cube Root be desired of 0,01860867, because (8) the *Index* of 7, cannot be divided by 3, I adjoyn a Cypher, or 4,7, or more Cyphers. But if the given Number had been 0,1860867, then the *Index* of 7 being (7), I must adjoyn 2 Cyphers to make the *Index* divisible by 3, and so increase the Cyphers proportionably to the Quantity of the Number given; for that the more Cyphers being adjoyned the nearer the truth will the Root be, as before in the *Third Chapter* of *Book* 2. *Part* 2. §. 6. was shewn.

```
      | 7                    x|208                          | 20
   668|46   Root          0032|926   Root             x208| 104
                                                 0032926|551  Root.
   . . .  | (4)           . . .   | (3)              . . . .  (4)
(8) x5x290 | 388    (9) x8608670 | 264     (12) x86086700000( 2649
   9 : :                8 : :                      8 : : :
   544 :                9576 :                     9576 : :
    6144                 832744                     823744 :
Remain 746           Remain 208926                  188821449
                                            Remain   20104551
   151290 Square        18608670 Cube
                                                 18608670000 Cube.
```

To extract the Root of a Compound Decimal; out of the Left Hand Number extract the Root of the Quantity desired, and then by your Divisor enquire for another Quotient Figure, and so proceed according to the instructions before given for Extraction of Roots. *Roots of Compound Decimals extracted.*

As to extract the Square Root of 1,44 —1,20 +,25, the Square of 1,44 is 1,2 which doubled is 2,4, the Divisor, by which 1,20 divided yieldeth 5 in the Quotient, which shall be —, because the sign of 1,20 was —, and of 1,2 the Divisor was +, and the *Index* of 5 shall be Primes; then the Square of 5 is 25, which substracted leaves 0; and the Root is 1,2 —0,5. *Example in the Square.*

Likewise to extract the Cube Root of 1,728 —2,160 +,900 —,125, the Cube of 1,728 is 1,2, the Divisor is 4,32, by which 2,160 divided giveth in the Quotient 5, which shall be —, because the Signs were unlike, and the *Index* Primes; then the Square of —0,5, which is +2,5, multiplyed into +3,6, the triple of 1,2, makes +,900 for the next Substraction. And lastly, the Cube of —0,5 is +0,125, which substracted leaves 0. *Example in the Cube.*

```
        Square.                                  Cube.
 . :              Root.            .   .                       Root.
 x,44 —x,20 +,25 | 1,2 —0,5        1,728 —2,160 +,900 —125 | 1,2 —0,5
   .                                 .
 1 :                               1 :
  4 :                              6 :
  4   +2,4 +25                    12 :  +4,32 +,900 —,125
      —0,5                          8   —0,5
      —1,20                             —2,160
```

Decimal Figurals like others prove their Production by Extraction, and Extraction by Production. Moreover if the Compound Decimals be turned into Simple, the Extractions of their Roots, when the Operations are right, by comparing together will be found to agree. *Proof of Decimal Figuration.*

```
        Square.                              Cube.
  +1,44 —1,20 +,25                 +1,728 —2,160 +,900 —,125
  +0,25                            + ,900 — ,125

  +1,69                            +2,628 —2,285
  —1,20

  + ,49 Root 7 = 1,2 —,5                ,343 Root 7 = 1,2 —,5
```

Partis Primæ & Libri Tertii.

FINIS.

THE

THE SECOND PART OF THE THIRD BOOK.

CHAP. I.

Of ASTRONOMICALS.

THE second sort of Numbers specially Contract are *Astronomicals*, sometime called *Astronomical Fractions*, and other while *Geodaticals* of Time and Motion, which indeed they are; but having some Operations more peculiar to them, with other Contract Numbers, then common to other Fractions, I reserved the handling of them to this place. Astronomicals *the next sort of Numbers specially Contract. How called.*

Time is measured by Years, Dayes, Hours, &c. And the Cœlestial Motions by Circles, Signs, Degrees, &c. Time *and* Motion *how measured.*

Annus, or a Year, was by the *Egyptians* in their *Hieroglyphicks*, emblem'd out by a *Snake* biting her Tail in her Mouth, and from *Anguis* therefore some conceive proceeded *Annus*; but others, and more truly, from *Annulus*, a Ring or a Circle, which returns again into it self. Year *how called and Emblemed.* Annus, *whence the word.*

Neither the beginning of the Year, nor the length thereof, hath had the hap alwayes in all Nations to be alike; nor is the former yet with us in *England* identified. For the *Commonalty* begin the Year the First Day of *January*, the *Lawyers* the Five and Twentieth Day of *March*, and the *Astronomers* when the *Sun* enters the first Scruple of *Aries*. *Divers beginnings of the* Year.

The length of the Year is now generally agreed to be that space of Time in which the *Sun* is running his course through the *Zodiack*, beginning at the first point of *Aries*, until his revolution or return thither again. *Length of the* Year.

This space of Time is divided into Twelve parts, called Moneths, but very unequally in our Calendars or Almanacks, 7 of them having 31 Dayes, 4 but 30, and one but 28. More evenly ordered by *Julius Cæsar*, as some say, but distorted in Honour of *Augustus*, to equal the Dayes in the Moneths bearing their Names. Year *how divided.* Moneths *now unequal, and by whom so made.*

Some think the Year at first was, or ought to be divided by the course of the *Moon*, who gave being to the Name *Moneth*: And by the Law of *England*, a Moneth is but 28 Dayes, agreeing to the visibility of the *Moon*, there being some time for her both before and after Conjunction with the *Sun* to be hidden under his Beams or Rayes. But neither hath this afforded a *Basis* for the exact limitation of a Moneth; since besides the time of the *Moons* visibility, called the *Apparition Moneth*, and the time when most proper to administer *Physick*, called the *Medicinal Moneth*, containing 26 Dayes, 12 Hours, according to *Johannes de Sacra Bosco*: There is the *Periodical Moneth*, and the *Synodical Moneth*; the former of which called also the Moneth of *Paragration*, is the space in which she makes her Revolution, and returns to the place she made her last ☌ with ☉, and the latter called by some the Moneth of *Consecution*, the whole space from her last Conjunction with the *Sun*, unto her next: Both which by reason of the sometime swift, and sometime slow Motion of both Luminaries, take up more or less time accordingly, then can be comprised in any precise Number of Dayes without surplusage. The *Periodical* containing 27 Dayes, 7 Hours, 43 Minutes, 7 Seconds; and the *Synodical* 29 Dayes, 12 Hours, 44 Minutes, 4 Seconds, by common Computation. Moneth *whence the word.* *Sorts of* Moneths. Apparition Moneth. Medicinal Moneth. Periodical *or Moneth of* Paragration. Synodical *or Moneth of* Consecution. *The length of the* Periodical *and* Synodical Moneths.

Moneth omitted by some, by others reckoned at 30 Dayes. Moneth of the Jews. Legal Moneth how much. Week the content. Legal Moneths in the Year.

Because of these differences about the Moneth, some omit them among the certain Denominations, others in their Operations accompt 30 Dayes to a Moneth, which some conceive to be the Old-Accompt of the *Jews*; for comparing *Gen.* 7. 11 with *Gen.* 8. 3, 4 the space from the seventeenth day of the second Moneth, to the seventeenth day of the seventh Moneth, is reckoned 150 Dayes, which being 5 Moneths, allowes 30 Dayes for every Moneth. And others use neither them nor the Calendar Moneths, but the Legal Moneth of 28 Dayes, as aforesaid, which divided into 4 parts called Weeks, makes every Week 7 Dayes, and by this accompt 13 Moneths in the Year.

Year certainly divided. Length of the Tropical Year. Bissextil or Leap Year.

The Year then for certainty is subdivided into Dayes, and containeth commonly 365 Dayes, and almost 6 Hours, but the true Tropical Year by late Writers is not above 365 Dayes, 5 Hours, 49 Minutes, 4 Seconds and 21 Thirds; for which odd time in *Arithmetick*, 6 Hours is usually taken, which in every Fourth Year called *Bissextil* or Leap-Year, maketh another Day, and it is added to the end of the Moneth of *February*, which then hath 29 Dayes, or else in the Common Years but 28; so hath the Leap Year 366 Dayes.

Day Natural and Artificial.

Dies, or a Day Natural, is in *Astronomers* accompt that space of Time run out while the *Sun* passeth from the *Meridian*, ere he can come thither again, and is divided into 24 equal parts called Hours, and comprehendeth both Day and Night But a Day Artificial is sometime limited between ☉ and ☉, or *Sun* Rising and *Sun* Setting, or to a quantity of Hours, as a Working-Day 12 Hours, from 6 at Morning till 6 at Night.

Hours how subdivided.

Every of these Hours is divided into 60 parts, called Minutes, and every Minute into 60 lesser divisions, called Seconds; every Second into 60 Thirds, and so lower into Fourths, Fifths, Sixths, *&c.* by 60.

Of Motion. Circle how divided. Degrees what part thereof how called. How many to a Sign.

Touching Motion, it is to be remembred, That the Circumference of the Heavens, as also every Circle, is parted into 360 divisions, called Degrees or Grades from the Latin *Gradus*, 30 of which make one Sign, so 12 Signs make the Circle; for 12 times 30 is 360, answerable to the division of Time into 30 Dayes for a Moneth, and 12 Moneths to the Year.

Degrees subdivided. Minutes and Miles how agree.

Every Degree contains in Longitude or Length 60 Minutes, which are not to be taken for Minutes of Time, but Minutes of Measure, called also Scruples, and sometime Primes. Every of these Minutes of Measure in the Heavens is generally accompted to answer to one Mile on the Earth. Nevertheless Mr. *Oughtred* in his *Circles of Proportion* is of Opinion, that one Degree or 60 Minutes answer more truly to $66\frac{1}{4}$ Miles.

These Minutes, as well as Minutes of Time, are subdivided into 60 Seconds, the Second into 60 Thirds, and so into Fourths, Fifths, Sixths, *&c.* of which 60 lesser are comprehended in one of the next greater.

The reason of the Subdivisions by 60.

That an exact Measure both of Time and Motion might be had, the Antients have thought fit to divide one Hour of Time, and one Degree of Motion into 60, and so downwards, unto very small subdivisions, as aforesaid. Forasmuch as alwayes an whole Year, Moneth or Day, *&c.* is not the subject of the Question, neither the moving of the Cœlestial Bodies to be alwayes measured by whole Circles, Signs, Degrees, *&c.* but sometimes Parts or Fractures of the whole are useful And the rather have they chosen 60 for the Integer or Denominator, because it is a Number that may receive more equal divisions than any Number under 100; for it may be parted by 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30.

Denominator 60 certain and omitted. Difference of Astronomicals from Decimals.

The Denominator 60 being certainly known is alway omitted, as the Decimal Denominators are, and the given Numerators only used. And as in Decimals the Numbers exceeding the Units place to the Left Hand pass for Integers; so these. Nevertheless here seems to be some difference in that beyond the Units place to the Left Hand (1 Day in Time, and 1 Degree in Motion being generally accompted the Integers) here are Fractions besides those at the Right Hand, as Signs in Motion, and Moneths in Operations of Time; for the utmost Integer or greatest Geodætical of Time is the whole Year, and of Motion the whole Circle, notwithstanding although sometime between the Unit and utmost Integer such Fractions be found, yet their Denominators being certain they are omitted, and accompted as Integers.

Astronomicals why so called. Sometime called Physical Fractions. Physical Signs what. Signs with Astronomers.

Because the use of these Divisions is most conversant about *Astronomy*, they are called *Astronomicals*, or *Astronomical Fractions*: Some Writers call them *Physical Fractions*, and with *Alphonsus* divide the Circle into 6 divisions, and then one Sign containeth 60 Degrees, and the other divisions are as before, which is certainly the most expedient way for Multiplication and Division; but *Astronomers* in all their Calculations generally reserve the Sign of 30 Degrees, and accordingly reckon all the Aspects both Old and New, Sinister and Dexter.

Aspects Old and New, how many Degrees therein.

Old Aspects						New Aspects				
Names.	*Marks.*	*Signs.*		*Degrees.*		*Names.*	*Marks.*	*Signs.*		*Degrees.*
Sextile	⚹	2	or	60		Semisextile	SS. or ⚺	1	or	30
Quartile	□	3		90		Semiquintile	♡.	$1\frac{1}{5}$		36
Trine	△	4		120		Semiquartile	∠.	$1\frac{1}{2}$		45
Opposition	☍	6		180		Quintile	Q. or ♡	$2\frac{2}{5}$		72
						Sesquiquintile	Td. or ⧖	$3\frac{3}{5}$		108
						Sesquiquadrate	⚼.	$4\frac{1}{2}$		135
						Byquintile	Bq. or ⧖	$4\frac{4}{5}$		144
						Quincunx	Vc. or ⚻	5		150

Denominations of Late use.

Late Writers generally use two kinds of Names or Denominations, the one called *Sexagenæ*, and the other *Sexagesimæ*, both from the *Latin* importing Sixties, because of the general Denominator 60.

Sexagenæ what.

Sexagenæ are the collection of Degrees all under 60, standing in the place of the Unit, and having a Cypher to their *Index*, then by Collection all above 60, and under 3600, fall in *Sexagenæ primis*, and have 1 for their *Index*; 2 is the *Index* of *Sexagenæ* Seconds, 3 of *Sexagenæ* Thirds, and so of other Numbers respecting Motion.

The Collection of Dayes under 60, resembling the former stand in the place of the Unit, and all above 60, and under 3600, possess the second place from the Unit to the Left Hand, and are called *Sexagenæ Primes*, and 60 times 60 Dayes, which is almost 10 Years, make one *Sexagenæ* Second, and so they are marshalled to Thirds and Fourths, *&c.* And that Hours may yield the place of Unity to Dayes, they are to be reduced to Minutes of a Day, as in the next *Chapter*. Nevertheless where there are no *Sexagenæ*, Hours may keep the place of Unity without Reduction.

Sexagesimæ what.

Sexagesimæ, both of Motion and Time, are set on the Right Hand of their Integers (Degrees or Dayes) like the Decimals on theirs, and are to the *Sexagenæ*, as Decimals to their Integers, and are the division of Minutes, Seconds, Thirds, *&c.* by 60, as aforesaid like the Decimals by Tens.

How both marked.

Both the *Sexagenæ* and *Sexagesimæ* are marked with exponent *Indices*, as the Decimals are, which accordingly shew the power of the Numerator either affirmatively or negatively, that is, either above or below the Unit.

Indices differently expressed.

These *Indices* are found differently expressed, and sometime the one, and sometime the other Form are used, which is not at all material whether.

Examples.

	Sexagena's.							*Sexagesima's.*							
Thus,	&c.	6	5	4	3	2	1	0	(1)	(2)	(3)	(4)	(5)	(6)	&c.	Common with
	&c.	6	5	4	3	2	1	0	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{6}$	*&c.*	Decimals.
or																
Thus,	&c	vj	v	\`\`\`\`	\`\`\`	\`\`	\`	0	′	″	‴	⁗	v	vj	&c.	Proper to
	&c.	æ6	æ5	æ4	æ3	æ2	æ1	0	′	″	‴	⁗	v	vj	*&c.*	Astronomicals

So if I see this Number 3\`\`\`, 1\`\`, 12\`, 15 Dayes, 16′, 10″, 12‴, I read it thus, 3 *Sexagenæ* Thirds, 1 *Sexagenæ* Second, 12 *Sexagenæ* Primes, 15 Dayes, 16 Minutes or Primes, 10 Seconds, and 12 Thirds of a Day; and so of others.

Astronomicals Simple and Compound.

Moreover if the Signs + and — be used, let the Astronomicals be counted Compound, but if not Simple.

Cyphers supply intermediate places.

If any intermediate Denominations in Numbers given be omitted, let the places be supplyed with Cyphers, as 2\`\` and 2′, thus, 2\`\`, 0\`, 0°, 12′.

CHAP. II.

Reduction of Astronomicals.

Astronomicals reduced how many wayes.

Reduction, called also Conversion of Astronomicals, is threefold, *viz.*

Either to convert { Geodæticals / Decimals / Astronomicals of one sort into another } into Astronomicals, or the contrary.

The

1\. *To reduce Common Signs into Physical and Years into Sexagenas, or contrary.*

The first sort serveth either to turn Common Signs into *Physical*, and Years or Moneths into *Sexagenas*, or the contrary.

To reduce Common Signs into Physical.

To turn Common Signs into Physical, half them, or reduce Geodætically by 30, the Signs into Degrees, and add the odd Degrees (if any) to them, then divide by 60, what Degrees remain place in the Units place, and the Numbers in the Quotient, if under 60, are Physical Signs, and to be set in the place of *Sexagenæ* Primes; if above 60, divide the Quotient again by 60, and place the Remainers as *Sexagenæ* Primes, and the Quotient of this Division for *Sexagenæ* Seconds, *&c.*

To reduce Physical Signs into Common.

On the contrary to turn Physical Signs into Common, or *Sexagenæ* of Motion into Circles and Signs; double or multiply the Physical Signs by 2, or reduce Geodætically all the *Sexagenæ* into Degrees, multiplying by 60, and then divide the summe by 30, and the Quotient, if under 12, shall be Signs; if above 12, divide by 12, and the Quotient of this Division shall be Circles.

Example of both.

As if 9 Signs, 12 Degrees, be given to be converted into Physical, reduced into Degrees they are 282, which divided by 60, gives 4 Physical Signs or *Sexagenæ* Primes, and 42 Degrees to be set in the place of Unity; which if on the contrary had been given to have been turned into Common Signs, the 4 Signs multiplyed by 60, make 240, to which the 42 Degrees added make 282 Degrees, which divided by 30 gives 9 Signs in the Quotient, and 12 Degrees remaining.

Common Signs. Degrees.
9 —— 12
30
270
12
282 Degrees.

Degrees. Signs Physical.
|4
28|2(4
60

Degrees. Signs Common.
(1
28(2 (9
30

` °
4 —— 42
60
240
42
282

To reduce Years &c. into Sexagenæ.

To turn Years or Moneths into *Sexagenæ*, reduce all Geodætically into Dayes accompting in, as in Reduction of Geodæticals was observed, the Dayes supernumerary for the Leap-Years, and adding in also the odd Dayes given in the Number, if any be, then divide by 60, and so Quotient after Quotient as far as you can, the Remain of the first Division being Dayes is to be placed as Integers in the Units place, the other Remains and last Quotient in their places orderly to the Left Hand, as *Sexagenæ* Primes, Seconds, Thirds, *&c.*

To reduce Sexagenæ into Years.

On the contrary to turn *Sexagenæ* of Dayes into Years, multiply Geodætically all the *Sexagenæ* into Dayes by 60, and then divide by the Dayes in one Year, and from the Remain substract the Dayes for the Leap-Years of the Quotienary Number, and the rest of the Remain shall be the odd Dayes, which if occasion be may be turned into Moneths by Division with the Dayes of one Moneth.

Example of both.

As if 1000 Years, 20 Dayes, were to be converted into *Sexagenæ*, the Common Dayes in 1000 Years by Geodætical are found to be 365000, to which 250 Dayes added, because there are so many Leap-Years in 1000, and the 20 Dayes given also added, make the Total 365270; then divided by 60, there is 1‴, 41″, 27′, 50°; that is, 1 *Sexagenæ* Third, 41 *Sexagenæ* Seconds, 27 *Sexagenæ* Primes, 50 Dayes; which if on the contrary were to be turned into Years, after Reduction by 60, and divided by 365, the Quotient will be 1000, and from the Remain 270, if 250 Dayes be substracted for the Leap-Years, there will remain 20 odd Dayes.

2
1000(250 } Leap Years
4

1000 Years.
365
365000
250
20
365270 Dayes.

4|5 °
36527|0(6 0
|2 `
608|7(6 0
|4 ``
10|1(1 ```
60

``` `` ` °
1 —— 41 —— 27 —— 50
60
60
41
101 ``
60
6060
27
6087 `
60
365220
50
365270 °

Dayes. Years.
365 | 270(1000
365 250
365 20
365
365

The
```

2. *To reduce* Astronomicals *into* Decimals, *or the contrary.* Sexagenæ *into* Integers, De-cimals *into* Sexagesimæ.

The second sort of Astronomical Reduction is to turn Astronomicals into Decimals, or Decimals into them, as *Sexagenæ* into Integers, and *Sexagesimæ* into Decimals, or the contrary.

To turn *Sexagenæ* into Integers, or Decimals into *Sexagesimæ*, multiply continually by 6, every time removing the *Seperatrix* one place, but on the contrary to turn Integers into *Sexagenæ*, or *Sexagesimæ* into Decimals, removing the *Seperatrix*, as before, divide continually by 6, proceeding both wayes till you come to the Units place: For Division by 60 removeth the *Seperatrix* one place towards the Left Hand, and divideth by 6; and Multiplication by 60, promoteth the *Seperatrix* one place towards the Right Hand, and multiplyeth by 6; and so the Cypher being cut off by the *Seperatrix*, the work is short and easie.

Variety.

Otherwise by common Geodætical Reduction bring all the Circles and Signs, or *Sexagenæ* thereof, into Degrees, also Years and Moneths, or *Sexagenæ* thereof, into Dayes by Multiplication, and them reserve for the Integers; then reduce by Multiplication with 60, all the *Sexagesimæ* into the lowest Denomination, and the last Product thereof adjoyning Cyphers if need be, divide by 60, Figurated to a Power of the Second, Third, Fourth, &c. Quantity, according to the Right Hand Denomination of the *Sexagesimæ*, and the Quotient shall be the Decimals. And to convert Decimals given into the other, multiply the Decimals as aforesaid, and divide the Integers by their proper Denominators.

Example of both.

As if 6‶, 30‵, 20°, 15′, 45″, were to be reduced into Integers and Decimals; by the first way I place them with the seperating Lines, as at *A.* and beginning at top, multiply the 6‶ by 6, the 36 produced is subscribed under the upper *Seperatrix*, as at *B.* which with the 30‵ multiplyed by 6, produceth 234, adjoyned to the 20°, makes in all 23420 Integers, as at *C.* then beginning with the *Sexagesimæ* at bottom, I divide 45″ by 6, (supposing 00 adjoyned) I superscribe over the lower *Seperatrix* the Quotient 75, as a Decimal to 15′, at *D.* And lastly, I divide 1575 (with 2 Cyphers also supposed to be adjoyned) and superscribe the Quotient 2625, as the Decimals desired to be adjoyned to the Integers; and the whole work stands compleat at *E.*

```
   A.          B.          C.           D.            E.
  6‶|         6‶|         6‶|          6‶|            6‶| ×6
 30‵|        36          36           36            36
  20°|       30‵|         30‵|         30‵|           30‵|
  ,15|        20°|       23420°|      23420°|     6) 23420|2625 ×6
  ″45         ,15|        ,15|         ,15|75          ,15|75
              ″45         ″45          ″45         6) ″45
```

Where there are many Species, this is much the shorter and more ingenuous way; the other for the Integers hath had its like often exemplified in Geodæticals, as at *F.*; for the 15′, 45″, after Reduction into 245″, I adjoyn 0000, and divide by 3600, which is the Square of 60, or the Power of the Second Quantity, the Number being ″ and the Quotient is 2625 at *G.* So is the Number as before 23420,2625.

```
   ‶   ‵   °   ′   ″                     (4)
   6  30  20  15  45   Reduced is 23420,2625.
  60          60
  ---         ---      (1)  60  Root.              x
  360         900      (1)  60                   2298  (6)    (4)
   30          45      ---------                 9450000 (2625   G.
F. ---         ---     (2) 3600 Square.          36    00
  390         945″     ---------                  36
   60         ---                                  36
  -----                                             36
  23400
     20
  -----
  23420 Integers.
  -----
```

And if 23420,2625 were given to be converted into Astronomicals this way, I continue Division of 23420, and Multiplication of ,2625 by 60, and obtain my desire.

Sexagenæ. *Sexagesimæ.*

```
 5   °  |3 `  "          ,2625
234|20( 39|0( 6          |   60
 6    0   60           ---|------
                       ' 15| 7500
                           |   60
                       ---|------
                       "45| 0000
```

But by the other way proceed, as before, by 6, to multiply the Decimals, and divide the Integers, cancelling, as some do, the Figures divided or multiplyed.

```
Numbers placed.      6"|  Sexagenæ.        6"|  Sexagenæ.
                     3 `                   3 `
                    390|°                 390|°
       |           23420,|2625           23420,|2625
23420,|2625                                 ,15|75
                                Sexagesimæ  " 45
```

3. Astronomicals reduced one into another.

The third sort of Astronomical Reduction is used to convert one Astronomical into another.

As { Degrees into { Decimals of a Day / Hours and Decimals of an Hour } or the contrary.
   { Hours into { Decimals of a Day / Minutes of a Day } or the contrary.

To reduce Degrees, &c. into Decimals of a Day. Example.

To turn Degrees with or without Decimals annexed to them, into Decimals of a Day, divide by 360, that is 6 × 60. And contrarily to turn Decimals of a Day into Degrees, multiply by 360, that is 6 × 60.

As to convert 236,4276 Deg. into Decimals of a Day, I first take the sixth part of the Number, or divide by 6, and subscribe, which is 39,4046, of which the 60th part is 0,6567433 for the Decimals of a Day desired. And if this Number had been given to procure Degrees, aud Decimals of a Degree, I had multiplyed first by 60, and the Number superscribed multiplyed again by 6, as at *H*, but by the common way, the one and the other is at *I.* and *K.* with Cyphers adjoyned, as needful, in the Division at *I*, and supply for the defect of the Decimal at *K*, making the 588(6), to be 6(4), or accompting them so, as most usual in Imperfect Decimals.

```
Divisors.      H.                           I.                          K.
{6 )    236|4276   Multiplyers.       2  xxx|1                     0,6567433
{60)    39|4046        × 6 }         20 265zz|2       (7)              |   360
        0|6567433      × 60}        236,4276000|0( 6567433         ----|------
                                     36       0                      39|4045980
                               (7)   36                             197|02299
                                0     36                            ---|------
                               ---     36                           236|427588
                               (7)      36
                                         36
                                          36
```

To reduce Degrees, &c. into Decimals of an Hour. Example.

To turn Degrees with or without Decimals annexed to them, into Hours and Decimals of an Hour, divide by 15, that is 3 × 5. And on the contrary to turn them into Degrees, multiply by 15, that is 3 × 5.

As to convert the former Number of Degrees, and Decimals of a Degree 236,4276 into Hours, and Decimals of an Hour, I first divide by 3, and the Number 78,8092 subscribed, divide by 5, and the subscribed 15,76184, as at *L.* is the desire; which if given by Multiplication first by 5, and the Product superscribed multiplyed by 3, would have produced 236,4276, as by Common Division and Multiplication by 15, at *M.* and *N.* appears.

Divisors.

```
Divisors.      L.                              M.                          N.
                                         1                             15,76184
{3)  236|4276    Multiplyers.            81 912 (5)       (5)              |    15
{5)   78|8092      ×3}                   236,42760 ( 15,75184          ----|------
      15|76184     ×5}                   15                              78|80920
                                           15                           157|6184
                                   (5)       15                        ----------
                                    0         15                       236,4276
                                   ---         15                      ----------
                                   (5)          15
                                                 15
```

To turn Hours with or without Decimals annexed to them, into Decimals of a Day, divide by 24, that is 4 × 6. And to turn them into the other, multiply by 24, that is 4 × 6.

To reduce Hours, &c. into Decimals of a Day.

Example.

As to convert the aforesaid 15 Hours, and 76184 Decimals of an Hour into Decimals of a Day, I first divide by 4, and subscribe the Quotient 3,94046, the which I divide by 6, and the Number subscribed is 0,6567433, the Decimals of a Day desired as at *O*. Contrariwise, if that same Number had been given, I would have multiplyed first by 6, and the Numbers superscribed on the *Seperatrix* would have been 3,94046 as before, which multiplyed by 4, would have produced the 15,76184. And so by Common Division or Multiplication with 24, the Numbers agree as at *P*. and *Q*. allowance being made for the Imperfect Decimal at *Q*.

```
Divisors.      O.                              P.                          Q.
                                          1 1                          0,6567433
{4)   15|76184   Multiplyers.            1 317088|8        (7)             |     24
{6)    3|94046     ×4}                   15,7618400 ( 6567433          ----|--------
       0|6567433   ×6}                   24                               2|6269732
                                           24                            13|134866
                                   (7)      24                         -----------
                                    0        24                        15,7618392
                                   ---        24                       -----------
                                   (7)         24
                                                24
```

To turn Hours, with or without Minutes and other smaller parts of an Hour, into Minutes of a Day, that is, into Sixtieth parts of a Day, which Hours are not, and so into other smaller parts of a Day, multiply every Number by 5, and half the Product, that is multiply by $2\frac{1}{2}$. And on the contrary to turn the parts of a Day into Hours, and parts of an Hour, double every Number, and divide by 5, that is divide by $2\frac{1}{2}$, because one Hour answers to 2′ 30″ of a Day, to make the Denominator 60, there being twice 24 and 12 therein.

To reduce Hours, &c. into Minutes of a Day.

Example.

As to convert 12 Hours and 40 Minutes of an Hour into parts of a Day: If I multiply by 5, the Products 60, 200, halfed are 30′, 100″, which 100″, because above 60, I carry a Minute to 30′, and subscribe the 40″ remaining, so the result 31′, 40″, of a Day, as at *R*. which if given to be turned into Hours, I first double them, and they are 62′, 80″; then dividing 62 by 5, I get 12 Hours, and the 2′, 80″, remaining turned into ″ make 200, which divided by 5 give 40′ of an Hour. The Multiplications and Divisions by $2\frac{1}{2}$ are at *S*. and *T*.

```
   Hours.  ,    R.                 S.               T.
     12   40    ″               12 . 40           ′    ″
 5 × Day  60   200  ) 5           2½              31 . 40
 2)       30   100  × 2        ---------            2
          ---------            24 | 80          --------- Hours ′
          31    40              6 | 20          62 . 80 ( 12 . 40
                               ---------        ---------
                               30 . 100            5
                               ---------
                               31 . 40
```

The Reductions of this *Chapter* being reciprocal serve for Proof each to other without further illustration.

Proof of Astronomical Reduction.

CHAP.

CHAP. III.

Addition of Astronomicals.

Astronomicals *added.* *Simple.*

ADdition of Astronomicals is either Simple or Compound.

Simple Addition differeth nothing from Geodætical Addition, for you begin at the Right Hand, and add the Numbers of like Denomination to their fellows: And if at any time the Numbers of that Denomination you are adding exceed 60, for every 60 carry an Unit to the next Left Hand Denomination, and subscribe the overplus: And so proceed in order to the Left Hand.

Example.

As to add 13‶, 26‵, 59°, 30′, 45″, to 2‶, 5‵, 17°, 15′, 30″, the Total will be 15‶, 32‵, 16°, 46′, 15″; which is so plain, explanation is needless.

	‶	‵	°	′	″
Addends	13	26	59	30	45
	2	5	17	15	30
Total	15	32	16	46	15

Compound.

Compound Astronomical Addition is like Compound Decimal Addition; for taking the Lesser Numbers out of the Greater, to the Remaining Total subscribe the Sign of the Greater.

Example.

As to add 13‶, 26‵, 59°, 30′, 45″, with —2‶, 5‵, 17°, 15′, 30″; these must be substracted from the other, and the Total remaining will be +11‶, 21‵, 42°, 15′, 15″. And if the Signs be intermixt, as to add 4‵+10° —40′, to 3‵—20° +10′, the Total will be +7‵—10° —30′. As the Examples shew sufficiently without further explanation.

	‶	‵	°	′	″		‵	°	′
Addends	+13	26	59	30	45		+4	+10	—40
	—2	5	17	15	30		+3	—20	+10
Totals	+11	21	42	15	15		+7	—10	—30

When the Signs are intermixed and Units carried to the next.

If where the Signs are intermingled in adding the Numbers of one Denomination together there amount to more than 60, for every of which an Unit be carried over to the next Left Hand Denomination, if the Sign there be changed, then substract an Unit for every 60 so carried, and set down the Total of the rest.

Examples.

As to 3′—50″, add 2′—40″, or —3′+50″ to —2′+40″; in both cases the 50″ and 40″ make 1′ and 30″, in the first instance —, in the second +, which 1′ carried over to the Left Hand ′, being of a contrary Sign, I therefore substract an Unit from the Total 5′, and subscribe the remaining 4′, as followeth;

	′	″		′	″
Addends	+3	—50		—3	+50
	+2	—40		—2	+40
Totals	+4	—30		—4	+30

Proof of Astronomical Addition.

Astronomical Addition hath the same benefit of being proved by Astronomical Substraction, as other Additions by their respective Substractions. And as Decimals may be also proved by reducing the Numbers into Geodæticals, and comparing the Totals of both Additions together, as equal in value, when the Operations are right after the manner used in Decimals.

As to instance in the 2 last Examples, thus;

′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
+3 —50	+2 —40	+4 —30	—3 +50	—2 +40	—4 +30
60	60	60	60	60	60
+180	+120	+240	—180	—120	—240
— 50	— 40	— 30	+ 50	+ 40	+ 30
+130	+ 80		—130	— 80	
Totals +210		Equal +210	Totals —210		Equal —210

CHAP.

CHAP. IV.

Substraction of Astronomicals.

Substraction of Astronomicals is either Simple or Compound.

Simple Substraction differeth nothing from Geodætical Substraction; for you begin at the Right Hand, and withdraw the under Number from the uppermost of like Denomination, and subscribe the Remain: And if the Number beneath be the greatest then borrow 60, and supposing the same to be added to the upper, make Substraction from the Total, and for every 60 borrowed pay an Unit in the next Left Hand Denomination; except where the Subtrahend is the greatest, and there, as in Decimals, the difference shall be taken with the contrary Signs; or else proceeding as before till the Left Hand Denomination, and the Sign there changed to the difference, the Signs of all the other Remains shall be as the given Numbers. Astronomicals *substracted. Simple.*

As to substract 13‶, 26‵, 59°, 30′, 45″, from 15‶, 32‵, 16°, 46′, 15″, the Remain shall be 2‶, 5‵, 17°, 15′, 30″; which needs no explanation. *Example.*

	‶	‵	°	′	″
Greater Number	15	32	16	46	15
Subtrahend	13	26	59	30	45
Remain	2	5	17	15	30

But if 15‶, 32‵, 16°, 46′, 15″, were to be substracted from 13‶, 26‵, 59°, 30′, 45″; here because the Subtrahend is the greatest Number in the ′, I borrow 60, which I pay again, by counting 16° one more than it is, or 59° an Unit less than it is. And in the ‵ I borrow 60 again, for which I count the 15‶ to be 16‶, or the 13‶ at top but 12‶; but now because I cannot take 16‶ from 13‶, or 15‶ from 12‶, but shall want 3‶, I set down the difference 3‶ with the contrary Sign, as at *A.* Otherwise, as in Decimals, I change the Sign to the difference of every Number in the Subtrahend too great to be substracted from the Numbers that stand over him respectively at top, as at *B.*

	‶	‵	°	′	″
	+13	26	59	30	45
A.	+15	32	16	46	15
	— 3	+54	+42	+44	+30

	‶	‵	°	′	″	
	+13	26	59	30	45	Upper Numbers
B.	+15	32	16	46	15	Subtrahends
	— 2	— 6	+43	—16	+30	Remains

Compound Astronomical Substraction is like Compound Decimal Substraction; for where Numbers of unlike Signs are to be substracted one from the other, the Numbers must be added together, and their Totals shall be the particular Remains; and their Signs shall be the upper Numbers Sign. *Compound.*

As if the Remain above at *B.* were to be substracted from the upper Number from which Substraction there is made, then must the Subtrahend there be the Remain here, as followeth; *Example.*

	‶	‵	°	′	″	
Upper Number	+13	+26	+59	+30	+45	Upper Number
Subtrahend here	— 2	— 6	+43	—16	+30	Remain above
Remain here	+15	+32	+16	+46	+15	Subtrahend above.

If where the Signs are intermixt in adding up the Numbers of contrary Signs, the summe exceed 60, then the overplus is to be subscribed under that Denomination where the summe ariseth, and for every 60 an Unit is to be carried over to the next Left Hand Denomination, which Unit shall have the Sign of the upper Number. And if the Number of the next Left Hand Denomination annexed to him be of a like Sign, then *When in contrary Signs an Unit is carried to the next.*

this Unit or Units so carried over shall be added thereto, but if of a contrary Sign substracted therefrom.

Examples. As if —2′ +50″, be substracted from +3′ —10″, the 50″ and 10″ of contrary Signs added make 60, for which 1, that is —1′, because 10″ the upper Number is — is carried to the +3′, which being contrary is substracted therefrom, and so leaves but +2′ to be added with —2′, which make +4 for the Remain, as at *C*.

But if +2′ —50″ be substracted from —3′ +10″, the 50″ and 10″ added make 60, as before, but + because 10″ the upper Number is + ; for which 60 is +1′ carried to the —3′, and being contrary and substracted therefrom, leaves but —2′ to be added with +2′, which make the Remain —4′, as at *D*.

Contrarywise, if —2′ —50″ be substracted from +3′ +10″, or +2′ +50″ from —3′ —10″, in the former the Unit carried over is +1′, and in the latter —1′, and to be added accordingly to the 5′ amounting of —2′ and +3′, or +2′ and —3′, making the Remains +6′ in the one, and —6′ in the other, as at *E*. and *F*.

	C.	*D.*	*E.*	*F.*
	′ ″	′ ″	′ ″	′ ″
Upper Numbers	+3 —10	—3 +10	+3 +10	—3 —10
Subtrahends	—2 +50	+2 —50	—2 —50	+2 +50
Remains	+4 —00	—4 +00	+6 +00	—6 —00

When Signs are intermixt, and the Subtrahend greatest. If among Numbers of intermixt Signs, some of the respective Species or Denominations, both in the Subtrahend and Number from which Substraction is to be made, be of like Signs, and it happen that the Number in the Subtrahend is the greater, then I am left at liberty, whether for the Remain I will change the Sign to the difference, or else as in Simple Substraction borrow 60. But if so, this must be remembred, that the Unit to be paid (for the 60 borrowed) in the next Left Hand Denomination, if the Sign thereof be contrary to the Sign of that Denomination where the 60 was borrowed, this Unit payable must be contrary; that is + with —, and — with + ; and although in both Remains the Numbers and Signs differ, they agree in value.

Examples. As to deduct —2′ +50″ from —3′ +10″, there after the former way, the Remain will be —1′ —40″; but after the latter way —2′ +20″; for to take 50″ from 10″, and 60″ borrowed to put thereto, the Remain will be +20″, for which 60, the Unit payable is +1′, and being affirmative therefore lessens the —2′ in the Subtrahend being of a contrary Sign, and makes it but —1′, which deducted from —3′ leaves —2′, as at *G*.

But to take +2′ —50″ from +3′ —10″, there after the former way, the Remain will be +1′ +40″, but after the latter +2′ —20″, because the 60 borrowed there is negative, and so being contrary to the +2′ in the Subtrahend is to be taken therefrom, and the remaining +1′ taken from +3′, leaves for the Remain +2′, as at *H*.

	G.	*H.*	
	′ ″	′ ″	
Upper Numbers, or Numbers from which Substraction is made	—3 +10	+3 —10	
Subtrahends	—2 +50	+2 —50	
Remains	—1 —40	+1 +40 by the former way	Equal.
Remains	—2 +20	+2 —20 by the latter way	

Proof of Astronomical Substraction. Astronomical Substraction will be proved by Astronomical Addition, as well as other Substractions by their respective Additions; and together with Decimals will abide the tryal by being turned into Geodæticals.

As in the last instance at *H*. will be plain.

′ ″	′ ″	′ ″	′ ″
+3 —10	+2 —50	+ 1 +40	+ 2 —20
60	60	60	60
180	120	60	120
— 10	—50	+ 40	— 20
+170 lacking	+70 is by the first Remain	+100 by the other	+100

CHAP.

CHAP. V.

Multiplication of Aſtronomicals.

Aſtronomicals multiplyed. Simple.

MUltiplication of Aſtronomicals is either Simple or Compound.

Simple Multiplication is like the fourth variety of the fourth Caſe of Geodætical Multiplication; for every Number of the Multiplicand is to be multiplyed by every Number of the Multiplyer. And to know the Denomination of the Products add the *Indices* together, as in Decimals, then collect the ſeveral Products or Multiplees into one Total Product. And in collection, if any file of Multiplees exceed 60, for every 60 carry 1 to the next Left Hand Denomination, and ſubſcribe the overplus.

Example in the Moons Diurnal Motion.

As ſuppoſe *Luna* in her ſwift Motion run in one Day 14 Degrees, 30′, and I would know according to that Diurnal Motion, how far ſhe will run in 3 Dayes, 6 Hours, and 40′ The 6 Hours, 40′, being reduced into Minutes and Seconds of a Day, according to the laſt kind of Aſtronomical Reduction mentioned in the *Second Chapter* before, make 16′, 40″. Then multiplying, as aforeſaid, Number by Number, the ſeveral Multiplees appear as at *A*, and collecting the ſumme, find 47 Degrees, 31′, 40″, or 1 Common Sign, 17 Degrees, 31 Minutes and 40 Seconds.

		°	′	″	‴	
	Multiplicand	14	30			*Index* ′
	Multiplyer	3	16	40		*Index* ″
A.	Multiplees	42	90			
			224	480		
				560	1200	*Index* ‴
	Total Product	47	31	40		

1200 / 60 (20″
480
560
—
10(4
106(0 / 60 (17′
90
224
—
3|31 / 60 (5°

Compound.

Compound Aſtronomical Multiplication is like Compound Decimal Multiplication; for every Number of the Multiplicand is to be multiplyed by every Number of the Multiplyer; and the Numbers of like Signs ſhall produce +, and unlike Signs —. And in collection of the Multiplees, by taking the + from the —, or — from the +, the Product may be contracted.

Example.

As to multiply 3′—20″, by 2′—30″, the Product ſhall be at large +6″—130‴+600⁗, as at *B*, which may be contracted by taking the 600⁗, that is +10‴ out of the —130‴, the Remain —120‴, that is —2″ taken out of +6″, will leave but 4″ at laſt.

		′	″	‴	⁗	
	Multiplicand	+3	—20			*Index* ″
	Multiplyer	+2	—30			*Index* ″
B.	Multiplees		+ 6	— 40		
				— 90	+600	*Index* ⁗
	Total Product		+ 6	—130	+600	
	Product contracted	+ 4″				

+600 / 60 (10‴
—130
+ 10
—
—120 / 60 (2″
+6″
—2
—
+4

Proof of Aſtronomical Multiplication

Aſtronomical Multiplication is to be proved by Aſtronomical Diviſion, as well as other Multiplications by their reſpective Diviſions; and together with Decimals will endure the tryal, if reduced into Geodæticals: As in the laſt Operation thus,

+ 3′ —20″	+ 2′ —30″	160″	
60	60	90″	
+180	+120	14400⁗	14400 / 60 (240‴ / 60 (4″
— 20	— 30		
+160″	+ 90″		

CHAP.

CHAP. VI.

Division of Astronomicals.

Astronomicals divided. Simple.

Division of Astronomicals is either Simple or Compound.

Simple Division is like the second variety of the fourth Case of Geodætical Division; for by the Number of the highest Denomination of the Divisor, the greater Denomination of the Dividend is to be divided, and thereby an apt Quotient Figure gotten, by which multiply the Divisor, and substract the Product from the Dividend, and set the Remains at top, which when to be brought to the Right Hand multiply by 60, or suppose it so done, and so continue the Division to the end of the work, or till a Quotient be gotten large enough for use. And to know the Denomination of the Quotient, substract the *Indices* as in Decimals.

[Si]mple Division 3 Cases.

This kind of Division may be throughly understood under the varieties in these three following Cases.

1. Case, When both Dividend and Divisor are single Numbers, though of a different *Index*.

2. Case, When the one of them is Single, and the other Plural.

3. Case, When both the given Numbers are Plural.

1. Data single.

First, Both the given Numbers Single may admit of two varieties; that is, either the Dividend greater than the Divisor, or less.

Dividend greatest and evenly. &c. Example.

If the Dividend be greater, and will be evenly divided by the Divisor, then the Numbers are divided as Integers, and the *Index* found as in Decimals.

As to divide 45'' by 5', the Quotient will be 9, and substracting 1 the *Index* of 5, from (2) the *Index* of 45, the Remain (3) is the *Index* of the Quotient 9, as below at *A*.

Dividend greatest and not evenly, &c.

If the Dividend be greater than the Divisor, and will not be evenly divided thereby, then after the first Quotient Figure gotten by dividing, as above, multiply the Remain by 60, and divide the Product by the Divisor, and add this Quotient to the former, and so proceed to the end of the work, or a Quotient large enough for your occasion.

Example.

As to divide 45'' by 12', the first Quotient Figure gotten will be 3''', the 9 which is left remaining multiplyed by 60 produceth 540, which divided by 12, giveth 45'''', as below at *B*.

Divisor greatest.

The other variety of this Case is when the Divisor is greater than the Dividend, and then multiply the Dividend by 60, and divide the Product by the Divisor; and if any thing remain, proceed as last abovementioned.

As to divide 9 Degrees by 10 Degrees. 9 Degrees multiplyed by 60 produce 540', which divided by 10 give in the Quotient 54', as at *C*.

A.

5') 45'' (9'''
(2)
1
Index (3)

B.

9
12') 45'' 00''' (3''' 45''''
(2) (3)
1 1
Indices (3) (4)

9''
60'
12') 540''' (45''''

C.

10°) 540' (54'
(1)
0
Index (1)

2. One Single and the other Plural. Divisor Single.

2. The one of the given Numbers Single, and the other Plural may also admit of a double variety, that is, either the Divisor Single, or the Dividend.

If the Divisor be Single, then thereby divide every Number of the Dividend, and carry over the Product of the Remains, if any be, reduced by 60, to the next Right Denomination, as above at *B*.

Example in the Hourly Motion of the Moon.

As suppose in 1 Day Natural, or 24 Hours, ☽ in her swift Motion runneth 14 Degrees, 58', 20'', and I would know her Hourly Motion; because 14 is less than 24, I multiply it by 60, and thereto add the 58', and the Total 898' divided by 24 giveth in the Quotient 37', and the 10' remaining multiplyed by 60, and carried to the 20'', make 620'', which divided by 24 gives 25'' in the Quotient, and there remaineth 20, which multiplyed as before, and the Product divided, addeth to the Quotient 50''', and the whole Quotient is 37', 25'', 50''', for the *Moons* Hourly Motion at the course aforesaid.

Divisor

Divisor Dividend Quotient

24°) 14° 58′ 20″ 00‴ (00° 37′ 25″ 50‴

24

14°	10′	20″
60′	60′	60′
840′	600″	24°) 1200‴ (50‴
58	20	00
24°) 898 (37′	24°) 620 (25″	
178	140	
10	20	

If the Dividend be single, as in the other varieties of this Case, then place Cyphers to the Right Hand as far as shall be needful, and then like the work of the former Examples, inquire with the first Figures of the Divisor for a Quotient Figure out of the Left Hand Numbers of the Dividend, and thereby multiply all the Divisor, and substract the Total of these Products from the Dividend, leaving the Remains at top, and then removing the Divisor inquire for another Quotient Figure, and so repeat this work till the Division be ended, or a Quotient large enough obtained. *Dividend Single.*

As suppose the mean Motion of the *Moon* from the *Sun* daily be 13 Degrees, 10′, 35″, and I would know when, according to that course, she will make her Revolution, or come to that Point of the *Zodiack* where she made her last ☌ with ☉. Then I divide 360 Degrees, which make the whole Circle, having adjoyned a convenient number of Cyphers, by 13°, 10′, 35″; and finding at first 27 times 13 may be had out of 360, I multiply the Divisor by 27, and the Products 355°, 45′, 45″, substracting leave remaining 4°, 14′, 15″; and then removing the Divisor, 13 may be had 19 times out of 4°, 14′, or 254′, the Divisor then multiplyed by 19, the Products to be substracted will be 4°, 10′, 21″, 5‴. And so in like manner proceeding to Thirds, which is far enough, the Revolution is 27 Dayes, 19′, 17″, 45‴ of a Day, and reduced into Hours is 27 Dayes, 7 Hours, 43′, 6″ of an Hour. And if occasion were, the Division might be continued, because there remaineth 2‴, 08⁗, 45ᵛ. *Example in the Revolution of Lunæ.*

|2 8 45

Index

9 55 5

(5) 3 53 55

(2) 4 14 15

(3) $13^{\text{Deg.}}$ 10′ 35″) 360° 00′ 00″ 00‴ 00⁗ 00ᵛ (27^{Dayes} 19′ 17″ 45‴ &c.

355 45 45

4 10 21 5

3 43 59 55

9 52 56 15

3. When both Dividend and Divisor are Plural, the Operation is like the last aforegoing; for the Cyphers adjoyned represent the Dividend there to be of Plural Denominations. *3. Data Plural.*

As in Case ☽ want 2 Degrees, 12′, 40″, 13‴, 20⁗, of *Aldebaran*, or some other fixed *Star*, and her Hourly Motion be 37′, 40″, I would know when she will be in Conjunction with the same *Star*: Then inquiring with 37′, out of 2°, 12′, that is reduced 132′, I find 3 may be taken in the Quotient, and multiplying the Divisor thereby substract the Total Product, and so continue the Division till I get 3 Hours, 31′, 20″, and nothing left remaining. *Example in ☌ of the Moon with the Bulls Eye.*

Index

12 33

(4) 19 40

(2) 37′ 40″) 2° 12′ 40″ 13‴ 20⁗ (3^{Hours} 31′ 20″

(2) 1 53 00

19 27 40

12 33 20

Compound. Compound Astronomical Division is like Compound Decimal Division; for Number is to be divided by Number. The *Index* of the Quotienary Numbers are got as above; and the Signs, as in Decimals, that is + with like Signs, and — with unlike.

Example. As if $6'' - 130''' + 600''''$, the Product of the Compound Multiplication in the last *Chapter*, were to be divided by $2' - 30'$, there divising with $2'$ out of $6''$, I get $3'$ for the Quotient with the Sign +, because 2 and 6 were both +, by which $3'$, multiplying the Divisor, the Product to be substracted is $6'' - 90'''$, which substracted leaves $-40''' + 600''''$. Then divising again by $+2'$ out of $-40'''$, I get $20''$ for the Quotient with the Sign —, because $2'$ was + but $40'''$ —, and the Divisor multiplyed by this $20''$, makes the Product equal to the Remain before, as is here to be seen.

$$
\begin{array}{llll}
 & & -\cancel{40} & \\
2' - 30'' & \big) & +\cancel{6}'' - \cancel{130}''' + \cancel{600}'''' & \big(3' - 20'' \\
3 - 20 & & +\cancel{6} - \cancel{90} & \\
\hline
6 - 90 & & -\cancel{40} + \cancel{600} & \\
-40 + 600 & & &
\end{array}
$$

Proof of Astronomical Division. Astronomical Division may be proved by Astronomical Multiplication, like other Divisions by their respective Multiplications; and like Decimals may be turned into Geodæticals, and tryed thereby.

As in the last Example thus;

$$
\begin{array}{ccc}
+6'' - 130''' + 600'''' & +2' - 30'' & +3' - 20'' \\
60 & 60 & 60 \\
\hline
+360''' & +120'' & +180 \\
-130 & -30 & -20 \\
\hline
+230 & +90 & +160 \\
60 & & \\
\hline
+13800'''' & & \\
+600 & & \\
\hline
+14400 & \cancel{1}\cancel{4}\cancel{4}\cancel{0}\cancel{0}\big(\,\cancel{1}\cancel{6}|0\,\big(\,2' & \cancel{1}\cancel{6}|0\,\big(\,2' \\
 & \text{(above: } 5,\ |4''\text{; below: } \cancel{9}\ \cancel{0},\ 60) & \text{(above: } |4''\text{; below: } \cancel{6}0)
\end{array}
$$

CHAP. VII.

Figuration of Astronomicals.

Figurate Astronomicals produced. **TO produce Figurate Astronomicals, is no other than to multiply any Astronomical Simple or Compound into it self, for the Square and the Square multiplyed by the Root produceth the Cube, *&c.* as other Figural Numbers are produced: And the same being done by Multiplication, the *Indices* and Signs of the Product are found as before declared.**

Examples

Examples.

Examples Simple

	°	′	″	‴	⁗	v	vj
Root	1	10	9				
	1	10	9				
	1	10	9				
		10	100	90			
			9	90	81		
Square	1	22	1	1	21		
	1	10	9				
	1	22	1	1	21		
		10	220	10	10	210	
			9	198	9	9	189
Cube	1	35	53	29	43	42	9

Compound

Root	1 ′	−10 ″		
	1 ′	−10 ″		
	1 ″	−10 ‴		
		−10 ‴	+100 ⁗	
Square	1 ″	−20 ‴	+100 ⁗	
	1 ′	−10 ″		
	1 ‴	−20 ⁗	+100 v	
		−10 ⁗	+200 v	−1000 vj
Cube	1 ‴	−30 ⁗	+300 v	−1000 vj

Roots of Simple Astronomicals extracted.

To extract the Root of a Simple Astronomical, prick the Number, and proceed in the same as in Extraction of Roots before taught, only observing, as before in Decimals, that when the Right Hand Denomination will not be parted evenly by the *Index* of the Quantity whose Root you would extract, you must adjoyn one or more Cyphers, that so it may be divisible accordingly, that is by 2 for the Square, 3 for the Cube, 4 for the Squared Square, *&c.*

Example in the Square.

As to extract the Square Root of 1°, 22′, 1″, 1‴, 21⁗, the Numbers pricked will be 21⁗, 1″, 1°, the Greatest Square in 1° is 1°, and the Root thereof 1°, the double whereof is 2, the Divisor to 22′, by which 10′ gotten for the next Quotient Figure, and 10 times 2 taken from 22, leaves 2′ behind. Then the Square of 10′ is 100″, that is 1′, 40″, which substracted in order from the next pricked Number, or added into a Gnomon, with the Multiplication of the Divisor, and substracted leaves 21″. Then doubling the Quotient 1°, 10′, the next Divisor will be 2°, 20′, by which 21″, 1‴, divided, 9″ will be gotten for the Quotient, and the Gnomon to be substracted cut off all the Remain.

21

Square) 1° 22′ 1″ 1‴ 21⁗ (1° 10′ 9″ Root

1

Gnomon { 20
1 40

Gnomon { 21
1 21

Example in the Cube.

Also to extract the Cube Root of 1°, 35′, 53″, 29‴, 43⁗, 42v, 9vi, the pricked Numbers will be 9vi, 29‴, 1°, the Greatest Cube in 1° is 1°, and the Root thereof 1°, the treble whereof, because 1 doth not multiply, is the Divisor to 35′, and thereby 10′ is gotten for the next Quotient Number, multiplyed by 3 makes 30, which with the Square of 10 increased by the triple of 1, and the Cube added into a Total, make the Gnomon 35′, 16″, 40‴; then will the next Divisor be 4°, 5′, 0″, and the Gnomon to be substracted cut off all the Remain.

Cube

36 49

Cube) $1^{\circ}\ 35'\ 53''\ 29'''\ 43''''\ 42^{v}\ 9^{vi}$ ($1^{\circ}\ 10'\ 9''$ Root

1

Gnomon { 30
5 00
16 40

Gnomon { 36 45 00
4 43 30
12 9

Roots of Compound Astronomicals *extracted.*

To extract the Root of a Compound Astronomical, prick the Number as before according to the Quantity, and out of the pricked Numbers to the Left Hand, having taken the greatest Figurate Number, whose Root you would extract, and placed the Root in the Quotient, you get the Divisor as before, and differ in nothing from the Extractions of the Simple, save as Compound Multiplication or Division differs from the Simple.

Example in the

As to extract the Square Root of $1''\ -20'''\ +100''''$, and the Cube Root of $1'''\ -30''''\ +300^{v}\ -1000^{vi}$, the Numbers pricked in the first are $100''''$ and $1''$, in the other 1000^{vi} and $1'''$, in both the Greatest Square and Cube of 1 is but 1, and the Root of both is but 1, which in the Square will be $'$ the half of $''$, and in the Cube $'$ the third part of $'''$, this 1 doubled is the Divisor to $20'''$ in the Square, and tripled, because 1 doth not multiply, makes 3 the Divisor to $30''''$ in the Cube, whereby $10''$ is gotten for the Quotient to both, and so proceeding the rest of the work is plain in the Operations following.

Square.

Square) $1''\ -20'''\ +100''''$ ($1'\ -10''$ Root

1

−20

+100

+ 2 Divisor −10 −20

−10 −10 +100

Cube.

Cube) $1'''\ -30''''\ +300^{v}\ -1000^{vi}$ ($1'\ -10''$ Root

1

−30

+300

−1000

+ 3 Divisor −10 −30

−10 −10 +100 +3 +300

−10 −10 +100 −10 −1000

These Numbers above Exemplary being of such Denominations as will be equally divided by the *Indices* of their Quantities need no Cyphers to be adjoyned, such as do having no other difference in their Extraction than to continue the Extraction to the end of the adjoyned Cyphers, need not be made exemplary here.

Proof of Astronomical Figuration.

Besides the Proof of Production by Extraction, and Extraction by Production, as in other Numbers; if the Astronomicals be reduced into Geodæticals of the Lowest Denomination, and the Compound turned into Simple, and their Figurations compared with the Figurations of the Numbers so reduced as Integers, the works will equally agree.

Examples in the Numbers above-mentioned in this *Chapter*.

Simple.

Simple.

```
        Root 1° 10' 9"        Square 1° 22' 1" 1''' 21''''     Cube 1° 35' 53" 29''' 43'''' 42v 9vj
                60                      60                                60
               ----                    ----                              ----
                70'                     82'                               95'
                60                      60                                60
               ----                    ----                              ----
     Reduced  4209"                   4921"                             5753"
              4209                      60                                60
              ----                    ------                            ------
             37881                    295261'''                         345209'''
            84180                         60                                60
           16836                     ----------                         ----------
          ----------                 17715681''''                      20712583''''
 Square   17715681''''               ----------                             60
              4209"                                                    ------------
          ----------                                                   1242755022v
         159441129                                                          60
        354313620                                                     -------------
       70862724                                                       74565301329vj
      -------------                                                   -------------
 Cube  74565301329vj
      -------------
```

```
           1 7                                        10477
           . . . . .  Root                            . . . . .  Root
 Square    17715681 | 4209              Cube     74565301329 | 4209
           16 : : :                              64 :   :   :
 Gnomon    164 : :                      Gnomon   10088  :   :
 Gnomon      000 :                      Gnomon      0000    :
 Gnomon      75681                      Gnomon   477301329
```

```
 53 3|2 ''''  51  '''  1  ''|2 '  °
 17715681|1 |29526|1 |492|1 8|2 |1 Square
 6         0  6     0  6   0 6 0

 12143  1 vj      13 2|4 v    23 |4 ''''   3|2 '''   3 ''  |3 '  °
 7456530132|9 |124275502|2 |2071258|3 |34520|9 |57|53 |9|5 |1 Cube
 6            0  6          0  6        0  6      0  6   0 6 0
```

Compound.

```
     Root 1' —10"         Square 1" —20'''+100''''      Cube 1''' —30''''+300v —1000vj
          60                     60                            60
         ----                   ----                          ----
          60"                    60'''                         60''''
         —10                    —20                           —30
         ----                   ----                          ----
 Reduced +50"                   +40'''                        +30''''
          50"                    60                            60
         ----                   ----                          ----
 Square  2500''''               2400''''                      1800v
           50"                + 100                          + 300
         ----                   ----                          ----
 Cube    125000vj               2500''''                      2100v
         ----                   ----                            60
                                                              ------
                                                              126000vj
                                                            —   1000
                                                              ------
                                                              125000vj
                                                              ------
```

Square 2500 | 50 Root   Cube 125000 | 50 Root
25   125
00   000

1 4'''' '''
250 | 0 | 41 Square $= 1'' - 20''' + 100'''' = 1'' - 19''' + 40''''$
6 0

2 2 vj 2 4 v ''''
125000 | 0 | 208 | 3 | 34 Cube $= 1''' - 30'''' + 300^{v} - 1000^{vj} = 1''' - 26'''' + 44^{v} - 40^{vj}$
6 0 6 0

CHAP. VIII.

Of the Sexagenary Table.

Sexagenary Table, its use, and why so called.

ENough hath been said of the Simple Elements of *Astronomicals*, to understand them; yet it is convenient to say something of the *Sexagenary Table*, because in Multiplication, Division and Extraction of Roots, before shewed, adding up the several Multiplees in Multiplication, and the Remains in Division and Extraction oftentimes need Reduction by 60, whereby the work is more tedious; therefore is the *Table* made to contain all the Products of any two Numbers under 60, multiplyed together and reduced into the next Denomination, where the Products will bear the same, and from thence called *The Sexagenary Table*.

Triangular Form sufficient.

The *Table* consists of a Quadrangular Form, yet if it were Triangular, as some make it, were sufficient: As may be seen by the upper part thereof above the Black Scale; for that 9 times 10, and 10 times 9, make the Product alike, *viz.* both 90.

Table explained.

The *Table* on each side hath 60 Columns, the outermost on the Left Hand, and the other on the Head, signifie sometimes Multiplicands and Multiplyers, sometimes Quotients and Roots. The other Columns serve to find out the Products of any two Numbers under 60, being multiplyed one by another, and signifie Dividends and Square Numbers, as occasion requireth, among which the Square Numbers are easily discernable, being only those that stand next above the Black Scale.

Those Angles that have 2 Numbers in them, imply that next the Right Hand to be the Number of the lowest Denomination the Product will afford, and the Number next the Left Hand one place higher.

As if I enter with $10''$ at the Left Hand, and $20''$ at the Head, the Common Angle is 3, 20, which 20 shall be $''''$, and the 3 shall be one place higher, that is $'''$.

And if I had entred with $10''$ and $20'''$, then should the Angular 3, 20, be $3''''$, 20^{v}, the like is to be understood of all others in finding the true *Index*.

The

Place the Sexagenary Table here.

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0	1
1	0. 1
2	0. 2
3	0. 3
4	0. 4
5	0. 5
6	0. 6
7	0. 7
8	0. 8
9	0. 9
10	0. 10
11	0. 11
12	0. 12
13	0. 13
14	0. 14
15	0. 15
16	0. 16
17	0. 17
18	0. 18
19	0. 19
20	0. 20
21	0. 21
22	0. 22
23	0. 23
24	0. 24
25	0. 25
26	0. 26
27	0. 27
28	0. 28
29	0. 29
30	0. 30
31	0. 31
32	0. 32
33	0. 33
34	0. 34
35	0. 35
36	0. 36
37	0. 37
38	0. 38
39	0. 39
40	0. 40
41	0. 41
42	0. 42
43	0. 43
44	0. 44
45	0. 45
46	0. 46
47	0. 47
48	0. 48
49	0. 49
50	0. 50
51	0. 51
52	0. 52
53	0. 53
54	0. 54
55	0. 55
56	0. 56
57	0. 57
58	0. 58
59	0. 59
60	1. 0

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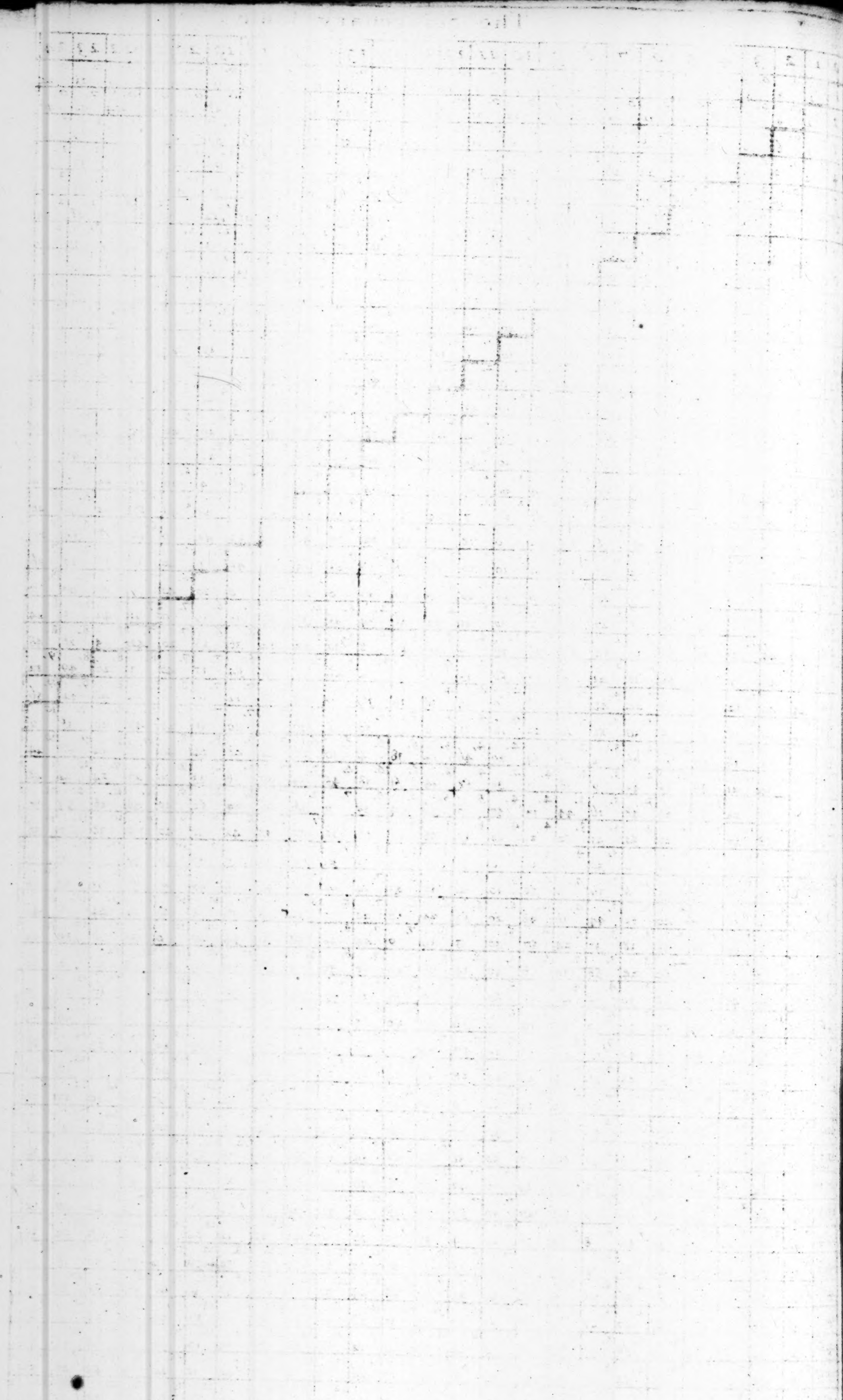
The Sexagenary Table.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
2	0.2	0.4	0.6	0.8	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46
3	0.3	0.6	0.9	0.12	0.15	0.18	0.21	0.24	0.27	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.51	0.54	0.57	1.0	1.3	1.6	1.9
4	0.4	0.8	0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.40	0.44	0.48	0.52	0.56	1.0	1.4	1.8	1.12	1.16	1.20	1.24	1.28	1.32
5	0.5	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	1.0	1.5	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.55
6	0.6	0.12	0.18	0.24	0.30	0.36	0.42	0.48	0.54	1.0	1.6	1.12	1.18	1.24	1.30	1.36	1.42	1.48	1.54	2.0	2.6	2.12	2.18
7	0.7	0.14	0.21	0.28	0.35	0.42	0.49	0.56	1.3	1.10	1.17	1.24	1.31	1.38	1.45	1.52	1.59	2.6	2.13	2.20	2.27	2.34	2.41
8	0.8	0.16	0.24	0.32	0.40	0.48	0.56	1.4	1.12	1.20	1.28	1.36	1.44	1.52	2.0	2.8	2.16	2.24	2.32	2.40	2.48	2.56	3.4
9	0.9	0.18	0.27	0.36	0.45	0.54	1.3	1.12	1.21	1.30	1.39	1.48	1.57	2.6	2.15	2.24	2.33	2.42	2.51	3.0	3.9	3.18	3.27
10	0.10	0.20	0.30	0.40	0.50	1.0	1.10	1.20	1.30	1.40	1.50	2.0	2.10	2.20	2.30	2.40	2.50	3.0	3.10	3.20	3.30	3.40	3.50
11	0.11	0.22	0.33	0.44	0.55	1.6	1.17	1.28	1.39	1.50	2.1	2.12	2.23	2.34	2.45	2.56	3.7	3.18	3.29	3.40	3.51	4.2	4.13
12	0.12	0.24	0.36	0.48	1.0	1.12	1.24	1.36	1.48	2.0	2.12	2.24	2.36	2.48	3.0	3.12	3.24	3.36	3.48	4.0	4.12	4.24	4.36
13	0.13	0.26	0.39	0.52	1.5	1.18	1.31	1.44	1.57	2.10	2.23	2.36	2.49	3.2	3.15	3.28	3.41	3.54	4.7	4.20	4.33	4.46	4.59
14	0.14	0.28	0.42	0.56	1.10	1.24	1.38	1.52	2.6	2.20	2.34	2.48	3.2	3.16	3.30	3.44	3.58	4.12	4.26	4.40	4.54	5.8	5.22
15	0.15	0.30	0.45	1.0	1.15	1.30	1.45	2.0	2.15	2.30	2.45	3.0	3.15	3.30	3.45	4.0	4.15	4.30	4.45	5.0	5.15	5.30	5.45
16	0.16	0.32	0.48	1.4	1.20	1.36	1.52	2.8	2.24	2.40	2.56	3.12	3.28	3.44	4.0	4.16	4.32	4.48	5.4	5.20	5.36	5.52	6.8
17	0.17	0.34	0.51	1.8	1.25	1.42	1.59	2.16	2.33	2.50	3.7	3.24	3.41	3.58	4.15	4.32	4.49	5.6	5.23	5.40	5.57	6.14	6.31
18	0.18	0.36	0.54	1.12	1.30	1.48	2.6	2.24	2.42	3.0	3.18	3.36	3.54	4.12	4.30	4.48	5.6	5.24	5.42	6.0	6.18	6.36	6.54
19	0.19	0.38	0.57	1.16	1.35	1.54	2.13	2.32	2.51	3.10	3.29	3.48	4.7	4.26	4.45	5.4	5.23	5.42	6.1	6.20	6.39	6.58	7.17
20	0.20	0.40	1.0	1.20	1.40	2.0	2.20	2.40	3.0	3.20	3.40	4.0	4.20	4.40	5.0	5.20	5.40	6.0	6.20	6.40	7.0	7.20	7.40
21	0.21	0.42	1.3	1.24	1.45	2.6	2.27	2.48	3.9	3.30	3.51	4.12	4.33	4.54	5.15	5.36	5.57	6.18	6.39	7.0	7.21	7.42	8.3
22	0.22	0.44	1.6	1.28	1.50	2.12	2.34	2.56	3.18	3.40	4.2	4.24	4.46	5.8	5.30	5.52	6.14	6.36	6.58	7.20	7.42	8.4	8.26
23	0.23	0.46	1.9	1.32	1.55	2.18	2.41	3.4	3.27	3.50	4.13	4.36	4.59	5.22	5.45	6.8	6.31	6.54	7.17	7.40	8.3	8.26	8.49
24	0.24	0.48	1.12	1.36	2.0	2.24	2.48	3.12	3.36	4.0	4.24	4.48	5.12	5.36	6.0	6.24	6.48	7.12	7.36	8.0	8.24	8.48	9.12
25	0.25	0.50	1.15	1.40	2.5	2.30	2.55	3.20	3.45	4.10	4.35	5.0	5.25	5.50	6.15	6.40	7.5	7.30	7.55	8.20	8.45	9.10	9.35
26	0.26	0.52	1.18	1.44	2.10	2.36	3.2	3.28	3.54	4.20	4.46	5.12	5.38	6.4	6.30	6.56	7.22	7.48	8.14	8.40	9.6	9.32	9.58
27	0.27	0.54	1.21	1.48	2.15	2.42	3.9	3.36	4.3	4.30	4.57	5.24	5.51	6.18	6.45	7.12	7.39	8.6	8.33	9.0	9.27	9.54	10.21
28	0.28	0.56	1.24	1.52	2.20	2.48	3.16	3.44	4.12	4.40	5.8	5.36	6.4	6.32	7.0	7.28	7.56	8.24	8.52	9.20	9.48	10.16	10.44
29	0.29	0.58	1.27	1.56	2.25	2.54	3.23	3.52	4.21	4.50	5.19	5.48	6.17	6.46	7.15	7.44	8.13	8.42	9.11	9.40	10.9	10.38	11.7
30	0.30	1.0	1.30	2.0	2.30	3.0	3.30	4.0	4.30	5.0	5.30	6.0	6.30	7.0	7.30	8.0	8.30	9.0	9.30	10.0	10.30	11.0	11.30
31	0.31	1.2	1.33	2.4	2.35	3.6	3.37	4.8	4.39	5.10	5.41	6.12	6.43	7.14	7.45	8.16	8.47	9.18	9.49	10.20	10.51	11.22	11.53
32	0.32	1.4	1.36	2.8	2.40	3.12	3.44	4.16	4.48	5.20	5.52	6.24	6.56	7.28	8.0	8.32	9.4	9.36	10.8	10.40	11.12	11.44	12.16
33	0.33	1.6	1.39	2.12	2.45	3.18	3.51	4.24	4.57	5.30	6.3	6.36	7.9	7.42	8.15	8.48	9.21	9.54	10.27	11.0	11.33	12.6	12.39
34	0.34	1.8	1.42	2.16	2.50	3.24	3.58	4.32	5.6	5.40	6.14	6.48	7.22	7.56	8.30	9.4	9.38	10.12	10.46	11.20	11.54	12.28	13.2
35	0.35	1.10	1.45	2.20	2.55	3.30	4.5	4.40	5.15	5.50	6.25	7.0	7.35	8.10	8.45	9.20	9.55	10.30	11.5	11.40	12.15	12.50	13.25
36	0.36	1.12	1.48	2.24	3.0	3.36	4.12	4.48	5.24	6.0	6.36	7.12	7.48	8.24	9.0	9.36	10.12	10.48	11.24	12.0	12.36	13.12	13.48
37	0.37	1.14	1.51	2.28	3.5	3.42	4.19	4.56	5.33	6.10	6.47	7.24	8.1	8.38	9.15	9.52	10.29	11.6	11.43	12.20	12.57	13.34	14.11
38	0.38	1.16	1.54	2.32	3.10	3.48	4.26	5.4	5.42	6.20	6.58	7.36	8.14	8.52	9.30	10.8	10.46	11.24	12.2	12.40	13.18	13.56	14.34
39	0.39	1.18	1.57	2.36	3.15	3.54	4.33	5.12	5.51	6.30	7.9	7.48	8.27	9.6	9.45	10.24	11.3	11.42	12.21	13.0	13.39	14.18	14.57
40	0.40	1.20	2.0	2.40	3.20	4.0	4.40	5.20	6.0	6.40	7.20	8.0	8.40	9.20	10.0	10.40	11.20	12.0	12.40	13.20	14.0	14.40	15.20
41	0.41	1.22	2.3	2.44	3.25	4.6	4.47	5.28	6.9	6.50	7.31	8.12	8.53	9.34	10.15	10.56	11.37	12.18	12.59	13.40	14.21	15.2	15.43
42	0.42	1.24	2.6	2.48	3.30	4.12	4.54	5.36	6.18	7.0	7.42	8.24	9.6	9.48	10.30	11.12	11.54	12.36	13.18	14.0	14.42	15.24	16.6
43	0.43	1.26	2.9	2.52	3.35	4.18	5.1	5.44	6.27	7.10	7.53	8.36	9.19	10.2	10.45	11.28	12.11	12.54	13.37	14.20	15.3	15.46	16.29
44	0.44	1.28	2.12	2.56	3.40	4.24	5.8	5.52	6.36	7.20	8.4	8.48	9.32	10.16	11.0	11.44	12.28	13.12	13.56	14.40	15.24	16.8	16.52
45	0.45	1.30	2.15	3.0	3.45	4.30	5.15	6.0	6.45	7.30	8.15	9.0	9.45	10.30	11.15	12.0	12.45	13.30	14.15	15.0	15.45	16.30	17.15
46	0.46	1.32	2.18	3.4	3.50	4.36	5.22	6.8	6.54	7.40	8.26	9.12	9.58	10.44	11.30	12.16	13.2	13.48	14.34	15.20	16.6	16.52	17.38
47	0.47	1.34	2.21	3.8	3.55	4.42	5.29	6.16	7.3	7.50	8.37	9.24	10.11	10.58	11.45	12.32	13.19	14.6	14.53	15.40	16.27	17.14	18.1
48	0.48	1.36	2.24	3.12	4.0	4.48	5.36	6.24	7.12	8.0	8.48	9.36	10.24	11.12	12.0	12.48	13.36	14.24	15.12	16.0	16.48	17.36	18.24
49	0.49	1.38	2.27	3.16	4.5	4.54	5.43	6.32	7.21	8.10	8.59	9.48	10.37	11.26	12.15	13.4	13.53	14.42	15.31	16.20	17.9	17.58	18.47
50	0.50	1.40	2.30	3.20	4.10	5.0	5.50	6.40	7.30	8.20	9.10	10.0	10.50	11.40	12.30	13.20	14.10	15.0	15.50	16.40	17.30	18.20	19.10
51	0.51	1.42	2.33	3.24	4.15	5.6	5.57	6.48	7.39	8.30	9.21	10.12	11.3	11.54	12.45	13.36	14.27	15.18	16.9	17.0	17.51	18.42	19.33
52	0.52	1.44	2.36	3.28	4.20	5.12	6.4	6.56	7.48	8.40	9.32	10.24	11.16	12.8	13.0	13.52	14.44	15.36	16.28	17.20	18.12	19.4	19.56
53	0.53	1.46	2.39	3.32	4.25	5.18	6.11	7.4	7.57	8.50	9.43	10.36	11.29	12.22	13.15	14.8	15.1	15.54	16.47	17.40	18.33	19.26	20.19
54	0.54	1.48	2.42	3.36	4.30	5.24	6.18	7.12	8.6	9.0	9.54	10.48	11.42	12.36	13.30	14.24	15.18	16.12	17.6	18.0	18.54	19.48	20.42
55	0.55	1.50	2.45	3.40	4.35	5.30	6.25	7.20	8.15	9.10	10.5	11.0	11.55	12.50	13.45	14.40	15.35	16.30	17.25	18.20	19.15	20.10	21.5
56	0.56	1.52	2.48	3.44	4.40	5.36	6.32	7.28	8.24	9.20	10.16	11.12	12.8	13.4	14.0	14.56	15.52	16.48	17.44	18.40	19.36	20.32	21.28
57	0.57	1.54	2.51	3.48	4.45	5.42	6.39	7.36	8.33	9.30	10.27	11.24	12.21	13.18	14.15	15.12	16.9	17.6	18.3	19.0	19.57	20.54	21.51
58	0.58	1.56	2.54	3.52	4.50	5.48	6.46	7.44	8.42	9.40	10.38	11.36	12.34	13.32	14.30	15.28	16.26	17.24	18.22	19.20	20.18	21.16	22.14
59	0.59	1.58	2.57	3.56	4.55	5.54	6.53	7.52	8.51	9.50	10.49	11.48	12.47	13.46	14.45	15.44	16.43	17.42	18.41	19.40	20.39	21.38	22.37
60	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0	16.0	17.0	18.0	19.0	20.0	21.0	22.0	23.0

22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.40	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50	0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59	1.0
0.44	0.46	0.48	0.50	0.52	0.54	0.56	0.58	1.0	1.2	1.4	1.6	1.8	1.10	1.12	1.14	1.16	1.18	1.20	1.22	1.24	1.26	1.28	1.30	1.32	1.34	1.36	1.38	1.40	1.42	1.44	1.46	1.48	1.50	1.52	1.54	1.56	1.58	2.0
1.6	1.9	1.12	1.15	1.18	1.21	1.24	1.27	1.30	1.33	1.36	1.39	1.42	1.45	1.48	1.51	1.54	1.57	2.0	2.3	2.6	2.9	2.12	2.15	2.18	2.21	2.24	2.27	2.30	2.33	2.36	2.39	2.42	2.45	2.48	2.51	2.54	2.57	3.0
1.28	1.32	1.36	1.40	1.44	1.48	1.52	1.56	2.0	2.4	2.8	2.12	2.16	2.20	2.24	2.28	2.32	2.36	2.40	2.44	2.48	2.52	2.56	3.0	3.4	3.8	3.12	3.16	3.20	3.24	3.28	3.32	3.36	3.40	3.44	3.48	3.52	3.56	4.0
1.50	1.55	2.0	2.5	2.10	2.15	2.20	2.25	2.30	2.35	2.40	2.45	2.50	2.55	3.0	3.5	3.10	3.15	3.20	3.25	3.30	3.35	3.40	3.45	3.50	3.55	4.0	4.5	4.10	4.15	4.20	4.25	4.30	4.35	4.40	4.45	4.50	4.55	5.0
2.12	2.18	2.24	2.30	2.36	2.42	2.48	2.54	3.0	3.6	3.12	3.18	3.24	3.30	3.36	3.42	3.48	3.54	4.0	4.6	4.12	4.18	4.24	4.30	4.36	4.42	4.48	4.54	5.0	5.6	5.12	5.18	5.24	5.30	5.36	5.42	5.48	5.54	6.0
2.34	2.41	2.48	2.55	3.2	3.9	3.16	3.23	3.30	3.37	3.44	3.51	3.58	4.5	4.12	4.19	4.26	4.33	4.40	4.47	4.54	5.1	5.8	5.15	5.22	5.29	5.36	5.43	5.50	5.57	6.4	6.11	6.18	6.25	6.32	6.39	6.46	6.53	7.0
2.56	3.4	3.12	3.20	3.28	3.36	3.44	3.52	4.0	4.8	4.16	4.24	4.32	4.40	4.48	4.56	5.4	5.12	5.20	5.28	5.36	5.44	5.52	6.0	6.8	6.16	6.24	6.32	6.40	6.48	6.56	7.4	7.12	7.20	7.28	7.36	7.44	7.52	8.0
3.18	3.27	3.36	3.45	3.54	4.3	4.12	4.21	4.30	4.39	4.48	4.57	5.6	5.15	5.24	5.33	5.42	5.51	6.0	6.9	6.18	6.27	6.36	6.45	6.54	7.3	7.12	7.21	7.30	7.39	7.48	7.57	8.6	8.15	8.24	8.33	8.42	8.51	9.0
3.40	3.50	4.0	4.10	4.20	4.30	4.40	4.50	5.0	5.10	5.20	5.30	5.40	5.50	6.0	6.10	6.20	6.30	6.40	6.50	7.0	7.10	7.20	7.30	7.40	7.50	8.0	8.10	8.20	8.30	8.40	8.50	9.0	9.10	9.20	9.30	9.40	9.50	10.0
4.2	4.13	4.24	4.35	4.46	4.57	5.8	5.19	5.30	5.41	5.52	6.3	6.14	6.25	6.36	6.47	6.58	7.9	7.20	7.31	7.42	7.53	8.4	8.15	8.26	8.37	8.48	8.59	9.10	9.21	9.32	9.43	9.54	10.5	10.16	10.27	10.38	10.49	11.0
4.24	4.36	4.48	5.0	5.12	5.24	5.36	5.48	6.0	6.12	6.24	6.36	6.48	7.0	7.12	7.24	7.36	7.48	8.0	8.12	8.24	8.36	8.48	9.0	9.12	9.24	9.36	9.48	10.0	10.12	10.24	10.36	10.48	11.0	11.12	11.24	11.36	11.48	12.0
4.46	4.59	5.12	5.25	5.38	5.51	6.4	6.17	6.30	6.43	6.56	7.9	7.22	7.35	7.48	8.1	8.14	8.27	8.40	8.53	9.6	9.19	9.32	9.45	9.58	10.11	10.24	10.37	10.50	11.3	11.16	11.29	11.42	11.55	12.8	12.21	12.34	12.47	13.0
5.8	5.22	5.36	5.50	6.4	6.18	6.32	6.46	7.0	7.14	7.28	7.42	7.56	8.10	8.24	8.38	8.52	9.6	9.20	9.34	9.48	10.2	10.16	10.30	10.44	10.58	11.12	11.26	11.40	11.54	12.8	12.22	12.36	12.50	13.4	13.18	13.32	13.46	14.0
5.30	5.45	6.0	6.15	6.30	6.45	7.0	7.15	7.30	7.45	8.0	8.15	8.30	8.45	9.0	9.15	9.30	9.45	10.0	10.15	10.30	10.45	11.0	11.15	11.30	11.45	12.0	12.15	12.30	12.45	13.0	13.15	13.30	13.45	14.0	14.15	14.30	14.45	15.0
5.52	6.8	6.24	6.40	6.56	7.12	7.28	7.44	8.0	8.16	8.32	8.48	9.4	9.20	9.36	9.52	10.8	10.24	10.40	10.56	11.12	11.28	11.44	12.0	12.16	12.32	12.48	13.4	13.20	13.36	13.52	14.8	14.24	14.40	14.56	15.12	15.28	15.44	16.0
6.14	6.31	6.48	7.5	7.22	7.39	7.56	8.13	8.30	8.47	9.4	9.21	9.38	9.55	10.12	10.29	10.46	11.3	11.20	11.37	11.54	12.11	12.28	12.45	13.2	13.19	13.36	13.53	14.10	14.27	14.44	15.1	15.18	15.35	15.52	16.9	16.26	16.43	17.0
6.36	6.54	7.12	7.30	7.48	8.6	8.24	8.42	9.0	9.18	9.36	9.54	10.12	10.30	10.48	11.6	11.24	11.42	12.0	12.18	12.36	12.54	13.12	13.30	13.48	14.6	14.24	14.42	15.0	15.18	15.36	15.54	16.12	16.30	16.48	17.6	17.24	17.42	18.0
6.58	7.17	7.36	7.55	8.14	8.33	8.52	9.11	9.30	9.49	10.8	10.27	10.46	11.5	11.24	11.43	12.2	12.21	12.40	12.59	13.18	13.37	13.56	14.15	14.34	14.53	15.12	15.31	15.50	16.9	16.28	16.47	17.6	17.25	17.44	18.3	18.22	18.41	19.0
7.20	7.40	8.0	8.20	8.40	9.0	9.20	9.40	10.0	10.20	10.40	11.0	11.20	11.40	12.0	12.20	12.40	13.0	13.20	13.40	14.0	14.20	14.40	15.0	15.20	15.40	16.0	16.20	16.40	17.0	17.20	17.40	18.0	18.20	18.40	19.0	19.20	19.40	20.0
7.42	8.3	8.24	8.45	9.6	9.27	9.48	10.9	10.30	10.51	11.12	11.33	11.54	12.15	12.36	12.57	13.18	13.39	14.0	14.21	14.42	15.3	15.24	15.45	16.6	16.27	16.48	17.9	17.30	17.51	18.12	18.33	18.54	19.15	19.36	19.57	20.18	20.39	21.0
8.4	8.26	8.48	9.10	9.32	9.54	10.16	10.38	11.0	11.22	11.44	12.6	12.28	12.50	13.12	13.34	13.56	14.18	14.40	15.2	15.24	15.46	16.8	16.30	16.52	17.14	17.36	17.58	18.20	18.42	19.4	19.26	19.48	20.10	20.32	20.54	21.16	21.38	22.0
8.26	8.49	9.12	9.35	9.58	10.21	10.44	11.7	11.30	11.53	12.16	12.39	13.2	13.25	13.48	14.11	14.34	14.57	15.20	15.43	16.6	16.29	16.52	17.15	17.38	18.1	18.24	18.47	19.10	19.33	19.56	20.19	20.42	21.5	21.28	21.51	22.14	22.37	23.0
8.48	9.12	9.36	10.0	10.24	10.48	11.12	11.36	12.0	12.24	12.48	13.12	13.36	14.0	14.24	14.48	15.12	15.36	16.0	16.24	16.48	17.12	17.36	18.0	18.24	18.48	19.12	19.36	20.0	20.24	20.48	21.12	21.36	22.0	22.24	22.48	23.12	23.36	24.0
9.10	9.35	10.0	10.25	10.50	11.15	11.40	12.5	12.30	12.55	13.20	13.45	14.10	14.35	15.0	15.25	15.50	16.15	16.40	17.5	17.30	17.55	18.20	18.45	19.10	19.35	20.0	20.25	20.50	21.15	21.40	22.5	22.30	22.55	23.20	23.45	24.10	24.35	25.0
9.32	9.58	10.24	10.50	11.16	11.42	12.8	12.34	13.0	13.26	13.52	14.18	14.44	15.10	15.36	16.2	16.28	16.54	17.20	17.46	18.12	18.38	19.4	19.30	19.56	20.22	20.48	21.14	21.40	22.6	22.32	22.58	23.24	23.50	24.16	24.42	25.8	25.34	26.0
9.54	10.21	10.48	11.15	11.42	12.9	12.36	13.3	13.30	13.57	14.24	14.51	15.18	15.45	16.12	16.39	17.6	17.33	18.0	18.27	18.54	19.21	19.48	20.15	20.42	21.9	21.36	22.3	22.30	22.57	23.24	23.51	24.18	24.45	25.12	25.39	26.6	26.33	27.0
10.16	10.44	11.12	11.40	12.8	12.36	13.4	13.32	14.0	14.28	14.56	15.24	15.52	16.20	16.48	17.16	17.44	18.12	18.40	19.8	19.36	20.4	20.32	21.0	21.28	21.56	22.24	22.52	23.20	23.48	24.16	24.44	25.12	25.40	26.8	26.36	27.4	27.32	28.0
10.38	11.7	11.36	12.5	12.34	13.3	13.32	14.1	14.30	14.59	15.28	15.57	16.26	16.55	17.24	17.53	18.22	18.51	19.20	19.49	20.18	20.47	21.16	21.45	22.14	22.43	23.12	23.41	24.10	24.39	25.8	25.37	26.6	26.35	27.4	27.33	28.2	28.31	29.0
11.0	11.30	12.0	12.30	13.0	13.30	14.0	14.30	15.0	15.30	16.0	16.30	17.0	17.30	18.0	18.30	19.0	19.30	20.0	20.30	21.0	21.30	22.0	22.30	23.0	23.30	24.0	24.30	25.0	25.30	26.0	26.30	27.0	27.30	28.0	28.30	29.0	29.30	30.0
11.22	11.53	12.24	12.55	13.26	13.57	14.28	14.59	15.30	16.1	16.32	17.3	17.34	18.5	18.36	19.7	19.38	20.9	20.40	21.11	21.42	22.13	22.44	23.15	23.46	24.17	24.48	25.19	25.50	26.21	26.52	27.23	27.54	28.25	28.56	29.27	29.58	30.29	31.0
11.44	12.16	12.48	13.20	13.52	14.24	14.56	15.28	16.0	16.32	17.4	17.36	18.8	18.40	19.12	19.44	20.16	20.48	21.20	21.52	22.24	22.56	23.28	24.0	24.32	25.4	25.36	26.8	26.40	27.12	27.44	28.16	28.48	29.20	29.52	30.24	30.56	31.28	32.0
12.6	12.39	13.12	13.45	14.18	14.51	15.24	15.57	16.30	17.3	17.36	18.9	18.42	19.15	19.48	20.21	20.54	21.27	22.0	22.33	23.6	23.39	24.12	24.45	25.18	25.51	26.24	26.57	27.30	28.3	28.36	29.9	29.42	30.15	30.48	31.21	31.54	32.27	33.0
12.28	13.2	13.36	14.10	14.44	15.18	15.52	16.26	17.0	17.34	18.8	18.42	19.16	19.50	20.24	20.58	21.32	22.6	22.40	23.14	23.48	24.22	24.56	25.30	26.4	26.38	27.12	27.46	28.20	28.54	29.28	30.2	30.36	31.10	31.44	32.18	32.52	33.26	34.0
12.50	13.25	14.0	14.35	15.10	15.45	16.20	16.55	17.30	18.5	18.40	19.15	19.50	20.25	21.0	21.35	22.10	22.45	23.20	23.55	24.30	25.5	25.40	26.15	26.50	27.25	28.0	28.35	29.10	29.45	30.20	30.55	31.30	32.5	32.40	33.15	33.50	34.25	35.0
13.12	13.48	14.24	15.0	15.36	16.12	16.48	17.24	18.0	18.36	19.12	19.48	20.24	21.0	21.36	22.12	22.48	23.24	24.0	24.36	25.12	25.48	26.24	27.0	27.36	28.12	28.48	29.24	30.0	30.36	31.12	31.48	32.24	33.0	33.36	34.12	34.48	35.24	36.0
13.34	14.11	14.48	15.25	16.2	16.39	17.16	17.53	18.30	19.7	19.44	20.21	20.58	21.35	22.12	22.49	23.26	24.3	24.40	25.17	25.54	26.31	27.8	27.45	28.22	28.59	29.36	30.13	30.50	31.27	32.4	32.41	33.18	33.55	34.32	35.9	35.46	36.23	37.0
13.56	14.34	15.12	15.50	16.28	17.6	17.44	18.22	19.0	19.38	20.16	20.54	21.32	22.10	22.48	23.26	24.4	24.42	25.20	25.58	26.36	27.14	27.52	28.30	29.8	29.46	30.24	31.2	31.40	32.18	32.56	33.34	34.12	34.50	35.28	36.6	36.44	37.22	38.0
14.18	14.57	15.36	16.15	16.54	17.33	18.12	18.51	19.30	20.9	20.48	21.27	22.6	22.45	23.24	24.3	24.42	25.21	26.0	26.39	27.18	27.57	28.36	29.15	29.54	30.33	31.12	31.51	32.30	33.9	33.48	34.27	35.6	35.45	36.24	37.3	37.42	38.21	39.0
14.40	15.20	16.0	16.40	17.20	18.0	18.40	19.20	20.0	20.40	21.20	22.0	22.40	23.20	24.0	24.40	25.20	26.0	26.40	27.20	28.0	28.40	29.20	30.0	30.40	31.20	32.0	32.40	33.20	34.0	34.40	35.20	36.0	36.40	37.20	38.0	38.40	39.20	40.0
15.2	15.43	16.24	17.5	17.46	18.27	19.8	19.49	20.30	21.11	21.52	22.33	23.14	23.55	24.36	25.17	25.58	26.39	27.20	28.1	28.42	29.23	30.4	30.45	31.26	32.7	32.48	33.29	34.10	34.51	35.32	36.13	36.54	37.35	38.16	38.57	39.38	40.19	41.0
15.24	16.6	16.48	17.30	18.12	18.54	19.36	20.18	21.0	21.42	22.24	23.6	23.48	24.30	25.12	25.54	26.36	27.18	28.0	28.42	29.24	30.6	30.48	31.30	32.12	32.54	33.36	34.18	35.0	35.42	36.24	37.6	37.48	38.30	39.12	39.54	40.36	41.18	42.0
15.46	16.29	17.12	17.55	18.38	19.21	20.4	20.47	21.30	22.13	22.56	23.39	24.22	25.5	25.48	26.31	27.14	27.57	28.40	29.23	30.6	30.49	31.32	32.15	32.58	33.41	34.24	35.7	35.50	36.33	37.16	37.59	38.42	39.25	40.8	40.51	41.34	42.17	43.0
16.8	16.52	17.36	18.20	19.4	19.48	20.32	21.16	22.0	22.44	23.28	24.12	24.56	25.40	26.24	27.8	27.52	28.36	29.20	30.4	30.48	31.32	32.16	33.0	33.44	34.28	35.12	35.56	36.40	37.24	38.8	38.52	39.36	40.20	41.4	41.48	42.32	43.16	44.0
16.30	17.15	18.0	18.45	19.30	20.15	21.0	21.45	22.30	23.15	24.0	24.45	25.30	26.15	27.0	27.45	28.30	29.15	30.0	30.45	31.30	32.15	33.0	33.45	34.30	35.15	36.0	36.45	37.30	38.15	39.0	39.45	40.30	41.15	42.0	42.45	43.30	44.15	45.0
16.52	17.38	18.24	19.10	19.56	20.42	21.28	22.14	23.0	23.46	24.32	25.18	26.4	26.50	27.36	28.22	29.8	29.54	30.40	31.26	32.12	32.58	33.44	34.30	35.16	36.2	36.48	37.34	38.20	39.6	39.52	40.38	41.24	42.10	42.56	43.42	44.28	45.14	46.0
17.14	18.1	18.48	19.35	20.22	21.9	21.56	22.43	23.30	24.17	25.4	25.51	26.38	27.25	28.12	28.59	29.46	30.33	31.20	32.7	32.54	33.41	34.28	35.15	36.2	36.49	37.36	38.23	39.10	39.57	40.44	41.31	42.18	43.5	43.52	44.39	45.26	46.13	47.0
17.36	18.24	19.12	20.0	20.48	21.36	22.24	23.12	24.0	24.48	25.36	26.24	27.12	28.0	28.48	29.36	30.24	31.12	32.0	32.48	33.36	34.24	35.12	36.0	36.48	37.36	38.24	39.12	40.0	40.48	41.36	42.24	43.12	44.0	44.48	45.36	46.24	47.12	48.0
17.58	18.47	19.36	20.25	21.14	22.3	22.52	23.41	24.30	25.19	26.8	26.57	27.46	28.35	29.24	30.13	31.2	31.51	32.40	33.29	34.18	35.7	35.56	36.45	37.34	38.23	39.12	40.1	40.50	41.39	42.28	43.17	44.6	44.55	45.44	46.33	47.22	48.11	49.0
18.20	19.10	20.0	20.50	21.40	22.30	23.20	24.10	25.0	25.50	26.40	27.30	28.20	29.10	30.0	30.50	31.40	32.30	33.20	34.10	35.0	35.50	36.40	37.30	38.20	39.10	40.0	40.50	41.40	42.30	43.20	44.10	45.0	45.50	46.40	47.30	48.20	49.10	50.0
18.42	19.33	20.24	21.15	22.6	22.57	23.48	24.39	25.30	26.21	27.12	28.3	28.54	29.45	30.36	31.27	32.18	33.9	34.0	34.51	35.42	36.33	37.24	38.15	39.6	39.57	40.48	41.39	42.30	43.21	44.12	45.3	45.54	46.45	47.36	48.27	49.18	50.9	51.0
19.4	19.56	20.48	21.40	22.32	23.24	24.16	25.8	26.0	26.52	27.44	28.36	29.28	30.20	31.12	32.4	32.56	33.48	34.40	35.32	36.24	37.16	38.8	39.0	39.52	40.44	41.36	42.28	43.20	44.12	45.4	45.56	46.48	47.40	48.32	49.24	50.16	51.8	52.0
19.26	20.19	21.12	22.5	22.58	23.51	24.44	25.37	26.30	27.23	28.16	29.9	30.2	30.55	31.48	32.41	33.34	34.27	35.20	36.13	37.6	37.59	38.52	39.45	40.38	41.31	42.24	43.17	44.10	45.3	45.56	46.49	47.42	48.35	49.28	50.21	51.14	52.7	53.0
19.48	20.42	21.36	22.30	23.24	24.18	25.12	26.6	27.0	27.54	28.48	29.42	30.36	31.30	32.24	33.18	34.12	35.6	36.0	36.54	37.48	38.42	39.36	40.30	41.24	42.18	43.12	44.6	45.0	45.54	46.48	47.42	48.36	49.30	50.24	51.18	52.12	53.6	54.0
20.10	21.5	22.0	22.55	23.50	24.45	25.40	26.35	27.30	28.25	29.20	30.15	31.10	32.5	33.0	33.55	34.50	35.45	36.40	37.35	38.30	39.25	40.20	41.15	42.10	43.5	44.0	44.55	45.50	46.45	47.40	48.35	49.30	50.25	51.20	52.15	53.10	54.5	55.0
20.32	21.28	22.24	23.20	24.16	25.12	26.8	27.4	28.0	28.56	29.52	30.48	31.44	32.40	33.36	34.32	35.28	36.24	37.20	38.16	39.12	40.8	41.4	42.0	42.56	43.52	44.48	45.44	46.40	47.36	48.32	49.28	50.24	51.20	52.16	53.12	54.8	55.4	56.0
20.54	21.51	22.48	23.45	24.42	25.39	26.36	27.33	28.30	29.27	30.24	31.21	32.18	33.15	34.12	35.9	36.6	37.3	38.0	38.57	39.54	40.51	41.48	42.45	43.42	44.39	45.36	46.33	47.30	48.27	49.24	50.21	51.18	52.15	53.12	54.9	55.6	56.3	57.0
21.16	22.14	23.12	24.10	25.8	26.6	27.4	28.2	29.0	29.58	30.56	31.54	32.52	33.50	34.48	35.46	36.44	37.42	38.40	39.38	40.36	41.34	42.32	43.30	44.28	45.26	46.24	47.22	48.20	49.18	50.16	51.14	52.12	53.10	54.8	55.6	56.4	57.2	58.0
21.38	22.37	23.36	24.35	25.34	26.33	27.32	28.31	29.30	30.29	31.28	32.27	33.26	34.25	35.24	36.23	37.22	38.21	39.20	40.19	41.18	42.17	43.16	44.15	45.14	46.13	47.12	48.11	49.10	50.9	51.8	52.7	53.6	54.5	55.4	56.3	57.2	58.1	59.0
22.0	23.0	24.0	25.0	26.0	27.0	28.0	29.0	30.0	31.0	32.0	33.0	34.0	35.0	36.0	37.0	38.0	39.0	40.0	41.0	42.0	43.0	44.0	45.0	46.0	47.0	48.0	49.0	50.0	51.0	52.0	53.0	54.0	55.0	56.0	57.0	58.0	59.0	60.0



The Use of the Table.

In Multiplication, enter the *Table* with the two Factors or multiplying Numbers, the one at the Head, and the other at the Left Side of the *Table*, and the Numbers or Number in the Common Angle is the desired Product. And if there be 2 Numbers, subscribe the Right Hand Number, and carry the Left Hand Number in mind to be added to the next Product, and so proceed till the Multiplicand be run through; and last of all set down the Left Hand Number, if there be any, ordering the *Indices* as above was directed, and adding together all the Multiplees, you have the Total Product. *Use of the Table in Multiplication.*

In Division enter with the Divisor at the Head, or the Left Side of the *Table*, and run along with your Eye in the same Column, till you espy a Number equal, or near to your Numbers in the Dividend standing over the first Left Hand Figures of your Divisor entred with: If they be not found exactly, take the lesser, alwayes having respect to the greatness or smallness of the other Figures in the Divisor, and the other outermost Number answering to the Common Angle, shall be the Quotient Figure. For if the Divisor be entred with at the Side, the Quotient is found at the Head, and if at the Head, the contrary. The Quotient Figure found, the Divisor is to be multiplyed thereby, and the Product substracted from the Dividend, as before; and then inquire for another Quotient Figure, and so proceed to the end of the work. *In Division.*

As if 06, 05, 20, be multiplyed by 18`, 15°, 10′. I enter the *Table* with 18 and 20, and find in the Common Angle 6, 0; subscribing the 0, I carry the 6 in mind. Then entring the *Table* with 18 and 5, I find 1, 30; to which 30 I add the 6 in mind, and subscribe the amounting 36, and bear the 1 in mind. Again, I enter the *Table* with 18 and 6, and find 1, 48; to which 1 added is 1, 49, to be set down; and so I have done with the first Multiplyer. And thus proceeding with the rest, and adding all the Multiplees together, the Product at last will be 1‴, 51″, 8`, 20°, 53′, 20″, as at *A*. *Example in Multiplication*

And if I divide this Product by one of the Factors, as suppose by 18`, 15°, 10′, first I enter the *Table* with 18, and running along in the Column, I look for the Dividend Numbers 1, 51, which I find not in the Column, but the next lesser Number is 1, 48, and over against the same, in the other outermost Column, is 6, which is to be set in the Quotient, and multiplying all the Divisor thereby, the Product 1, 49, 31, 0, I substract and leave remaining 1, 37 for the next inquiry. Then against the divising 18, I look in the *Table* for 1, 37, and find the next lesser Number to be 1, 30, and in the other outermost Column, answering to 18, is found 5, which set in the Quotient, and the Divisor multiplyed thereby, produceth 1, 31, 15, 50′; this substracted leaveth 6, 5, for the next inquiry, which in the Column against 18 is not found in the *Table*, but 6, 0, the next lesser, which hath 20 for the Quotient answering thereto, the Product of which multiplyed into the Divisor, and substracted leaveth 0 behind, as at *B*. *Example in Division.*

			`	°	′			
	Multiplicand		6	5	20			(1)
			`	°	′			
	Multiplyer		18	15	10			(1)
		1	49	36	00			
A	Multiplees		1	31	20	00		
				1	00	53	20	(2)
		‴	″	`	°	′	″	
	Total Product	1	51	08	20	53	20	

				6					
	Divisor		1	37	5	3			Quotient
	18` 15° 10′)	1‴	51″	08`	20°	53′	20″		(6` 5° 20′
B		1	49	31	00				
			1	31	15	50			(2)
				6	5	3	20		(1)
		1	51	08	20	53	20		(1)

In

Table *useful in Extraction of the* Square Root.

What to be done for Higher Roots.

In Extraction of Roots, forasmuch as Division and Multiplication is used therein, the *Table* is not a little helpful to Astronomical Extraction : Only if a Root higher than the Square be extracted, those higher Quantities substracted at first out of the Numbers belonging to the Left Hand Prick, must be gotten by ordinary Multiplication, no other Figural Numbers but Squares being in the *Table*.

Partis Secundæ Libri Tertii

FINIS.

THE

THE THIRD PART OF THE THIRD BOOK.

CHAP. I.

Of LOGARITHMES.

I Have with all possible brevity transited *Decimals* and *Astronomicals*, and shall now apply my self to overlook *Logarithmes*, martialled in the beginning of this *Book*, in the third rank of Numbers specially Contract. Logarithmes next ranked in Numbers specially Contract.

Logarithmes, are Numbers artificially prepared for other Numbers, first invented by the Honourable *John Nepeir* Baron of *Marchiston* in *Scotland*, and afterward transformed, and their foundation and use illustrated by the truly Ingenuous Mr. *Henry Briggs*, (from whose Labours I acquainted my self with them) but need say the less of them here, because their excellent use in the *Mathematicks* hath made them familiar to many; for by them, and with much expedition, all troublesome Multiplications and Divisions in *Arithmetick* are avoided, and performed only by Addition instead of Multiplication, and by Substraction instead of Division, The Curious and Laborious Extractions of Roots, are also performed with great Ease, as hereafter shall be shewed. Proportions Disjunct and Continued, Double, Triple, and what else, are thereby made more facil than otherwise can be possible. All Triangles, of what kind soever, with facility resolved. Also not only in *Arithmetick*, but generally in *Geometry*, *Geography*, *Navigation*, *Astronomy*, *&c.* their use is such, as a Volumn of it self is little enough to give Example. Logarithmes by whom first invented and illustrated. How excellently useful.

They are called *Logarithmes* from the *Greek* word λόγος, which signifieth Reason or Proportion, and ἀριθμός, another *Greek* word signifying Numbers. So as the word *Logarithmes* implyeth Rational or Proportional Numbers. Whence the word, and what implyed thereby.

They have the same Foundation with Decimal and Astronomical *Arithmetick*, as the *Table* in the *First Chapter* of *Decimals* well understood will clearly testifie, because as was there hinted, the uppermost Numbers are in Arithmetical Proportion, and are *Indices* or *Logarithmes*, and the lower in Geometrical Progression or Proportion, and do perform by Multiplication and Division, what the other by Addition and Substraction. For if $3+2=5$. *Ergo* $1000 \times 100 = 100000$; 3 being the *Index* of 1000, and 2 of 100, the Total of both 5, shall be the *Index* of the Product of 100 multiplyed into 1000, that is 100000; and so of others. Their foundation one with the Decimals, &c.

The *Indices* of Numbers being thus useful, gave rise to the Invention of Logarithmes, which are indeed nothing else but the *Indices* or Numbers of places in the Higher Powers of Figural Numbers. For if all Integers be advanced into one and the same Quantity of a very High Power, as suppose to the Ten Thousandth Million, then the number of places contained in those Figural Quantities shall be the several Logarithmes for those Integers so advanced. Their rise from Decimal Indices. Logarithmes, what they are, and how made.

To verifie this, you may make tryal with any Number, as suppose 100, the *Index* of which is 2: Let then 100 be multiplyed Figurately to the 10^{th} Power, the Figural Number thereof will be 100000000000000000000, that is an Unit and 20 Cyphers. Then shall the 100^{th} Power be 1 and 200 Cyphers; the 1000^{th} Power 1, and 2000 Cyphers: And so consequently the 10000000000^{th} Power 1, and 20000000000 Cyphers. The Number of places then being so many lacking 1, because the *Index* of To make the Log. of 100.

the Units place is 0, that Number of Cyphers shall be the Logarithme of 100, *viz.* 20000000000. All this is plain by the following Operation.

Example.

	Number of Quantities.	Decimal Indices
Root 100	1	2
Square 10000	2	4
Cube 1000000	3	6
&c. 100000000	4	8
10000000000	5	10
1000000000000	6	12
100000000000000	7	14
10000000000000000	8	16
1000000000000000000	9	18
100000000000000000000	10	20
1. 40	20	40
1. 80	40	80
1. 120	60	120
1. 160	80	160
1. 200	100	200
1. 2000	1000	2000
1. 20000	10000	20000
1. 200000	100000	200000
1. 2000000	1000000	2000000
1. 20000000	10000000	20000000
1. 200000000	100000000	200000000
1. 2000000000	1000000000	2000000000
1. 20000000000	10000000000	20000000000
Figural Quantities.	Figural Indices.	Logarithmes.

What being certain is omitted in Logarithmes.

In the use of Logarithmes both the Figural Numbers themselves, and the Figural Indices which declare the Number of their Quantities or Figurate Multiplications, are omitted, and only the Decimal Indices or number of places in such Figural Numbers used. And because these Indices used for Logarithmes have reference to some certain Quantity or Power to which all absolute Integers are to be contracted, therefore they are rightly placed among Contract Numbers. And though the Figural Quantities, and their Figural Indices be omitted, yet they are certain, and may certainly be known by the number of places in the Logarithme.

Explained.

As because the Logarithme of 100 is 20000000000, in which there is 11 places; I know that 100 was multiplyed Figurately 10 Thousand Million of times; for that 10 Thousand of Millions is the 11^{th} place in Numeration. And if I use the Logarithme of 100 multiplyed to such an High Quantity, all the other Logarithmes I use must be equivalent; that is, the other Numbers must be multiplyed to the same Power or Quantity of 10 Thousand of Millions, and the number of places, or Decimal Indices, taken for their several Logarithmes.

Tables of Logarithmes *to be gotten ready for use.* *What called the* Canon of Logarithmes. *Tables of Mr.* Briggs *large.* *Large* Tables *best.* *The* Table *following transcribed from his.*

To have Logarithmes to seek when they should be used, is inconvenient; to make them as above is more tedious than to resolve the Question otherwise without them. To ease the Artist therefore in his work with Logarithmes, *Tables* are to be prepared, which some call, *The Canon of Logarithmes.* Mr. *Briggs* hath fitted Logarithmes for all Whole Numbers from 1 to 100000, which are sufficient enough to serve for any use, seeing thereby the Logarithme of any Number betwixt 100000 and 1000000000 may be found out. And every Practitioner will find a large *Table* most beneficial. But lest those *Tables* may not be in hand, that the Learner may not be altogether unfurnished wherewith to make experiment, and prove the truth of the Exemplary Operations following, I have therefore in the end of this *Chapter* transcribed from Mr. *Briggs*, a *Table* of Logarithmes for all Integers, from an Unit to 1000, which will be sufficient to give Example by.

The Table *explained.*

The *Table* consists of two Columns, in that towards the Left Hand under the Title Numbers, you have the Absolute Numbers from 1 to 1000, in the Right Hand Column, over against the several Numbers you have their Logarithmes.

The

The Logarithme of 1 is 0; of 10 is 1, with Cyphers; of 100 is 2, with Cyphers; of 1000 is 3, with Cyphers, and so the *Table* may be increased; for the Logarithme of 10000 is 4, with Cyphers; of 100000 is 5, with Cyphers, and so infinitely.

Characteristique *or* Index *of the* Logarithme *what, and how made.*

The Left Hand Figure or Cypher of every Logarithme, may fitly be called the *Characteristique* or *Index* of the Logarithme; for it shews how many places the Absolute Number doth consist of, because it is alwayes less by an Unit than the places of the Absolute Number. As the *Index* of all Absolute Numbers under 10 shall be 0, for they have but 1 place, and Unit substracted from Unit leaves 0. But from 10 to 100, the *Index* shall be 1, because the Numbers are but 1 place distant from the Unit. And for the like Reason Numbers from 100 to 1000, shall have 2 for their *Index*; and Numbers that have 4 places, as 1000 hath, and all Numbers from 1000 to 10000 shall have 3 for their *Index*, and so infinitely; as in the *Tables* in the *First Chapter* of *Decimals* is plainly to be seen: Wherefore in some *Tables* the *Characteristique* being so generally known is omitted.

The like is to be observed in Decimal Fractions; from 1 to Tenth Parts the *Index* is 0; from Tenth Parts to 100 Parts, it's 1; from Hundred Parts to 1000 Parts, it's 2. And so decreasing infinitely.

Cõma, *of what use in* Logarithmes.

The *Characteristique* is commonly seperated from the rest of the Logarithme by a Cõma, as also the next 5 places from the Remainder by another Cõma, and every 10 Logarithmes in the *Table* from the ensuing by a Line. All which is for no other service, than to help the Eye more readily to discern and find them out. Some omit 2 or 3 of the Right Hand Figures, which breeds no material Error.

Logarithme *of a* Fraction *how known from the* Logarithme *of an* Integer.

For distinction between the Logarithme of Integers, and the Logarithme of Fractions; it is to be noted, that as the Logarithme of a Proper Fraction is defective or negative, so it is to be marked with — the sign of Substraction, either over the Head, or at the Left Hand of the *Index*. And yet the Logarithme of an Improper Fraction, or Mixt Number, that consists of an Whole Number and a Fraction, is not defective, but affirmative or abundant.

Numbers

Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
1	0,00000,00000	51	1,70757,01761	101	2,00432,13738	151	2,17897,69473
2	0,30102,99957	52	1,71600,33436	102	2,00860,01718	152	2,18184,35879
3	0,47712,12547	53	1,72427,58696	103	2,01283,72247	153	2,18469,14308
4	0,60205,99913	54	1,73239,37598	104	2,01703,33393	154	2,18752,07208
5	0,69897,00043	55	1,74036,26895	105	2,02118,92991	155	2,19033,16982
6	0,77815,12504	56	1,74818,80270	106	2,02530,58653	156	2,19312,45984
7	0,84509,80400	57	1,75587,48557	107	2,02938,37777	157	2,19589,96524
8	0,90308,99870	58	1,76342,79936	108	2,03342,37555	158	2,19865,70870
9	0,95424,25094	59	1,77085,20116	109	2,03742,64979	159	2,20139,71243
10	1,00000,00000	60	1,77815,12504	110	2,04139,26852	160	2,20411,99827
11	1,04139,26852	61	1,78532,98350	111	2,04532,29788	161	2,20682,58760
12	1,07918,12460	62	1,79239,16895	112	2,04921,80227	162	2,20951,50145
13	1,11394,33523	63	1,79934,05495	113	2,05307,84435	163	2,21218,76044
14	1,14612,80357	64	1,80617,99740	114	2,05690,48513	164	2,21484,38480
15	1,17609,12591	65	1,81291,33566	115	2,06069,78404	165	2,21748,39442
16	1,20411,99827	66	1,81954,39355	116	2,06445,79892	166	2,22010,80880
17	1,23044,89214	67	1,82607,48027	117	2,06818,58617	167	2,22271,64711
18	1,25527,25051	68	1,83250,89127	118	2,07188,20073	168	2,22530,92817
19	1,27875,36010	69	1,83884,90907	119	2,07554,69614	169	2,22788,67046
20	1,30102,99957	70	1,84509,80400	120	2,07918,12460	170	2,23044,89214
21	1,32221,92947	71	1,85125,83487	121	2,08278,53703	171	2,23299,61104
22	1,34242,26808	72	1,85733,24964	122	2,08635,98307	172	2,23552,84469
23	1,36172,78360	73	1,86332,28601	123	2,08990,51114	173	2,23804,61031
24	1,38021,12417	74	1,86923,17197	124	2,09342,16852	174	2,24054,92483
25	1,39794,00087	75	1,87506,12634	125	2,09691,00130	175	2,24303,80487
26	1,41497,33480	76	1,88081,35923	126	2,10037,05451	176	2,24551,26678
27	1,43136,37642	77	1,88649,07252	127	2,10380,37210	177	2,24797,32664
28	1,44715,80313	78	1,89209,46027	128	2,10720,99696	178	2,25042,00023
29	1,46239,79979	79	1,89762,70913	129	2,11058,97103	179	2,25285,30310
30	1,47712,12547	80	1,90308,99870	130	2,11394,33523	180	2,25527,25051
31	1,49136,16938	81	1,90848,50189	131	2,11727,12957	181	2,25767,85749
32	1,50514,99783	82	1,91381,38524	132	2,12057,39312	182	2,26007,13880
33	1,51851,39399	83	1,91907,80924	133	2,12385,16410	183	2,26245,10897
34	1,53147,89170	84	1,92427,92861	134	2,12710,47984	184	2,26481,78230
35	1,54406,80444	85	1,92941,89257	135	2,13033,37685	185	2,26717,17284
36	1,55630,25008	86	1,93449,84512	136	2,13353,89084	186	2,26951,29442
37	1,56820,17241	87	1,93951,92526	137	2,13672,05672	187	2,27184,16065
38	1,57978,35966	88	1,94448,26722	138	2,13987,90864	188	2,27415,78493
39	1,59106,46070	89	1,94939,00066	139	2,14301,48003	189	2,27646,18042
40	1,60205,99913	90	1,95424,25094	140	2,14612,80357	190	2,27875,36010
41	1,61278,38567	91	1,95904,13923	141	2,14921,91127	191	2,28103,33672
42	1,62324,92904	92	1,96378,78273	142	2,15228,83444	192	2,28330,12287
43	1,63346,84556	93	1,96848,29486	143	2,15533,60375	193	2,28555,73090
44	1,64345,26765	94	1,97312,78536	144	2,15836,24921	194	2,28780,17299
45	1,65321,25138	95	1,97772,36053	145	2,16136,80022	195	2,29003,46114
46	1,66275,58317	96	1,98227,12330	146	2,16435,28558	196	2,29225,60714
47	1,67209,78579	97	1,98677,17343	147	2,16731,73347	197	2,29446,62262
48	1,68124,12374	98	1,99122,60757	148	2,17026,17154	198	2,29666,51903
49	1,69019,60800	99	1,99563,51946	149	2,17318,62684	199	2,29885,30764
50	1,69897,00043	100	2,00000,00000	150	2,17609,12591	200	2,30102,99957

Numb.

Nu	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
201	2,30319,60574	251	2,39967,37215	301	2,47856,64956	351	2,54530,71165
202	2,30535,13694	252	2,40140,05408	302	2,48000,69430	352	2,54654,26635
203	2,30749,60379	253	2,40312,05212	303	2,48144,26285	353	2,54777,47054
204	2,30963,01674	254	2,40483,37166	304	2,48287,35836	354	2,54900,32620
205	2,31175,38611	255	2,40654,01804	305	2,48429,98393	355	2,55022,83531
206	2,31386,72204	256	2,40823,99653	306	2,48572,14265	356	2,55144,99980
207	2,31597,03455	257	2,40993,31233	307	2,48713,83755	357	2,55266,82161
208	2,31806,33350	258	2,41161,97060	308	2,48855,07165	358	2,55388,30266
209	2,32014,62861	259	2,41329,97641	309	2,48995,84794	359	2,55509,44486
210	2,32221,92947	260	2,41497,33480	310	2,49136,16938	360	2,55630,25008
211	2,32428,24553	261	2,41664,05073	311	2,49276,03890	361	2,55750,72019
212	2,32633,58609	262	2,41830,12913	312	2,49415,45940	362	2,55870,85705
213	2,32837,96034	263	2,41995,57485	313	2,49554,43376	363	2,55990,66250
214	2,33041,37733	264	2,42160,39269	314	2,49692,96481	364	2,56110,13836
215	2,33243,84599	265	2,42324,58739	315	2,49831,05538	365	2,56229,28645
216	2,33445,37512	266	2,42488,16366	316	2,49968,70826	366	2,56348,10854
217	2,33645,97338	267	2,42651,12614	317	2,50105,92622	367	2,56466,60643
218	2,33845,64936	268	2,42813,47940	318	2,50242,71200	368	2,56584,78187
219	2,34044,41148	269	2,42975,22800	319	2,50379,06831	369	2,56702,63662
220	2,34242,26808	270	2,43136,37642	320	2,50514,99783	370	2,56820,17241
221	2,34439,22737	271	2,43296,92909	321	2,50650,50324	371	2,56937,39096
222	2,34635,29745	272	2,43456,89040	322	2,50785,58717	372	2,57054,29399
223	2,34830,48630	273	2,43616,26470	323	2,50920,25223	373	2,57170,88318
224	2,35024,80183	274	2,43775,05628	324	2,51054,50102	374	2,57287,16022
225	2,35218,25181	275	2,43933,26938	325	2,51188,33610	375	2,57403,12677
226	2,35410,84391	276	2,44090,90821	326	2,51321,76001	376	2,57518,78449
227	2,35602,48572	277	2,44247,97691	327	2,51454,77527	377	2,57634,13502
228	2,35793,48470	278	2,44404,47959	328	2,51587,38437	378	2,57749,17998
229	2,35983,54823	279	2,44560,42033	329	2,51719,58979	379	2,57863,92100
230	2,36172,78360	280	2,44715,80313	330	2,51851,39399	380	2,57978,35966
231	2,36361,19799	281	2,44870,63199	331	2,51982,79938	381	2,58092,49757
232	2,36548,79849	282	2,45024,91083	332	2,52113,80837	382	2,58206,33629
233	2,36735,59210	283	2,45178,64355	333	2,52244,42335	383	2,58319,87740
234	2,36921,58574	284	2,45331,83400	334	2,52374,64668	384	2,58433,12244
235	2,37106,78623	285	2,45484,48600	335	2,52504,48070	385	2,58546,07295
236	2,37291,20030	286	2,45636,60331	336	2,52633,92774	386	2,58658,73047
237	2,37474,83460	287	2,45788,18967	337	2,52762,99009	387	2,58771,09650
238	2,37657,69571	288	2,45939,24878	338	2,52891,67003	388	2,58883,17256
239	2,37839,79009	289	2,46089,78428	339	2,53019,96982	389	2,58994,96013
240	2,38021,12417	290	2,46239,79979	340	2,53147,89170	390	2,59106,46070
241	2,38201,70426	291	2,46389,29890	341	2,53275,43790	391	2,59217,67574
242	2,38381,53660	292	2,46538,28514	342	2,53402,61061	392	2,59328,60670
243	2,38560,62736	293	2,46686,76204	343	2,53529,41200	393	2,59439,25504
244	2,38738,98263	294	2,46834,73304	344	2,53655,84426	394	2,59549,62218
245	2,38916,60844	295	2,46982,20160	345	2,53781,90951	395	2,59659,70956
246	2,39093,51071	296	2,47129,17111	346	2,53907,60988	396	2,59769,51859
247	2,39269,69533	297	2,47275,64493	347	2,54032,94748	397	2,59879,05068
248	2,39445,16808	298	2,47421,62641	348	2,54157,92439	398	2,59988,30721
249	2,39619,93471	299	2,47567,11883	349	2,54282,54270	399	2,60097,28957
250	2,39794,00087	300	2,47712,12547	350	2,54406,80444	400	2,60205,99913

Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
401	2,60314,43726	451	2,65417,65419	501	2,69983,77259	551	2,74115,15989
402	2,60422,60531	452	2,65513,84348	502	2,70970,37171	552	2,74193,90777
403	2,60530,50461	453	2,65609,82020	503	2,70156,79851	553	2,74272,51313
404	3,60638,13651	454	2,65705,58529	504	2,70243,05364	554	2,74350,97647
405	2,60745,50232	455	2,65801,13967	505	2,70329,13781	555	2,74429,29831
406	2,60852,60336	456	2,65896,48427	506	2,70415,05168	556	2,74507,47916
407	2,60959,44092	457	2,65991,62001	507	2,70500,79593	557	2,74585,51952
408	2,61066,01631	458	2,66086,54780	508	2,70586,37123	558	2,74663,41990
409	2,61172,33080	459	2,66181,26855	509	2,70671,77823	559	2,74741,18079
410	2,61278,38567	460	2,66275,78317	510	2,70757,01761	560	2,74818,80270
411	2,61384,18219	461	2,66370,09254	511	2,70842,09001	561	2,74896,28613
412	2,61489,72160	462	2,66464,19756	512	2,70926,99610	562	2,74973,63156
413	2,61595,00517	463	2,66558,09910	513	2,71011,73651	563	2,75050,83949
414	2,61700,03411	464	2,66651,79806	514	2,71096,31190	564	2,75127,91040
415	2,61804,80967	465	2,66745,29529	515	2,71180,72290	565	2,75204,84478
416	2,61909,33306	466	2,66838,59167	516	2,71264,97016	566	2,75281,64312
417	2,62013,60550	467	2,66931,68806	517	2,71349,05431	567	2,75358,30589
418	2,62117,62818	468	2,67024,58531	518	2,71432,97597	568	2,75434,83357
419	2,62221,40230	469	1,67117,28427	519	2,71516,73578	569	2,75511,22664
420	2,62324,92904	470	2,67209,78579	520	2,71600,33436	570	2,75587,48557
421	2,62428,20958	471	2,67302,09071	521	2,71683,77233	571	2,75663,61082
422	2,62531,24510	472	2,67394,19986	522	2,71767,05030	572	2,75739,60288
423	2,62634,03674	573	2,67486,11407	523	2,71850,16889	573	2,75815,46220
424	2,62736,58566	474	2,67577,83417	524	2,71933,12870	574	2,75891,18924
425	2,62838,89301	475	2,67669,36096	525	2,72015,93034	575	2,75966,78447
426	2,62940,95991	476	2,67760,69527	526	2,72098,57442	576	2,76042,24834
427	2,63042,78750	477	2,67851,83750	527	2,72181,06152	577	2,76117,58132
428	2,63144,37690	478	2,67942,78966	528	2,72263,39225	578	2,76192,78384
429	2,63245,72922	479	2,68033,55134	529	2,72345,56720	579	2,76267,85637
430	2,63346,84556	480	2,68124,12374	530	2,72427,58696	580	2,76342,79936
431	2,63447,72702	481	2,68214,50764	531	2,72509,45211	581	2,76417,61324
432	2,63548,37468	482	2,68304,70382	532	2,72591,16323	582	2,76492,29846
433	2,63648,78964	483	2,68394,71308	533	2,72672,72090	583	2,76566,85548
434	2,63748,97295	484	2,68484,53616	534	2,72754,12570	584	2,76641,28471
435	2,63848,92570	485	2,68574,17386	535	2,72835,37820	585	2,76715,58661
436	2,63948,64893	486	2,68663,62693	536	2,72916,47897	586	2,76789,76160
437	2,64048,14370	487	2,68752,89612	537	2,72997,42857	587	2,76863,81012
438	2,64147,41105	488	2,68841,98220	538	2,73078,22757	588	2,76937,73261
439	2,64246,45202	489	2,68930,88591	539	2,73158,87652	589	2,77011,52948
440	2,64345,26765	490	2,69019,60800	540	2,73239,37598	590	2,77085,20116
441	2,64443,85895	491	2,69108,14921	541	2,73319,72651	591	2,77158,74809
442	2,64542,22693	492	2,69196,51028	542	2,73399,92865	592	2,77232,17067
443	2,64640,37262	493	2,69284,69193	543	2,73479,98296	593	2,77305,46934
444	2,64738,27901	494	2,69372,69489	544	2,73559,88997	594	2,77378,64450
445	2,64836,00110	495	2,69460,51989	545	2,73639,65023	595	2,77451,69657
446	2,64933,48587	496	2,69548,16765	546	2,73719,26427	596	2,77524,62597
447	2,65030,75231	497	2,69635,63887	547	2,73798,73263	597	2,77597,43311
448	2,65127,80140	498	2,69722,93428	548	2,73878,05585	598	2,77670,11840
449	2,65224,63410	499	2,69810,05456	549	2,73957,23445	599	2,77742,68224
450	2,65321,25138	500	2,69897,00043	550	2,74036,26895	600	2,77815,12504

Numb.

Nu	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
601	2,77887,44720	651	2,81358,09886	701	2,84571,80180	751	2,87563,99370
602	2,77959,64913	652	2,81424,75957	702	2,84633,71121	752	2,87621,78406
603	2,78031,73121	653	2,81491,31813	703	2,84695,53250	753	2,87679,49762
604	2,78103,69386	654	2,81557,77483	704	2,84757,26591	754	2,87737,13459
605	2,78175,53747	655	2,81624,13000	705	2,84818,91170	755	2,87794,69516
606	2,78247,26242	656	2,81690,38394	706	2,84880,47011	756	2,87852,17955
607	2,78318,86911	657	2,81756,53696	707	2,84941,94138	757	2,87909,58795
608	2,78390,35793	658	2,81822,58936	708	2,85003,32577	758	2,87966,92056
609	2,78461,72926	659	2,81888,54145	709	2,85064,62352	759	2,88024,17759
610	2,78532,98350	660	2,81954,39355	710	2,85125,83487	760	2,88081,35923
611	2,78604,12102	661	2,82020,14595	711	2,85186,96007	761	2,88138,46568
612	2,78675,14221	662	2,82085,79894	712	2,85247,99936	762	2,88195,49713
613	2,78746,04745	663	2,82151,35284	713	2,85308,95299	763	2,88252,45380
614	2,78816,83711	664	2,82216,80794	714	2,85369,82118	764	2,88309,33586
615	2,78887,51158	665	2,82282,16453	715	2,85430,60418	765	2,88366,14352
616	2,78958,07122	666	2,82347,42292	716	2,85491,30223	766	2,88422,87696
617	2,79028,51640	667	2,82412,58339	717	2,85551,91557	767	2,88479,53639
618	2,79098,84751	668	2,82477,64625	718	2,85612,44442	768	2,88536,12200
619	2,79169,05490	669	2,82542,61178	719	2,85672,88904	769	2,88592,63398
620	2,79239,16895	670	2,82607,48027	720	2,85733,24964	770	2,88649,07252
621	2,79309,16002	671	2,82672,25202	721	2,85793,52647	771	2,88705,43781
622	2,79379,03847	672	2,82736,92731	722	2,85853,71976	772	2,88761,73003
623	2,79448,80467	673	2,82801,50642	723	2,85913,82973	773	2,88817,94939
624	2,79518,45897	674	2,82865,98965	724	2,85973,85662	774	2,88874,09607
625	2,79588,00173	675	2,82930,37728	725	2,86033,80066	775	2,88930,17025
626	2,79657,43332	676	2,82994,66959	726	2,86093,66207	776	2,88986,17213
627	2,79726,75408	677	2,83058,86687	727	2,86153,44109	777	2,89042,10188
628	2,79795,96437	678	2,83122,96939	728	2,86213,13793	778	2,89097,95970
629	2,79865,06454	679	2,83186,97743	729	2,86272,75283	779	2,89153,74577
630	2,79934,05495	680	2,83250,89127	730	2,86332,28601	780	2,89209,46027
631	2,80002,93592	681	2,83314,71119	731	2,86391,73770	781	2,89265,10339
632	2,80071,70783	682	2,83378,43747	732	2,86451,10811	782	2,89320,67531
633	2,80140,37100	683	2,83442,07037	733	2,86510,39746	783	2,89376,17621
634	2,80208,92579	684	2,83505,61017	734	2,86569,60599	784	2,89431,60627
635	2,80277,37253	685	2,83569,05715	735	2,86628,73391	785	2,89486,96567
636	2,80345,71156	686	2,83632,41157	736	2,86687,78143	786	2,89542,25460
637	2,80413,94323	687	2,83695,67371	737	2,86746,74879	787	2,89597,47324
638	2,80482,06787	688	2,83758,84382	738	2,86805,63618	788	2,89652,62175
639	2,80550,08582	689	2,83821,92219	739	2,86864,44384	789	2,89707,70032
640	2,80617,99740	690	2,83884,90907	740	2,86923,17197	790	2,89762,70913
641	2,80685,80295	691	2,83947,80474	741	2,86981,82080	791	2,89817,64835
642	2,80753,50281	692	2,84010,60945	742	2,87040,39053	792	2,89872,51816
643	2,80821,09729	693	2,84073,32346	743	2,87098,88138	793	2,89927,31873
644	2,80888,58674	694	2,84135,94705	744	2,87157,29355	794	2,89982,05024
645	2,80955,97146	695	2,84198,48046	745	2,87215,62727	795	2,90036,71287
646	2,81023,25180	696	2,84260,92396	746	2,87273,88275	796	2,90091,30677
647	2,81090,42807	697	2,84323,27781	747	2,87332,06018	797	2,90145,83214
648	2,81157,50059	698	2,84385,54226	748	2,87390,15979	798	2,90200,28914
649	2,81224,46968	699	2,84447,71757	749	2,17448,18177	799	2,90254,67793
650	2,81291,33566	700	2,84509,80400	750	2,87506,12634	800	2,90308,99870

Numb.

Nu.	Logarithmes.	Nu	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
801	2,90363,25161	851	2,92992,95601	901	2,95472,47910	951	2,97818,05169
802	2,90417,43683	852	2,93043,95948	902	2,95520,65375	952	2,97863,69484
803	2,90471,55453	853	2,93094,90312	903	2,95568,77503	953	2,97909,29006
804	2,90525,60487	854	2,93145,78707	904	2,95616,84305	954	2,97954,83747
805	2,90579,58804	855	2,93196,61147	905	2,95664,85792	955	2,98000,33716
806	2,90633,50418	856	2,93247,37647	906	2,95712,81977	956	2,98045,78923
807	2,90687,35347	857	2,93298,08219	907	2,95760,72871	957	2,98091,19378
808	2,90741,13608	858	2,93348,72878	908	2,95808,58485	958	2,98136,55091
809	2,90794,85216	859	2,93399,31638	909	2,95856,38832	959	2,98181,86072
810	2,90848,50189	860	2,93449,84512	910	2,95904,13923	960	2,98227,12330
811	2,90902,08542	861	2,93500,31515	911	2,95951,83770	961	2,98272,33877
812	2,90955,60292	862	2,93550,72658	912	2,95999,48383	962	2,98317,50720
813	2,91009,05456	863	2,93601,07958	913	2,96047,07775	963	2,98362,62871
814	2,91062,44049	864	2,93651,37425	914	2,96094,61957	964	2,98407,70339
815	2,91115,76087	865	2,93701,61075	915	2,96142,10941	965	2,98452,73133
816	2,91169,01588	866	2,93751,78920	916	2,96189,54737	966	2,98497,71264
817	2,91222,20565	867	2,93801,90975	917	2,96236,93357	967	2,98542,64741
818	2,91275,33037	868	2,93851,97252	918	2,96284,26812	968	2,98587,53573
819	2,91328,39018	869	2,93901,97765	919	2,96331,55114	969	2,98632,37771
820	2,91381,38524	870	2,93951,92526	920	2,96378,78273	970	2,98677,17343
821	2,91434,31571	871	2,94001,81550	921	2,96425,96302	971	2,98721,92299
822	2,91487,18175	872	2,94051,64849	922	2,96473,09211	972	2,98766,62649
823	2,91539,98352	873	2,94101,42437	923	2,96520,17010	973	2,98811,28403
824	2,91592,72117	874	2,94151,14326	924	2,96567,19712	974	2,98855,89569
825	2,91645,39485	875	2,94200,80530	925	2,96614,17327	975	2,98900,46157
826	2,91698,00473	876	2,94250,41062	926	2,96661,09867	976	2,98944,98177
827	2,91750,55096	877	2,94299,95934	927	2,96707,97341	977	2,98989,45637
828	2,91803,03368	878	2,94349,45159	928	2,96754,79762	978	2,99033,88548
829	2,91855,45306	879	2,94399,88751	929	2,96801,57140	979	2,99078,26918
830	2,91907,80924	880	2,94448,26722	930	2,96848,29486	980	2,99122,60757
831	2,91960,10238	881	2,94497,59084	931	2,96894,96810	981	2,99166,90074
832	2,92012,33263	882	2,94546,85851	932	2,96941,59124	982	2,99211,14878
833	2,92064,50014	883	2,94596,07036	933	2,96988,16437	983	2,99255,35178
834	2,92116,60506	884	2,94645,22650	934	2,97034,68762	984	2,99299,50984
835	2,92168,64755	885	2,94694,32707	935	2,97081,16109	985	2,99343,62305
836	2,92220,62774	886	2,94743,37219	936	2,97127,58487	986	2,99387,69149
837	2,92272,54580	887	2,94792,36198	937	2,97173,95909	987	2,99431,71527
838	2,92324,40186	888	2,94841,29658	938	2,97220,28384	988	2,99475,69446
839	2,92376,19608	889	2,94890,17610	939	2,97266,55923	989	2,99519,62916
840	2,92427,92861	890	2,94939,00066	940	2,97312,78536	990	2,99563,51946
841	2,92479,59958	891	2,94987,77040	941	2,97358,96234	991	2,99607,36545
842	2,92531,20915	892	2,95036,48544	942	2,97405,09028	992	2,99651,16722
843	2,92582,75746	893	2,95085,14589	943	2,97451,16927	993	2,99694,92485
844	2,92634,24466	894	2,95133,75188	944	2,97497,19943	994	2,99738,63844
845	2,92685,67089	895	2,95182,30353	945	2,97543,18085	995	2,99782,30807
846	2,92737,03630	896	2,95230,80097	946	2,97589,11364	996	2,99825,93384
847	2,92788,34103	897	2,95279,24430	947	2,97634,99790	997	2,99869,51583
848	2,92839,58523	898	2,95327,63367	948	2,97680,83373	998	2,99913,05413
849	2,92890,76902	899	2,95375,96917	949	2,97726,62124	999	2,99956,54882
850	2,92941,89257	900	2,95424,25094	950	2,97772,36053	1000	3,00000,00000

CHAP.

CHAP. II.

Reduction of Logarithmes.

TO operate by Logarithmes, two things are necessary.

What necessary to work with Logarithmes.

First, That for every Integer from an Unit upward, and for every Fraction from an Unit downward, *ad infinitum*, we know how to fit a Logarithme thereto; only an Unit, which neither multiplyeth nor divideth, needeth no Logarithme. 1.

Secondly, That for every Logarithme we be able to find out the Integer, or Fraction, Common or Decimal, that answers thereto. Both these are to be found under the *Propositions* following in this *Chapter* of *Reduction*, called therefore by some, *Invention of Logarithmes*. 2.

Reduction sometime called Invention of Logarithmes.

Proposition I. *To find a Logarithme for any Absolute Number under* 1000, *expressed in the* Table

Prop. 1. *To find a* Logarithme *for a Number under* 1000.

Seek in the Left Hand Column of the *Table*, under the Title Numbers, for the Number whose Logarithme is desired; and over against the same, in the Right Hand Column, you shall find the Logarithme answering thereto

As if the Logarithme of 12 be desired; over against 12 stands in the *Table*, 1,07918,12460, which is the Logarithme thereof. So the Logarithme of 340 is found to be 2,53147,89170. *Example.*

Prop. II. *To find a Logarithme for any Absolute Number above* 1000.

Prop. 2. *To find a* Logarithme *for a Number above* 1000.

Where the Tables of Logarithmes are large enough, the Logarithme is to be found, as the Logarithme of any Number under 1000 is to be found in the *Table* foregoing.

But where the Tables are too short, proceed thus;

1. According to the first Proposition find the first 3, or more Left Hand Figures of your given Number, as far as your Tables will serve.

2. Instead of the *Index* of the Logarithme found, place another which shall fit the Number given.

3. Take the difference between the Logarithme found, and the next ensuing, and take the rest of the Figures that remain to the Number given, after the 3 or more Left Hand Figures be cut off.

4. By these get a proportional part thus: As an Unit and so many Cyphers, as there be Figures remaining, to the same Figures, so shall the difference between the Logarithme found, and that which follows, be to another Number, which found is to be added to the Logarithme before found; and the summe you may take for the quesited Logarithme. And though there will be some little alteration from the true Logarithme, yet the difference being inconsiderable makes the Error immaterial.

Example. If the Logarithme of 99945, be sought, then by the *Table* the Logarithme of 999, the first 3 Left Hand Figures is seen to be 2,99956,54882. The *Index* of 99945, must be 4, therefore I alter the *Index*, and make the Logarithme 4,99956,54882. The difference between the Logarithme of 999, and the Logarithme of 1000, the next is 43,45118. The Figures that remain besides 999 are 45. Then the Analogy is thus: As 100. 45 :: 4345118. 1955303; for by multiplying 45 into 4345118, and dividing the Product by 100, that Number of 1955303 is gotten, which added to the Logarithme before found makes 4,99976,10185 the Logarithme desired; and differs but little with the true Logarithme in the Tables of Mr. *Briggs*, found there to be 4,99976,10723. *Example.*

Prop. III. *To find the Logarithme of a Common Fraction, or Integer and Fraction Mixt.*

Prop. 3. *To find the* Logarithme *of a Common Fraction.*

Vulgar Fractions in the *Second Part* of the *First Book*, were considered, as Proper, Equal, and Improper.

Equal Fractions being alwayes an Unit, have 0 for their Logarithme. *Equal.*

The Logarithme of Proper or Improper Fractions are found by substracting the Logarithme of the Lesser Term out of the Logarithme of the Greater. The remain shall be the Logarithme of the Fraction, which shall be affirmative if the Fraction be Improper, but negative if the Fraction be Proper. *Proper or Improper.*

As to find the Logarithme of $\frac{3}{4}$ or $\frac{4}{3}$, the Logarithme of each is 0,12493,87366, but of the Proper Fraction negative, of the other affirmative. *Examples.*

Proper Fraction $\frac{3}{4}$.		Improper Fraction $\frac{4}{3}$.	
0,60205,99913	Log. of 4.	0,60205,99913	Log. of 4.
0,47712,12547	Log. of 3.	0,47712,12547	Log. of 3.
—0,12493,87366	Log. of $\frac{3}{4}$ differ.	0,12493,87366	Log. of $\frac{4}{3}$.

Mixt Numbers first to be reduced into an Improper Fraction. Example.

If the Logarithme of a Mixt Number be sought, reduce the same into an Improper Fraction, and seek the Logarithme thereof as before.

As to get the Logarithme of $2\frac{1}{2}$, I reduce it into $\frac{5}{2}$, and taking the Logarithme of 2 from the Logarithme of 5, find the Log. desired of $\frac{5}{2}$ or $2\frac{1}{2}$ to be 0,39794,00086.

Arithmetical Complement what. Logarithme of the Decimal Fraction how got. Example.

If the Logarithme of the Denominator of any Common Fraction be substracted out of the Logarithme of the Numerator, the Remain will differ nothing, save in the *Index*, from the Arithmetical Complement (which is the Remainder of any Logarithme substracted out of 10 with Cyphers); and this Complement, with the true *Index*, may be taken for the Logarithme of the Fraction, but is properly the Logarithme of the Decimal Fraction; for so the Logarithme of a Decimal Fraction may be gotten.

As in the former instance of $\frac{3}{4}$, taking the Logarithme of 4 from the Logarithme of 3, in the *Index* there will lack an Unit, therefore marked negative. The Logarithme by this way is found to be —1,87506,12634, which Logarithme in the *Table* answers to 75, being here negative shall be the Logarithme of '75, or $\frac{75}{100}$, which is $\frac{3}{4}$: For 75 is $\frac{3}{4}$ of 100. And this Logarithme and the former make up the Logarithme of 1, as at *A* may be seen, by adding them together, and differs not from the Arithmetical Complement, except in the *Index*, as following may be seen at *B*.

Proper Fraction $\frac{3}{4}$.	
0,47712,12547	Log. of 3.
0,60205,99913	Log. of 4.
—1,87506,12634	Log. of $\frac{3}{4}$ of 100, or '75.

A.		*B.*	
—1,87506,12634	Log. of '75.	10,00000,00000	10, and Cyphers.
—0,12493,87366	Log of $\frac{3}{4}$.	0,12493,87366	Log. of $\frac{3}{4}$.
0,00000,00000	Log. of 1.	9,87506,12634	Arith. Compl.

Notice to be taken how the Logarithme of the Fraction is gotten.

But great notice is to be taken, whether the Logarithme of the Fraction be gotten this or the other way: For that in Addition and Substraction, Multiplication and Division of Logarithmes hereafter, the Case will differ between the Logarithme of the Decimal, and the Logarithme of the Common Fraction.

Prop. 4. *To find the Logarithme of a Decimal Pure or Mixt.*

Prop. IV. *To find the Logarithme of a Decimal, or Integers mixt with Decimals.*

Besides the way of getting the Decimal Logarithme of a Fraction last mentioned, take this General Rule.

Suppose the Numbers whose Logarithme is sought to be an Integer, and find the Logarithme thereof accordingly, then prefix an *Index* thereto according to the distance of the first Left Hand Figure of the given Number from unity: For the Characteristique alwayes differs according to the nature of the Number, though the rest of the Logarithme may be the same. Examples.

Examples.

	Numbers.	Logarithmes.
Integers	485	2,68574,17386.
Mixt	48,5	1,68574,17386.
	4,85	0,68574,17386.
Decimals	,485	—1,68574,17386.
	,0485	—2,68574,17386.

Prop. 5. *To find the Number for a Logarithme given.*

Prop. V. *To find the Absolute Number corresponding to a Logarithme given, be it integral or mixed.*

Those Logarithmes that answer to Integers or Mixt Numbers may be found in the *Tables*.

1. Either

1. Either the Logarithme with the *Index*. Or,
2. The Logarithme with another *Index* Greater or Lesser. Or,
3. The *Index* with another Logarithme. Or else,
4. Neither *Index* nor Logarithme exactly. And therefore,

If the Logarithme be in the Tables. If the Logarithme be expressed in the Tables, then by the orderly increase or decrease of the Logarithmes, seeking in the Column under the Title Logarithmes, you will soon find the Logarithme sought, and just against it under the Title Numbers in the Left Hand Column, you find the Absolute Number that answers thereto.

Example. As if the Logarithme 2,15836,24921 be given, and the Absolute Number belonging thereto be desired, I look in the *Table* and find the Logarithme, and in the Left Hand Column 144 to be the corresponding Number.

If the Logarithme have an Index Greater or Lesser. When found with a Lesser. If the Logarithme given have a greater *Index* than is to be found in the *Tables*, then considering that the Logarithme of 2 is the Logarithme of 20, only altering the *Index*; and so the Logarithme of 3 the same with 30, of 4 with 40, &c. the Logarithme of 11 the same with 110; the Logarithme of 12 with 120, &c. It is easie having found the Logarithme with the least *Index* in the *Tables* to produce the true Number corresponding to the Greatest *Index*, by adding to the Right Hand of the Number answering the Logarithme found with the least *Index*, so many Cyphers as there are Units in the *Index* of the given Logarithme more than in the *Index* of the Logarithme found.

Example. As if 4,30102,99957, be the Logarithme given, neglecting the *Index* I look in the *Tables*, and find the Logarithme against 2, 20, 200, &c. but all of a different *Index*. I take that of 2, being the least, and adjoyn to 2 the Number corresponding 4 Cyphers, because the *Index* of the Logarithme given was 4, and the *Index* of 2 was 0. So is 20000 the Number answering to the Logarithme 4,30102,99957.

When found with a Greater Index. If the Logarithme given be not precisely found in the *Table*, in the proper place, according to the *Index* thereof, or with a lesser *Index*, then the same may be sought for among the Logarithmes of a greater *Index*. And if found there you shall have the absolute Number thereof in more Figures than the *Index* of the given Logarithme requires. Wherefore cut off so many of the Right Hand Figures as are superfluous; for the Numerator of a Fraction, whose Denominator shall be an Unit with so many Cyphers as there be Figures in the Numerator, or it may be set as a Decimal.

Example. *Example.* Let the Logarithme given be 1,0969 ,00 30, which sought for among the Logarithmes whose *Index* is 1, cannot be found exactly, but is found among the Logarithmes that have 2 for their *Index*, and over against the same the absolute Number 125, which consists of 3 Figures, whereas the *Index* of the given Logarithme being but 1, required but 2 Figures in the absolute Number, therefore I cut off the last Right Hand Figure of 125, and leave 2, *viz.* 12, and the 5 is Numerator of a Fraction to 10 the Denominator; so shall the absolute Number be $12\frac{5}{10}$, or 12,5.

And if the given Logarithme had been found with a greater *Index* than 2, as it happeneth oftentimes in large Tables, such as those of Mr. *Briggs* are: As suppose 1,04328,36656, the Logarithme given and found under the *Index* 4, and the corresponding Number 11048, then should the Number be 11,048, or $11\frac{048}{1000}$. And if the *Index* of the given Logarithme had been 2, then $110\frac{48}{100}$. If 3, then $1104\frac{8}{10}$, &c.

If the Index be found with another Logarithme. If the *Index* be found with another Logarithme than that given, and the *Table* not large enough to find it with a greater *Index*, then enter the *Table* with the *Index* of the Logarithme given, and find the next lesser Logarithme to the given Logarithme, and you have the Integer answering thereto, to which a Fraction is to be adjoyned, which is thus gotten.

Substract the Logarithme found from the Logarithme given, the Remain shall be the Numerator, and the difference between the Logarithme found, and that which next follows in the *Table* shall be the Denominator of the Fraction.

Example. As in the former instance; if 1,09691,00130, be the given Logarithme, the next lesser Logarithme found in the *Table* with the *Index* 1, is 1,07918,12460, and the absolute Number answering thereto is 12; then substracting the Logarithme of 12 from the Logarithme given, the Remain is 1772,87670, which shall be the Numerator to 3476,21063, the difference between the Logarithme of 12 and the Logarithme of 13. And being near $\frac{1}{2}$, by cutting off many of the Right Hand Figures, (without sensible Error) may be reduced to $\frac{17}{34}$ equivalent to $\frac{5}{10}$ or $\frac{1}{2}$, as before.

Best to be turned into a Decimal. Or rather, turn it into a Decimal Fraction, which is thus done; adjoyn Cyphers to the Right Hand of the Difference or Remain after the next lesser Logarithme in the *Table* is substracted from the given Logarithme: And divide this Number with the Cyphers so adjoyned by the Difference between the next Lesser and next Greater Logarithmes found in the *Table*; wherefore if to 1772,87670, there be but 2 Cyphers *Example.*

phers adjoyned, and the same be divided by 3476,21063, the Quotient will be 51, to be added to 12 as a Decimal, and so the Logarithme given, shall be the Logarithme for 12,51, and by adjoyning more Cyphers, and continuing the Division, the Decimal will be greater.

| 9278
347 1385|7
1772,8767000 (51
3476,210633
3476,2106

If Index *nor* Logarithme *be found.*

If neither *Index* nor Logarithme be found exactly in the *Table*, proceed thus; under the Greatest *Index* your Tables will afford, find the next lesser Logarithme to the Logarithme given, neglecting the *Indices* of both, and reserve the Number answering to the Logarithme found apart, and note the true *Index* of that Number. Then substract the Logarithme found from the Logarithme given, and with the Difference between the Logarithme found and that which next follows in the *Table*, you may get a proportional part by this Analogy. As the Difference between the Logarithme found and the following, to the Difference between the Logarithme found, and the Logarithme given; so is an Unit with so many Cyphers as there are Absolute Numbers wanting in the Number found to make up the *Index* of the Logarithme given, to the same Numbers wanting, which Numbers gotten adjoyn to the Right Hand.

Example.

As if the Logarithme given be 4,99976,10185, under the greatest *Index* in the foregoing *Table* 2, I find 99956,54882, the next Logarithme to the Logarithme given, and the corresponding Number to be 999, the true *Index* whereof is 2, the Logarithme of 999 taken from the given Logarithme leaves 19,55303, the Difference between the Logarithme of 999, and the Logarithme of 1000 is 43,45118, the *Index* of the given Logarithme is 4, and of 999 but 2, therefore 2 places are wanting: Then I say, As 43,45118, to 19,55303, so is 100 to 45, almost; which 45 adjoyned to 999, makes 99945, for the Number corresponding to the given Logarithme.

| 434510
2 172558|8
19,5530300 (44 and above.
43,451188
43,4511

And in like manner the Absolute Number answering to Logarithmes given that have 6, 7, 8, or more Units in their *Index* may be had exact enough for Operation.

Prop. 6. *To find a Fraction for a Logarithme.*

Prop. VI. *To find a Fraction for a given Logarithme.*

Fractions desired for a given Logarithme may be either Common or Decimal. The Decimal much the easier to be found, and therefore in common use the Logarithmes of them are best to be chosen; yet sometime the Logarithme of a Vulgar Fraction may be necessary: The way to get the Logarithmes of both, see in the *Sections* following.

Logarithme *of a Common Fraction found with the* Index *and desired Common.*

§. 1. When a Common Fraction is desired for the Logarithme thereof given. If the given Logarithme with the proper *Index* be found in the *Table*, then the absolute Number corresponding shall be the Denominator of the Fraction, to which 1 shall be Numerator.

Example.

As if —0,30102,99957, be the Logarithme given, the absolute Number answering thereto is 2, which shall be the Denominator; so shall $\frac{1}{2}$ be the Fraction of the given Logarithme.

Logarithme *of a Common Fraction found, and desired to be a Decimal.*

§. 2. When the Logarithme of a Common Fraction with the given *Index* is found in the *Table*, and the Fraction is desired to be a Decimal; substract the given Logarithme from the Logarithme of the Decimal Denominator given, the corresponding Number to the remaining Logarithme shall be the Numerator to the given Denominator.

Example.

As if —0,30102,99957, be given, and the Decimal thereof be sought in Primes or Tenths, then I take the Logarithme given from the Logarithme of 10, the Remain

is 0,69897,00043, which is the Logarithme of 5; so shall 5 be the Numerator to 10, and the Decimal Fraction $\frac{5}{10}$.

If the Denominator given had been 100, and so the Decimal set in Seconds, then the Logarithme given taken from the Logarithme of 100, leaves 1,69897,00043, the Logarithme of 50, which shall be the Numerator to 100, and the Fraction $\frac{50}{100}$, or $\frac{5}{10}$ as before.

The like is to be done with others: Neverthelefs it often happens, the greater the Denominator given, the more easie to find the correfponding Number to the Logarithme of the Remain, to be an Integer.

The Greater the Denominator the more easie to find.

§. 3. When the given Logarithme of a Common Fraction is not found in the *Table*, yet if the Fraction be defired to be a Decimal, the work is the fame with the laft above-mentioned.

Logarithme of a Common Fraction not found, and defired to be a Decimal.

As if —0,12493,87366, be the Logarithme given, and I defire to know the Decimal Fraction fignified thereby, whofe Denominator fhall be 100; or Seconds, after Subftraction of the given Logarithme from the Logarithme of 100, the Remain is 1,87506,12634, the Logarithme of 75, which fhall be the Numerator to 100, as was feen before.

Example.

Here may be noted what was mentioned above, that the greater the Denominator, the more exact the Whole Number may be found. For if to this Logarithme 0,12493,87366, the Denominator had been given 10, then would the Remain have been 0,87506,12634, the neareft Integer anfwering which is 7, which is 7 Primes or Tenths; but then for that the Remain is greater than the Logarithme of 7, the Numerator is not exact, but I lofe the 5 Seconds, unlefs I work for them by fome of the varieties in the *Fifth Propofition.*

The Greater the Denominator the nearer the truth.

§. 4. When the Logarithme of a Common Fraction is given, which is not to be found in the *Table*, and the Fraction thereof defired Vulgar and not Decimal: Then after the Decimal Fraction thereof is found as above, reduce the fame to the leaft Termes.

Logarithme of a Common Fraction not found, and defired Common.

As if $\frac{75}{100}$ be found as above, for the Logarithme of —1,87506,12634, I abbreviate $\frac{75}{100}$ to $\frac{3}{4}$ the Common Fraction, whofe Log. as above was —0,12493,87366.

Example.

§. 5. When the Logarithme of a Decimal Fraction is given, look out the correfponding Number: As if the given Logarithme were the Logarithme of an Integer, and then place the *Seperatrix* according to the Units contained in the *Index*, or which is all one, prefix before the Number found fo many Cyphers lacking one, as there be Units in the *Index* of the Logarithme given.

Decimal found for the Logarithme thereof.

As if —2,68574,17386, be the Logarithme of a Decimal, which I find in the *Table* againft the Integers 485, then I place Unity two places before the Left Hand Figure thereof, for that the *Index* here is 2 defective, fo is the Decimal 0,0485.

Examples.

But if the Logarithme given were —1,68574,17386, then fhould the Decimal be 0,485. If the Logarithme were 0,68574,17386, the Integers and Decimal would be thus 4,85, &c. as was feen in the *Fourth Propofition* of this Chapter, and the like is to be underftood of others.

Nothing need be added to prove the Invention of Logarithmes for Numbers, or Numbers for Logarithmes, feeing both are to be examined and proved by the foregoing *Table* of *Logarithmes.*

Proof by the Tables.

CHAP. III.

Addition of Logarithmes.

ADdition of Logarithmes is equivalent to Multiplication of other Numbers. If therefore the Logarithme of the Multiplyer be added to the Logarithme of the Multiplicand, the Total fhall be the Logarithme of the Product.

Logarithmes added what equivalent to.

To add two Logarithmes, confider whether the given Logarithmes be both of one Nature or not, that is, Affirmative, Negative or Mixt; for Affirmative will produce Affirmative, and Negative Negative; but if they be Mixt, the Product will be fome-

The Key of Addition of Logarithmes.

time the one, and sometime the other; for one of them is in nature substractive from the other.

As if $4+2=6$, then is $10000 \times 100 = 1000000$. And as $\overline{4}+\overline{2}=\overline{6}$, so is $,0001 \times ,01 = ,000001$. But $3+\overline{2}=1$, or $1000 \times ,01 = 10$, is as $1000 \div 100 = 10$. And as $\overline{3}+2=\overline{1}$, or $,001 \times 100 = 0,1$, so is $100 \div 1000 = 0,1$. This Key may unlock Addition of Logarithmes, but the particular Cases following make all plain.

1. *If both be Affirmative.*

1. *Case.* If the given Logarithmes be both affirmative, then add the Numbers, as if they were Integers, and the summe shall be a Logarithme of the same kind, and his Absolute Number the Product.

Example.

As to add the Logarithme of 90 to the Logarithme of 9, the Total is the Logarithme of 810, the Product of both.

Multiplicand	1,95424,25094	Log. of	90
Multiplyer	0,95424,25094	Log. of	9
Product	2,90848,50188	Log. of	810

2. *If both be Negative.*

2. *Case.* If the given Logarithmes be both Negative, as before, add the Numbers as Integers, and the summe shall be a Logarithme of the same kind; only in the Logarithmes of Decimals, if from the Left Hand place next the *Index*, any Tens be carried onward to the *Index*, they are alwayes Affirmative, although the *Index* be Negative: And then the Affirmative must be substracted out of the Negative *Indices*, as was taught in Addition of Decimal *Indices*.

Example in a Fraction.

As to add —0,30102,99957, the Logarithme of $\frac{1}{2}$ to it self, the Total shall be —0,60205,99914, the Logarithme of $\frac{1}{4}$, which is the Product of $\frac{1}{2}$ multiplyed by it self, and herein is no difficulty.

Example in a Decimal.

But if the Fraction had been a Decimal, that is 0,5, or $\frac{50}{100}$, the Logarithme of which is —1,69897,00043, then in adding them, when I come to the Figures next the *Indices*, an Unit is to be carried over to the *Indices* for the 10 which is there, and this Affirmative I take from the summe of both the Negative *Indices*, and the Remain is 1 Negative; so is the Total —1,39794,00086, which is the Logarithme of 0,25, that is the Product of 5′ by 5′, and agrees to the other in value, though in other Terms, because 25 is $\frac{1}{4}$ of 100.

Multiplicand	—0,30102,99957	Log. of $\frac{1}{2}$		—1,69897,00043	Log. of	0,5′
Multiplyer	—0,30102,99957	Log. of $\frac{1}{2}$		—1,69897,00043	Log. of	0,5′
Product	—0,60205,99914	Log. of $\frac{1}{4}$		—1,39794,00086	Log. of	,25″

3. Data *of divers kinds, and the* Logarithme *defective be the* Logarithme *of a Fraction. Examples.*

3. *Case.* If the given Logarithmes be of different kinds, that is, the one Affirmative and the other Negative, and the defective Logarithme be the Logarithme of a Common Fraction; then substract the Lesser Logarithme out of the Greater, the Remainder shall be the Logarithme of the Product required, and shall alwayes be of the same kind with the Greater Logarithme.

As if 5 were to be multiplyed by $\frac{1}{2}$, then —0,30102,99957, the Logarithme of $\frac{1}{2}$ taken from 0,69897,00043, the Logarithme of 5, leaveth 0,39794,00086, the Logarithme of $\frac{5}{2}$ or $2\frac{1}{2}$, the Product not defective because the Logarithme of 5 the Greater Logarithme was abundant.

But if 20 be multiplyed by $\frac{1}{40}$, then 1,30102,99957, the Logarithme of 20 being the Lesser, is to be substracted from —1,60205,99913, the Logarithme of $\frac{1}{40}$, and the Remain will be —0,30102,99956, the Logarithme of the Product $\frac{20}{40}$ or $\frac{1}{2}$ defective, because the Greater Logarithme is Negative.

Multiplicand	0,69897,00043	Log. of 5		1,30102,99957	Log. of	20
Multiplyer	—0,30102,99957	Log. of $\frac{1}{2}$		—1,60205,99913	Log. of	$\frac{1}{40}$
Product	0,39794,00086	Log. of $\frac{5}{2}$		—0,30102,99956	Log. of	$\frac{20}{40}$

4. *Case.*

4. *Case.* If the given Logarithmes be of divers kinds, and the Negative Logarithme be the Logarithme of a Decimal Fraction, then add the Logarithmes together till you come to the *Index*, and there substract the Lesser *Index* from the Greater, remembring the 10 carried over, if any be, is Affirmative, as before noted in the *Second Case*, and to be ordered accordingly.

4. Data of divers kinds, and the Logarithme defective be the Logarithme of a Decimal. Examples.

As if —1,69897,00043, the Logarithme of $\frac{50}{100}$, or 0,5, which is $\frac{1}{2}$, be added to the Logarithme of 5, that is 0,69897,00043, the Total Product is 0,39794,00086, the Logarithme of 2,5 and Affirmative; for there the Unit carried for the 10 next the *Index* being Affirmative, substracted from the Negative *Index* 1, leaves 0 to the Product abundant.

But if the defective Logarithme of the Decimal 0,025, that is —2,39794,00087 be added to the Logarithme of 20, the Total will be —1,69897,00044, the Logarithme of 5 Primes or ,50″; for the *Index* 1 Affirmative taken from 2 Negative, leaves the Remain defective, because the *Index* of that kind was the Greater.

Multiplicand	—1,69897,00043	Log. of 0,5′		1,30102,99957	Log. of 20
Multiplyer	0,69897,00043	Log. of 5		—2,39794,00087	Log. of ,025‴
Product	0,39794,00086	Log. of 2,5′		—1,69897,00044	Log. of ,5′

5. *Case.* If several Numbers be given to be multiplyed one into another, add all their Logarithmes together, according to the Directions foregoing, and the Total thereof shall be the Product desired, whereby in dispatch of great Multiplications, a wonderful expedition is attained. View the Examples following.

5. Data many Numbers.

Examples.

Integers.

0,47712,12547	Log. of 3	0,30102,99957	Log. of 2	Numbers multiplyed one into another.
1,04139,26852	Log. of 11	0,69897,00043	Log. of 5	
1,23044,89214	Log. of 17	1,90308,99870	Log. of 80	
2,74896,28613	Log. of 561	2,90308,99870	Log. of 800	Product.

Common Fractions.

—0,60205,99913	Log. of $\frac{1}{4}$	—0,17609,12590	Log. of $\frac{2}{3}$	Fractions multiplyed together.
—0,60205,99913	Log. of $\frac{1}{4}$	—0,12493,87366	Log. of $\frac{3}{4}$	
—0,30102,99957	Log. of $\frac{1}{2}$	—0,07918,12461	Log. of $\frac{5}{6}$	
—1,50514,99783	Log. of $\frac{1}{32}$	—0,38021,12417	Log. of $\frac{5}{12}$	Product.

Decimals.

—1,00000,00000	Log. of 0,1′	—2,47712,12547	Log. of 0,03″
—1,17609,12591	Log. of 0,15″	—3,90308,99870	Log. of 0,008‴
—1,95424,25094	Log. of 0,9′	—3,60205,99913	Log. of 0,004‴
—2,13033,37685	Log. of 0,0135⁗	—7,98227,12330	Log. of 0,00000096^{viii}

Mixt Numbers.

0,30102,99957	Log. of 2	1,22530,92817	Log. of 16,8′
2,15836,24921	Log. of 144	—1,87506,12634	Log. of 0,75″
—1,87506,12634	Log. of 0,75″	—2,69897,00043	Log. of 0,05″
2,33445,37512	Log. of 216,00″	—1,79934,05494	Log. of 0,63000^{v}

The Product of the Numbers multiplyed answering to the Number corresponding to the Total Logarithme of the Logarithmes added, is Proof sufficient of Logarithmical Addition.

Proof of Addition of Logarithmes.

CHAP.

CHAP. IV.

Substraction of Logarithmes.

Logarithmes *substracted what it performeth.*

SUbstraction of Logarithmes performeth as much as Division of other Numbers, and therefore to substract the Logarithme of the Divisor from the Logarithme of the Dividend, the Remain shall be the Logarithme of the Quotient.

The Key of Substraction *of* Logarithmes.

But as before in Addition, the Logarithmes being some defective, and others abundant, consideration must be had in the Substraction, that the Remain or Quotient be rightly Denominate, Affirmative or Negative, according as the Natures of the given Logarithmes will admit.

For as in *Indices* $4 - 2 = 2$: And in multiplyed Numbers corresponding $10000 \div 100 = 100$. And $\overline{4} - \overline{2} = \overline{2}$; that is in Decimals thus, $,0001 \div ,01 = ,01$. So if $0 - \overline{2}$ be taken from $0 + 3$, the Remain shall be $= 5$, that is in Numbers. If $1 \div ,01$ be taken from 1×1000, the Remain shall be $= 100000$. For there as the *Indices* are added, 1000 shall be multiplyed by 100, the Quotient of the Division. But if $0 + 2$ be taken from $0 - \overline{3}$, the Remain shall be $= \overline{5}$, which in Numbers is as 1×100 taken from $1 \div ,001$, the Remain shall be $= ,00001$. This is the Key of Substraction, and may be fully understood in the Cases following.

1. *If both be Affirmative.*

1. *Case.* If the given Logarithmes be both affirmative, and that of the Divisor less than that of the Dividend, then substract the lesser Logarithme from the greater, as if they were Integers; and the Remain shall be a Logarithme of the same kind, and the absolute Number answering thereto, the Quotient.

Example.

As to divide 810 by 9, the Quotient is 90; so to substract 0,95424,25094, the Logarithme of 9 from 2,90848,50189, the Logarithme of 810, the Remain is 1,95424,25095, the Logarithme of 90, the Quotient.

Dividend	2,90848,50189	Log. of	810
Divisor	0,95424,25094	Log. of	9
Quotient	1,95424,25095	Log. of	90

2. *If both be Affirmative, and the Divisor greatest.*

2. *Case.* If both the given Logarithmes are affirmative, and that of the Divisor greater than that of the Dividend, then after Substraction of all the rest of the Logarithme of the Divisor from the Logarithme of the Dividend, except the *Index*, take the lesser *Index* from the greater, and change the Sign of the Remain. And in substracting, if in the next Figure to the *Index* you have occasion to borrow 10, then accompt the *Index* of the Dividend 1 less than it is; as if 3, accompt it 2, if 2 but 1, *&c.* Examples of both.

Examples.

As if 9 be divided by 90, the Quotient will be $\frac{1}{10}$ or 0,1; so to substract 1,95424,25094 the Logarithme of 90, from 0,95424,25094, the Logarithme of 9, the Remain will be —1,00000,00000, the Logarithme of $\frac{1}{10}$ or 0,1, as at *A.*

And if 36 be divided by 90, the Quotient by abbreviation will be $\frac{2}{5}$: So 1,95424,25094 the Logarithme of 90, taken from the Logarithme of 36, which is 1,55630,25008, the Remain will be the Logarithme of 0,4′, or abbreviated $\frac{2}{5}$; where 1 for the 10 borrowed next the *Index* abates the Characteristique 1 of the upper Logarithme; and so 0 being left for the lesser *Index* taken from the lower, changes the Sign, as at *B.*

	A.			*B.*		
Dividend	0,95424,25094	Log. of	9	1,55630,25008	Log. of	36
Divisor	1,95424,25094	Log. of	90	1,95424,25094	Log. of	90
Quotient	—1,00000,00000	Log. of	0,1′	—1,60205,99914	Log. of	0,4′

4. *Case.*

3. *Case.* If the given Logarithmes are both Negative, of Vulgar Fractions, and that of the Divisor the lesser; then as if they were Integers, substract the Logarithme of the Divisor from the Logarithme of the Dividend, and the Remain shall be a defective Logarithme of the same kind. But if the Logarithme of the Divisor be the greater, take the lesser Logarithme from the greater, and change the Sign of the Remain, for in this case it shall be abundant. 3. *If both be Negative of Fractions.*

As if $\frac{1}{32}$ be divided by $\frac{1}{4}$, the Quotient will be $\frac{1}{8}$; so if the Logarithme of $\frac{1}{4}$, that is —0,60205,99913, be taken from —1,50514,99783, the Logarithme of $\frac{1}{32}$, the Remain will be —0,90308,99870, the Logarithme of $\frac{1}{8}$, as at *C.* *Examples.*

But if $\frac{1}{2}$ be divided by $\frac{1}{4}$ the Quotient will be 2; for the Divisor having the greater Logarithme, the *Index* which was negative is now changed, and the Remain affirmative, as at *D.*

	C.			*D.*		
Dividend	—1,50514,99783	Log. of	$\frac{1}{32}$	—0,30102,99957	Log. of	$\frac{1}{2}$
Divisor	—0,60205,99913	Log. of	$\frac{1}{4}$	—0,60205,99913	Log. of	$\frac{1}{4}$
	—0,90308,99870	Log. of	$\frac{1}{8}$	0,30102,99956	Log. of	2

4. *Case.* If the Logarithmes given are both of Decimals, then substract them as Integers till you come to the *Index*; and there if in the next place to the *Index* you borrow 10, accompt the *Index* of the Dividend 1 less than it is; as if —1, accompt it —2, if —2 then —3, *&c.* and then take the lesser *Index* from the greater: And if the upper *Index*, which is of the Dividend, be the least, then change the Sign of the Remain, which shall be the Logarithme of the Quotient. 4. Data *Decimals.*

As to divide 0,0135'''' by 0,1', the Quotient will be 0,135''': So if the Logarithme of 0,1' be taken from —2,13033,37685, the Logarithme of 0,0135'''', the Remain will be —1,13033,37685, the Logarithme of 0,135''', as at *E.* *Examples.*

And if 0,0135'''' be divided by 0,9', the Quotient will be 0,015''': So if the Logarithme of 0,9', which is —1,95424,25094, be taken from —2,13033,37685, the Logarithme of 0,0135'''', there will be left —2,17609,12591, the Logarithme of 0,015''', as at *F.*; where by borrowing the 10, next the *Index*, the upper *Index* is accompted —3.

	E.			*F.*		
Dividend	—2,13033,37685	Log. of	0,0135''''	—2,13033,37685	Log. of	0,0135''''
Divisor	—1,00000,00000	Log. of	0,1'	—1,95424,25094	Log. of	0,9'
Quotient	—1,13033,37685	Log. of	0,135'''	—2,17609,12591	Log. of	0,015'''

Other Examples of the varieties that may happen under this Case here follow.

Dividend	—1,87506,12634	Log. of	0,75''	—1,65321,25138	Log. of	0,45''
Divisor	—2,69897,00043	Log. of	0,05''	—1,95424,25094	Log. of	0,9'
Quotient	—1,17609,12591	Log. of	15,0'	—1,69897,00044	Log. of	0,5'

Dividend	—2,13033,37685	Log. of	0,0135''''	—1,30102,99957	Log. of	0,2'
Divisor	—2,69897,00043	Log. of	0,05''	—2,60205,99913	Log. of	0,04''
Quotient	—1,43136,37642	Log. of	0,27''	0,69897,00044	Log. of	5,0'

Dividend	—1,95424,25094	Log. of	0,9'	—2,07918,12460	Log. of	0,012'''
Divisor	—1,17609,12591	Log. of	0,15''	—1,17609,12591	Log. of	0,15''
Quotient	—0,77815,12503	Log. of	6,0	—2,90308,99869	Log. of	0,08''

5. *Data of divers kinds.*

5. *Case.* If the Logarithmes given are of divers kinds; that is one affirmative and the other negative, and the defective Logarithme be the Logarithme of a Common Fraction; add them together, and the summe shall be the Logarithme of the Quotient, and of the same kind with the Dividend. But if the defective Logarithme be the Logarithme of a Decimal Fraction, then substract the Logarithme of the Divisor out of the Logarithme of the Dividend till you come to the *Index*, and then add the *Indices* together: And if 10 be borrowed, accompt the *Index* of the Dividend one less, as before.

Examples.

Examples of both sorts follow so plainly that illustration thereof is needless.

Integers with Common Fractions.

Dividend	0,77815,12504	Log. of 6	—0,12493,87366	Log. of $\frac{3}{4}$
Divisor	—0,12493,87366	Log. of $\frac{3}{4}$	0,77815,12504	Log. of 6
Quotient	0,90308,99870	Log. of 8	—0,90308,99870	Log. of $\frac{1}{8}$
Dividend	1,20411,99827	Log. of 16	—1,90308,99870	Log. of $\frac{1}{80}$
Divisor	—1,90308,99870	Log. of $\frac{1}{80}$	1,20411,99827	Log. of 16
Quotient	3,10720,99697	Log. of 1280	—3,10720,99697	Log. of $\frac{1}{1280}$

Integers with Decimals.

Dividend	0,60205,99913	Log. of 4	—3,90308,99870	Log. of 0,008'''
Divisor	—3,90308,99870	Log. of 0,008'''	0,60205,99913	Log. of 4
Quotient	2,69897,00043	Log. of 500	—3,30102,99957	Log. of 0,002'''
Dividend	0,69897,00043	Log. of 5	—1,39794,00087	Log. of 0,25''
Divisor	—1,39794,00087	Log. of 0,25''	0,69897,00043	Log. of 5
Quotient	1,30102,99956	Log. of 20	—2,69897,00044	Log. of 0,05''

Mixt Numbers.

Dividend	0,68574,17386	Log. of 4,85''	0,69897,00043	Log. of 5
Divisor	—1,79588,00173	Log. of 0,625'''	—2,90308,99870	Log. of 0,08''
Quotient	0,88986,17213	Log. of 7,76''	1,79588,00173	Log. of 62,5'

Proof of Substraction of Logarithmes.

The Quotient of the Numbers divided, answering to the Number of the Remaining Logarithme of the Logarithmes substracted, is Proof enough of the truth of Logarithmical Substraction.

CHAP. V.

Multiplication of Logarithmes.

Logarithmes multiplyed produceth Figurals.

Multiplication of Logarithmes resembleth Multiplication of Ratio's, hereafter treated of; for that it maketh the Product a Figurate Number. As to multiply a Logarithme by 2, produceth the Logarithme of the Square. And to multiply by 3 the Logarithme of the Cube, *&c.* according to the Figural *Index* of the Quantity used for Multiplyer.

1. *Case.*

1. *Case.* There is no difficulty therein, if the *Index* of the given Logarithme be affirmative, or the defective Logarithme be the Logarithme of a Common Fraction; for then the Multiplication is as in Integers.

1. *If the* Index *be affirmative, or the defective* Logarithme *be the* Logarithme *of a Fraction. Examples.*

For as 9 multiplyed by 9 giveth 81; so the Logarithme of 9 multiplyed by 2 produceth the Logarithme of 81. And the Fraction $\frac{2}{9}$ squared is $\frac{4}{81}$; so the Logarithme of $\frac{2}{9}$ multiplyed by 2 shall give the Logarithme of $\frac{4}{81}$. And the like may be done for any other power.

	Integers.		Common Fractions.	
Root	0,95424,25094	Log. of 9	—0,65321,25137	Log. of $\frac{2}{9}$
	2		2	
Square	1,90848,50188	Log. of 81	—1,30642,50274	Log. of $\frac{4}{81}$
Root	0,69897,00043	Log. of 5	—0,17609,12590	Log. of $\frac{2}{3}$
	3		3	
Cube	2,09691,00129	Log. of 125	—0,52827,37770	Log. of $\frac{8}{27}$
Root	0,60205,99913	Log. of 4	—0,69897,00043	Log. of $\frac{1}{5}$
	4		4	
Sq Squa.	2,40823,99652	Log. of 256	—2,79588,000172	Log. of $\frac{1}{625}$

2. *Case.* If the given Logarithme be the Logarithme of a Decimal, there is no difference between the Multiplication thereof and others, save when in multiplying the Figure next the *Index* if any Tens arise, the Units carried over for them are affirmative, and to be substracted from the Product of the negative *Index*.

2. *If the* Logarithme *be the* Logarithme *of a Decimal. Examples.*

As in squaring 0,05″, the 10 carried from the Multiplication of 6 in the Logarithme, shall abate 1 from the Product of the *Index*, and leave but —3. The like also happeneth in other Examples.

Root	—2,69897,00043	Log. of 0,05″	—1,30102,99957	Log. of 0,2′
	2		2	
Square	—3,39794,00086	Log. of 0,0025⁗	—2,60205,99914	Log. of 0,04″
Root	—2,69897,00043	Log. of 0,05″	—2,30102,99957	Log. of 0,02″
	3		3	
Cube	—4,09691,000129	Log. of 0,000125 vj	—6,90308,99871	Log. of 0,000008 vj
Root	—1,69897,00043	Log. of 0,5′	—1,30102,99957	Log. of 0,2′
	4		4	
Sq.Squ.	—2,79588,00172	Log. of 0,0625⁗	—3,20411,99828	Log. of 0,0016⁗

The Product of the Numbers multiplyed Figurately answering to the Number of the produced Logarithmes, serveth for a sufficient Proof of the truth of Logarithmical Multiplication.

Proof of Multiplication *of* Logarithmes.

CHAP.

CHAP. VI.

Division of Logarithmes.

Logarithmes divided, is Extraction of Roots.

AS Multiplication of Logarithmes produceth Figural Numbers, so Division of Logarithmes answereth to Extraction of Roots, and is much like Division of Ratio's hereafter spoken to in the *Fourth Book*: For the Quotient is the Ratio of the Figurate Number whose Logarithme is the Dividend, according to the Ratio in the Divisor. So as if the Divisor be 2, the Quotient is the Logarithme of the Square Root; if 3, the Logarithme of the Cube Root, *&c.* according to the Figural *Index* of the Quantity used for Divisor.

1. *If the* Index *be affirmative, or the defective* Logarithme *be the* Logarithme *of a Fraction.*

1. *Case.* Division is easie, if the *Index* of the given Logarithme be affirmative, or the defective Logarithme be the Logarithme of a Common Fraction; for then the Division is performed as in Integers. As in the Examples of the former *Chapter* may be seen.

Examples.

	Integers.		Fractions.	
Square	1,90848,50188 2	Log. of 81	—1,30642,50274 2	Log. of $\frac{4}{81}$
Root	0,95424,25094	Log. of 9	—0,65321,25137	Log. of $\frac{2}{9}$
Cube	2,09691,00129 3	Log. of 125	—0,52827,37770 3	Log. of $\frac{8}{27}$
Root	0,69897,00043	Log. of 5	—0,17609,12590	Log. of $\frac{2}{3}$

2. *If the* Logarithme *be the* Logarithme *of a Decimal.*

2. *Case.* If the Logarithme given to be divided be the Logarithme of a Decimal, and the *Index* will be evenly divided by the Divisor, then there is no difference between the Division thereof and others: But when the *Index* of the Logarithme will not be evenly divided by the Divisor, then add to the *Index* so many Units till it may be evenly divided thereby, and setting the Quotient down for a new *Index*, keep the Units added in mind, multiply them by 10, and add the Product thereof to the next Right Hand Figure of the Logarithme, and then divide the rest of the Logarithme, as others, where nothing was borrowed.

Examples.

As among the former Examples in the last *Chapter* foregoing, where —6,90308,99871, was found to be the Logarithme of the Cube 0,000008^{vi}; because —6 the Characteristique will be evenly divided by 3, the Division is as in Integers, at *A*. But in dividing —4,09691,00129, the Logarithme of the Cube 0,000125^{vi}, because —4 the *Index*, will not be evenly divided by 3, I add in mind 2 Units to 4, to make it so divisible, and so get —2 for the new *Index*; which added 2, multiplyed by, make 20, and to this should have been added the next Figure of the Logarithme, but there being 0, leaves 20, only to be divided by 3, which gives 6, and so continuing the Division, the Logarithme of 0,05'' is gotten, as at *B*.

	A.		*B.*	
Cube	—6,90308,99871 3	Log. of 0,000008^{vi}	—4,09691,00129 3	Log. of 0,000125^{vi}
Root	—2,30102,99957	Log. of 0,02''	—2,69897,00043	Log. of 0,05''

For the better understanding of the Division of Decimal Logarithmes whose Characteristiques will not be evenly divided, Mr. *Oughtred* at the end of the *Resolution of Affected Æquations*, hath presented us with part of a *Table* to be increased at pleasure; which though the foregoing Rule may supply the use of, yet because some by

Occular

Occular inspection in a *Table* receive a firmer impression in their Minds of the Numbers used therein, I have here transcribed it a little altered, with the Explanation following.

A Table for Division of Decimal Logarithmes.

	2)				1	2	1
					3	4	2
					5	6	3
					7	8	4
					9	10	5
						&c.	
Divisors	3)			1	2	3	1 Indices.
				4	5	6	2
				7	8	9	3
						&c.	
	4)		1	2	3	4	1
			5	6	7	8	2
						&c.	
	5)	1	2	3	4	5	1
		6	7	8	9	10	2
						&c.	
&c.		40	30	20	10	0	

The Table *explained.*

In this *Table* the Numbers that stand to the Left Hand within a bowed Line are Divisors, or the Numbers of Quantity, as 2, 3, 4, 5, and may be increased as the *&c.* denotes.

The Numbers next the Right Hand are the *Indices* of the Quotient, answering collaterally to such Divisors with such *Indices* of the Dividends as stand along with them in the same Line from the Right Hand to the Left.

The Numbers next the *Indices* of the Quotient are such *Indices* of the Dividends as will be evenly divided by the Divisors standing against them, and so need no Units to be added to them. These may also be increased at pleasure, as the several *&c.* signifie.

But the Numbers next them, if they happen to be the Characteristiques of the Logarithmes given to be divided, must have 1 Unit added, and the next File of Numbers must have 2 Units added, the next File 3, and the next 4; and so of others, if the *Table* be enlarged. Every of which multiplyed by 10, as before noted in the Rule, makes the 10, 20, 30, 40, at the bottom of the *Table*, to be added to the Figure next the *Index* of the Logarithme to be divided.

So if I were to divide a Logarithme whose *Index* is —7 by 2, I need add but 1 to it, and the *Index* of the Quotient shall be —4; but if a Logarithme whose *Index* is —7 by 3, I must add 2, and the *Index* of the Quotient shall be —3. The like is to be seen of others, and all to be understood of Decimal Logarithmes.

Proof of Division *of* Logarithmes.

The Root of any Number extracted agreeing with the Number answering to the Quotient of the divided Logarithme, will be Proof enough of the truth of Logarithmical Division.

Partis Tertiæ Libri Tertii

FINIS.

THE FOURTH PART OF THE THIRD BOOK.

CHAP. I.

Of COSSICKS.

Numbers specially Contract, that have their Denominators implyed, and because of their certainty omitted, are already handled in the three former Parts of this *Third Book*. The next in order are those special Contract Numbers, whose Denominations are uncertain, and therefore needful to be expressed: As *Cossicks*, *Surdes*, and *Species*. *Numbers specially contract whose Denominators are uncertain.*

Cossicks, are Figural Numbers-Compound, either *per se*, or *inter se*. Cossicks *what.*

Those Compound *per se*, or by themselves, are such Figural Numbers of some single Species as have annexed to them some Absolute Number. As 3 Roots, 8 Squares, 5 Cubes, or the like. *Compound* per se.

Those Compound *inter se*, or among themselves, are such Figural Numbers of different Species as are annexed one to another, and have to each of them some Absolute Numbers also adjoyned. As 2 Roots and 3 Squares, or 4 Cubes lacking 2 Roots, or the like. *Compound* inter se.

So as Cossicks are a Number of Figural Numbers, or a Quantity of Quantities; and in both sorts the Numbers are Contract to the Figural Quantities whereto they are adjoyned. And when any Abstract Number is used with them besides the Number of the Quantities, this Abstract Number is kept distinct and marked accordingly. *Abstract Numbers used with* Cossicks *are marked distinct.*

The Figural Names or Denominations of the Quantities, as well to avoid prolixity in the often rescription, as for conveniency in working, are usually expressed by Marks or Characters, under the cover whereof the Numbers got the Name of *Cossicks*, derived, as is thought, from the *Hebrew* כָּסָה, signifying to cover, hide or conceal; And from thence both the *Latin*, *Cossa* and *Cossicus*, and the *Italian*, *Cossica*, whence it seems the Name came to us. But as the Denominations are various, and therefore must be exprest; so the Stenographical Mantles in which they are wrapt up, are not certain, but arbitrary at the pleasure of the Operator, as he conceiveth most expeditious or commodious for his use, and therefore are to be sought in the respective *Nomenclatura* of every Author. In the ensuing Operations let the following Characters be thus understood. *Characters used for what end.* *Whence the Name.* *Characters uncertain.*

Indices.

A Table *of the* Cossical *Characters used in this Book.*

Indices.	*Characters.*	*Signification of the Characters.*
0	N	An Absolute Number, as if it had no Mark.
1	ℨ	The Root of any Number.
2	ʒ	A Square.
3	φ	A Cube.
4	ʒʒ	A Squared Square, or Zenzizenzike.
5	ʃβ	A Sursolide.
6	ʒφ	A Squared Cube, or Zenzicube.
7	Bʃβ	A Second Sursolide.
8	ʒʒʒ	A Zenzizenzizenzike, or Square of Squared Square.
9	φφ	A Cubed Cube.
10	ʒʃβ	A Square of Sursolids.
11	Cʃβ	A Third Sursolide.
12	ʒʒφ	A Zenzizenzicube, or Square of Squared Cubes.
13	Dʃβ	A Fourth Sursolide.
14	ʒBʃβ	A Square of Second Sursolids.
15	φʃβ	A Cube of Sursolids.
16	ʒʒʒʒ	A Zenzizenzizenzizenzike, or Square of Squares Squaredly (Squared.
17	Eʃβ	A Fifth Sursolide.
18	ʒφφ	A Zenzicubicube, or Square of Cubick Cubes.
19	Fʃβ	A Sixth Sursolide.
20	ʒʒʃβ	A Square of Squared Sursolids.
21	φBʃβ	A Cube of Second Sursolids.
22	ʒCʃβ	A Square of Third Sursolids.
23	Gʃβ	A Seventh Sursolide.
24	ʒʒʒφ	A Square of Squares of Squared Cubes, or a Zenzizenzizenzicube.
&c.	&c.	

What Indices *the* Indices *of* Cossicks *are.*

The Reader may not take these *Indices* either for Decimal *Indices*, or the *Indices* of Logarithmes, nor confound the one with the other, for these are no other than the Figural *Indices*, or Numbers of the Quantities as they exceed in Power, mentioned before in the *Second Part* of the *Second Book* among Simple Figural Numbers.

How to increase the Characters of Cossicks.

As the Powers may be increased, so may the Cossical Characters be formed higher, though it will be rare that so many as above set down need be used. But if any desire to know how to form the Characters of Higher Powers than these; let him first set down the Figural *Indices* of the Powers in their Natural Order or Progression as far as he pleaseth, then observe, that all the Characters after that of the Root, are but of 3 sorts, ʒ, φ, ʃβ, *viz.* those for the Square, Cube and Sursolide. For whereas B. C. D. *&c.* are used with the Sursolids of different Powers, it is but Numerically according as they stand in the Alphabet, to shew they are the second, third, fourth, *&c.* of that sort; so shall the Eighth Sursolide be Hʃβ, the Ninth Iʃβ, the Tenth Kʃβ, and so on. This being observed it remains then, that as the Figural *Indices* are Numbers Uncompound or Compound, so are the Characters, and consequently the Arithmetical Names of these Higher Powers signified by the Cossical Characters.

For Indices *Uncompound.*

In the *First Book*, *Chap.* 2. it was shewed that Uncompound Numbers are procreated by collection of Units, but cannot be made by Multiplication, as 5, 7, 11, 13, 17, 19, 23, 29, 31, *&c.* under all which Uncompound *Indices* after 3, fall Sursolids; therefore under the first of them is to be placed the Sursolide Character; under the second, the same Character, only with B. the second Letter in the Alphabet added to denote it is a Second Sursolide. Wherefore (as above) against 11, the third of these uncompound *Indices* is Cʃβ, betokening the Third Sursolide, against 13, Dʃβ, against 17, Eʃβ, *&c.* And if the *Table* were enlarged, against 29 would be Hʃβ, and 31, Iʃβ, *&c.*

For Indices *Compound.*

Numbers Compound must be compound of 2 or 3, or of 2 and 3, or of 2 or 3, and some other Number.

Of 2.

If the *Index* be compound of 2, set down ʒ so often as 2 is in the composition. As 16 being compound of 2 four times, shall have ʒʒʒʒ for its Character. So 32 having 2 five times in the composition, shall have ʒ five times; and 64, six times, *&c.*

Of 3.

If the *Index* be compound of 3 only, then set down the Character of the Cube as often as 3 is used to compound the *Index*. As 9, compounded of 3, twice shall be φφ; so 27, because it is compounded of 3, thrice shall have φφφ for the Character, *&c.*

If

If the *Index* be compounded of 2 and 3, then for every time that 2 is multiplyed in the composition, set down ʒ; and for every time 3 is multiplyed set down φ, minding still to set ʒ before φ to the Left Hand, because ʒ is the foremost Power. Thus against 6, made of 2 and 3, is ʒφ; and against 12, compounded of 2 by 2, which makes 4, and then by 3, the Character ʒʒφ; so 18 hath ʒφφ, because twice 3 is 6, and thrice 6 is 18. *Of 2 and 3.*

If the *Index* be compound of 2 or 3, and some other Number, then joyn their Characters together in such order, as the Lesser Figural Powers may precede the Greater. As 10, compounded of 2 and 5, is ʒß, where the Mark of the second Quantity being the Lesser, stands before ß, the Mark of the fifth Quantity; 15 likewise compounded of 3 and 5, is φß. *Of 2 or 3, and some other.*

In like manner for Higher *Indices*; as 20, compounded of 4 and 5, shall be a ʒʒß; 50, of 5 and 10, shall be a ʒßß, *&c.*

According to the Characters so are their Arithmetical Names, save that some to shorten and facilitate the long Names of such Higher Powers, as have the Square or Cube often ingeminated, borrow some Names bordering on the *Latin*, and call ʒʒ a Biquadrate, ʒʒʒ a Triquadrate, ʒʒʒʒ a Quaquadrate, ʒʒʒʒʒ a Quinquadrate, *&c.* Also φφ a Bicube, φφφ a Tricube, *&c.* and their Compounds accordingly; as ʒʒφ a Biquadratcube, ʒʒʒφ a Triquadratcube, *&c* But having kept the older Names in the former Treatise of Figural Numbers, I have here retained them to their Cossical Characters. *How some shorten the Names.*

As to the Nature of Cossicks, they are either Whole or Broken, and both either Simple or Compound: For though all Cossicks are Compound Figural Numbers (as aforesaid), yet are none counted Compound Cossicks, unless they admit of various Denominations, and have their Characters connexed by the Signs + or —. *Nature of Cossicks.*

Of Broken Cossicks, see the 6, 7, 8, 9, 10 and 11th *Chapters* following.

Simple Cossicks then that are Whole or Integral, are Homogeneal, or Quantities of one sort: As 4 ℨ, or 10ʒ, or 8φ, *&c.* to be read, 4 Roots, 10 Squares, 8 Cubes, and shall be understood with no more relation one to another, than 4 Inches, 10 Planks, 8 Trees, because they are not knit together by any Sign to bring them under an obligation of relation to one and the same Root, from whence they should all spring: Yet if Simple Cossicks are to be added or substracted, multiplyed or divided with other Simple Cossicks, then they are understood to have all one Root. *Simple Whole Cossicks.*

Compound Whole Cossicks, connexing together divers Simple Cossicks, are Heterogeneal. As 8φ + 4ℨ, read thus, 8 Cubes more [or and] 4 Roots: So 10ʒ — 4ℨ, that is 10 Squares lacking 4 Roots, whose Quantities are ever considered in summe, according to the mixture of the Signs + or —: For they are increased by +, and diminished by —. *Compound Whole Cossicks*

Compound Cossicks are of 3 sorts. *Compound of 3 sorts.*

Those connexed by the Additional Sign +, are called *Binomials.* Binomials.

Those connexed by the Sign of Substraction —, are called *Residuals*, and sometime *Apotomes.* Residuals *or* Apotomes.

Those knit together by both Signs are called *Medials*, and by some *Multinomials*, or *Polynomials*, that is, many named. And yet more Cossicks than 2 joyned with + or —, deserve as well as others the name of a *Polynomial*, and are improperly called *Binomials* or *Residuals.* Medials, *how otherwise called.*

Examples of Binomials. *Examples.*

6ʒ + 4ℨ that is, 6 Squares and 4 Roots.
5φ + 3ʒ + 10ℨ is 5 Cubes and 3 Squares and 10 Roots.

Examples of Residuals.

12φ — 5ʒ that is, 12 Cubes lacking 5 Squares.
10ß — 3φ — 2ʒ is 10 Sursolids lacking 3 Cubes and 2 Squares.

Examples of Medials or Polynomials.

8ʒʒ + 2φ — 4ʒ that is, 8 Squared Squares and 2 Cubes, lacking 4 Squares.
5φ — 4ʒ + 5ℨ is 5 Cubes and 5 Roots, lacking 4 Squares.

Cossicks Simple and Compound, being all Rooted Numbers, admit of this difference, that the Simple having not relation one to another, may have different Roots indefinitely; *Difference of Simple and Compound Cossicks.*

nitely; but Compound Cossicks in one Question alwayes imply one and the same definite Root to all the Quantities joyned together. Neverthelesss several Compound Cossicks, or the same Cossicks in several Questions or Operations may have several Roots. For 3 ℨ, 10 ʒ, the Simple Cossicks; suppose one Root be 3, the 3 ℨ shall be 9, in Absolute Number: But the 10 ʒ are not tyed to the Root 3, but may have any other Number for the Root: Except it were 10 ʒ + 3 ℨ, or 10 ʒ — 3 ℨ; then if the ℨ be 3, the Root of the ʒ shall also be 3, and the 10 ʒ in Absolute Numbers 90. Yet in another Operation the 10 ʒ + 3 ℨ are capable of any other Absolute Number for the Root thereof.

Sign + where understood. Cossicks how placed.

In Cossicks, where the sign — is not, the sign + is understood, and on the outmost to the Left Hand of Compound Cossicks commonly omitted. And in all Polynomials it is most usual, though not essential except in Multiplication and Division, to set all the Cossicks connexed with + to the Left Hand of those connexed with —, though these are Quantities of Higher Powers than the other. Some promiscuously use the word Sign, as well for the Cossick Quantity, as for the sign + or —. But it is most orderly to reserve the name of Sign only to these. And this is to be remembred both in *Surdes* and *Species*. And also that these signs + and — be used, where a Simple Quantity or Magnitude is affirmative or negative to another Simple Quantity or Magnitude: But when an Asterisque is set over either of them, then the Quantity or Magnitude-Compound is affirmed or denyed of a Simple, or a Simple of a Compound. As

Word Sign how used.

Signs used with or without Asterisques *what noted thereby.*

5 φ $\overset{*}{—}$ 4 ʒ + 5 ℨ shall signifie, that the 5 Cubes shall want 4 ʒ and 5 ℨ, which without the Asterisque, the 5 ℨ affirmative and the 5 φ, shall be added together, and from the summe only the 4 ʒ shall be deducted.

Absolute Numbers used with Cossicks, *sometime not marked.*

Cossicks Simple and Compound, Whole and Broken, take into their society Absolute Numbers, marked as before with N. and sometime used without any Character.

CHAP. II.

Addition of Whole Cossicks.

Addition *of Whole* Cossicks *Simple and Homogeneal.*

To add Simple Cossicks, if they be Homogeneal, add the Numbers together as Integers, and to the Total adjoyn the Cossical Character common to the given Numbers.

As 5 ℨ added to 10 ℨ, make 15 ℨ. So 15 ʒ to 40 ʒ are 55 ʒ.

Examples.

	Homogeneal.			
Simple	5 ℨ 10 ℨ	15 ʒ 40 ʒ	30 φ 10 φ	Addends.
	15 ℨ	55 ʒ	40 φ	Totals.

Simple and Heterogeneal.

If the Simple Cossicks to be added are Heterogeneal, then place the Highest Power to the Left Hand, and connex the other thereto by the sign of Addition +.

As to add 14 ʒ to 10 φ; or 30 φ to 4 ʒ and 15 ʒʒ; they are set thus,

Examples.

		Heterogeneal.	
Simple	10 φ 14 ʒ	15 ʒʒ 30 φ 4 ʒ	Addends.
	10 φ + 14 ʒ	15 ʒʒ + 30 φ + 4 ʒ	Totals

Compound of like Signs.

To add Compound Cossicks, add together like Cossicks with like, as ℨ with ℨ, and ʒ with ʒ, *&c.* also + with +, and — with —. But if any Cossick be odd, or Heterogeneal to the others given to be added, adjoyn him to the Total with his proper Sign.

As

As to add 12ʒ + 10ℨ to 5ʒ + 6ℨ: Or 12ℨ — 10ℨ to 5ʒ — 6ℨ; the Total of the Binomials will be 17ʒ + 16ℨ, and of the Reſiduals 17ʒ — 16ℨ. But if 10φ + 10ʒ were added to 4φ + 3ʒ + 2ℨ; or to 4φ + 3ʒ — 2ℨ, there the Totals ſhall be 14φ + 13ʒ + 2ℨ, or 14φ + 13ʒ — 2ℨ. *Examples.*

Binomials.	Reſiduals.	Medials or Polynomials.	
12ʒ + 10ℨ	12ʒ — 10ℨ	10φ + 10ʒ	10φ + 10ʒ
5ʒ + 6ℨ	5ʒ — 6ℨ	4φ + 3ʒ + 2ℨ	4φ + 3ʒ — 2ℨ
17ʒ + 16ℨ	17ʒ — 16ℨ	14φ + 13ʒ + 2ℨ	14φ + 13ʒ — 2ℨ

If the Compound Coſſicks have their Signs unlike, then take the Leſſer Number out of the Greater, and to the Remain, which ſhall be the Total, ſubſcribe the Sign that belongeth to the Greater Number, whether it be + or — accordingly. *Compound of unlike Signs.*

As to add 12ʒ + 10ℨ to 5ʒ — 6ℨ, or 12ʒ — 10ℨ with 5ʒ + 6ℨ, the Total of the former will be 17ʒ + 4ℨ, of the latter 17ʒ — 4ℨ, as at *A.* and *B.* Other Examples of Polynomials follow at *C. D. E.* *Examples.*

Binomials and Reſiduals. / Polynomials.

	A.	B.	C.
	12ʒ + 10ℨ	12ʒ — 10ℨ	46ʒʒ + 10φ + 2ʒ
	5ʒ — 6ℨ	5ʒ + 6ℨ	16ʒʒ + 8φ — 5ʒ
	17ʒ + 4ℨ	17ʒ — 4ℨ	62ʒʒ + 18φ — 3ʒ

	D.	E.
	18φ + 16ʒ — 9N.	4ß + 16ʒʒ + 19ʒ
	4φ — 10ʒ + 4N.	3ß — 13φ — 14ʒ
	22φ + 6ʒ — 5N.	7ß + 16ʒʒ — 13φ + 5ʒ

To prove Coſſical Addition, beſides the tryal by Coſſical Subſtraction, reſolve your Coſſicks into Abſtract Numbers, by ſuppoſing 2, 3, or ſome other Number for a Root, and ſo accordingly getting the ſumme of the ʒ, φ, *&c.* of the other Coſſicks, and then compare the Numbers to be added with the Total of the Addition, and the ſummes will be parallel when the Operation is right. *Proof of* Coſſical Addition.

As in the former inſtance at *A*, ſuppoſe 2 be a Root, then is 10ℨ 20, and 4 being the Square of 2, the 12ʒ ſhall be 48; which 48 and 20 make 68; then 5ʒ more is 20 lacking 6ℨ which is 12, leave 8 remaining, this added to 68 makes the Total 76. And ſo much is 17ʒ and 4ℨ, ſuppoſing 2 for a Root.

12ʒ + 10ℨ	48 + 20 = 68
5ʒ — 6ℨ	20 — 12 = 8
17ʒ + 4ℨ	68 + 8 = 76

CHAP. III.

Subſtraction of Whole Coſſicks.

TO ſubſtract Simple Coſſicks, if they be Homogeneal, take the Leſſer Number out of the Greater, and to the Remain ſubſcribe the Coſſical Character common to both the given Numbers, when the Subtrahend is the leaſt, but if it be the greateſt of the two, then change the Sign to the Remain. Subſtraction *of Whole* Coſſicks. *Simple and Homogeneal.*

As to abate 3φ from 10φ, there will remain 7φ: But if 10φ were to be taken from 3φ, there will lack 7φ, therefore the Sign ſhall be — to the Remain. *Examples.*

Homogeneal.

	Homogeneal.		
Simple	10φ	3φ	Numbers from which Substraction is made.
	3φ	10φ	Subtrahends.
	7φ	—7φ	Remains.

Simple and Heterogeneal.

If the Simple Cossicks to be substracted are Heterogeneal, then place the Highest Power to the Left Hand, and connex the other thereto with the Sign of Substraction —.

Examples.

As to take 4ℨ out of 10ʒ, or 5φ from 20ʒʒ + 2ℨ, they are set thus ;

	Heterogeneal.		
Simple	10ʒ	20ʒʒ + 2ℨ	Numb. from which Substraction is made.
	4ℨ	5φ	Subtrahends.
	10ʒ — 4ℨ	20ʒʒ — 5φ + 2ℨ	Remain.

Compound of like Signs.

To substract Compound Cossicks, take like Cossicks from like, as ℨ from ℨ, and ʒ from ʒ, &c. also + from +, and — from —, and to the Remain subscribe the same Sign, except the Number to be substracted be the greater, then change the Sign, and set down thereto the difference of the Numbers. And if any single Cossick have none to fellow him, adjoyn him to the Remain with the contrary Sign if he belonged to the Subtrahend, but with the same Sign he hath if he belonged to the Number from which Substraction is made.

Examples.

As to take 5ʒ + 6ℨ from 12ʒ + 10ℨ, the Remain shall be 7ʒ + 4ℨ, as at *A.* So 5ʒ — 6ℨ from 12ʒ — 10ℨ, shall leave 7ʒ — 4ℨ, as at *B.* But to substract 5ʒ + 10ℨ from 12ʒ + 6ℨ, the Remain shall be 7ʒ — 4ℨ, as at *C.* And 5ʒ — 10ℨ from 12ʒ — 6ℨ, shall leave 7ʒ + 4ℨ, as at *D.* For in both these latter the Roots to be substracted being the Greater Number changeth the Sign to the difference of the Numbers.

Also if 3φ + 4ʒ + 10N, be taken from 16φ + 18ʒ, the Remain shall change the Sign to N, as at *E.* But if N had not been in the Subtrahend, he shall keep his Sign as at *F.*

	Binomials.		Residuals.		Medials or Polynomials.
A.	12ʒ + 10ℨ	*B.*	12ʒ — 10ℨ	*E.*	16φ + 18ʒ
	5ʒ + 6ℨ		5ʒ — 6ℨ		3φ + 4ʒ + 10N
	7ʒ + 4ℨ		7ʒ — 4ℨ		13φ + 14ʒ — 10N
C.	12ʒ + 6ℨ	*D.*	12ʒ — 6ℨ	*F.*	16φ + 18ʒ + 10N
	5ʒ + 10ℨ		5ʒ — 10ℨ		3φ + 4ʒ
	7ʒ — 4ℨ		7ʒ + 4ℨ		14φ + 14ʒ + 10N

Compound of unlike Signs.

If the Compound Cossicks have contrary Signs, add the Cossicks with their fellows of unlike Signs, and to the Total, which is the Remain, adjoyn the Sign of the upper Number, that is that from which Substraction is to be made.

Examples.

As to take 5ʒ — 6ℨ from 12ʒ + 10ℨ, or 5ʒ — 10ℨ from 12ʒ + 6ℨ, in both the Remain shall be 7ʒ + 16ℨ, as at *G.* and *H.* which 16ℨ is the summe of — 6ℨ and + 10ℨ, or — 10ℨ and + 6ℨ. But if 5ʒ + 6ℨ be taken from 12ʒ — 10ℨ, or 5ʒ + 10ℨ from 12ʒ — 6ℨ, in both these Cases the Remain shall be 7ʒ — 16ℨ, as at *I.* and *K.* because the Sign of the upper Number was —. Other Examples of Polynomials follow at *L.* and *M.*

Binomials

Binomials and Residuals.				Polynomials.	
G.	12ʒ + 10ℨ 5ʒ − 6ℨ	*I.*	12ʒ − 10ℨ 5ʒ + 6ℨ	*L.*	3φ + 10ʒ + 100N 9φ + 10ʒ − 30N
	7ʒ + 16ℨ		7ʒ − 16ℨ		130N − 6φ
H.	12ʒ + 6ℨ 5ʒ − 10ℨ	*K.*	12ʒ − 6ℨ 5ʒ + 10ℨ	*M*	3ʒ + 19ℨ − 10N 18ℨ − 3ʒ + 4N
	7ʒ + 16ℨ		7ʒ − 16ℨ		6ʒ + 1ℨ − 14N

To prove Cossical Substraction, as before in Addition, by some fit Root convert the Cossicks into Abstract Numbers, and making Substraction as in Integers, the Remains will be left equal, if the Operations be right. *Proof of Cossical Substraction.*

As in the Example above at *G*, supposing the Root 2, then shall 10ℨ be 20, and 12ʒ be 48, in all 68. And the Subtrahend 5ʒ shall be 20 lacking 12, which is 6ℨ, that is 8, which 8 taken from 68, leaves 60 equal to 7ʒ, that is 28, and 16ℨ which is 32.

12ʒ + 10ℨ	48 + 20 = 68
5ʒ − 6ℨ	20 − 12 = 8
7ʒ + 16ℨ	28 + 32 = 60

Besides this Proof, if you add 7ʒ + 16ℨ to 5ʒ − 6ℨ, the Total will be 12ʒ + 10ℨ, as before, and shew the alternate Proof of Cossical Addition by Substraction, and Substraction by Addition.

CHAP. IV.

Multiplication of Whole Cossicks.

TO multiply Simple Cossicks, multiply Number by Number as in Integers, and add the respective *Indices* of the Cossical Quantities, and the Total shall be the *Index* of the Product, whose Character is to be affixed thereto. *Multiplication of Whole* Cossicks. *Simple. Examples.*

As to multiply 3φ by 9ʒ, the Numbers 3 and 9 multiplyed produce 27, and 2 the *Index* of ʒ, added to 3 the *Index* of φ, make 5, which is the *Index* of ß; so is the Product 27ß.

So 10ʒʒ multiplyed by 8φ, produce 80Bß; for the *Index* of φ 3, added to 4 the *Index* of ʒʒ, make together 7, which hath Bß for his Character.

Muitiplicands	3φ	3	10ʒʒ	4	30ß	5	
Multiplyers	9ʒ	2	8φ	3	10ʒʒ	4	Indices.
Product	27ß	5	80Bß	7	300φφ	9	

If Compound Cossicks be multiplyed, let the Cossicks be placed according to their Quantities, and every Number in the Multiplicand be multiplyed by every Number in the Multiplyer, and the respective *Indices* thereof gotten, as if they were Simple Cossicks. And for the Signs, as before in other Contract Numbers, observe like Signs multiplyed together produce +, and unlike −. *Compound.*

As to multiply 3φ + 2ʒ − 4ℨ by 2ʒ + 3ℨ − 8N, the Product will be 6ß + 13ʒʒ − 26φ − 28ʒ + 32ℨ, as by the Operation it self beginning at the Left Hand, and after Multiplication adding the several Lines of Production, or Multiplees together, appeareth plainly thus, *Example.*

Multiplicand	3φ + 2ʒ − 4ℨ
Multiplyer	2ʒ + 3ℨ − 8N
Multiplees	6ß + 4ʒʒ − 8φ
	9ʒʒ + 6φ − 12ʒ
	− 24φ − 16ʒ + 32ℨ
Product	6ß + 13ʒʒ − 26φ − 28ʒ + 32ℨ

Proof of Cossical Multiplication.

To prove Cossical Multiplication, besides the tryal by Cossical Division, turn the Cossicks into Abstract Numbers, taking at pleasure some fit Number for a Root, and after Multiplication of the Numbers as Integers, compare the Product with the summe of the Cossical Product, for without Error they exactly agree.

As if 3ʒ + 4ℨ be multiplyed by 2ʒ − 3ℨ, the Product will be 6ʒʒ − 1φ − 12ʒ. To prove which I suppose 2 the ℨ, then shall 4ℨ be 8, and 3ʒ, 12, in summe 20. And 2ʒ, 8, lacking 3ℨ, 6, makes the summe of the Multiplyer but 2, which multiplying 20, produceth 40, and so much is the summe of 6ʒʒ − 1φ − 12ʒ.

3ʒ + 4ℨ	12 + 8 ═══ 20
2ʒ − 3ℨ	8 − 6 ═══ 2
6ʒʒ + 8φ	96 + 64 ═══ 160
− 9φ − 12ʒ	− 72 − 48 ═ − 120
6ʒʒ − 1φ − 12ʒ	96 − 8 − 48 ═ 40

CHAP. V.

Division of Whole Cossicks.

Division *of Whole* Cossicks

Simple.

TO divide Simple Cossicks, divide Number by Number, as in Integers, and substract the *Index* of the Divisor from the *Index* of the Dividend, and the Character belonging to the Remaining *Index* adjoyn to the Quotient. And if the Numbers will not evenly be divided without a Remainer, or the Divisor have the Greater Cossick, then are they to be set, as Cossical Fractions.

Examples.

As to divide 40ʒʒ by 10φ, the Quotient will be 4ℨ: For if 3, the *Index* of φ, be abated from 4, the ʒʒ *Index*, the Remain will be 1, the *Index* of ℨ.

So 60ß, divided by 5ʒ, gives in the Quotient 12φ.

	Indices.		*Indices.*		
Dividends	4	40ʒʒ (4ℨ	5	60ß (12φ	Quotients
Divisors	3	10φ	2	5ʒ	
	1		3		

Compound.

If Compound Cossicks be divided, then as before in Compound Decimal and Astronomical Division, let the Numbers in the Dividend be divided by the Numbers in the Divisor. The *Indices* of the Quotienary Numbers are got as in Division of Simple Cossicks. And the Signs alike give +, and unlike −, as in Multiplication of Compound Cossicks and other Contract Numbers. If any Cossical Quantity be omitted in the given Numbers, express the same with Cyphers, and the Sign +, and place all Cossicks according to their Powers, whether their Signs be + or −. And if the Numbers will not be evenly divided, or the Divisor have the Greater Cossicks, then the Divisor is placed beneath to represent it as a Fraction.

Example.

As to divide 6ß + 13ʒʒ − 26φ − 28ʒ + 32ℨ by 2ʒ + 3ℨ − 8N, after by the first application of the Divisor to the Dividend 3φ is gotten in the Quotient, the Divisor is multiplyed thereby, and the Product 6ß + 9ʒʒ − 24φ substracted from the

the Dividend; by the second application of the Divisor 2Ʒ is gotten in the Quotient, the Product of the Divisor multiplyed thereby is 4ƷƷ + 6φ — 16Ʒ. And after this substracted the last application of the Divisor gives 4ℨ in the Quotient, and the Divisor multiplyed thereby produceth a Cossick Quantity equal to what was left on the Dividend after the former Substractions.

Divisor — Dividend — Quotient

— 8φ
4ƷƷ — 2φ — 12Ʒ
2Ʒ + 3ℨ — 8N) 6ß + 13ƷƷ — 26φ — 28Ʒ + 32ℨ (3φ + 2Ʒ — 4ℨ
6ß + 9ƷƷ — 24φ
4ƷƷ + 6φ — 16Ʒ
— 8φ — 12Ʒ + 32ℨ

To prove Cossical Division as before in the other Elementary Operations, let the Cossicks be exchanged for Abstract Numbers, at pleasure taking some fit Number for a Root, and the Division as in Integers performed, the Quotient will parallel the summe of the Cossical Quotient so exchanged, when the Division is well wrought. *Proof of Cossical Division.*

As if 8φ + 64N, be divided by 2ℨ + 4N, the Quotient will be thus, 4Ʒ — 8ℨ + 16N. I then suppose the Root 2, the Dividend then shall be 128, for so much is 8φ and 64N. The Divisor is 8, that is 2ℨ which are 4, and 4 Numbers over. And the Quotient of 128 divided by 8 is 16, which agreeth exactly with the Cossical Quotient: For 4Ʒ is 16, and 16N make 32, from which 8ℨ taken which are 16, there is left but 16 for the summe of the Quotient.

— 16Ʒ + 32ℨ
2ℨ + 4N) 8φ + 0Ʒ + 0ℨ + 64N (4Ʒ — 8ℨ + 16N
8φ + 16Ʒ
— 16Ʒ — 32ℨ
+ 32ℨ + 64N

— 64 + 64
4 + 4) 64 + 0 + 0 + 64 (16 — 16 + 16
64 + 64
— 64 — 64
+ 64 + 64

128 / 8 (16

Besides this Proof, if you multiply 4Ʒ — 8ℨ + 16N by 2ℨ + 4N, the Product will be the Dividend 8φ + 0Ʒ + 0ℨ + 64N, as before, and thereby shew the alternate Proof of Multiplication by Division, and Division by Multiplication in Cossicks, as well as other Numbers.

CHAP. VI.

Of Broken Cossicks.

AS Whole Cossicks (of which the foregoing Five *Chapters* have sufficiently spoken) are of two sorts, *viz.* Simple and Compound; so are their Fractions. *Broken Cossicks.*

Simple Cossical Fractions are some Broken Parts, or a part only of some Simple Cossick, and are expressed like Vulgar Fractions with the Cossical Character or Denomination annexed. As $\frac{2}{3}$ℨ signifieth $\frac{2}{3}$ of a Root, let the Root be what it will. So $\frac{3}{5}$Ʒ importeth $\frac{3}{5}$ of a Square, &c. And hereby is understood that the Cossical Quantity is divided into so many parts as the Denominator denoteth, and a certain number of those parts to be taken as the Numerator denoteth. And hence it may happen *Simple how expressed.* *Examples.*

happen sometime that the Cossical Fraction may in value be an Integer, and no Fraction. For if the Cossical Fraction be $\frac{3}{4}$ʒ, if the Square be 16, then shall $\frac{3}{4}$ of that Square be 12 Integers. Also $\frac{2}{3}$φ shall be 18 Integers, if the Cube be 27. All these Simple Cossical Fractions may be set as Compound, by placing the Cossical Denomination to the Numerator, and N to the Denominator; or else leaving the Denominator as an Integer: For $\frac{2}{3}$ʒ is as $\frac{2ʒ}{3N}$ or $\frac{2ʒ}{3}$.

Compound of 2 sorts. Dual.

Compound Cossical Fractions are of two sorts, Dual or Plural.

Examples.

Dual, when the Fraction consisteth only of two Cossical Denominations, and look like Simple Fractions. As $\frac{3ʒʒ}{2φ}$ which is as much as 3ʒʒ to be divided by 2φ. So $\frac{4ſβ}{9ʒφ}$ import that 9ʒφ must divide 4ſβ, *&c.*

Plural.

Example.

Plural Cossical Fractions are when the Numerator or Denominator, or both, consist of more than two Cossical Quantities. As $\frac{4φ + 3ℨ - 10N}{3ℨ + 12N}$ that implyeth that 4 Cubes and 3 Roots lacking 10 Numbers are to be divided by 3 Roots and 12 Numbers added together; and so of others.

Proper and Improper how known.

Cossical Fractions are also Proper and Improper, when the Denominator is the Greater Cossick, it is a Proper Fraction, but Improper when the contrary.

CHAP. VII.

Reduction of Broken Cossicks.

Broken Cossicks reduced.

To their least Termes.

THE Reduction of Cossical Fractions, is either to reduce them to their least Terms, or to like Denominators.

The first sort, as in Vulgar Fractions, may be called Abbreviation; for the Simple Cossical Fractions may be reduced lower sometime in their Numbers, and the Compound sometime both in their Numbers and Quantities. But if the Numbers be incommensurable in either, or any one Quantity of the Compound be N, then each of them respectively must be kept as they happen, unaltered, unless alterable by the subsequent Proposition.

Examples.

As $\frac{24}{28}$ʒ may be reduced in its Numbers, by the Rules of abbreviating Fractions seen before in the *Second Part* of the *First Book*, *Chap.* 2. to $\frac{6}{7}$ʒ, as at *A*.

So $\frac{30φ}{36ʒʒ}$ may be abbreviated in its Numbers to $\frac{5φ}{6ʒʒ}$, and in its Quantities to $\frac{5N}{6ℨ}$, there being alike Quantities abated from the Cossical Numerator and Denominator; ℨ being as far distant from ʒʒ, as N from φ. See below at *B*.

Also in Plural Fractions, as $\frac{3ʒʒ + 6ʒ}{3ſβ + 12φ}$ the Numbers being all commensurable by 3, may be reduced first in its Numbers to $\frac{1ʒʒ + 2ʒ}{1ſβ + 4φ}$, and in the Quantities till one of them be an Absolute Number, thus, $\frac{1ʒ + 2N}{1φ + 4ℨ}$, as at *C*.

		Abbreviated,			
Simple Cossical Fraction		$\frac{24}{28}$ʒ	$\frac{12}{14}$ʒ	$\frac{6}{7}$ʒ	*A.*
Compound Cossical	Dual Fraction	$\frac{30φ}{36ʒʒ}$	$\frac{15ʒ}{18φ}$	$\frac{5ℨ}{6ʒ}$	$\frac{5N}{6ℨ}$ *B.*
			in Numbers	in Quantities	
	Plural Fraction	$\frac{3ʒʒ + 6ʒ}{3ſβ + 12φ}$	$\frac{1ʒʒ + 2ʒ}{1ſβ + 4φ}$	$\frac{1ʒ + 2N}{1φ + 4ℨ}$	*C.*

The

The Proposition above-mentioned.

Sometimes, and but sometimes, it happeneth, when yet the Numbers are incommensurable, and the Quantities reduced as low as N, that some part of the composition may be abated, and yet the Remain be in like proportion. *Cossicks in their least Terms sometime shortned.*

As $\frac{4\varphi \ +12ʒ}{8ʒ\varphi+24ß}$ reduced in its Numbers is $\frac{1\varphi \ +3ʒ}{2ʒ\varphi+6ß}$ then in its Quantities is $\frac{1ℨ \ +3N}{2ʒʒ+6\varphi}$. But now if I abate the Quantities that follow the Sign of Composition +, yet the remaining Fraction will retain the same proportion, and $\frac{1ℨ}{2ʒʒ}$ or $\frac{1N}{2\varphi}$ shall be equal. *Examples.*

Also Residual Numbers, as $\frac{4\varphi \ -12ʒ}{8ʒ\varphi-24ß}$ may be reduced to $\frac{1ℨ-3N}{2ʒʒ-6\varphi}$, and consequently to $\frac{1N}{2\varphi}$ as before.

For the better understanding of this kind of Reduction, observe; *When this happeneth what must be.*

1. That the Signs must be *Synonima's*, that is both + or both —; for if one be more, and the other less, it will not suffer this Reduction.

2. The Numbers taken away must be in like proportion to them that remain. As in the former Examples, 3 to 6, is as 1 to 2, or 4 to 8, as 12 to 24

3. As the Numbers, so the Cossical Quantities must also be in like proportion. For in the former Examples, N to φ is as ℨ to ʒʒ, the difference 3, being between their several *Indices*.

So that if the difference of the abatement unto abatement, be as the whole is in proportion to the whole, then shall the residue be in like proportion to the residue, as the whole is to the whole, by *Euclid.* 5 *Lib.* 19 *Prop.* *The Reason thereof.*

The second sort of Cossical Reduction, to bring Cossicks of different Denominations into one, comprehendeth, *To reduce Cossicks to one Denominator. Simple.*

1. To reduce Simple Fractions to one Denomination, which is effected as Abstract Fractions, without altering the Cossical Quantities.

And so $\frac{2}{3}ʒ+\frac{3}{4}\varphi$ reduced, shall stand as at *D*, and be $\frac{8ʒ \quad 9\varphi}{12}$. *Example.*

$$D. \quad \frac{8ʒ}{\frac{2}{3}ʒ} \times \frac{9\varphi}{\frac{3}{4}\varphi} \quad 12$$

2. To reduce Dual Fractions to one Denomination. And this differs nothing from the other, but in increasing the Cossical Quantities according to the nature of Cossical Multiplication. *Dual.*

And so $\frac{3ʒ}{4\varphi}$ reduced with $\frac{1ℨ}{3N}$ shall stand as at *E*, and be $\frac{9ʒ \quad 4ʒʒ}{12\varphi}$. *Example.*

$$E. \quad \begin{array}{c} 9ʒ \\ \hline 3ʒ \\ \hline 4\varphi \end{array} \times \begin{array}{c} 4ʒʒ \\ \hline 1ℨ \\ \hline 3N \end{array} \quad 12\varphi$$

3. To reduce Plural Fractions to one Denomination, which *mutatis mutandis* is like the last. *Plural.*

And so $\frac{1ʒ+2ℨ}{2\varphi+3N}$ reduced with $\frac{2\varphi \ -4N}{3ʒʒ-5ℨ}$, shall stand as at *F.*; and be $\frac{3ʒ\varphi+6ß-5\varphi-10ʒ \qquad 4ʒ\varphi-2\varphi-12N}{6Bß-1ʒʒ-15ℨ}$, as by the respective Multiplications of Denominator by Denominator, and then alternately the Denominator of each into the others Numerator will appear. *Example.*

$$F. \quad \begin{array}{c} 3ʒ\varphi+6ß-5\varphi-10ʒ \\ \hline 1ʒ+2ℨ \\ \hline 2\varphi+3N \end{array} \times \begin{array}{c} 4ʒ\varphi-2\varphi-12N \\ \hline 2\varphi-4N \\ \hline 3ʒʒ-5ℨ \end{array} \quad 6Bß-1ʒʒ-15ℨ$$

 4. To

Mixt turned into Improper Fractions.

4. To reduce Mixt Numbers, *viz.* Whole and Broken Cossicks into an Improper Fraction, or to set an Whole Cossick in form of a Fraction, both like as was shewed in Abstract Fractions.

Examples.

And so $2\text{ʒ}\frac{2N}{3\text{ℨ}}$ shall be reduced into $\frac{6φ + 2N}{3\text{ℨ}}$ by multiplying 2ʒ, the Whole Cossick, into 3ℨ the Denominator of the Fraction, which make $6φ$, and then adding thereto 2 N, makes the Numerator $6φ + 2N$, to which the 3ℨ shall be the Denominator.

Absurd Numbers what.

And if $\frac{2N}{3\text{ℨ}} - 2\text{ʒ}$, which indeed is an absurd Number, or less than nothing were to be reduced, the Reduction shall be $\frac{2N - 6φ}{3\text{ℨ}}$.

And if $2\text{ʒ} + 3N\frac{3φ - 2N}{4\text{ʒʒ}}$ be reduced, the Improper Fraction will stand thus, $\frac{8\text{ʒ}φ + 12\text{ʒʒ} + 3φ - 2N}{4\text{ʒʒ}}$.

Whole Cossick set as a Fraction.

But if any Whole Cossick be set Fraction wise, there is only 1 N to be subscribed.

And so $3φ + 4\text{ʒ} - 2\text{ℨ}$ shall be set thus, $\frac{3φ + 4\text{ʒ} - 2\text{ℨ}}{1N}$.

Improper turned back to Integral.

5. To reduce Improper Cossical Fractions back into Whole Cossicks, or an Whole and Broken Cossick, divide the Numerator by the Denominator, whereby sometime the Denominator is wholly discharged, when nothing remains upon the Division, or the Integer turned into a Mixt Number.

Examples.

And so $\frac{63φ}{21\text{ʒ}}$ will be reduced into 3ℨ. And $\frac{30\text{ß} + 24\text{ʒʒ} + 18φ}{20\text{ʒ}}$ into $1\frac{1}{2}φ + 1\frac{1}{5}\text{ʒ} + \frac{9}{10}\text{ℨ}$.

Many Fractions to one Denominator.

6. To reduce several sorts of Fractions, or many of one sort, into one Denomination, by the Reduction proper to each of them.

Examples.

As $\frac{3}{4}φ$ reduced with $\frac{2\text{ʒ}}{3φ}$ shall be $\frac{9\text{ʒ}φ \quad 8\text{ʒ}}{12φ}$.

And so $\frac{1\text{ℨ}}{2N}$ and $\frac{2N}{3\text{ℨ}}$ reduced with $\frac{3\text{ʒ} - 2N}{5φ}$ shall be $\frac{15\text{ß} \quad 20φ \quad 18φ - 12\text{ℨ}}{30\text{ʒʒ}}$ and abbreviated, $\frac{15\text{ʒʒ} \quad 20\text{ʒ} \quad 18\text{ʒ} - 12N}{30φ}$.

Proof of Cossical Reduction.

One part of Cossical Reduction hath the same faculty with other Reductions, to prove the other part reciprocal thereto, as may easily be discerned without Example. And moreover, by resolving the Cossical Fractions into abstract Numbers, as before in the works of Whole Cossicks, every part of Reduction may be fully proved.

As in the last Example, supposing 2 be a Root, then shall each of the Cossical Fractions be as at *G*, and the reduced Fractions found to agree in value without and with abbreviation.

$15\text{ß} = 480$ | $20φ = 160$ | $18φ - 12\text{ℨ} = 144 - 24$

G. $\frac{1\text{ℨ}}{2N} = \frac{2}{2}$ X $\frac{2N}{3\text{ℨ}} = \frac{2}{6}$ X $\frac{3\text{ʒ} - 2N}{5φ} = \frac{12 - 2}{40}$

$30\text{ʒʒ} = 480$

Abbreviated.

$15\text{ʒʒ} = 240$ | $20\text{ʒ} = 80$ | $18\text{ʒ} - 12N = 72 - 12$

$30φ = 240$

CHAP. VIII.

Addition of Broken Cossicks.

Broken Cossicks added. Simple and Homogeneal.

IN adding Cossical Fractions; first if they be Simple, and the Numbers and Cossicks be of one Denomination, then add the Numerators, and subscribe the Common Denominator with the Cossical Character: But if the Numbers be not of one Denomination,

mination, reduce them as Common Fractions, and then add their Numerators as above.

As to add $\frac{3}{5}ʒ$ to $\frac{1}{5}ʒ$, make the Total $\frac{4}{5}ʒ$, as at *A*. *Examples.*

But $\frac{3}{4}\varphi$ added to $\frac{1}{3}\varphi$ must first be reduced to $\frac{9\ \ 4}{12}\varphi$, and then added, make together $\frac{13}{12}\varphi$, or $1\frac{1}{12}\varphi$, as at *B*.

$$A.\quad \cfrac{\overline{\ \ 4\ \ }}{\cfrac{3}{5}ʒ+\cfrac{1}{5}ʒ}\ \Big/\ 5 \;=\; \frac{4}{5}ʒ \qquad\qquad B.\quad \cfrac{\overline{13}}{\cfrac{9}{\frac{3}{4}\varphi} \;+\; \cfrac{4}{\frac{1}{3}\varphi}}\ \Big/\ 12 \;=\; \frac{13}{12}\varphi \text{ or } \left(1\frac{1}{12}\varphi\right.$$

2. If the Cossical Quantities of Simple Fractions to be added, be unlike or Heterogeneal, then connex them by the Sign of Addition, or reduce them. *Simple and Heterogeneal.*

As to add $\frac{3}{5}ʒ$ to $\frac{1}{2}\varphi$, I set them as at *C*. or *D*. *Example.*

$$C.\quad \frac{3}{5}ʒ+\frac{1}{2}\varphi \qquad\qquad D.\quad \cfrac{6ʒ \;+\; 5\varphi}{\frac{3}{5}ʒ \times \frac{1}{2}\varphi}\ \Big/\ 10 \;=\; \frac{5\varphi+6ʒ}{10}$$

3. If the Fractions be Compound, and of like Denominations in Numbers and Cossicks, then add the Numerators as Cossicks are to be added, and subscribe the Common Denominator. *Compound and Homogeneal.*

As $\frac{3N}{10ℨ}$ added to $\frac{6N}{10ℨ}$ shall make the Total $\frac{9N}{10ℨ}$, as at *E*. *Examples.*

And $\frac{3ℨ+2N}{20ʒ}$ added to $\frac{2ℨ+1N}{20ʒ}$ shall make together $\frac{5ℨ+3N}{20ʒ}$, as at *F*.

And $\frac{3ℨ+2N}{20ʒ}$ added to $\frac{2ℨ-5N}{20ʒ}$ shall make the Total $\frac{5ℨ-3N}{20ʒ}$, as at *G*.

$$E.\quad \cfrac{9N}{\cfrac{3N}{10ℨ}+\cfrac{6N}{10ℨ}}\ \Big/\ 10ℨ \;=\; \frac{9N}{10ℨ} \qquad\qquad F.\quad \cfrac{5ℨ+3N}{\cfrac{3ℨ+2N}{20ʒ}+\cfrac{2ℨ+1N}{20ʒ}}\ \Big/\ 20ʒ \;=\; \frac{5ℨ+3N}{20ʒ}$$

$$G.\quad \cfrac{5ℨ-3N}{\cfrac{3ℨ+2N}{20ʒ}+\cfrac{2ℨ-5N}{20ʒ}}\ \Big/\ 20ʒ \;=\; \frac{5ℨ-3N}{20ʒ}$$

4. If the Numbers or Compound Cossicks be not alike, first reduce them, and then add their Numerators, as last above-mentioned. *Compound and Heterogeneal.*

As to add $\frac{8\varphi+9ʒ}{10ʒʒ}$ to $\frac{6\varphi-3ʒ}{10\varphi}$, they are first reduced to the Denomination of $100Bſδ$, and then added and abbreviated, as at *H*. and *I*. *Examples.*

$$H.\quad \cfrac{\overline{60Bſδ+50ʒ\varphi+90ſδ}}{\cfrac{80ʒ\varphi+90ſδ}{\frac{8\varphi+9ʒ}{10ʒʒ}} \;+\; \cfrac{60Bſδ-30ʒ\varphi}{\frac{6\varphi-3ʒ}{10\varphi}}}\ \Big/\ 100Bſδ \;=\; \frac{60Bſδ+50ʒ\varphi+90ſδ}{100Bſδ}$$

Abbreviated in Numbers and Quantities.

$$I.\quad \frac{60Bſδ+50ʒ\varphi+90ſδ}{100Bſδ} = \frac{6Bſδ+5ʒ\varphi+9ſδ}{10Bſδ} = \frac{6ʒ+5ℨ+9N}{10ʒ}$$

Also

Also $\frac{2ʒ + 3ℨ}{3φ + 4N}$ is added to $\frac{4ℨ - 2N}{2ʒ + 2ℨ}$ in like manner at *K*. The Total whereof may be abbreviated to $\frac{8ʒʒ + 2φ + 3ʒ + 8ℨ - 4N}{3ß + 3ʒʒ + 4ʒ + 4ℨ}$.

$$16ʒʒ + 4φ + 6ʒ + 16ℨ - 8N$$

$$4ʒʒ + 10φ + 6ʒ \qquad 12ʒʒ + 16ℨ - 6φ - 8N$$

$$K. \quad \frac{2ʒ + 3ℨ}{3φ + 4N} \times \frac{4ℨ - 2N}{2ʒ + 2ℨ}$$

$$6ß + 6ʒʒ + 8ʒ + 8ℨ$$

Proof of Cossical Addition of Fractions. Addition of Cossical Fractions is proved both by Substraction, as in the next *Chapter*, and by taking some fit Number for a Root, and accordingly turning the Cossical Fractions into Abstract, and after the Addition to parallel the Totals.

As in the last Example, supposing 2 be a Root, the Fractions given to be added will be in their least Terms, $\frac{1}{2}$ and $\frac{1}{2}$, which added make 1 Integer; and so much is the added Cossick.

$$\frac{2ʒ + 3ℨ}{3φ + 4N} = \frac{8 + 6}{24 + 4} = \frac{14}{28} = \frac{1}{2}$$

$$\frac{4ℨ - 2N}{2ʒ + 2ℨ} = \frac{8 - 2}{8 + 4} = \frac{6}{12} = \frac{1}{2}$$

Numerator.		Denominator.	
16ʒʒ =	256	6ß =	192
+ 4φ =	32	+ 6ʒʒ =	96
+ 6ʒ =	24	+ 8ʒ =	32
+ 16ℨ =	32	+ 8ℨ =	16
	344		336
− 8N	8		
	336		

And as $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = \frac{1}{1}$, or 1, so is $\frac{336}{336}($ 1

CHAP. IX.

Substraction of Broken Cossicks.

Broken Cossicks substracted. Simple and Homogeneal. IN substracting Cossical Fractions; first if they be Simple, and the Numbers and Cossicks be of like Denominations, then take the Lesser Numerator from the Greater, and to the Remain subscribe the Common Denominator with the Cossical Character. But when the Subtrahend is the Greater change the Sign to the difference.

Examples. As to take $\frac{2}{5}φ$ from $\frac{3}{5}φ$, the Remain shall be $\frac{1}{5}φ$, as at *A*.

But to take $\frac{3}{5}φ$ from $\frac{2}{5}φ$, the Remain shall be $-\frac{1}{5}φ$, as at *B*.

$$A. \quad \frac{3}{5}φ - \frac{2}{5}φ = \frac{1}{5}φ \qquad\qquad B. \quad \frac{2}{5}φ - \frac{3}{5}φ = -\frac{1}{5}φ$$

Simple and Heterogeneal. 2. If the Numbers be not of one Denomination, or the Simple Cossicks Heterogeneal, then reduce the Numbers as Common Fractions, and then substract the Numerators of the Homogeneal, as above: But let the unlike Cossicks be connexed with the Sign of Substraction.

Example. As to take $\frac{1}{2}ʒ$ from $\frac{2}{3}ʒ$, the Remain after Reduction of the Fractions will be $\frac{1}{6}ʒ$ as at *C*.

But if $\frac{1}{2}ʒ$ were to be taken from $\frac{2}{3}φ$, I set them as at *D*, or *E*.

C.

$$C.\ \overset{1}{\overline{\overset{4}{\tfrac{2}{3}ʒ} \;\overset{-}{\mathbf{X}}\; \overset{3}{\tfrac{1}{2}ʒ}}}\underset{6}{} = \tfrac{1}{6}ʒ \qquad D.\ \tfrac{2}{3}φ - \tfrac{1}{2}ʒ \qquad E.\ \overset{4φ \;-\; 3ʒ}{\tfrac{2}{3}φ \;\mathbf{X}\; \tfrac{1}{2}ʒ}\underset{6}{} = \frac{4φ - 3ʒ}{6}$$

3. If the Fractions be Compound, and of like Denominations both in Numbers and Cossicks, then substract the Lesser Numerator out of the Greater, as Cossicks are to be substracted, and subscribe the Common Denominator: But when the Subtrahend is the Greater, change the Sign to the Difference, as before. *Compound and Homogeneal.*

As to take $\frac{1ℨ}{3ʒ}$ from $\frac{2ℨ}{3ʒ}$, the Remain will be $\frac{1ℨ}{3ʒ}$, as at *F*. *Examples.*

But if $\frac{2ℨ}{3ʒ}$ be taken from $\frac{1ℨ}{3ʒ}$, the Remain will be $-\frac{1ℨ}{3ʒ}$, as at *G*.

So if $\frac{3ℨ + 2N}{9ʒ}$ be taken from $\frac{6ℨ + 7N}{9ʒ}$, the Remain will be $\frac{3ℨ + 5N}{9ʒ}$, as at *H*.

And if $\frac{3ʒ + 5ℨ}{4φ}$ be abated out of $\frac{5ʒ - 6ℨ}{4φ}$, the Remain will be $\frac{2ʒ - 11ℨ}{4φ}$, as at *I*.

Also $\frac{4ℨ - 3N}{10ʒ - 2ℨ}$ deducted out of $\frac{7ℨ - 2N}{10ʒ - 2ℨ}$, the Remain will be $\frac{3ℨ + 1N}{10ʒ - 2ℨ}$, as at *K*.

$$F.\ \overset{1ℨ}{\overline{\frac{2ℨ}{3ʒ} - \frac{1ℨ}{3ʒ}}}\underset{3ʒ}{} = \frac{1ℨ}{3ʒ} \qquad G.\ \overset{-1ℨ}{\overline{\frac{1ℨ}{3ʒ} - \frac{2ℨ}{3ʒ}}}\underset{3ʒ}{} = -\frac{1ℨ}{3ʒ}$$

$$\overset{3ℨ + 5N}{\overline{\frac{6ℨ + 7N}{9ʒ} - \frac{3ℨ + 2N}{9ʒ}}}\underset{9ʒ}{}\ H. = \frac{3ℨ + 5N}{9ʒ} \qquad \overset{2ʒ - 11ℨ}{\overline{\frac{5ʒ - 2ℨ}{4φ} - \frac{3ʒ + 5ℨ}{4φ}}}\underset{4φ}{}\ I. = \frac{2ʒ - 11ℨ}{4φ}$$

$$K.\ \overset{3ℨ + 1N}{\overline{\frac{7ℨ - 2N}{10ʒ - 2ℨ} - \frac{4ℨ - 3N}{10ʒ - 2ℨ}}}\underset{10ʒ - 2ℨ}{} = \frac{3ℨ + 1N}{10ʒ - 2ℨ}$$

4. If the Numbers or Compound Cossicks be of unlike Denominations; first reduce them, and then substract their Numerators, as last above-mentioned. *Compound and Heterogeneal.*

As to abate $\frac{2ℨ}{3ʒ}$ from $\frac{9N}{10ℨ}$, they are first reduced to the Denomination of $30φ$, and the Remain then is $\frac{7ʒ}{30φ}$, and abbreviated $\frac{7N}{30ℨ}$, as at *L*. *Examples.*

And so to take $\frac{2}{3}ʒ + \frac{9}{10}ℨ$ from $\frac{3}{4}ʒ + \frac{2}{3}ℨ$, they are first to be reduced severally, then jointly; and then substracted the Remain is $\frac{5ʒ - 14ℨ}{60N}$, as at *M*.

Also $\frac{2ʒ + 3ℨ}{3φ + 4N}$ substracted, as at *N*, from $\frac{4ℨ - 2N}{2ʒ + 2ℨ}$, will leave remaining, $\frac{8ʒʒ - 16φ - 6ʒ + 16ℨ - 8N}{6ß + 6ʒʒ + 8ʒ + 8ℨ}$ which will be abbreviated to $\frac{4ʒʒ - 8φ - 3ʒ + 8ℨ - 4N}{3ß + 3ʒʒ + 4ʒ + 4ℨ}$.

$$L.\ \overset{7ʒ}{\overline{\overset{27ʒ}{\frac{9N}{10ℨ}} \;\overset{-}{\mathbf{X}}\; \overset{20ʒ}{\frac{2ℨ}{3ʒ}}}}\underset{30φ}{} = \frac{7ʒ}{30φ} \text{ or } \frac{7N}{30φ}$$

M.

$$\frac{\dfrac{9\text{ʒ} + 8\text{ꝣ}}{\dfrac{3}{4}\text{ʒ} + \dfrac{2}{3}\text{ꝣ}}}{12} - \frac{\dfrac{20\text{ʒ} + 27\text{ꝣ}}{\dfrac{2}{3}\text{ʒ} + \dfrac{9}{10}\text{ꝣ}}}{30} \qquad \frac{\dfrac{45\text{ʒ} + 40\text{ꝣ}}{9\text{ʒ} + 8\text{ꝣ}}}{12} - \frac{\dfrac{40\text{ʒ} + 54\text{ꝣ}}{20\text{ʒ} + 27\text{ꝣ}}}{30} = \frac{5\text{ʒ} - 14\text{ꝣ}}{60\text{N}}$$

$$\underbrace{5 \qquad\qquad 2}_{60}$$

N.

$$\frac{4\text{ꝣ} - 2\text{N}}{2\text{ʒ} + 2\text{ꝣ}} \times \frac{2\text{ʒ} + 3\text{ꝣ}}{3\varphi + 4\text{N}}$$

$$12\text{ʒʒ} + 16\text{ꝣ} - 6\varphi - 8\text{N} \qquad 4\text{ʒʒ} + 10\varphi + 6\text{ʒ}$$

$$8\text{ʒʒ} - 16\varphi - 6\text{ʒ} + 16\text{ꝣ} - 8\text{N}$$

$$6\text{ß} + 6\text{ʒʒ} + 8\text{ʒ} + 8\text{ꝣ}$$

Proof of Cossical Substraction of Fractions.

Substraction of Cossical Fractions is proved, as Addition before, by converting the Cossicks into Abstract Fractions, by some apt Root taken at pleasure, and after Substraction made therewith, the Remains will be alike valuable.

As in the last Example, taking 2 for the Root, the Fractions given in their least Terms will be $\frac{1}{2}$ and $\frac{1}{2}$, which substracted the one from the other, leave o for the Remain. And so is the Cossical Remain $8\text{ʒʒ} - 16\varphi - 6\text{ʒ} + 16\text{ꝣ} - 8\text{N}$, because the negative Quantities equally counterballance the affirmative.

$$\frac{4\text{ꝣ} - 2\text{N}}{2\text{ʒ} + 2\text{ꝣ}} = \frac{8 - 2}{8 + 4} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{2\text{ʒ} + 3\text{ꝣ}}{3\varphi + 4\text{N}} = \frac{8 + 6}{24 + 4} = \frac{14}{28} = \frac{1}{2}$$

Numerator.

$8\text{ʒʒ} = 128$	$-16\varphi = 128$
$+16\text{ꝣ} = 32$	$-6\text{ʒ} = 24$
	$-8\text{N} = 8$
160	160

$$\text{And as } \frac{\dfrac{1}{2} - \dfrac{1}{2}}{2}^{\,0} = \frac{0}{2}, \text{ or } 0; \text{ so } 160 - 160 = 0.$$

Besides, if the Remain $8\text{ʒʒ} - 16\varphi - 6\text{ʒ} + 16\text{ꝣ} - 8\text{N}$ be Cossically added to $4\text{ʒʒ} + 10\varphi + 6\text{ʒ}$, the Number substracted, the total will be $12\text{ʒʒ} + 16\text{ꝣ} - 6\varphi - 8\text{N}$, as above; whereby Addition and Substraction of Cossical Fractions are seen to be alternate Proofs of each other.

CHAP. X.

Multiplication of Broken Cossicks.

Broken Cossicks multiplyed. Simple.

IN multiplying Cossical Fractions; first if they be Simple, increase Numerator by Numerator, and Denominator by Denominator, as Common Fractions are multiplyed, and annex to the Product the Cossical Character due to the Total of both their *Indices.*

Example.

As $\frac{1}{2}\text{ʒ}$ multiplyed by $\frac{5}{6}\varphi$, shall make the Product $\frac{5}{12}\text{ß}$, as at *A.*

And $\frac{1}{3}\varphi$ with $\frac{2}{3}\text{ʒ}$, and $\frac{4}{5}\text{ꝣ}$ multiplyed together shall make the Product $\frac{8}{45}\text{ʒ}\varphi$, as at *B.*

A. $\dfrac{\overset{5}{\dfrac{1}{2}\text{ʒ} \times \dfrac{5}{6}\varphi}}{12} = \dfrac{5}{12}\text{ß}$ *Indices.* 2, 3, sum 5

B. $\dfrac{\overset{8}{\dfrac{1}{3}\varphi \times \dfrac{2}{3}\text{ʒ} \times \dfrac{4}{5}\text{ꝣ}}}{45} = \dfrac{8}{45}\text{ʒ}\varphi$ *Indices.* 3, 2, 1, sum 6

Compound.

2. If the Fractions be Compound, then multiply after the manner of Compound Cossicks,

Cossicks, Numerator by Numerator, and Denominator by Denominator; and both Cossicks and Signs + and — of the produced Numbers are known, as if they were Whole Cossicks.

As to multiply $\frac{1N}{3\text{ℨ}}$ by $\frac{1\text{ℨ}}{2\text{ʒ}}$, the Product will be $\frac{1\text{ℨ}}{6\varphi}$, and abbreviated $\frac{1N}{6\text{ʒ}}$, as at *C.* *Examples.*

And to multiply $\frac{2\text{ʒ}+3\text{ℨ}-2N}{4\varphi}$ by $\frac{4\text{ℨ}+2N}{3\varphi-5\text{ʒ}}$, the Numerators and Denominators being multiplyed as at *D*, the Product of the new Fraction is $\frac{8\varphi+16\text{ʒ}-2\text{ℨ}-8N}{12\text{ʒ}\varphi-20\text{ß}}$ and abbreviated $\frac{4\varphi+8\text{ʒ}-1\text{ℨ}-2N}{6\text{ʒ}\varphi-10\text{ß}}$.

$$C.\quad \overset{1}{\overline{\frac{1N}{3\text{ℨ}}\times\frac{1\text{ℨ}}{2\text{ʒ}}}}\!\!\underset{6}{} = \frac{1\text{ℨ}}{6\varphi}$$

$$D.\quad \overset{8\varphi+16\text{ʒ}-2\text{ℨ}-4N}{\overline{\underline{\frac{2\text{ʒ}+3\text{ℨ}-2N}{4\varphi}\times\frac{4\text{ℨ}+2N}{3\varphi-5\text{ʒ}}}}}\underset{12\text{ʒ}\varphi-20\text{ß}}{} = \frac{8\varphi+16\text{ʒ}-2\text{ℨ}-4N}{12\text{ʒ}\varphi-20\text{ß}}$$

$$\begin{array}{rl} & 2\text{ʒ}+3\text{ℨ}-2N \\ & 4\text{ℨ}+2N \\ \hline & 8\varphi+12\text{ʒ}-8\text{ℨ} \\ & \quad\;\; 4\text{ʒ}+6\text{ℨ}-4N \\ \hline & 8\varphi+16\text{ʒ}-2\text{ℨ}-4N \text{ Numerator.} \end{array} \qquad \begin{array}{l} 3\varphi-5\text{ʒ} \\ 4\varphi \\ \hline 12\text{ʒ}\varphi-20\text{ß} \text{ Denominator.} \end{array}$$

Multiplication of Cossical Fractions, is proved as well by Division noted in the next *Chapter*, as by converting the Cossical Fractions into Abstract; and after Multiplication of them, comparing the equality in their Products. *Proof of Cossical Multiplication of Fractions.*

As in the last Example, the given Cossicks converted into Abstract Fractions supposing the Root 2, are $\frac{12}{32}$ and $\frac{10}{4}$, which multiplyed make the Product without abbreviation $\frac{120}{128}$ and with it $\frac{15}{16}$, which agree with the new Cossical Fraction in equal value.

$$\frac{2\text{ʒ}+3\text{ℨ}-2N}{4\varphi} = \frac{8+6-2}{32} = \frac{12}{32} = \frac{3}{8}$$

$$\frac{4\text{ℨ}+2N}{3\varphi-5\text{ʒ}} = \frac{8+2}{24-20} = \frac{10}{4} = \frac{5}{2}$$

$$\text{And as } \overset{120\;=\!=\;15}{\underset{128\;=\!=\;16}{\frac{12}{32}\times\frac{10}{4} \text{ or } \frac{3}{8}\times\frac{5}{2}}}, \text{ so is } \frac{120}{128}\Big|\frac{60}{64}\Big|\frac{30}{32}\Big|\frac{15}{16}$$

Numerator.		Denominotor.	
8φ =	64	$12\text{ʒ}\varphi$ =	768
$+16\text{ʒ}$ =	64	-20ß =	640
	128		128
-2ℨ =	4		
$-4N$ =	4		
	8		
	120		

CHAP. XI.

Division of Broken Cossicks.

IN dividing Cossical Fractions; first if they be Simple, multiply cross wise, as in Common Fractions, the Numerator of the Dividend by the Denominator of the Divisor for the Numerator of the Quotient, and the Denominator of the Dividend by the Numerator of the Divisor for the Denominator of the Quotient: And thereto annex the Cossical Character due to the Remain of the *Index* of the Divisor deducted from the *Index* of the Dividend. *Broken Cossicks divided. Simple.*

As

Examples. As to divide $\frac{1}{2}\text{ß}$ by $\frac{8}{9}\text{ʒ}$, the Quotient shall be $\frac{9}{16}\varphi$, set as at *A.* or *B.*

And $\frac{3}{4}\varphi$ divided by $\frac{3}{4}\text{ʒ}$, shall make the Quotient $1\,\mathfrak{z}$, as at *C.* or *D.*

$$A.\ \tfrac{1}{2}\text{ß} \overset{9}{\underset{16}{\times}} \tfrac{8}{9}\text{ʒ} = \tfrac{9}{16}\varphi \qquad B.\ \tfrac{8}{9}\text{ʒ}\big)\,\tfrac{1}{2}\text{ß}\,\big(\tfrac{9}{16}\varphi \qquad C.\ \tfrac{3}{4}\varphi \overset{1}{\underset{1}{\times}} \tfrac{3}{4}\text{ʒ} = \tfrac{1}{1}\mathfrak{z} \qquad D.\ \tfrac{3}{4}\text{ʒ}\big)\,\tfrac{3}{4}\varphi\,\big(1\,\mathfrak{z}$$

Compound. 2. If the Fractions be Compound, then after the manner of Compound Cossicks, multiply the Numerator of the Dividend into the Denominator of the Divisor, and contrariwise, the Denominator of the Dividend into the Numerator of the Divisor; and the Cossicks and Signs + and —, of the produced Quotienary Numbers, shall be as if they were Whole Cossicks.

Examples. As to divide $\frac{4\,N}{5\,\mathfrak{z}}$ by $\frac{5\,N}{6\,\mathfrak{z}}$ giveth in the Quotient $\frac{24\,\mathfrak{z}}{25\,\mathfrak{z}}$ or $\frac{24\,N}{25\,N}$, as at *E.* or *F.*

And $\frac{12\,\text{ʒʒ} - 16\,\mathfrak{z}}{2\,\text{ʒ} - 5\,N}$ divided by $\frac{9\,\text{ʒ} - 3\,N}{4\,\text{ʒ} + 5\,N}$, maketh the Quotient to be $\frac{48\,\text{ʒ}\varphi + 60\,\text{ʒʒ} - 64\,\varphi - 80\,\mathfrak{z}}{18\,\text{ʒʒ} - 51\,\text{ʒ} + 15\,N}$, as at *G.* or *H.*

$$E.\ \frac{4\,N}{5\,\mathfrak{z}} \overset{24\,\mathfrak{z}}{\underset{25\,\mathfrak{z}}{\times}} \frac{5\,N}{6\,\mathfrak{z}} = \frac{24\,\mathfrak{z}}{25\,\mathfrak{z}} \qquad F.\ \frac{5\,N}{6\,\mathfrak{z}}\Big)\,\frac{4\,N}{5\,\mathfrak{z}}\,\Big(\frac{24\,\mathfrak{z}}{25\,\mathfrak{z}} \text{ or } \frac{24\,N}{25\,N}$$

$$G.\ \frac{12\,\text{ʒʒ} - 16\,\mathfrak{z}}{2\,\text{ʒ} - 5\,N} \overset{48\,\text{ʒ}\varphi + 60\,\text{ʒʒ} - 64\,\varphi - 80\,\mathfrak{z}}{\underset{18\,\text{ʒʒ} - 51\,\text{ʒ} + 5\,N}{\times}} \frac{9\,\text{ʒ} - 3\,N}{4\,\text{ʒ} + 5\,N} = \frac{48\,\text{ʒ}\varphi + 60\,\text{ʒʒ} - 64\,\varphi - 80\,\mathfrak{z}}{18\,\text{ʒʒ} - 51\,\text{ʒ} + 15\,N}$$

$$H.\ \frac{9\,\text{ʒ} - 3\,N}{4\,\text{ʒ} + 5\,N}\Big)\,\frac{12\,\text{ʒʒ} - 16\,\mathfrak{z}}{2\,\text{ʒ} - 5\,N}\,\Big(\frac{48\,\text{ʒ}\varphi + 60\,\text{ʒʒ} - 64\,\varphi - 80\,\mathfrak{z}}{18\,\text{ʒʒ} - 51\,\text{ʒ} + 15\,N}$$

$$\begin{array}{rl}
 & 12\,\text{ʒʒ} - 16\,\mathfrak{z} \\
 & 4\,\text{ʒ} + 5\,N \\ \hline
 & 48\,\text{ʒ}\varphi - 64\,\varphi \\
 & \qquad 60\,\text{ʒʒ} - 80\,\mathfrak{z} \\ \hline
\text{Numerator} & 48\,\text{ʒ}\varphi + 60\,\text{ʒʒ} - 64\,\varphi - 80\,\mathfrak{z}
\end{array}
\qquad
\begin{array}{ll}
2\,\text{ʒ} - 5\,N & \\
9\,\text{ʒ} - 3\,N & \\ \hline
18\,\text{ʒʒ} - 45\,\text{ʒ} & \\
\qquad - 6\,\text{ʒ} + 15\,N & \\ \hline
18\,\text{ʒʒ} - 51\,\text{ʒ} + 15\,N & \text{Denominator}
\end{array}$$

Proof of Cossical Division of Fractions. Division of Cossical Fractions is proved, as other Elementary Operations thereof, by resolving the Cossical Fractions into Abstract, taking some fit Number for a Root at pleasure, and after Division made, as in Common Fractions, the Quotients will be found equal.

As in the last Example, supposing 2 for the Root, the given Fractions will be $\frac{160}{3}$ and $\frac{33}{21}$, and the Quotient thereof $\frac{3360}{99}$, and so is the Cossical Quotient $\frac{48\,\text{ʒ}\varphi + 60\,\text{ʒʒ} - 64\,\varphi - 80\,\mathfrak{z}}{18\,\text{ʒʒ} - 51\,\text{ʒ} + 15\,N}$.

$$\frac{12\,\text{ʒʒ} - 16\,\mathfrak{z}}{2\,\text{ʒ} - 5\,N} = \frac{192 - 32}{8 - 5} = \frac{160}{3}$$

$$\frac{9\,\text{ʒ} - 3\,N}{4\,\text{ʒ} + 5\,N} = \frac{36 - 3}{16 + 5} = \frac{33}{21}$$

$$\begin{array}{rr}
\multicolumn{2}{c}{\text{Numerator.}} \\
48\,\text{ʒ}\varphi = & 3072 \\
+60\,\text{ʒʒ} = & 960 \\ \hline
 & 4032 \\
-64\,\varphi = & 512 \\
-80\,\mathfrak{z} = & 160 \\ \hline
 & 672 \\ \hline
 & 3360
\end{array}
\qquad
\begin{array}{rr}
\multicolumn{2}{c}{\text{Denominator.}} \\
18\,\text{ʒʒ} = & 288 \\
+15\,N = & 15 \\ \hline
 & 303 \\
-51\,\text{ʒ} = & 204 \\ \hline
 & 99
\end{array}$$

And so $\frac{33}{21}\Big)\,\frac{160}{3}\,\Big(\frac{3360}{99}$ or $\frac{1120}{33}$ being abbreviated.

Besides

Befides if the Quotient be multiplyed by the Divifor, the Dividend of the Simple Fractions will be returned or reduced thereto by abbreviation. But in Compound Fractions other Coffical Fractions in thew will be produced, yet the fame in value; which ferveth to evidence the alternate Proof of Multiplication and Divifion of Coffical Fractions one by the other.

For if $\frac{4\text{φ} + 9\text{ʒ}}{12\,N}$ be divided by $\frac{3\text{ʒ} + 2\text{ℨ}}{3\text{ℨ}}$, the Quotient will be $\frac{12\text{ʒʒ} + 27\text{φ}}{36\text{ʒ} + 24\text{ℨ}}$ and by abbreviation $\frac{4\text{φ} + 9\text{ʒ}}{12\text{ℨ} + 8N}$; wherefore if this Quotient be multiplyed by $\frac{3\text{ʒ} + 2\text{ℨ}}{3\text{ℨ}}$ the Divifor, the Product abbreviated will be $\frac{12\text{ʒʒ} + 35\text{φ} + 18\text{ʒ}}{36\text{ℨ} + 24N}$, which neverthelefs is but equal in value to the Dividend $\frac{4\text{φ} + 9\text{ʒ}}{12\,N}$, as by the Root 2 appeareth.

$$\frac{4\text{φ} + 9\text{ʒ}}{12N} = \frac{32 + 36}{12N} = \frac{68}{12} = 5\tfrac{2}{3}$$

12ʒʒ = 192
35φ = 280
18ʒ = 72
544

36ℨ = 72
24N = 24
96

16
544 ($5\frac{2}{3}$
96

CHAP. XII.

Figuration of Cofficks.

TO Figurate any Coffick is Coffically to multiply the fame, be it Simple or Compound by it felf to produce the Square, and that again by the Root to produce the Cube, *&c.* as other Figural Numbers are produced. *Figurate* Cofficks *produced.*

As to produce the Cube of 2ℨ, or the Square of 2ℨ + 3N, their Figurate Multiplications bring forth 8φ for the one, and 4ʒ + 12ℨ + 9N for the other. *Examples.*

	Simple		Compound
Root	2ℨ	Root	2ℨ + 3N
	2ℨ		2ℨ + 3N
Square	4ʒ		4ʒ + 6ℨ
	2ℨ		6ℨ + 9N
Cube	8φ		4ʒ + 12ℨ + 9N

In extracting the Root of a Coffick feveral things are to be noted. For firft, every Coffick is not Coffically Rooted, no more than every Abfolute Number is a Rooted Number. But thofe Cofficks are Rooted, which have a Root agreeable to the Figure or Character of his Quantity; and therefore no Coffick may be properly called Square, Cubick, or otherwife Rooted, except the Root of the Coffick agreeth with his Character. So 8φ is a Cubick Coffick, and his Root 2ℨ; becaufe 8 is a Cubick Number agreeable to his Character φ. But 8ʒ hath no Coffical Root, becaufe 8 hath no Square Root agreeable to the Character ʒ, neither is it a Cubick Coffick, although the Number have a Cubick Root, becaufe the Cubick Root is difagreeable to the Character ʒ. Likewife 16φ, is no Rooted Coffick, becaufe 16 hath no exact Cubick Root; but 16ʒ is Coffically Rooted, and hath 4ℨ for the Root thereof. *What to be noted in extracting of their Roots.* 1. *Which* Cofficks *are Rooted.*

2. Simple Cofficks Compound in their Characters are not Rooted, unlefs the Number annexed will yield a Root according to the compofition of the Character. So 16ʒʒ becaufe compound of the Square twice, may either have a ʒℨ, or a ʒʒℨ, the one 4ʒ, the other 2ℨ. And 4ʒʒ hath a Square Root 2ʒ, but no ʒʒℨ agreeable to the whole Character. And 9ʒφ hath a ʒℨ, which is 3φ agreeable to part of his Character, but no ʒφ Root anfwerable to the compounded Character. 2. *Which Simple* Cofficks *Compound in their Characters are Rooted.*

And therefore if any Coffick Compound in his Character have a Root agreeable to his whole Character, then may he have alfo as many Roots as their be parts in that compofition: For fo 4096ʒʒφ hath not only a ʒʒφ Root, which is 2ℨ, but alfo a ʒφ *How many Roots fuch may have.*

Root which is 4ʒ, and thirdly a ʒʒ Root which is 8φ, and fourthly a φ Root which is 16ʒʒ, and lastly a ʒ Root which is 64ʒφ.

3. *What proper to Compound Rooted Cossicks.* 3. Compound Cossicks, if they be Cossically Rooted, must first have their greatest Cossick a Rooted Cossick, and the Number annexed to the least Denomination of that Compound Cossick, must be a Rooted Number of the same kind or denomination with the greatest Cossick. As in the Example above of 4ʒ + 12ℨ + 9N, the greatest Cossick 4ʒ is Rooted, and the least Denomination 9N is a ʒ Number.

4. *What in the Compounds the part not Rooted is.* 4. In a Compound Rooted Cossick, every part that is not a Rooted Cossick is a mean between the greatest Cossick and the least Denomination in that Cossick. And if ℨ be one Denomination, then N shall be another: As appeareth in the said Example, where also is 12ℨ to be seen a mean proportional between 9N and 4ʒ.

Others not with these properties what they are. Other Cossicks that suit not with these particulars, are either Surdes, or such affected Cossicks, whose Roots are not properly Cossical, but being equal to some other Number, the investigation of their Roots are to be sought hereafter among *Æquations*.

Root of a Simple Cossick extracted. Examples. These things premised, to extract the Root of a Simple Cossick, extract the Root of the Number according to the Character of the Quantity, and let the Character of the Root be set for the Denomination.

As 9ʒ hath for his Root 3ℨ; and 64φ hath 4ℨ for his Root.

And if the Simple Cossick be Compound in the Character, then as in the former *Book* in *Figural Numbers*, search out the Root from the Number, and affix thereto the Cossical Character.

As if from 256, a Number of the 8th Quantity from the Root 2, I would search out the ʒʒʒ Root, I find it 2ℨ, as at *A*. If out thereof I would seek the ʒʒ Root, I find it 4ʒ, as at *B*. And if the ʒ Root, I find it 16ʒʒ, as at *C*.

So also 729ʒφ gives the ʒφ Root 3ℨ, as at *D*. And the φ Root 9ʒ, as at *E*. And the ʒ Root 27φ, as at *F*.

ʒʒʒ ʒʒ ʒ
A. 256 | 16 | 4 | 2ℨ
1 16 4
156

ʒʒʒ ʒʒ
B. 256 | 16 | 4ʒ
1 16
156

ʒʒʒ
C. 256 | 16ʒʒ
1
156

ʒφ φ
D. 729 | 27 | 3ℨ
27
4
329

ʒφ
E. 729 | 9ʒ

ʒφ
F. 729 | 27φ
4
329

Root of the Compounds extracted. But if the Cossick be Compound, then prick your Cossick according to the Quantity whose Root you would extract, as was before taught in *Figural Numbers* in the *Second Part* of the *Second Book*; and out of the Left Hand pricked Cossick, take the Cossical Root, and place in the Quotient, with half his Cossical Denomination, for the Square; the third part for the Cube, *&c.* as was noted before in *Extraction of the Roots of Astronomicals*, and proceed with the rest of the work as before in *Figural Numbers*; Cossical Extraction differing therefrom no otherwise than as Cossical Addition and Substraction, Multiplication and Division, differs from that of Integers.

Example in the Square. As to extract the Cossical Square Root of 16ʒʒʒ — 16Bſſ — 20ʒφ + 20ſſ + 5ʒʒ — 6φ + 1ʒ. The pricked Numbers will be 1ʒ, 5ʒʒ, 20ʒφ, and 16ʒʒʒ, in which last the greatest Cossical Square Root is 4ʒʒ; for 16 being a Square Number hath 4 for the Root, and half the *Index* of ʒʒʒ which is 8, is 4, whose Character is ʒʒ. This Root doubled is 8ʒʒ for the first Divisor, which dividing — 16Bſſ gets — 2φ for the Quotient, and the Square of — 2φ is + 4ʒφ to be substracted from the next Right Hand Prick: And then doubling the Root, and proceeding to the end of the work, the whole Root obtained is 4ʒʒ — 2φ — 3ʒ + 1ℨ, as is obvious at *G*.

Example in the Cube. Also to extract the Cossical Cube Root of 27φφ + 54ʒʒʒ — 72Bſſ — 136ʒφ + 96ſſ + 96ʒʒ — 64φ. The pricked Numbers will be 64φ, 136ʒφ, and 27φφ, in which the greatest Cube Root is 3φ; for 3 is the Cube Root of 27, and the *Index* of φφ is 9, whose third part is 3, the Character of which *Index* is φ: This tripled is 9φ, and multiplyed by the Root 3φ is 27ʒφ for the first Divisor; which dividing + 54ʒʒʒ gets + 2ʒ for the Quotient. The other parts of the Gnomon and proceedings in the work are plain at *H*.

G.

G. Square Root

−24zφ + 8ß − 4zz

16zzz − 16Bß − 20zφ + 20ß + 5zz − 6φ + 1z (4zz − 2φ − 3z + 1ʐ

16zzz

Gnom. { −16Bß + 4zφ
. . . . . . −24zφ + 12ß + 9zz
. . . . + 8ß − 4zz − 6φ + 1z }

H. Cube Root

−108Bß − 144zφ

27φφ + 54zzz − 72Bß − 136zφ + 96ß + 96zz − 64φ (3φ + 2z − 4ʐ

27φφ

Gnomons { 54zzz + 36Bß + 8zφ
. . . −108Bß − 144zφ + 96ß + 96zz − 64φ }

The one part of Cossical Figuration serves for Proof of the other; that is, Production by Extraction, and Extraction by Production, as other Figural Numbers before exemplified clearly shew; and here may further be seen at *I.* and *K.*; where the Productions of the Simple Cossicks, whose Roots were before extracted at *A. B. C.* and *D. E. F.*; and at *L.* and *M.* are the Productions of the Compound Cossicks, whose Roots are extracted at *G.* and *H.* *Proof of Cossical Figuration.*

But besides, if the Cossicks be resolved into Abstract Numbers, and the Figural Roots thereof be taken, the same will not differ with the Cossical Root, unless some mistake occasion it.

Proof of the Simple Cossical Extraction.

I.		Root supposed 2.	K.		Root supposed 2.
2ʐ	1	4	3ʐ	1	6
2ʐ	1	4	3ʐ	1	6
4z	2	16	9z	2	36
4z	2	16	3ʐ	1	6
16zz	4	96	27φ	3	216
16zz	4	16	27φ	3	216
96		256	189		1296
16		256	54		216
	8			6	432
256zzz		1536	729zφ		46656
		1280			
		512			
		65536			

65536 = 256zzz by the Root 2. 46656 = 729zφ by the Root 2.

Proof of the Compound Cossical Extraction.

L.

Root 4zz − 2φ − 3z + 1ʐ
4zz − 2φ − 3z + 1ʐ

16zzz − 8Bß − 12zφ + 4ß
− 8Bß + 4zφ + 6ß − 2zz
− 12zφ + 6ß + 9zz − 3φ
4ß − 2zz − 3φ + 1z

Square 16zzz − 16Bß − 20zφ + 20ß + 5zz − 6φ + 1z

Root

M.

Root	3φ + 2ʒ − 4ℨ
	3φ + 2ʒ − 4ℨ
	9ʒφ + 6ß − 12ʒʒ
	6ß + 4ʒʒ − 8φ
	− 12ʒʒ − 8φ + 16ʒ
Square	9ʒφ + 12ß − 20ʒʒ − 16φ + 16ʒ
	3φ + 2ʒ − 4ℨ
	27φφ + 36ʒʒʒ − 60Bß − 48ʒφ + 48ß
	18ʒʒʒ + 24Bß − 40ʒφ − 32ß + 32ʒʒ
	− 36Bß − 48ʒφ + 80ß + 64ʒʒ − 64φ
Cube	27φφ + 54ʒʒʒ − 72Bß − 136ʒφ + 96ß + 96ʒʒ − 64φ

So in the tryal by Numbers, supposing the Root 2, then,

L.

4ʒʒ = 64		16ʒʒʒ = 4096	
+ 1ℨ = 2		+ 20ß = 640	
	66	+ 5ʒʒ = 80	
		+ 1ʒ = 4	
− 2φ = 16			
− 3ʒ = 12			4820
	28	− 16Bß = 2048	
True value of the Root	38	− 20ʒφ = 1280	
	38	− 6φ = 48	
	304		3376
	114		
Square	1444	====	1444

M.

3φ = 24		27φφ = 13824	
+ 2ʒ = 8		+ 54ʒʒʒ = 13824	
	32	+ 96ß = 3072	
− 4ℨ = 8		+ 96ʒʒ = 1536	
ℨ	24		32256
	24		
	96		
	48	− 72Bß = 9216	
ʒ	576	− 136ʒφ = 8704	
	24	− 64φ = 512	
	2304		18432
	1152		
φ	13824	====	13824

Partis Quartæ Libri Tertii

FINIS.

THE

THE FIFTH PART OF THE THIRD BOOK.

CHAP. I.

Of SURDES.

Surdes placed as the fifth of special Contracts, &c.

NEXT after *Cossicks* follow in order *Surdes*, as the fifth sort of Numbers especially Contract, and the second of those whose Denominations are expressed, because of the variety and uncertainty thereof.

Surdes what they are, and why so called.

Surdes are Irrational Numbers, as before in *Book* 1. *Par.* 1. *Chap.* 2. and *Book* 2. *Par.* 2. *Chap.* 3. was noted; that is, Numbers set for Roots that cannot be expressed by any Absolute Number: Or Numbers whose Roots cannot certainly be expressed by Integers, but besides the Integers contain some broken part or parts thereof. As the Square Root of 2, 3, 5, 6, or of any other Number that is not a Square Number. So the Cube Root of 2, 3, 4, 5, or of any other Number that is not Cubick. And in like manner any other Root of any Number that hath no such Root exactly to be measured by Whole Numbers, causeth that Number to be called a *Surde Number*. And perhaps the Reason why so called, it being absurd or irrational to attribute to any thing, or seek out thereof a Root or other thing not there to be had. *Latus inexplicabile* (saith *Alsted*) *dicitur, asymmetrum, incommensurabile, incommunicabile, irrationale, & surdum, quia ejus explicatio à nobis, quasi exaudiri nequeat, ut surdam buccinam, surdos ictus, dicimus qui difficulter & obscurè audiantur.* Encyclop. lib. 14. pag. 830.

Denominators why expressed, and how. Characters arbitrary.

The Denominations of these Roots being different according to the Powers of Numbers before-mentioned in Figural Numbers, maketh it necessary to express them. And for the Reason before rendred in the *First Chapter* of *Cossicks*, by some known Characters to distinguish them the one from the other. Which Characters, as in *Cossicks*, are arbitrary and mutable at the pleasure of the Operator.

Characters used in this Book.

In the ensuing survey of *Surdes*, the following Characters are used with their significations, thus;

Characters.	*Significations.*
√	Root. Divers set this for the Square Root.
√: or V or V√	Universal Root.
w or √ʒ	Square Root.
ww or √φ	Cube Root.
www or √ʒʒ	Squared Square Root.
wwww or √ß	Sursolide Root.

Cossical Characters best for the Higher Powers.

And so increasing the Minnoms according to the *Index* of the Figural Number, or adjoyning the Cossical Character of the Power to the Character for the Root, √, you have a Character for any Surde Denomination. Wherefore if I see √ʒφ 8, I read it the Zenzicube Root of 8. And so I understand by this Character wwww 8. And the like is to be done with others; only because the Minnoms increasing in the Higher Powers take up more room, and are not so soon made as the Cossical Characters, it is better to use them altogether for the Powers beyond the Zenzicube.

Surdes have society with Rational Numbers,

Besides the taking Denominations from Figural Numbers, and borrowing the Cossical Characters for them, Surdes admit into their society Rational Numbers; that is, such Numbers whose Roots may be expressed by Integers. As w4, the Square Root of 4; so ww8, the Cube Root of 8; and such others: For the Square Root of 4 is 2; and so is the Cube Root of 8; and so consequently no Surdes, but often set thus for the more apt Operation. And so also Absolute Numbers not Rational are used with them as well as Cossicks. And Cossicks themselves I have seen wrought together with Surdes.

and Absolute Numbers.

Nature of Surdes.

Surdes are Simple or Compound, Integral or Fracted. Of Fractionary Surdes, see *Chap.* 11. following.

Integral and Simple.

The Integral Simple Surdes consist of one Species or Denomination: As w5, which is to be read, the Square Root of 5; so ww4, the Cube Root of 4, and the like of others.

Integral and Compound. These of 2 sorts.

Compound Surdes consist of different Species, or divers Simple Surdes, or some Simple Surde with another Number set for a Surde, and are of two sorts, to wit, Particular or Universal.

Particular.

Examples.

Particular, when the different Denominations compounded by the signs + or —, or both, are to be considered distinct as to their Roots. As w5 + w6, which signifieth, the Square Root of 5, and the Square Root of 6. So w6 — w5 denoteth the Square Root of 6, lacking the Square Root of 5. In both which, and such others, their Roots are considered as two distinct Numbers.

Universal.

Examples.

Universal, when though the Quantities consist of different Species, yet the Root of the whole compounded Number is to be understood thereby. As √: 5 + √36; here it signifieth, that the Square Root of 6 is to be added to 5, and then the Square Root of that summe is to be taken for the Root Universal and summe of that compounded Surde. So √: 6 — √35; here the Square Root of 5 is to be taken from 6, and the Square Root of the Remain is the Root Universal. See further of these *Chap.* 7. *Addition of Compound Surdes.*

Compounds otherwise considered.

These Compound Surdes fall again under a threefold Consideration: 1. In their Signs. 2 In their Characters. 3. In their Numbers.

1.
In Signs, as Binomials *or* Bimedials. Residuals *or* Apotomes. Polynomials *or* Multinomials. *Examples of* Binomials.

1 As to their Signs, two Surdes, or a Simple Surde with a Rational or other Number conjoyned with the sign +, are called *Binomials*, and sometime *Bimedials*; but conjoyned with the sign —, they are called *Residuals* or *Apotomes*. If three Quantities be conjoyned, and but three, they are sometime called *Trinomials*. But generally where the composition hath more than two parts, the Compound is called a *Polynomial* or a *Multinomial*, that is a many named Number, as was before noted in *Cossicks*.

Examples of Binomials.

3 + w8	That is, 3 more the Square Root of 8.
w24 + 4	Is, the Square Root of 24 more 4.
w6 + w2	Is, the Square Root of 6, and the Square Root of 2.
ww9 + www8	Signifieth, the Cubick Root of 9, and the Squared Square Root of 8.

Residuals.

Examples of Residuals.

25 — w80	That is, 25 lacking the Square Root of 80.
w160 — 9	Is, the Square Root of 160 wanting 9.
w180 — w6	Is, the Square Root of 180 wanting the Square Root of 6.
ww100 — w30	Signifieth, the Cubick Root of 100 abating the Square Root of 30.

Polynomials.

Examples of Polynomials.

3 + w10 + ww9	That is, 3 more the Square Root of 10, and the Cube Root of 9.
100 — w20 — ww5	Is, 100 lacking the Square Root of 20, and the Cube Root of 5.
4 + ww30 — w6	Is, 4 and the Cube Root of 30, wanting the Square Root of 6.
w8 + 100 — www7 — ww40	Signifieth, the Square Root of 8, added to 100, lacking the Squared Square Root of 7, and the Cube Root of 40.

2.
In Characters, as Homogeneal, Heterogeneal.

2. Compound Surdes are considered in their Characters, and so they are divided into *Homogeneal* and *Heterogeneal*.

Homogeneal, when their Characters or Denominations are one and the same: *Heterogeneal* when contrary.

Examples

Examples of Homogeneals. *Examples of Homogeneals.*

Binomials	$\sqrt{5} + \sqrt{6}$	$\sqrt[3]{9} + \sqrt[3]{10}$	$\sqrt[4]{19} + \sqrt[4]{18}$
Residuals	$\sqrt{6} - \sqrt{5}$	$\sqrt[3]{10} - \sqrt[3]{9}$	$\sqrt[4]{19} - \sqrt[4]{18}$
Polynomials	$\sqrt[3]{5} + \sqrt[3]{6} + \sqrt[3]{4}$		$\sqrt[4]{6} + \sqrt[4]{15} - \sqrt[4]{20}$

Examples of Heterogeneals. Heterogeneals.

Binomials	$\sqrt{5} + \sqrt[3]{6}$	$\sqrt[3]{5} + \sqrt[4]{6}$	$\sqrt[3]{14} + \sqrt[4]{25}$
Residuals	$\sqrt{5} - \sqrt[3]{6}$	$\sqrt[3]{9} - \sqrt[4]{12}$	$\sqrt[4]{14} - \sqrt[3]{15}$
Polynomials	$\sqrt[4]{5} + \sqrt[3]{6} + \sqrt{3}$		$\sqrt[4]{6} + \sqrt{3} - \sqrt[3]{5}$

3. Their consideration in their Numbers divides them into commensurable and incommensurable. 3.

Commensurable, called also *Symmetral*, is when the given Numbers have a Common Divisor, that will reduce them into less Terms of like nature. *In Numbers as Commensurable or* Symmetral.

Examples of Symmetrals. *Examples.*

Binomials	$\sqrt{8} + \sqrt{32}$ $\sqrt[3]{81} + \sqrt[3]{24}$	Common Divisor	2 3
Residuals	$\sqrt{8} - \sqrt{32}$ $\sqrt[3]{81} - \sqrt[3]{24}$	Common Divisor	2 3
Polynomials	$\sqrt{48} + \sqrt{75} - \sqrt{27}$ $\sqrt[3]{24} + \sqrt[3]{81} - \sqrt[3]{192}$	Common Divisor	3

Because by these Common Divisors, the Square Surdes will be brought into Square Numbers, and the Cubical Surdes into Cube Numbers, they are therefore called Commensurable. For by the Divisor 2, will 8 and 32 in the upper Binomial and Residual be brought into 4 and 16, which are both Square Numbers. And by the Divisor 3, both 81 and 24 in the lower Binomial and Residual will be brought into 27 and 8, which are both Cube Numbers. And by the same Common Divisor 3, will the upper Polynomial be brought into $16 + 25 - 9$, which are all Squares or Rational Numbers, and the lower into $8 + 27 - 64$, which are all Cubical Numbers.

Incommensurable, or Asymmetral Surdes, are those which have no such Common Divisors. *Incommensurable or* Asymmetral. *Examples.*

Examples of Asymmetrals.

Binomials	$\sqrt{5} + \sqrt{6}$	$\sqrt[3]{9} + \sqrt{8}$	$\sqrt[3]{12} + \sqrt[4]{19}$
Residuals	$\sqrt{6} - \sqrt{5}$	$\sqrt{8} - \sqrt[3]{9}$	$\sqrt[4]{12} - \sqrt[3]{19}$
Polynomials	$\sqrt{6} + \sqrt[3]{5} + \sqrt[4]{3}$		$\sqrt{6} + \sqrt[3]{5} - \sqrt[4]{3}$

Numbers thus Commensurable or Incommensurable are said to be Commensurable or Incommensurable in Power; to difference this measure of Numbers from plain Commensuration, spoken of in Fractions before. For Numbers may be Commensurable, as 2 and 12, yet Incommensurable in Power: But 3 and 12 are as well Commensurable in Power as otherwise, seeing 12 divided by 3 gives in the Quotient 4, a Square Number. *Commensurable in Power how different from other Commensuration.*

Symmetral Surdes are discovered from Asymmetral thus: Divide the greater given Number by the lesser, and if 0 remain, then shall the Quotient be a Number of the same nature with the given Surdes, that is Square, Cube, or other like Quantity accordingly, if the Surdes are Commensurable. But if any thing remain upon the Division, reduce the Fraction into its least Terms, and then reduce all into an Improper Fraction, and this shall represent two Figural Numbers of like Quantity with the given Surdes, if they are Commensurable. And to find the Common Divisor do thus: If 0 remain upon the Division as aforesaid, then by the Quotient of this Division, divide the least of the two given Surdes, and this last Quotient shall be the Common Divisor. But if the Division left a Remain, which is to be brought, as aforesaid, into an Improper Fraction, then by the Numerator thereof divide the greatest given Surde, or by the Denominator the least, and this Quotient shall be the Common Divisor. *How Symmetral Surdes are discovered.*

As $\sqrt{8} + \sqrt{32}$ was before counted Commensurable, and the Common Divisor 2; because if 8 divide 32, the Quotient will be 4, a Square Number agreeable to the Surde and 0 remain; and by this 4, if 8 the least of the two Surdes be divided, 2 the Common Divisor appears in the Quotient, as at *A*. But if $\sqrt{12} + \sqrt{147}$, be given, *Examples.*

then

then after Division, because 3 remains, I reduce 3 with 12 the Divisor to $\frac{1}{4}$, and then 12 in the Quotient and this $\frac{1}{4}$ into an Improper Fraction, which is $\frac{49}{4}$, and being both Square Numbers of the nature with the given Surdes, shew them to be Symmetral; and by dividing 147 by 49, or 12 by 4, the Common Divisor is found in the Quotient to be 3, as at *B*. Examples of Cubes see at *C.* and *D.*

A. $\frac{32}{8}($4 A Square and o left: *Ergo*, $\sqrt{8}+\sqrt{32}$ are Commensurable.

$\frac{8}{4}($2 Common Divisor.

B. $\frac{147}{12}($$12\frac{1}{4}$ or $\frac{49}{4}$ Squares: *Ergo* $\sqrt{12}+\sqrt{147}$ are Commensurable.

$\frac{147}{49}($3 $\frac{12}{4}($3 Common Divisor.

C. $\frac{128}{16}($8 A Cube and o remaining: *Ergo*, $\sqrt[3]{16}+\sqrt[3]{128}$ are Commensurable.

$\frac{16}{8}($2 Common Divisor.

D. $\frac{81}{24}($$3\frac{3}{8}$ or $\frac{27}{8}$ Cubes: *Ergo*, $\sqrt[3]{24}+\sqrt[3]{81}$ are Commensurable.

$\frac{81}{27}($3 $\frac{24}{8}($3 Common Divisor.

Symmetral Surdes may have more Common Divisors than one.

Some Symmetral Surdes may have more Common Divisors than one, which is thus known: Divide one of the given Surdes, according to his Quantity, by any Number of like nature that will part it exactly without leaving a Remain; and by this Quotient divide the other given Surde, and if this second Quotient be a Number of like nature, then those given Surdes have more Common Divisors than one. And so proving with all, less than the given Surdes; so many Quotients as will hold this tryal, so many Common Divisors have those Commensurable Surdes.

Examples.

As in the Square Surdes above at *A*, if 32 be divided by 4, it giveth 8 in the Quotient; and if this 8, divide 8 the other given Surde, the Quotient is 1 another Square Number, therefore shall 8 be another Common Divisor to $\sqrt{8}+\sqrt{32}$ besides 2 found out as above.

So in the Cube Surdes above at *C*, if 128 be divided by 8, it giveth 16 in the Quotient: And if this 16 divide 16, the other given Surde, the Quotient is 1, another Cube Number; therefore shall 16 be another Common Divisor to $\sqrt[3]{16}+\sqrt[3]{128}$, besides 2, found as above.

$\sqrt{}\ \frac{32}{4}($8 $\frac{8}{8}($1 $\sqrt[3]{}\ \frac{128}{8}($16 $\frac{16}{16}($1

Surdes not so strictly used to place the greatest foremost. Use of Signs as in Cossicks.

All further needful to this *Chapter* is, That Authors do not so strictly observe to place the Greatest Surdes foremost, as they do the Cossicks; but sometime the Lesser Surdes are set to the Left Hand of the Greater. But agreeable to Cossicks in the use of the signs with or without Asterisques. And where — is not set + is understood.

CHAP. II.

Reduction of Surdes.

Surdes reduced. To their least Termes.

Under Reduction of Surdes is comprehended, To lessen their Terms, and alter their different Denominations into one: Both sometime called, *Alteration of Surdes.*

1. To lessen the Terms of a Surde is but abbreviation. And as all Common Fractions

ctions will not be abbreviated; so neither will all Surdes have their Terms lessened. But when the Denomination or Character is a Compound Cossick, and the annexed Number hath a Root that may be expressed by part of that Cossical Character, then reduce the Number and Character thereto accordingly, by extracting the Root of the Number, and clearing the Character of that part of the Composition.

Examples.

As √ʒʒ 25 and √ʒφ 81, the Characters being compounded of ʒ with ʒ in the first, and ʒ with φ in the second, and the Numbers 25 and 81 being both Square Numbers, the Square Roots thereof are to be taken, and Square in the Characters abated from both their Quantities, and both are to be expressed in less Terms by √ʒ 5 and √φ 9, or w 5 and ww 9.

So also √ʒφ 27 may be reduced to the √ʒ 3, being discharged of φ. And √ʒß 32 may be abbreviated to the √ʒ 2, and discharged of ß.

To reduce Surdes to one Denominator. Surdes and Absolute Numbers.

2. To bring the different Denominations of Surdes into one, belongs to Heterogeneal Surdes, or one Surde with an Absolute Number.

(1.) If an Absolute Number be given to be reduced into the Denomination of a Surde, then multiply the Absolute Number according to the Denomination of the Surde, and set before it the like Character.

Examples.

As if w 8 and 2 be reduced into one Denomination; 2 must be multiplyed Squarely, because the Denomination of the Surde 8 is such. And so will the Numbers stand thus, w 8 and w 4.

And if ww 9 and 4 be reduced into one Denomination, 4 must be multiplyed Cubically: And thus reduced the Surde shall be ww 9 and ww 64.

Surdes with uncompound Indices.

(2.) If two different Surdes be given to be reduced into one Denomination, and their *Indices* be uncompounded; then alternately multiply the Number of the one Surde according to the Denomination of the other, and to both the Products adjoyn both the Characters for the Common Denominator.

Example.

As if w 8 and ww 10 were to be reduced to one Denominator; then must 10 be multiplyed Squarely and 8 Cubically, and each production shall be Square Cube, set after the manner of Surdes thus, √ʒφ 100 and √ʒφ 512; or thus, www 100 and www 512.

√ʒ 8 × 8 × 8 = 512 } ʒφ
√φ 10 × 10 = 100 }

√ʒφ 512		√ʒφ 100
√ʒ 8	X	√φ 10
	√ʒφ	

Surdes with Compound Indices.

But (3.) If the *Indices* of the given Quantities be Numbers Compound, then by the greatest Common Divisor divide them, and then by the least Terms of the *Indices* multiply alternately as well the *Indices* of the one by the other for a new *Index*, as the Numbers given by the Powers of these alternate *Indices* for the reduced Surdes.

Example.

As √ʒʒ 10 and √ʒφ 7, thus reduced shall be √ʒʒφ 1000 and √ʒʒφ 49. For the *Index* of ʒʒ is 4, and the *Index* of ʒφ is 6, the Common Divisor of 4 and 6 is 2, which reduceth the one to 2 the *Index* of ʒ, and the other to 3 the *Index* of φ; therefore alternately Squaring 7 it is 49, and Cubing 10 it is 1000, and multiplying 6 by 2, or 4 by 3, the Product 12 is the *Index* of the reduced Powers, which is ʒʒφ.

	√ [12] 1000	√ [12] 49
2)	√ [4] 10	√ [6] 7
	[2]	[3]

Proof of Reduction of Surdes.

Besides the Proof of that sort of Reduction which lesseneth the Terms, by exalting the lessened Surdes into the Powers from whence they were abated; and reciprocally that sort of Reduction which increaseth their Terms and Denominations, by extracting the Roots, and abating the Characters of the Surdes accordingly; all sorts of Reduction of Surdes may be proved, by supposing Rational Numbers instead of the Surdes, and working with them as if they were Surdes.

As in the former Example, where √ʒʒ 25 was reduced to w 5; therefore if w 5 be multiplyed Squarely, it shall be ww 25: And in Rational Numbers it is evident that w 4 and the ww 16 are equal, being in each but 2.

Again in the second sort of Reduction, and first Example, there 2 reduced to a Square Denomination shall be 4, the Rational Number, and w 8 and w 4, is all one as w 8 and 2; for they are both equal: And so is the ww 64 or 4 Absolute Numbers in the next Example.

Likewise if √4 and the √√27, both Rational Numbers, be reduced to one Denomination, they shall be √3φ 64 and √3φ 729, agreeable to the next Example of Reduction. And the √3φ of 64 being but 2, equal to the √4, and the √3φ of 729 being 3, as the √√27 shew the Reduction right.

Moreover, agreeable to the last Example of Reduction, if I take √33 16 and the √3φ 64, that are both Rational Numbers, and have 2 for their Roots, and reduce them to one Denomination, they shall be √33φ 4096 and √33φ 4096, and be equal to the other. For the √33φ of 4096 is 2; whence may be also observed, that if the Roots of the given Surdes be equal, the reduced Surdes will be equal.

CHAP. III.

Addition of Simple Surdes.

Addition of Simple Surdes.

TO understand Addition of Surdes the better, it is meet to Analyse them.

Into { Simple { Homogeneal and Commensurable. / Homogeneal and Incommensurable. / Heterogeneal. } ; Compound { Particular. / Universal. } }

1. Homogeneal and Commensurable.

1. To add two Simple Surdes that are both Homogeneal and Commensurable, divide them by the Common Divisor, and extract the Roots of the Quotients, then multiply the Total of the Roots Figurately according to their Quantities, and this Product multiplyed by the Common Divisor shall be the Total of the added Surdes.

Examples.

As to add √12 and √147; both Numbers divided by 3, the Common Divisor, giveth 4 and 49, which are Square Numbers, and their Roots 2 and 7, the Total whereof 9 multiplyed Squarely produceth 81; this multiplyed by 3 is 243. So is √243 the Total of √12 + √147, as at *A*.

Also to add √√81 and √√24, Commensurable also by 3; after Division thereby the Cubes are 27 and 8, and their Roots 3 and 2, which together make 5, the Cube whereof is 125; this multiplyed by 3 produceth 375. So is √√375 the summe of √√81 + √√24, as at *B*.

An Example of Squared Square Surdes is set at *C*.

	A.	*B.*	*C.*
Addends	√12 + √147	√√81 + √√24	√√√648 + √√√5000
	3) 43 493	3) 27φ 8φ	8) 81 33 625 33
	2√ + 7√	3√ + 2√	3√ + 5√
	9	5	8
	9 ♃	25 3	512φ
	81 3	125φ	4096 33
	3	3	8
Totals	√243	√√375	√√√32768

Where the Data have many Common Divisors.

If the given Surdes have many Common Divisors, any one of them may be used. As in √1152 + √288, there are 5 Common Divisors, *viz.* 2, 8, 18, 32 and 72, the Addition by all which agree to be √2592, as in the Operations following.

Examples.

√1152 + √288	√1152 + √288	√1152 + √288
2) 576 3 144 3	8) 144 3 36 3	18) 64 3 16 3
24√ + 12√	12√ + 6√	8√ + 4√
36	18	12
36 ♃	18 ♃	12 ♃
1296 3	324 3	144 3
2	8	18
√2592	√2592	√2592

√

```
     √1152 + √288                  √1152 + √288
32)————————————              72)————————————
     36 3      9 3                  16 3      4 3
     6√   +   3√                    4√   +   2√
          9                              6
          9 z                            6 z
        —————                          —————
         81 3                           36 3
         32                             72
        —————                          —————
        162                             72
        243                            252
       —————                          —————
       √2592                          √2592
```

2. Simple Square Surdes Homogeneal and Incommensurable, if the Product of both multiplyed together bring forth a Figural Number of the same kind (as many times happeneth when the Numbers are Commensurable, or Rational Numbers used as Surdes) then are they called reducible, and are added thus: To the Total of the Surdes given prefix their proper Character, afterward either extract the Root of their Product and multiply this Root by the *Index*, and add this Product to the former Total; or else multiply the Product of both Surdes by the double *Index*, and extract the Root of this last Product, and add this Root to the Total first reserved; and this Number with his Character shall be the summe of the added Surdes. *2. Square Surdes Incommensurable. Reducible what, and how added.*

As to add √3 and √12; because 3 and 12 multiplyed is 36, a Square Number agreeable to the given Surdes, they are reducible; then 3 and 12 added are 15 to be set apart with their Character thus, √15; afterward the Root of 36, which is 6, is to be multiplyed by 2, the *Index* of Squares, and the Product 12 is to be added to the 15 before reserved; or else 36 is to be multiplyed by 4 the double *Index*, and the Root of 144 which is 12, added to 15, as before, makes the Total of this Addition √27, as at *D.* and *E.* *Example.*

```
               D.                              E.
Addends  √3 + √12                        √3 + √12
        ——————————                   ——————————————
           |  3                            |   3
           | 12                            |  12
    √15    |√36 is 6              √15      | ————
           |     2 Index                   |  36
        ——————————                         |   4
         √15 + √12                         | √144
Totals    √27                      ——————————————————————
        ——————————                  √15 + 12 or √15 + √144
                                   ——————————————————————
                                         √27
```

If Square Surdes are not thus reducible, they with others of higher Denominations Incommensurable, although Homogeneal, are to be joyned together with the sign of Addition +. *When not Reducible.*

As to add √6 and √7, they are set thus, √6 + √7. *Examples.*

And to add √√6 and √√7, they are set thus, √√6 + √√7.

Yet some set the Square, although Incommensurable, after the form of Addition abovementioned; whereby √6 + √7 is brought to √13 + √168; for that by this form they come to be Commensurable with other Numbers with whom they occasionally may be used. *How set by some.*

Example.

```
        √6 + √7
      ——————————
          |  6
          |  7
   √13    | ————
          | 42
          |  4
      ——————————
      √13 + √168
```

3. Simple Heterogeneal Surdes are first to be reduced, and then if by their Reduction they prove Commensurable add them as such; if otherwise, conjoyn them by the sign +. *3. Heterogeneal.*

As to add √3 and 9, Absolute Numbers together, being reduced they are √3 and √81, and because Incommensurable abide so; unless set after the form of Addition last mentioned, and then they stand thus, √84 + √972. *Examples.*

And if √3 be given to be added to √√2, after Reduction to √3φ27 + √3φ4, because Incommensurable they are left so without further work.

But

But if the Rational Number w 16 be given to be added to ww 8, another Rational Number, by Reduction they are brought to √ʒφ 4096 + √ʒφ 64, and being Commensurable, and added as before, make √ʒφ 46656, which is also a Rational Number, and hath 6 for Zenzicube Root thereof.

```
                                  √ʒφ4096 + √ʒφ64        4 + 2 = 6
                              64)—————————————————
                                  64ʒφ       1ʒφ
√ʒφ4096      √ʒφ64                2√    +    1√
—————————————————————                    3
  w 16    X   ww 8                    243 ß
—————————————————————              —————————
         √ʒφ                          729 ʒφ
                                       64
                                   —————————
                                      2916
                                     4374
                                  ——————————————
                                  √ʒφ 46656 is 6
```

4. *Different Signs*

4. If two Simple Surdes of different signs be given to be added together, then conjoyn them by the sign of Substraction —.

As to add w 3 with — w 5, they shall be set thus, w 3 — w 5.

To add any Surde *to it self.*

By what hath been said of Addition of Simple Surdes, appeareth this Consectary; That to add any Surde to it self is but to multiply the Square Surdes by 4, Cube Surdes by 8, Squared Squares by 16, *&c.* See the Examples following.

Examples.

```
Addends  w8 + w8           ww9 + ww9              www10 + www10
       8)————————        9)—————————          10)———————————————
          13    13           13    13               133      133
          1√ + 1√            1√ + 1√                1√   +   1√
             2                  2                        2
            2ʓ                 43                       8φ
         ————————           ———————                ——————————
            43      w8         8φ      ww9            1633      www10
            8        4         9        8              10          16
         ————————  ————     —————    ————           ————————    ————————
Totals   w 32      w 32     ww 72    ww 72          www 160     www 160
```

Addition of Compounds why deferred.

Addition of Compound Surdes is deferred to the *Seventh Chapter*, till Substraction of Simple Surdes be learned; because the knowledge thereof is necessary to the Addition of some Compound Surdes.

Proof of Addition of Simple Surdes.

Substraction of Simple Surdes is the Proof of Addition of Simple Surdes, and there to be seen. But the truth of Addition will be clear, if instead of the Surdes, Rational Numbers be taken, and operation made therewith.

For in the last Example of reduced Surdes, the Zenzicube Root of 4096 is 4, and of 64 is 2, which 4 and 2 make 6, and the Total of the Addition bringeth forth 46656 a Zenzicube Number that hath 6 for the Root, as was before noted.

```
        ʒφ    φ  ʓ               6ʓ
        25                        6                        Value
       46656 |216|6             ————        Indices.         4
        . . .                    363                         2
        4 : :                     6                        ————
Gnomon{ 4: :                    ————                       6 Total
        1 :                      216φ           3          ————
                                 216            3
Gnomon{ 252:                    ————          ————
        36                       1296
                                 216
                                432
                                ———————
                                46656 ʒφ         6
```

CHAP. IV.

Substraction of Simple Surdes.

Substraction *of Simple* Surdes.

1. *Homogeneal and Commensurable.*

IN the same method with Addition doth Substraction proceed.

1. Then to substract one Homogeneal and Commensurable Surde from another, after dividing them by the Common Divisor, and extracting the Roots of the Quotients,

tients, take the Lesser Root from the Greater, then multiply the Remainer Figurately according to their Quantities, and this Product multiplyed by the Common Divisor shall be the remaining Surde desired with the same sign. But if the Subtrahend be the Greater, then the sign + shall be changed into —.

As to substract √12 from √243 by the Common Divisor 3, they are reduced into the Squares 81 and 4, whose Roots are 9 and 2, the difference 7, this multiplyed Squarely is 49, which increased by the Common Divisor is √147, the Remainer desired, as at *A*. *Examples.*

Likewise if √147 were taken from √243, there would remain √12, as at *B*.

But if √243 were to be taken from √147 or √12, in this there would want √147, and in that √12, and then to be marked with —, as at *C.* and *D.*

```
Greater Surde. Subtrahend.        Greater Surde. Subtrahend.
     √243 — √12                        √243 — √147
  3)-----------                     3)------------
     81ℨ    4ℨ                         81ℨ    49ℨ
      9√ — 2√                           9√ —  7√
A.     7                          B.     2
       7ℨ                                2ℨ
     -----                             -----
      49ℨ                                4ℨ
       3                                 3
    -----                             -----
   Remain √147                       Remain √12

Lesser Surde. Subtrah.  Remain.    Lesser Surde. Subtrah.  Remain.
C. √147 — √243 = — √12             D. √12 — √243 = — √147
```

Commensurable Surdes of Higher Powers, are likewise thus to be substracted.

Examples to take √√24 from √√375, the Remain will be √√81, but will want √√81, if √√375 be taken from √√24.

So √√648 taken from √√32768, will leave √√5000; but the Greater substracted from the Lesser, the Remain will be so much too short.

```
Greater Surde. Subtrahend.        Greater Surde. Subtrahend.
    √√375 — √√24                     √√32768 — √√648
 3)-------------                  8)-----------------
    125φ     8φ                       4096ℨℨ — 81ℨℨ
      5√ — 2√                             8√ — 3√
        3                                   5
        9ℨ                                125φ
      -----                              -----
       27φ                               625ℨℨ
        3                                   8
      -----                             -----
   Remain √√81                      Remain √√5000

Lesser Surde. Subtrah.  Remain.    Lesser Surde. Subtrah.   Remain.
√√24 — √√375 = — √√81              √√648 — √√32768 = — √√5000
```

If the given Surdes have many Common Divisors; any one of them may be used in Substraction, as before in Addition. *If the Data have many Common Divisors.*

2. One Simple Square Surde Homogeneal and Incommensurable to another, may be substracted therefrom, if the Product of them both multiplyed together produce a Figural Number of the same kind: For then to the Total of the Surdes prefix their proper Character; afterward either extract the Root of their Product, and multiply this Root by the *Index*, and deduct this Product from the former Total: Or else multiply the Product of both the given Surdes by the double *Index*, and extract the Root of this last Product, and substract this Root from the Total first reserved. And this Number with his Character shall be the Remain desired, with the sign changed, as before noted, if the Subtrahend be the Greater. *2. Square Surdes Incommensurable. Reducible how added.*

As to extract √12 from √27, being multiplyed they produce 324, the Square Number of 18; therefore 18 multiplyed by the *Index* 2, or 324 by 4 the double *Index*, and the Root of this Product, or the Product 36 substracted from √39, the Total of 12 and 27 added, leaves remaining √3, as at *E.* and *F.* But if √27 were to be taken from √12, because the Subtrahend is greatest, the sign or places of the Surdes shall be changed, as at *G.* *Example.*

Greater Surde √27 — √12		√27 — √12 Subtrahend
E. √39	27 12 54 27 √324 is 18 2 *Ind.*	*F.* √39	27 12 54 27 324 4 Double *Index.* √1296 is 36
	√39 — 36	√39 — 36 or √39 — √1296
Remain	√3	√3

$$\sqrt{12} - \sqrt{27} = \left\{ \begin{array}{c} \sqrt{1296} - \sqrt{1521} \\ \text{or} \\ 36 - 39 \end{array} \right\} = -\sqrt{3} \quad G.$$

Not Reducible. If Square Surdes be not thus reducible to Square Numbers by Multiplication, then they with other Surdes Incommensurable of Higher Powers, although Homogeneal, are to be joyned together with the sign of Substraction —.

Examples. As to substract √6 from √7, or √7 from √6; they are set thus;

√7 — √6 √6 — √7

And to substract √√6 from √√7, or √√7 from √√6; they are set thus;

√√7 — √√6 √√6 — √√7

How set by some. Nevertheless for the Reason before rendred in Addition, some set the Square Surdes, though Incommensurable, after the form of Substraction above-mentioned; whereby √7 — √6 is brought to √13 — √168, and √6 — √7 to √168 — 13.

Examples.

√7 — √6		√6 — √7	
√13	7 6 42 4	√13	6 7 42 4
√168		√168	
√13 — √168		√168 — 13	

3. *Heterogeneal.* 3. Simple Heterogeneal Surdes are first to be reduced, and then, if by their Reduction they happen to be Commensurable, substract them as such: But if otherwise, conjoyn them accordingly with the sign —.

Examples. As to take √3 from 9, Absolute Numbers after Reduction, they stand, because Incommensurable, √81 — √3, unless after the form of Substraction last above-mentioned, and then they stand thus, √84 — √972.

And if √√2 be substracted from √3, they are reduced, and because Incommensurable left thus, √3φ 27 — √3φ 4.

But if √√8 be taken from √16, they being reduced, and both Rational Numbers and Commensurable, will be √3φ 4096 — √3φ 64, that is in conclusion √3φ 64, which is also a Rational Number, and hath 2 for the Zenzicube Root thereof.

√3φ 4096 — √3φ 64 4 — 2 = 2

64) ——————

643φ 13φ

2√ — 1√

1

1 ƒ3

——

13φ

64

——

√3φ 64

√3φ 4096	√3φ 64
√16 —	√√8
√3φ	

And if √16 had been given to have been substracted from √√8, then had there wanted √3φ 64, and the Remain should have been — √3φ 64.

4. *Different Signs* 4. If the given Surdes be of different signs, the Surdes are to be added, and the sign of the Remain, as in Cossicks, shall be contrary to the sign of the Subtrahend, or Number substracted, that is as the upper Number, if the Surdes be Commensurable; but if Incommensurable, then conjoyn them by the sign of Addition +.

Examples. As to substract — √13 from √52, they are added because the signs are unlike, the one + and the other —, the Total, which is √117, is the Remain, as at *H.*

And √5 — √3 and √5 — √3 being Incommensurable, have their Remains, as at *I.* and *K,* √5 + √3 and √5 + √3.

√52

w 52 — w 13
13)——————
43 13
2√ + 1√
3

H. 3 ♃
——
93
13
——
Total w 117 Remain.

I.

w 5 — w 3
——————
w 5 + w 3
——————

K.

w 5 — w 3
——————
w 5 + w 3
——————

From what hath been said of Substraction of Simple Surdes, these two Consectaries are apparent. *2 Consectaries hence.*

1. That to substract any Surde from it self leaves o remaining, as in other Numbers. As to take w 10 from w 10, the Remain is w o. *A Surde taken from it self leaves o.*

w 10 — w 10
10)——————
13 13
1√ — 1√
o
o ♃
——
o3
10
——
w o

w 10
w 10
——
w o
——

2. That to take half any Surde is but to divide Square Surdes by 4, Cube Surdes by 8, Squared Squares by 16, &c. as is plain by the Examples following. *To take half a Surde.*

Greater Surde. Subtrah.
w 32 — w 8
8)——————
43 13
2√ — 1√
1
1 ♃
——
13
8
——
Remain w 8
——
w 32/4 (w 8

Greater Surde. Subtrah.
ww 72 — ww 9
9)——————
8φ 1φ
2√ — 1√
1
13
——
1φ
9
——
Remain ww 9
——
ww 72/8 (ww 9

Greater Surde. Subtrah.
www 160 — www 10
10)——————
1633 133
2√ — 1√
1
1φ
——
133
10
——
Remain www 10
——
www 160/16 (www 10

Substraction of Compound Surdes is referred to the *Eighth Chapter*, after their Addition hath been inspected. *Substraction of Compounds why deferred.*

Forasmuch as several of the Examples in this *Chapter* from which Substraction is made, are the Totals of the Additions in the foregoing *Chapter*, and the Subtrahends here are one of the Surde Numbers added, and the Remains the other, it will apparently manifest the Proof of Substraction of Surdes by Addition, and Addition by Substraction. *Proof of Substraction of Simple Surdes.*

But for a full demonstration of all substractionary Operations, let Rational Numbers be set instead of the Surdes, and after Substraction made therewith, the truth will appear by the equal value of the Remain.

For in the Rational Numbers above-mentioned √3φ 4096 is 4, from which if √3φ 64 which is 2, be substracted, the Remain is √3φ 64, which is 2, and answers to the Remain of 4 — 2, which is but 2.

3φ φ ♃
64 | 8 | 2

2 ♃
2
——
43 *Indices.*
2
——
8φ 3
8 3
——
643φ 6

Value
4
2
——
2 Remain
——

CHAP.

CHAP. V.

Multiplication of Simple Surdes.

Multiplication of Simple Surdes. Homogeneal.

TO multiply Simple Surdes observe their Homogenity or Heterogeniety. For,

1. If the Surdes given to be multiplyed be Homogeneal, then multiply Number by Number, Integers as Integers, and Fractions as Fractions, and to the Product prefix the Character common to the given Surdes.

Examples.

As to multiply √ 15 by √ 5, the Product is √ 75, set as at *A.* or *B.*
And to multiply √ $12\frac{1}{2}$ by √ $4\frac{1}{2}$, the Product is √ $56\frac{1}{4}$, as at *C.*

	Multiplicand	√ 15		Md. Mr. Prod.
A.	Multiplyer	√ 5	*B.*	√ 15 × √ 5 = √ 75
	Product	√ 75		

	Multiplicand	√ $12\frac{1}{2}$		225
C.	Multiplyer	√ $4\frac{1}{2}$		$\frac{25}{2} \times \frac{9}{2} = \frac{225}{4}$ ($56\frac{1}{4}$
	Product	√ $56\frac{1}{4}$ or $7\frac{1}{2}$		4

1 Root
225 | 15
1
10
25
4 | 2
($7\frac{1}{2}$

Other Examples.

Multiplicands	√√ 48	√√√ 12	√√√√ 30
Multiplyers	√√ 5	√√√ 6	√√√√ 3
Products	√√ 240	√√√ 72	√√√√ 90

Heterogeneal.

2. If the Surdes given to be multiplyed be Heterogeneal, or one Surde with an Absolute Number; then first reduce them to one Denomination, and then multiply Number by Number, as before.

Examples.

As to multiply √ 10 by 3; first 3 is squared, and then by 9 is 10 multiplyed; so is the Product √ 90, as at *D.*

And to multiply √ 8 by √√ 10, being reduced they are √3φ 512, and √3φ 100, and 512 multiplyed by 100 produce 51200; so is the Product √3φ 51200, as at *E.*

		D.			*E.*
√ 10	√ 9	√ 10	√3φ 512	√3φ 100	√3φ 512
√ 10 X	3	√ 9	√ 8 X	√√ 10	√3φ 100
√		√ 90	√3φ		√3φ 51200

Consectaries hence.

Out of Multiplication of Simple Surdes issue these ensuing Consectaries.

1. To double, triple, &c. a Surde, what.

1. That to multiply any Surde, is to increase him by the Power of a Root Homogeneal: And so to double any Square Surde is to multiply him by 4, which is the Square Power of the Root 2, as before noted in Addition. Likewise to triple any Square Surde is to multiply him by 9, *&c.* And to double any Cube Surde is to multiply him by 8, the Cube of 2. As also to triple him is to multiply him by 27, *&c.*

Examples.

	Doubled.	Tripled.	Quadrupled.
Square Surdes	√ 3	√ 3	√ 3
	4	9	16
	√ 12	√ 27	√ 48
Cube Surdes	√√ 3	√√ 3	√√ 3
	8	27	81
	√√ 24	√√ 81	√√ 243
Squared Squares	√√√ 3	√√√ 3	√√√ 3
	16	81	243
	√√√ 48	√√√ 243	√√√ 729

2. Product is often Rational.

2. That Multiplication of Surdes oftentimes produceth Rational Numbers, whose Roots are Absolute Numbers, and may be so expressed.

Examples.

Examples in Surdes	Square.	Cubed	Squared Square.
	√ 48	√√ 9	√√√ 8
	√ 3	√√ 3	√√√ 2
	√ 144 √ 12	√√ 27 √ 3	√√√ 16 √ 2

3. That

3. When the Character may be cancelled.

3. That to multiply the side of any Power according to the exigency of his own kind; the Character or Note of the side may be cancelled, and the Number left Absolute. For W 3 multiplyed by the W 3, produceth W 9, which is the Absolute Number 3. So the Square of the Square Root of 64, and the Cube of the Cube Root of 64, is 64.

Examples.

Examples	W 3	Also W 64 × W 64 = 64
	W 3	And ww 64 × ww 64 × ww 64 = 64
	W 9 √ 3	

4. What produced by the Side of a Power multiplyed, &c. Examples.

4. That the side of a Power whose *Index* is a Compounded Number, multiplyed into one of the Compounding Powers, produceth a Surde answerable to the Quotient of the Greater *Index* divided by the Lesser, and may be set alone accordingly. As 33 whose *Index* is 4, compounded of 3 whose *Index* is 2; it shall be therefore, that if ww 10 were so to be multiplyed, the Product ww 100 shall be equal to W 10, because W answers to the *Index* 2, brought out by the Division of 4 by 2. So the Square of √3φ 64 is √φ 64; and the Cube of √3φ 64 is √3 64; for the √3φ is √ of 2 multiplyed by 3.

ww 10
ww 10
ww 100 = W 10

Ergo ww 10 × ww 10 = W 10 because

3 33 3
2) 4 (2

√3φ 64
√3φ 64 — A Rational Number, and hath the Root 2.

Ergo √3φ 64 × √3φ 64 = √φ 64 because

3 3φ φ
2) 6 (3

256
384
√3φ 4096 = √φ 64 which is a Rational Number, and the Root 4.
√3φ 64

Ergo √3φ 64 × √3φ 64 × √3φ 64 = √3 64 because

φ 3φ 3
3) 6 (2

16384
24576
√3φ 262144 = √3 64, also a Rational Number, and the Root 8.

5. Homogeneal Figurals multiplyed what produced. Examples.

5. That if a Figural Number be multiplyed by an Homogeneal Figural Number, the Product shall be a Figural Number of the same kind, whose Side or Root shall be equal to the Product of the sides of the Numbers multiplyed. As 4 and 9, both Squares, produce 36, whose Root is 6, equal to 2 × 3, the sides of 4 and 9. So 343 the Cube of 7, if multiplyed into 27, the Cube of 3, shall produce 9261, the Cube of 21, equal to the Product of 7 × 3.

Squares	4	√ 2	Cubes	343	√ 7
	9	3		27	3
Square	36	√ 6		2401	
				686	
			Cube	9261	√ 21

6. Sides of Homogeneals what they produce. Examples.

6. That the sides of Homogeneal Surdes multiplyed procreateth sides of Homogeneal Surdes.

Ergo W 2 × W 3 begetteth W 6. And ww 7 × ww 3 begetteth ww 21.

Multiplication of Compounds why deferred.

Multiplication of Compound Surdes is remitted to the *Ninth Chapter* of this *Fifth Part* of the *Third Book*, that it may follow in order Compound Addition and Substraction.

Proof of Multiplication of Simple Surdes.

Division of Simple Surdes is the Proof of Multiplication of Simple Surdes, and there set forth. Yet besides, the truth of this Simple Multiplication will appear by taking Rational Numbers and multiplying them, and extracting the Roots of the Product, which will equalize the Product of the Roots of the Factors multiplyed in Absolute Numbers.

As if W 9 be multiplyed by W 16, the Product will be W 144, whose Root is 12, and so will be the Product of 3, the Root of 9, multiplyed into 4, the Root of 16, as at *F*.

And so, if W 9, which is 3, be multiplyed by ww 8, which is 2, the Product will be 6, agreeable to the √3φ 46656, as at *G*.

F.

W 9 √3
W 16 4

W 144 12

3 ꝶ
144 (12
. .
1
4
4

G.

√3φ 729 √3
√3φ 64 2

2916
4374

√3φ 46656 √6

3φ φ ꝶ
25
46656 (216 (6
. . .
4
41
2556

√3φ 729 √3φ 64

W 9 X ww 8

√3φ

CHAP. VI.

Division of Simple Surdes.

Division *of Simple* Surdes. *Homogeneal.*

After Multiplication of Simple Surdes followeth their Division, and with like ease and order, as they are Homogeneal or Heterogeneal.

1. If they be Homogeneal, then divide the Number of the Dividend by the Number of the Divisor, Integers as Integers, and Fractions as Fractions, and to the Quotient annex the Common Character of the Surde.

Examples.

As to divide W 75 by W 5, the Quotient will be W 15, as at *A.* or *B.*

And to divide W $56\frac{1}{4}$ by W $4\frac{1}{2}$, the Quotient will be W $12\frac{1}{2}$, as at *C.* or *D.*

A.

Dividend W 75 (W 15 Quotient
Divisor W 5

B.

Divisor. Dividend. Quotient.
W 5) W 75 (W 15

C.

Dividend. Divisor. Quotient.

$$\frac{225}{W\,56\frac{1}{4}} \; \overset{450}{\underset{36}{X}} \; \frac{9}{W\,4\frac{1}{2}} = W\,\frac{450}{36} \left(W\,12\tfrac{1}{2} \right.$$

D.

Divisor. Dividend. Quotient.

$$\frac{9}{W\,4\frac{1}{2}} \Big) \; \frac{225}{W\,56\frac{1}{4}} \; \Big(W\,\frac{450}{36} \; \Big(W\,12\tfrac{1}{2}$$

Other Examples.

Dividend ww 240 (ww 48 Quotient
Divisor ww 5

Dividend www 72 (www 12 Quotient
Divisor www 6

Heterogeneal.

2. If the Surdes be Heterogeneal, or one Surde be given to be divided with, or to divide an Absolute Number; then first reduce them to one Denomination, and then divide the Dividend by the Divisor, as above.

Examples.

As to divide W 90 by 3; first 3 is squared, and then by 9 is 90 divided, so will the Quotient be W 10, as at *E.*

And to divide √3φ 51200 by W 8, being reduced they are √3φ 51200 and √3φ 512, and then divided, the Quotient will be √3φ 100, as at *F*, and may be depressed to √φ 10.

W 90 W 9
W 90 X 3
W

E.

W 90 (W 10
W 9

√ [6] 512 √ [6] 51200
2) √ [2] 8 √ [6] 51200
[1] [3]

F.

√3φ 51200 (√3φ 100
√3φ 512

Consectaries hence.

The following Consectaries flow from Division of Simple Surdes.

1. To take the half, third part, &c. of a Surde.

1. That to divide any Surde is to diminish him by the Power of a Root Homogeneal. So as to take the half of any Square Surde is to divide him by 4, the Power of the Root 2, as was before noted in Substraction. Also to take the third part of any Square Surde is to divide him by 9, *&c.* And to take the half of any Cube Surde is to divide him by 8, the Cube of 2. Likewise to take the third part thereof is to divide him by 27, *&c.*

Examples.

Examples of	The Half.	The Third Part.	The Fourth part.
Square Surdes	W $\frac{12}{4}$ (W 3	W $\frac{27}{9}$ (W 3	W $\frac{48}{16}$ (W 3
Cube Surdes	ww $\frac{24}{8}$ (ww 3	ww $\frac{81}{27}$ (ww 3	ww $\frac{192}{64}$ (ww 3

Squared

Squared Squares www 48/16 (www 3   www 243/81 (www 3   www 729/243 (www 3

2. That Division of Surdes sometimes bringeth forth Rational Numbers in the Quotient: The Roots whereof being Absolute Numbers, may be so expressed.

2. *Quotient is often a Rational. Examples.*

Examples in Surdes	Square.	Cubed	Squared Square.
	w 27 / w 3 (w 9	ww 24 / ww 3 (ww 8	www 48 / www 3 (www 16
	√ 3	√ 2	√ 2

3. That Division of any Surde by himself, giveth in the Quotient a Surde Unit.

3. *Surde dividing himself gives* 1. *Examples.*

Examples in	Square Surdes.	Cube Surdes.	Squared Square Surdes.
	w 5 / w 5 (w 1	ww 9 / ww 9 (ww 1	www 15 / www 15 (www 1

4. That a Power whose *Index* is compounded, divided by one side of the Compounding Powers, shall give the Quotient higher or lower according to the divising Power. For Division made by the Root, or lowest Quantity of the Dividend, the Root of the Quotient shall be equal to the Root of the higher compounding Power of the Divisor; and if by the Higher Power, the contrary.

4. *Quotient of a Power, &c. divided by the Side.*

As if √3φ 262144 which is 8, be divided by √3φ 64 which is 2, the Quotient will be 4, the √3φ of 4096. And because 64 is the Root, the √φ 64 which is 4, may be taken, (Cube being the higher compounding Power in 3φ). But if √3φ 262144 be divided by √3φ 4096, the Quotient will be √3φ 64 which is 2, and may be taken in stead thereof.

Examples.

√2) √8 (√4
√3φ 64) √3φ 262144 (√3φ 4096 = √φ 64, that is 4.

√4) √8 (√2
√3φ 4096) √3φ 262144 (√3φ 64 = √2

5. That if a Figural Number be divided by a Figural Number Homogeneal, the Quotient shall be a Figural Number of the same kind, whose side is equal to the Quotient of the side of the Dividend applyed to the side of the Divisor, or the Greater divided by the Lesser, and the contrary.

5. *Homogeneal Figurals divided, what the Quotient. Examples.*

As if 36 be divided by 4 (both Squares) the Quotient will be 9, whose Root 3 is equal to 6 divided by 2, the sides of 36 and 4. And if 36 be divided by 9, the Quotient 4, whose Root 2, is equal to 6 divided by 3, the sides of 36 and 9.

So 9261, the Cube of 21, divided by 27 and 243, the Cubes of 3 and 7, gives alternately in the Quotient the Powers whose Roots are equal to the Division of 21 by 3 or 7 accordingly.

Square.		Cube.	
√2) √6 (√3	√3) √6 (√2	√3) √21 (√7	√7) √21 (√3
4) 36 (9	9) 36 (4	27) 9261 (343	343) 9261 (27

6. That the sides of Homogeneal Surdes divided procreateth sides of Homogeneal Surdes.

6. *Sides of such divided what begotten. Examples.*

Ergo w 2 dividing w 6 begetteth w 3. And ww 3 dividing ww 21 begetteth ww 7.

Division of Compound Surdes is to be found in its proper place, in the *Tenth Chapter* following of this *Book*.

Division of Compounds where.

Because most of the Divisions of this *Chapter* are the Products of the Multiplications in the foregoing *Chapter* divided by one of the Factors, it will serve sufficiently to prove the truth of Surde Multiplication by Division, and Division by Multiplication.

Proof of Division of Simple Surdes.

Yet to make all clear, take Rational Numbers for Surdes, and proceed in their Division as if they were Surdes, and the Quotients of such Divisions will be equal to the Division of their Roots in Absolute Numbers.

For if w 144 be divided by w 16, the Quotient will be w 9, whose Root is 3, agreeable to the Quotient of 12 the √ of 144 divided by 4 the √ of 16, as at *G*.

And so if √3φ 46656 which is 6, be divided by w 9 which is 3, the Quotient will be 2, as at *H*.

G. √4) √12 (√3
w 16) w 144 (w 9

√ [6] 46656   √ [6] 729
2) √ [6] 46656   √ [2] 9   46656/729 (64
H. [3]   [1]
√3) √6 (√2
√3φ 729) √3φ 46656 (√3φ 64 = 2

CHAP.

CHAP. VII.

Addition of Compound Surdes.

Compound Surdes *added.*

IN the Addition of Compound Surdes, let them be considered as they are Particular or Universal.

[A]s the [C]ompound Surdes are made of the Simple, or else of Rational Numbers with Surdes; so the work of the Compound dependeth on the work of the Simple, and to be wrought alike. And the signs + and — to be ordered, as in Addition of Compound Cossicks.

Particular.

[I]n particular Compound Surdes, as the parts given to be added be, so shall the Addition be. For like Surdes and Symmetral are to be added with like as Simple, and unlike and Asymmetral with the sign of Addition +.

Because Examples are very demonstrative, the varieties of Examples following with their explanations, are to be born with.

Examples of Binomials.

Examples of Binomials.

Addends $\begin{cases} 9 + \sqrt{40} \\ 30 + \sqrt{10} \end{cases}$

Total $39 + \sqrt{90}$

After Addition of the Absolute Numbers 9 and 30, the Surdes $\sqrt{40}$ and $\sqrt{10}$, are added as before in Simple Surdes.

$\sqrt{40} + \sqrt{10}$
10)
43 13
2√ + 1√
3
3ʒ
93
10
$\sqrt{90}$

Addends $\begin{cases} 7 + \sqrt{8} \\ 5 + \sqrt{3} \end{cases}$

Total $12 + \sqrt{8} + \sqrt{3}$

The Absolute Numbers 7 and 5 make 12. The Surdes being Incommensurable are conjoyned by +. And after the second form of Addition of Simple Surdes, may be set thus, $12 + \sqrt{11} + \sqrt{96}$.

Addends $\begin{cases} \sqrt{1264} + 8 \\ 28 + 316 \end{cases}$

Total $\sqrt{2844} + 36$

In this Example the Absolute Numbers make up 36, and the Surdes added as Simple are $\sqrt{2844}$.

$\sqrt{1264} + \sqrt{316}$
79)
163 43
4√ + 2√
6
6ʒ
363
79
$\sqrt{2844}$

Addends $\begin{cases} \sqrt[3]{32} + \sqrt{10} \\ \sqrt[3]{4} + \sqrt{19} \end{cases}$

Total $\sqrt[3]{108} + \sqrt{29} + \sqrt{760}$

The Cube Surdes added as Simple make $\sqrt[3]{108}$, but the Square Surdes set after the second form of Addition being Incommensurable might have stood thus, $\sqrt{10} + \sqrt{19}$; and the whole Total thus, $\sqrt[3]{108} + \sqrt{10} + \sqrt{19}$.

Addends $\begin{cases} \sqrt[3]{48} + \sqrt{5} \\ \sqrt[3]{243} + \sqrt{45} \end{cases}$

Total $\sqrt[3]{1875} + \sqrt{80}$

ww added.
$\sqrt[3]{48} + \sqrt[3]{243}$
3)
1633 8133
2√ + 3√
5
62533
3
$\sqrt[3]{1875}$

w added.
$\sqrt{5} + \sqrt{45}$
5)
13 93
1√ + 3√
4
163
5
$\sqrt{80}$

Examples of Residuals.

Examples of Residuals.

Addends $\begin{cases} 14 - \sqrt{3} \\ 12 - \sqrt{12} \end{cases}$

Total $26 - \sqrt{27}$

After Addition of the Absolute Numbers which make 26, the Surdes added make $\sqrt{27}$, which is to be adjoyned to 26 with —, because the Numbers to be added had the same sign.

$\sqrt{3} + \sqrt{12}$
3)
13 43
1√ + 2√
3
93
3
$\sqrt{27}$

Addends $\begin{cases} 8 - \sqrt{5} \\ 6 - \sqrt{3} \end{cases}$

Total $14 - \sqrt{5} - \sqrt{3}$

The Absolute Numbers added make 14. The Surdes are Incommensurable, and so annexed with their proper signs; or else after the second form of Simple Addition set thus, $14 - \sqrt{8} - \sqrt{60}$.

Addends

Addends { √50 — 4
          { √18 — 3

Total √128 — 7

In this Example the work is like the first Example of Residuals above; for the Surdes added make √128, and 4 and 3 the Absolute Numbers make 7 to be set with —, because they were of the same Nature.

√50 + √18
2)
25ʒ 9ʒ
5√ + 3√
8
64ʒ
2
√128

Addends { √128 — 6
          { 8 — √72

Total 2 + √8

Addends { √128 — 8
          { 6 — √72

Total √8 — 2

Because one of the Absolute Numbers is + and the other —, 6 taken from 8 leaves 2, but in the one Example +, in the other —, according to the nature of the greater Number. Then the √72 being —, and the other Surde +, the Lesser is taken from the Greater, and the Remain is + √8, according to the sign of √128.

√128 — √72
2)
64ʒ 36ʒ
8√ — 6√
2
4ʒ
2
√8

Addends { 250 — √99
          { √44 — 76

Total 174 — √11

The work in this Example is like the last; for — 76 taken from + 250 leaves + 174, and √44 which is +, taken from √99 which is —, leaves √11.

√99 — √44
11)
9ʒ 4ʒ
3√ — 2√
1
1ʒ
11
√11

Addends { √44 — √27
          { √99 — √3

Total √275 — √48

Here √44 + √99 and √27 + √3, are added severally, and their Totals conjoyned by their proper sign.

√44 + √99
11)
4ʒ 9ʒ
2√ + 3√
5
25ʒ
11
+ √275

√27 + √3
3)
9ʒ 1ʒ
3√ + 1√
4
16ʒ
3
— √48

Addends { √√72 — √96
          { √√9 — √6

Total √√243 — √150

The Cube Surdes added by themselves, and the Squares by themselves, make the Total √√243 — √150.

√√72 + √√9
9)
8ɸ 1ɸ
2√ + 1√
3
27ɸ
9
+ √√243

√96 + √6
6)
16ʒ 1ʒ
4√ + 1√
5
25ʒ
6
— √150

Addends { √√32 — √5
          { √√32 — √24

Total √√512 — √24 — √5
or √√512 — √29 — √480

This Example is like the second of the Residual Examples above in the Square Surdes, and so being Asymmetral may be set differently. The √√ being added to himself, is as 32 × 16.

Examples of Binomials *with* Residuals.

Examples of Mixt.

Addends { 8 — √27
          { 11 + √3

Total 19 — √12

Addends { 8 + √27
          { 11 — √3

Total 19 + √12

In both these Examples adding 8 and 11, they make 19 absolute Numbers; and √3 taken from √27, because their Signs are contrary, there rests √12, which in the one Total is +, in the other —, according to the nature of the Greater Surde.

√27 — √3
3)
9ʒ 1ʒ
3√ — 1√
2
4ʒ
3
√12

Addends $\begin{cases}\sqrt{12}-8\\ \sqrt{3}+6\end{cases}$

Total $\sqrt{27}-2$

Addends $\begin{cases}\sqrt{12}+8\\ \sqrt{3}-6\end{cases}$

Total $\sqrt{27}+2$

Both these Examples adding the Surdes make their Total $\sqrt{27}$, and taking 6, one of the Absolute Numbers, from 8, the other leave 2, which in the one is —, and in the other +, according to the sign of the greater Number.

$$\begin{array}{l} \sqrt{12}+\sqrt{3} \\ 3)\overline{\quad\quad\quad\quad} \\ 4\square \quad 1\square \\ 2\surd + 1\surd \\ \quad 3 \\ \quad 9\square \\ \quad 3 \\ \sqrt{27} \end{array}$$

Addends $\begin{cases}\sqrt{50}+\sqrt{7}\\ \sqrt{52}-\sqrt{7}\end{cases}$

Total $\sqrt{242}$

Here $\sqrt{7}$ being found with + and —, both are cancelled, and the Total of the other Surdes only set down.

$$\begin{array}{l} \sqrt{50}+\sqrt{72} \\ 2)\overline{\quad\quad\quad\quad} \\ 25\square \quad 36\square \\ 5\surd + 6\surd \\ \quad 11 \\ \quad 121\square \\ \quad 2 \\ \sqrt{242} \end{array}$$

Addends $\begin{cases}\sqrt{63}+\sqrt{20}\\ \sqrt{7}-\sqrt{5}\end{cases}$

Total $\sqrt{112}+\sqrt{5}$

In this Addition $\sqrt{5}$ is taken from $\sqrt{20}$ because the Signs are contrary, and there rests $\sqrt{5}$, which is +, because the Lesser Surde was substracted.

$$\begin{array}{l} \sqrt{63}+\sqrt{7} \\ 7)\overline{\quad\quad\quad\quad} \\ 9\square \quad 1\square \\ 3\surd + 1\surd \\ \quad 4 \\ \quad 16\square \\ \quad 7 \\ \sqrt{112} \end{array} \qquad \begin{array}{l} \sqrt{20}-\sqrt{5} \\ 5)\overline{\quad\quad\quad\quad} \\ 4\square \quad 1\square \\ 2\surd - 1\surd \\ \quad 1 \\ \quad 1\square \\ \quad 5 \\ \sqrt{5} \end{array}$$

Addends $\begin{cases}\sqrt{80}-\sqrt[3]{160}\\ \sqrt{5}+\sqrt[3]{20}\end{cases}$

Total $\sqrt{125}-\sqrt[3]{20}$

After Addition of the Square Surdes, the lesser Cube Surde is taken from the greater, their Signs being contrary.

$$\begin{array}{l} \sqrt{80}+\sqrt{5} \\ 5)\overline{\quad\quad\quad\quad} \\ 16\square \quad 1\square \\ 4\surd + 1\surd \\ \quad 5 \\ \quad 25\square \\ \quad 5 \\ \sqrt{125} \end{array} \qquad \begin{array}{l} \sqrt[3]{160}-\sqrt[3]{20} \\ 20)\overline{\quad\quad\quad\quad} \\ 8\phi \quad 1\phi \\ 2\surd - 1\surd \\ \quad 1 \\ \quad 1\phi \\ \quad 20 \\ \sqrt[3]{20} \end{array}$$

Addends $\begin{cases}\sqrt{63}+\sqrt{20}+\sqrt[3]{10}\\ \sqrt{7}-\sqrt{5}\end{cases}$

Total $\sqrt{112}+\sqrt{5}+\sqrt[3]{10}$

This Example is like the last above save one, only the odd Surde $\sqrt[3]{10}$, is adjoyned to the Total of the others added.

Addends $\begin{cases}\sqrt[3]{40}+\sqrt{24}\\ \sqrt[3]{320}-\sqrt{56}\end{cases}$

Total $\sqrt[3]{1080}-\sqrt{80}-\sqrt{5376}$

Addends $\begin{cases}\sqrt[3]{40}-\sqrt{24}\\ \sqrt[3]{320}+\sqrt{56}\end{cases}$

Total $\sqrt[3]{1080}+\sqrt{80}-\sqrt{5376}$

These two last Examples make the Cube Surdes in both $\sqrt[3]{1080}$, in the Square Surdes the Total is $-\sqrt{80}$ in one, and $+\sqrt{80}$, because in this the Greater Surde to be added was +, but in the other —. The Totals may be thus set,

$$\sqrt[3]{1080}+\sqrt{24}-\sqrt{56}$$
$$\sqrt[3]{1080}-\sqrt{24}+\sqrt{56}$$

$$\begin{array}{l} \sqrt[3]{40}+\sqrt[3]{320} \\ 40)\overline{\quad\quad\quad\quad} \\ 1\phi \quad 8\phi \\ 1\surd + 2\surd \\ \quad 3 \\ \quad 27\phi \\ \quad 40 \\ \sqrt[3]{1080} \end{array} \qquad \begin{array}{l} \sqrt{24}\mp\sqrt{56} \\ \quad 24 \\ \quad 56 \\ \quad 144 \\ \sqrt{80}\quad 120 \\ \quad 1344 \\ \quad 4 \\ \sqrt{80}\mp 5376 \end{array}$$

Universal. In Addition of Universal Surdes respect is to be had to the Mark prefixed; for so properly is the Addition to be: So as if there be given to be added thus differently marked;

$\surd 3\ 12$

√ʒ12 to √ʒ12 or W12 + W12 — Then are the Surdes looked on as Compounds, only Particular and not Universal, and the Total of both their Roots added as before is W48, which is almost 7 absolute Numbers. *How differently marked and taken.*

√:12 to √ʒ12 or √:12 + W12 — Then is to be understood that the Square Root of the Dexter 12 is to be added to the Sinister 12, and the Square Root of that summe to be taken for the Universal Root.

√:ʒ12 to √ʒ12 or √:W12 + W12 — This form is understood by some as the last above; but others more strictly, after the √ʒ of the Dexter 12 is added to the √ʒ of the Sinister 12, take the √ʒ of that Total for the Universal Root.

To make all plain let Rational Numbers be taken; as suppose W81 which is 9, and W361 which is 19, these added together make 28, and so they are considered as Compound only, and their Roots particular and distinct; yet as both Roots are added the Total may seem to be the Universal Root of both Surdes. But usually considered as Universal, then must 19 be added to 81, which make 100, and the Square Root thereof taken which is but 10. And this *Record* in his *Whetstone of Wit*, *p.* 11. counts most aptly the Universal Root. And according to him, and other good Authors, let Universal Roots be so understood here. Otherwise more strictly, if the Root of the Dexter Surde be added to the Root of his Sinister Surde alike denominate, and the Root thereof taken for the Universal Root; then after 19 is added to 9, the Root of 28, the Total taken for the Universal Root will be 5, and somewhat more. *Explained by Rationals.*

Addends √ʒ81 + √ʒ361
$\underbrace{9\surd \quad 19\surd}$
Total 28 Particular.

Addends √:81 + √ʒ361
19 19√
Total W100 (10√ Universal.

Addends √:W81 + W361
Total strictly 5 and + Universal.

√ʒ81 + √ʒ361

	361
	81
W442	361
	2888 √
	29241 (171
	2

√:W442 + W342
Total W784 = 28
$\underbrace{\surd 9 + \surd 19}$
(3
√28(5

ʒ (3 √
784 28(5

4
{32
{64

Hence it is apparent, that the Totals of Simple or Particular Compound Surdes, as before added, if considered as Rooted after a sort may be taken for the Universal Root of the Surdes given to be added. But if from the Total the Character of the Universal Surdes or √: be removed or cancelled, and the Sinister Number left absolute, the Surdes are commonly considered as distinct, and their Roots particular.

To the right understanding of the Addition of Universal Roots, it is meet to proceed in the following steps. *Steps to the Addition of Universals.*

1. If the Numbers or Surdes given be Incommensurable or Heterogeneal, then conjoyn them with +, and before them prefix √:, the Mark that signifies the Universal Root. *1. If the Data be Incommensurable or Heterogeneal*

As to add the Universal Root of 39, absolute Numbers, the WW9 and the W8 together, the Total shall be √:39 + WW9 + W8; or thus, √:WW9 + W8 + 39, of which this latter form is the better, to set the absolute Numbers to the Right Hand of the Surdes; lest standing next the Character √: they be taken for a Surde: For there sometime the Denomination is omitted, and the Number valued as his next Dexter Surde. *Example.*

2. If the one be an Universal Surde, and the other a Particular, then add the Particular Surde to the Sinister part of the Universal, as Particulars are added; and to the Dexter part of the Universal, add double the Root of the Particular (or the Square multiplyed by 4, which is all as one) if the Root of that Dexter part of the Universal added to the Sinister part, make up the next Square Number thereto. *2. One Universal, and the other Particular, two Varieties.*

As to add W16 to √:36 + W169, the Square Roots of 16 and 36 added, which are 4 and 6, make 10, the Square whereof is 100; for the Sinister work to the Dexter W169, whose Root is 13, must be added 8, the double of 4, which is 21; or 16 multiplyed by 4 makes W64, which added as Surdes to W169, makes the Dexter part W441, whose Root is 21. Another Example is at *B.* *Examples of the first variety.*

Roots

	Roots.	Rationals.		*B.*
Addends	7	√: 36 + W 169	4	√: 9 + W 49
	4	√3 16	2	√3 4
Total	11	√: 100 + W 441	6	√: 25 + W 121

Other variety. But if the Root of the Dexter part of the Universal added to the Sinister part, make not up the next Square Number, then if one be omitted, quadruple the Root of the Particular which before you doubled, if 2 be omitted then sexcuple the Root, &c.

Examples. As to add W 25 to √: 4 + W 144; because 12 the Root of 144, added to 4, make 16, which is not the next Square to 4, but 9 is omitted; therefore 5 the Root of 25 must be quadrupled which is 20, and the Square therefore added to 144.

So to add W 9 to √: 4 + W 441; because 21, the Root of 441, added to 4 make 25, which is not the next Square to 4, but 9 and 16, two Squares are omitted, therefore 3, the Root of 9, must be sexcupled, which is 18, and the Square thereof added to 441.

Addends	4	√: 4 + W 144	5	√: 4 + W 441
	5	√3 25	3	√3 9
Total	9	√: 49 + W 1024	8	√: 25 + W 1521

3. *Data both Universals, and o in the Sinister part of one.*

3. If both Numbers given to be added be Universals, and in the Sinister part of one be o, then subscribe the Sinister part as it is, without any alteration, and double the Root Universal of that Number wherein the Cypher is not, multiply the other Root thereby, and the Square thereof add with both the Dexter Squares.

But if the Number that hath the Cypher be a Rational, the Square Root thereof may be taken and placed under the Sinister part, and added as last before.

Example. As to add √: 4 + W 25 to √: 0 + W 256; because 256 is a Rational, and hath 16 for its Root, this may be added with √: 4 + W 25, as before.

But otherwise 4 is subscribed in the sinister part only, and 3 the Root Universal doubled is 6, which multiplying 4, the other Root makes 24, whose Square is to be added with both the other Dexter Squares. The Totals of both Additions are here set down.

Addends	3	√: 4 + W 25	3	√: 4 + W 25
	4	√: 0 + W 256	4	√3 16
Total	7	√: 4 + W 2025	7	√: 36 + W 169

4. *Data both Universals, without o.*

4. If both Universals be without Cyphers, then as before, add the Sinister Surdes as Particular: And the Dexter must be increased to such a Number, that the Root thereof added to the Sinister may be equal to the Roots of the Numbers given to be added. But herein great circumspection is to be used; for though the Roots be Universal, no Universal Rule can be given that I know of to work by, but sometime the double of the Dexters, sometime the √33 of the Dexters severally taken, added, and then figurate accordingly and set as Squares, sometime the method used in the Pathway, but sometime neither will serve.

Examples. As in the two Examples following, the Dexters in both have their 33 Roots, yet the Dexters at *D.* require 169, a greater Number than their Roots 33 will give, which will be but 81.

		C.		*D.*
Addends	8	√: 48 + W 256	7	√: 48 + W 1
	4	√: 12 + W 16	4	√: 12 + W 16
Total	12	√: 108 + W 1296	11	√: 108 + W 169

Other Examples.

Addends	7	√: 48 + W 1	4	√: 12 + W 16
	2	√: 3 + W 1	2	√: 3 + W 1
Total	9	√: 75 + W 36	6	√: 27 + W 81

5. *Residual Universals. Examples.*

5. If the Universals be Residuals, then add them as above, and keep the sign Residual to the Total.

As to add √: 48 — W 144 which is 6, to √: 12 — W 9 which is 3, the Total will be √: 108 — W 729, whose Root is 9. Another Example is set at *F.*

E.

		E.		F.
Addends	6	$\surd{:}48 - \mathrm{W}144$	6	$\surd{:}48 - \mathrm{W}144$
	3	$\surd{:}12 - \mathrm{W}9$	5	$\surd{:}27 - \mathrm{W}4$
Total	9	$\surd{:}108 - \mathrm{W}729$	11	$\surd{:}147 - \mathrm{W}676$

6. If the Univerſals be mixt then after Addition of the Siniſters, one of the Dexters is to be taken from the other, as in Addition of Particulars, yet the Remain muſt be left valuable to make the Root of the Siniſters ſufficient for the ſumme of the Addition. See the Examples following. *6. Mixt Univerſals.* *Examples.*

Addends	8	$\surd{:}48 + \mathrm{W}256$	6	$\surd{:}28 + \mathrm{W}64$
	5	$\surd{:}27 - \mathrm{W}4$	2	$\surd{:}7 - \mathrm{W}9$
Total	13	$\surd{:}147 + \mathrm{W}484$	8	$\surd{:}63 + \mathrm{W}1$
Addends	6	$\surd{:}48 - \mathrm{W}144$	7	$\surd{:}50 - \mathrm{W}1$
	4	$\surd{:}12 + \mathrm{W}16$	2	$\surd{:}2 + \mathrm{W}4$
Total	10	$\surd{:}108 - \mathrm{W}64$	9	$\surd{:}72 + \mathrm{W}81$

7. If any Univerſal Square Surde be added to himſelf, the Siniſter may be multiplyed by 4, the next Dexter by 16, and the next by 256, &c. *7. Square Univerſals added to themſelves.*

But if any Abſolute Number be intermixed amongſt the Surdes, they are to be multiplyed by the next Siniſter Multiplyers before them.

As to add $\surd{:}3 + \mathrm{W}30 + \mathrm{W}36$ to it ſelf, that is 3 to 3, the Total will be $\surd{:}12 + \mathrm{W}480 + \mathrm{W}9216$, whoſe Root Univerſal is 6: For 96, the Root of 9216, added to 480, and the Root of 576, the ſumme which is 24, brought and added to 12, makes 36, whoſe Root is equal to the other Roots of the Addends. *Example.*

		Multiplyers 4 16 256
Addends	3	$\surd{:}3 + \mathrm{W}30 + \mathrm{W}36$
	3	$\surd{:}3 + \mathrm{W}30 + \mathrm{W}36$
Total	6	$\surd{:}12 + \mathrm{W}480 + \mathrm{W}9216$

8. If any Square Univerſal be added to himſelf, the Multiplications may be ſhortned thus: Let the Siniſter Number be multiplyed by 2, the next Dexter by 4, and the next Dexter by 16, &c. and the Abſolute Numbers accordingly: For if the outmoſt Dexter be Abſolute and not Figurate, then multiply that by 4; if the middlemoſt be Abſolute, then multiply that by 2; and the outmoſt Dexter by 4, though Figurate. Alſo in getting the abſolute value of the Total ſo added, when the Root is brought to be added to or ſubſtracted from the Siniſter, the ſumme or difference is accordingly to be multiplyed again by 2. *8. To ſhorten the Multiplications in ſuch Additions.*

As in the laſt Example the Total will be $\surd{:}6 + \mathrm{W}120 + \mathrm{W}576$; but after 24, the Root of 576, is added to 120, and the Root of 144, the ſumme which is 12, brought and added to 6, then 18, the ſumme there is to be multiplyed by 2, the Root of which Product 36, is 6, the Total as before. *Example.*

		Multiplyers 2 4 16
Addends	3	$\surd{:}3 + \mathrm{W}30 + 36$
	3	$\surd{:}3 + \mathrm{W}30 + 36$
Total	6	$\surd{:}6 + \mathrm{W}120 + \mathrm{W}576$

$$\begin{array}{r|l} 576 & 24\surd \\ +120 & \\ \hline 144 & 12\surd \\ +6 & \\ \hline 18 & \\ \times 2 & \\ \hline 36 & 6\surd \text{ value} \end{array}$$

Other Examples follow with their Multiplyers at top, and the ſumme of their Roots under the Totals. *Other Examples.*

	2 4 16	2 4 16	2 4 16
Addends	$\surd{:}5 + \mathrm{W}12 + \mathrm{W}16$	$\surd{:}5 + \mathrm{W}20 - \mathrm{W}16$	$\surd{:}5 - \mathrm{W}20 - \mathrm{W}16$
	$\surd{:}5 + \mathrm{W}12 + \mathrm{W}16$	$\surd{:}5 + \mathrm{W}20 - \mathrm{W}16$	$\surd{:}5 - \mathrm{W}20 - \mathrm{W}16$
Total	$\surd{:}10 + \mathrm{W}48 + \mathrm{W}256$	$\surd{:}10 + \mathrm{W}80 - \mathrm{W}256$	$\surd{:}10 - \mathrm{W}80 - \mathrm{W}256$
	8 16	8 −16	−8 −16
	18 64(8√	18 64(8√	2 64(8√
	36(6√ value	36(6√ value	4(2√ value

Addends

	2 4 4	2 4 4	2 2 4
Addends	√:5 + W20 — 16	√:2 + W20 — 16	√:25 — 12 + W16
	√:5 + W20 — 16	√:2 + W20 — 16	√:25 — 12 + W16
Total	√:10 + W80 — 64	√:4 + W80 — 64	√:50 — 24 + W64
	4 —64	4 —64	—32 8
	14 16 (4 √	8 16 (4 √	18 32
	W28 value	16 (4 √ value	36 (6 √ value

Whence this work last mentioned proceeds. This and the precedent work take original from the *First Consectary* in *Chap.* 5. *Multiplication of Simple Surdes*, and according thereto may alſo Univerſal Surdes of Higher Powers than Squares be doubled, tripled, *&c.* but little uſe being of any higher Univerſals than Squares, the foregoing Operations are fitted for them.

Proof of Addition *of Compound* Surdes. The Proof of Addition of Compound Surdes is like the Proof of Simple Addition, either by Subſtraction, Particular by Particular, and Univerſal by Univerſal; or by taking Rational Numbers, and working therewith inſtead of the Surdes. And ſeeing ſeveral of the Examples are of Rational Numbers, they may ſerve without further inſtance.

CHAP. VIII.

Subſtraction of Compound Surdes.

Compound Surdes *ſubſtracted. Particular.* IN the Subſtraction of Compound Surdes, as in Addition, let them be conſidered as they are Particular or Univerſal.

In Particular Compound Surdes, to Heterogeneal and Incommenſurable ſet the ſign of Subſtraction —; and for Commenſurable, as the parts given to be ſubſtracted, ſo ſhall the Subſtraction be. Like is to be ſubſtracted from like; and the uſe of the ſigns + and — is here as in Subſtraction of Compound Coſſicks.

Examples of Binomials.

Examples of Binomials.

Greater Surde	10 + W48	
Subtrahend	7 + W27	
Remain	3 + W3	
Leſſer Surde	10 + W27	
Subtrahend	7 + W48	
Remain	3 — W3	

After Subſtraction of the Abſolute Number 7 from 10, and the Remain 3 ſet down, in both theſe Examples the W27 and W48 are ſubſtracted as before in Simple Surdes, and the Greater of them being taken from the Leſſer, makes the Remain to be —.

W48 — W27
3) ——————
16 3 9 3
4 √ — 3 √
1
13
3
W3

Greater Surde	12 + W8 + W3		Greater Surde	12 + W11 + W96
Subtrahend	5 + W3		Subtrahend	5 + W3
Remain	7 + W8		Remain	7 + W8

The Abſolute Numbers being ſubſtracted in both theſe Examples, there remains 7; then in the one finding W3 in the Greater Surde and Subtrahend both, they are both cancelled, for taking one from the other, 0 remains; ſo W8 is only ſet down. But in the latter Example, where the Greater Surde is ſet as the Total of the ſecond form of Simple Addition, 96 is to be divided by 4, and the Quotient by W3, and ſo W8 is gotten for the Remain.

W 96/4 (24/W3 (W8

Greater Surde	W2844 + 36
Subtrahend	28 + W316
Remain	W1264 + 8

In this Example taking W316 from W2844, the Remain is W1264. And 28 Abſolute Numbers taken from 36, there Remains + 8.

W2844 — W316
316) ——————
9 3 1 3
3 √ — 1 √
2
4 3
316
W1264

Greater

Greater Surde	$\surd\surd108 + \surd29 + \surd760$	
Subtrahend	$\surd\surd4 + \surd19$	
Remain	$\surd\surd32 + \surd10$	

Here taking $\surd\surd4$ from $\surd\surd108$ remains $\surd\surd32$, then taking $\surd19$ from $\surd29$, rests $\surd10$, rejecting $\surd760$, or dividing as before by 4, and the Quotient by 19, the $\surd10$ is obtained, rejecting $\surd29$.

$\surd760$ / 4 (190 / $\surd19$ ($\surd10$

$\surd\surd108 - \surd\surd4$
4)
27φ 1φ
3√ − 1√
2
8φ
4
$\surd\surd32$

Greater Surde	$\surd\surd1875 + \surd5$	
Subtrahend	$\surd\surd48 + \surd80$	
Remain	$\surd\surd243 - \surd45$	

substracted.
$\surd\surd1875 - \surd\surd48$
3)
62533 1633
5√ − 2√
3
8133
3
$\surd\surd243$

substracted.
$\surd5 - \surd80$
5)
13 163
1√ − 4√
−3
−93
5
$-\surd45$

Examples of Residuals.

Examples of Residuals.

Greater Surde	$26 - \surd27$	
Subtrahend	$12 - \surd12$	
Remain	$14 - \surd3$	
Lesser Surde	$26 - \surd12$	
Subtrahend	$12 - \surd27$	
Remain	$14 + \surd3$	

After Substraction of 12 from 26, the Absolute Number 14 is left in both these Examples. And $\surd12$ taken from $\surd27$ leaves $\surd3$, but in the one −, in the other +, according as the Subtrahend was the Greater or Lesser Surde.

$\surd27 - \surd12$
+
3)
93 43
3√ − 2√
+
1
13
3
$\surd3$

Greater Surde	$14 - \surd5 - \surd3$	
Subtrahend	$8 - \surd5$	
Remain	$6 - \surd3$	

The Absolute Number 6, is left when 8 is taken from 14. And $\surd5$ taken from $\surd5$ leaves 0; so both cancelled, $\surd3$ is brought down to the Remain: But if the Surde had been set thus, $14 - \surd8 - \surd60$, after the second form of Simple Substraction, then 60 divided by 4, and the Quotient 15 divided by 5, gives $\surd3$ for the Remain, as before.

Greater Surde	$\surd275 - \surd48$	
Subtrahend	$\surd99 - \surd3$	
Remain	$\surd44 - \surd27$	

Here taking $\surd99$ from $\surd275$, and $\surd3$ from $\surd48$, the Remain appears thus,

$\surd275 - \surd99$
11)
253 93
5√ − 3√
2
43
11
$\surd44$

$\surd48 - \surd3$
3)
163 13
4√ − 1√
3
93
3
$\surd27$

Greater Surde	$\surd\surd243 - \surd150$	
Subtrahend	$\surd\surd72 - \surd96$	
Remain	$\surd\surd9 - \surd6$	

Both Cube Surdes and Square Surdes severally substracted as Simple their Remains are thus;

$\surd\surd243 - \surd\surd72$
9)
27φ 8φ
3√ − 2√
1
1φ
9
$\surd\surd9$

$\surd150 - \surd96$
6)
253 163
5√ − 4√
1
13
6
$\surd6$

Greater Surde	$\surd\surd512 - \surd24 - \surd5$	
Subtrahend	$\surd\surd32 - \surd24$	
Remain	$\surd\surd32 - \surd5$	
Greater Surde	$\surd\surd512 - \surd24$	
Subtrahend	$\surd\surd32 - \surd24 - \surd5$	
	$\surd\surd32 + \surd5$	

In both these Examples the Remain of the Squared Square Surdes is $\surd\surd32$. The Remain of the Square Surdes in the one is +, in the other −, as the odd $\surd5$ in the one was in the Subtrahend, in the other in the Number from which Substraction is made. And if the Square Surdes had been set after the second form of Addition, that is $\surd29 - \surd480$, then dividing 480 by 4, and the Quotient 120 by 24, the Remain will be $\surd5$, as before.

$\surd\surd512 - \surd\surd32$
2)
25633 1633
4√ − 2√
2
1633
2
$\surd\surd32$

Ex-

Examples of Binomials *with* Residuals.

Examples of Mixt.

Greater Surde	19 + √12	In both these Examples the Absolute Numbers 11 taken from 19, leaves 8. And √3 added to √12 because their Signs are contrary, makes the Remain √27, in the one +, in the other —, as the sign of the Number from which Substraction is made.
Subtrahend	11 — √3	
Remain	8 + √27	
Greater Surde	19 — √27	
Subtrahend	11 + √3	
Remain	8 — √27	
Greater Surde	√72 — 2	Both Surdes and Absolute Numbers being of contrary Signs, the respective sums of both are taken.
Subtrahend	6 — √8	
Remain	√128 — 8	
Greater Surde	174 — √11	The sum of the Absolute Numbers, and the Total of the Surdes, is the Remain of this Substraction, the signs of both being contrary.
Subtrahend	√44 — 76	
Remain	250 — √99	
Greater Surde	√242	In both these Examples the work is alike; for √72 taken from √242 leaves √50, and √50 taken out of √242 leaves √72. Then √7 in both having none to be taken from doth remain; but the Signs are changed; for where + is substracted — shall remain, and where less, more.
Subtrahend	√72 — √7	
Remain	√50 + √7	
Greater Surde	√242	
Subtrahend	√50 + √7	
Remain	√72 — √7	
Greater Surde	√112 + √5	Here √7 taken from √112 leaves √63, and — √5 added to + √5 makes √20, whose sign is as the upper Surde.
Subtrahend	√7 — √5	
Remain	√63 + √20	
Greater Surde	√125 — √√20	The work in this is like that in the last Example above.
Subtrahend	√5 + √√20	
Remain	√80 — √√160	

3) √12 + √3
4 3   1 3
2√ + 1√
3
9 3
3
√27

8) √72 + √8
9 3   1 3
3√ + 1√
4
16 3
8
√128

11) √44 + √11
4 3   1 3
2√ + 1√
3
9 3
11
√99

2) √242 — √72
121 3   36 3
11√ — 6√
5
25 3
2
√50

7) √112 — √7
16 3   1 3
4√ — 1√
3
9 3
7
√63

5) √5 + √5
1 3   1 3
1√ + 1√
2
4 3
5
√20

5) √125 — √5
25 3   1 3
5√ — 1√
4
16 3
5
√80

20) √√20 + √√20
1 φ   1 φ
1√ + 1√
2
8 φ
20
√√160

Greater Surde	√√1080 — √80 — √5376	The Operations in both these last Examples are much alike; for √√320 substracted from √√1080 leaves √√40, and therefore √√40 substracted must leave √√320. In the one √56 may be substracted from √80, and the Remain set down with the sign of 80; in the other it may be added to √80. Or else 5376 divided by 4, and the Quotient divided by 56 in the one, or 24 in the other, gives in the Quotient the Remain accordingly.
Subtrahend	√√320 — √56	
Remain	√√40 + √24	
Greater Surde	√√1080 — √80 — √5376	
Subtrahend	√√40 + √24	
Remain	√√320 — √56	

40) √√1080 — √√320
27 φ   8 φ
3√ — 2√
1
1 φ
40
√√40

40) √√1080 — √√40
27 φ   1 φ
3√ — 1√
2
8 φ
40
√√320

√5376 / 4 (1344 / 24 (√56

√5376 / 4 (1344 / 56 (√24

In

Universal.

In *Substraction of Universal Surds*, proceed with respect to the Mark prefixed, as in *Addition of Universals* was before noted. For so accordingly must the *Substraction* be.

Explained by Rationals.

As in Rational Numbers it may be clearly demonstrated, that if √ʒ25+√ʒ121, which are 5 and 11, be substracted from √ʒ81+√ʒ361, which are 9 and 19, they being marked only as *Compounds*, shall make the particular Remain as at A, 12 Absolute Numbers. But marked as Universal, the Remain shall be but 4 as at B. And if strictly taken, as before shewed in the foregoing Chapter, it shall not amount to 4; but be W12, which is a single *Surd*, and hath 3 and somewhat more for his Root, as at C.

Greater Surd.	√ʒ81+√ʒ361	A	W81+W361=9+19=28
Subtrahend.	√ʒ25+√ʒ121		W25+W121=5+11=16
Remain.	12	Particular.	W16+W 64=4+ 8=12

			W81
Greater Surd.	√:81+√ʒ361	B	W361= 19
Subtrahend.	√:25+√ʒ121		W100=10
Remain.	4	Universal.	W 25
			W121= 11
			W 36=6
			4

Greater Surd.	√:W81+W361	C.	
Subtrahend.	√:W25+W121		
Remain.	W12	Strictly Universal.	√9+19=28
			√5+11=16
			W12

Hereby it appeareth that the Remains of the Substraction of Simple or Particular *Compound Surds* as before substracted, if considered as rooted, are after a sort to be taken for the Universal Root. But if the Character of the *Universal Surds* be removed or cancelled, or the *Remain* but a single Number, then they cease to be *Universal*, but are understood as Simple or Particular *Compound Surds*.

Steps to the Substraction of Universals.

Further to understand the Substraction of Universal Roots, it may be safe to tread in the like Steps as before in their *Addition*.

1. If the Data be Incommensurable or Heterogeneal.

1. If the Numbers or *Surds* given be Incommensurable or Heterogeneal; then conjoin them with —, and prefix before them √: to signify the Root Universal.

Example.

As to take W8 from √:WW9+W8+39, the Remain shall be √:WW9+39.

2. One Particular, and the other Universal, 2 Varieties.

2. If a Particular *Surd* be to be substracted from some Universal; then take the Particular *Surd* from the finister Part of the Universal, as Particulars are substracted: And from the dexter Part of the Universal, take the Square of double the Root of the Particular, when the Root of that dexter Part of the Universal added to the finister Part make up the next square Number thereto; but if one be omitted, then quadruple the Root; if two be omitted, sexcuple the Root, &c.

Example of the first Variety.

As to take W16 from √:36+W169, the Root of 16, which is 4 doubled, is 8; so must 64 the Square thereof be taken from 169, after the manner of *Surds*.

Other Variety.

But if W25 be taken from √:36+W784, there 5 the Root of 25 must be multiplied by 4; because 28, the Root of 784, added to 36, makes 64; which is not the next square Number to 36, but one is omitted, to wit 49.

		Roots.		*Roots.*
Greater Surds.	√:36+W169	7	√:36+W784	8
Subtrahends.	√ʒ16	4	√ʒ25	5
Remains.	√:4+W25	3	√:1+W64	3

Example.

3. Data both Universals, and 0 in the Sinister Place of one.

3. If both Numbers given be Universals, and the finister Part of the *Subtrahend* be a Cipher; then if the Dexter thereof be Rational, take the Root and place in the finister Place of the Cipher, and proceed as in the last Direction.

Example.

As to take √:0+W81 from √:49+W225, the Root of 81 being 9, if taken from √:49+W225 as above, leaves the Remain √:16+W81.

Greater Surd.	√:49+W225		8
Subtrahend.	√39	for √:0+W81	3
Remain.	√:16+W81		5

4. Data both Universals without 0. 4. If both Universals be without Ciphers, then take the Sinister of the *Subtrahend* from the Sinister of the other, as particular *Surds* are substracted. And let the Dexter be diminished, that the *Remain* may be just. But herein lies the Skill, as before in *Addition*, wherein the Operator must have under consideration several Directions at once.

Examples. As to take √:12+W16 from √:48+W256, after 12 is taken from 48, and the Remain 12 subscribed, the Dexter can be but 16, to make 12 half the other *Surd* as it is.

But if √:12+W16 be taken from √:48+W1, the Sinister Part of the Remain will be as before; but the Dexter will be —W9, because W1 was not sufficient to substract W16 from, and therefore the Sign is changed.

Greater Surds.	√:48+W256	8	√:48+W1	7
Subtrahends.	√:12+W16	4	√:12+W16	4
Remains.	√:12+W16	4	√:12—W9	3

5. *Residual Universals.* 5. If the Universals be Residuals, they are to be substracted as above, and the Residual Sign kept to the Remain, or changed as the Case requires.

Examples. As to take √:12—W9 from √:48—W144, the Remain shall be √:12—W9. But √:2—W4 taken from √:50—W1, shall leave the Remain √:32+W289. where the Sign is changed, because W1 was too little to substract W4 from.

Greater Surds.	√:48—W144	6	√:50—W1	7
Subtrahends.	√:12—W9	3	√:2 —W4	0
Remains.	√:12—W9	3	√:32+W289	7

6. *Mixt Universals.* 6. If the Universals be mixt, then after substraction of the Sinisters, the Dexters are to be added; yet so as the Dexter Remain must have respect to the Sinister, and not exceed its due Proportion. See the Examples following.

Examples.

Greater Surds.	√:4+W144	4	√:48+W256	8
Subtrahends.	√:4—W16	0	√:27—W4	5
Remains.	√:0+W256	4	√:3 +W36	3
Greater Surds.	√:48—W144	6	√:50—W1	7
Subtrahends.	√:12+W16	4	√:2 +W4	2
Remains.	√:12—W64	2	√:32—W49	5

7. *Sq. Universals halfed.* 7. If any Square Universal be to be halved, divide the Sinister by 4, the next Dexter by 16, and the next by 256, &c. And if Absolute Numbers be intermixed, they are to be divided by the next Sinister Divisors before them.

Example. As to half √:12+W480+W9216, dividing accordingly by 4.16.256. there will be brought forth the √:3+W30+W36, for the half of the former.

8. *To shorten the Division in such Substractions.* 8. If any Square Universal be divided by 2.4.16, &c. orderly, and the Absolute Numbers, if any, as the Sinisters next before them, the Quotient shall be an Universal, like those in the last Direction mentioned in *Addition*. And if the Root be gotten, the Sum or Difference at last must be divided by 2, and the Root of the Quotient taken for the Remain.

Example. As if √:12+W480+W9216 be thus divided, the Quotient will be thus, √:6+W120+W576: then 24 the Root of 576 brought and added to 120, make 144, whose Root 12 added to 6 make 18; but this 18 must be divided by 2, and the Root of 9 the Quotient taken.

Whence this last Work proceeds. This and the precedent Work take their Original from the first Consectary in Chap. 6. *Division of Simple Surds*; and according thereto may higher Universal *Surds* be halfed, &c. But these are fitted only for Squares, others being seldom used, as before noted in *Addition*.

Proof of Substraction of Compound Surds. The Proof of *Substraction of Compound Surds*, is like the Proof of *Simple Substraction*, either by *Addition*, or by taking Rational Numbers instead of the *Surds*, and working therewith. For making the Total of any *Addition*, the Number

from

from which *Substraction* is made, and one of the Numbers added the Subtrahend: then shall the Remain be the other Addend, and so *vice versa*; remembring Particular to try Particular, and Universal, Universal; of which Instances are spared here, forasmuch as many of the Examples in this Chapter are so ordered, as they prove the Additions of the former: And divers of the Examples being of Rational Numbers, may be Instances sufficient without farther explanation.

CHAP. IX.

Multiplication of Compound Surds.

TO multiply *Compound Surds*, let them be considered as they are, Particular or Universal. *Compound Surds multiplied.*

Multiply the Numbers of Particular *Compound Surds*, as Simple *Surds*, like with like, or reduced thereunto; and let the Signs + and — be ordered as in *Multiplication of Compound Cossicks*, for like Signs give +, and unlike —. *Particular.*

Example of Binomials. *Examples of Binomials.*

Multiplicand. $\sqrt{26}+\sqrt{3}$
Multiplier. $\sqrt{5}$
Product. $\sqrt{130}+\sqrt{15}$

The Multiplier being a Simple *Surd*, tho the Multiplicand a Compound, there is no Difficulty or Difference from Simple Multiplication.

Multiplicand. $5+\sqrt{10}$
Multiplier. $5+\sqrt{10}$
$25+\sqrt{250}$
$\sqrt{250}+10$
Product. $35+\sqrt{1000}$

The Absolute Numbers multiplied make 25, and $\sqrt{10}$ by $\sqrt{10}$ make 10 Absolute Nnmbers, the 5 squared is 25; which multiplied by 10 is 250, and adding $\sqrt{250}$ to $\sqrt{250}$, the Total is $\sqrt{1000}$, that is 250 multiplied by 4.

Multiplicand. $23+\sqrt{15}$
Multiplier. $6+\sqrt{8}$
$138+\sqrt{540}$
$\sqrt{4232}+\sqrt{120}$
Product. $138+\sqrt{540}+\sqrt{4232}+\sqrt{120}$

After Multiplication of the Absolute Numbers 23 and 6, and the two *Surds* which make $\sqrt{120}$, the Absolute Numbers are squared; and so multiplied alternately into the other *Surd*, all which collected make the Total Product.

Multiplicand. $\sqrt{120}+\sqrt{12}$
Multiplier. $\sqrt{12}+\sqrt{7}$
$\sqrt{1440}+12$
$\sqrt{840}+\sqrt{84}$
Product. $12+\sqrt{1440}+\sqrt{840}+\sqrt{84}$

Here $\sqrt{120}$ multiplied by $\sqrt{12}$, the Product is $\sqrt{1440}$; and $\sqrt{12}$ by $\sqrt{12}$, gives 12 Absolute Numbers by cancelling $\sqrt{}$, as was taught in Simple *Surds*; the other Products are plain.

Example of Residuals. *Examples of Residuals.*

Multiplicand. $\sqrt{26}-\sqrt{3}$
Multiplier. $\sqrt{5}$
Product. $\sqrt{130}-\sqrt{15}$

The Multiplicand being a Compound, but the Multiplier Simple, there is no Difficulty nor Difference from Simple Multiplication.

Multiplicand. $10-\sqrt{5}$
Multiplier. $10-\sqrt{5}$
$100-\sqrt{500}$
$-\sqrt{500}+5$
Product. $105-\sqrt{2000}$

The Absolute Numbers multiplied, produce 100, and $\sqrt{5}$ by $\sqrt{5}$, make 5. Absolute Numbers $\sqrt{}$ being cancelled, the rest of the Work is like the second Example of *Binomials* above.

Multiplicand. $23-\sqrt{15}$
Multiplier. $6-\sqrt{8}$
$138-\sqrt{540}$
$-\sqrt{4232}+\sqrt{120}$
Product. $138-\sqrt{540}-\sqrt{4232}+\sqrt{120}$

The Work in this Example is like that in the third Example of *Binomials* above, only altered in the Signs. And $\sqrt{120}$ which is + may be set next to 138; which is of the same Nature, and so the Product will stand thus;

$138+\sqrt{120}-\sqrt{540}-\sqrt{4232}$.

Multiplicand.

Multiplicand. $\sqrt{24}-\sqrt{20}$
Multiplier. $\sqrt{30}-\sqrt{24}$
$\sqrt{720}-\sqrt{600}$
$-24+\sqrt{480}$
Product. $\sqrt{720}+\sqrt{480}-24-\sqrt{600}$

In this Example $\sqrt{24}\times\sqrt{30}$ produceth $\sqrt{720}$, and $\sqrt{30}\times\sqrt{20}$, makes $\sqrt{600}$; but $\sqrt{24}\times\sqrt{24}$, giveth 24 Absolute Numbers, cancelling the Character; and $\sqrt{24}\times\sqrt{20}$, makes $\sqrt{480}$.

Examples of mixt.

Examples of Binomials with Residuals.

Multiplic. $6+\sqrt{3}$
Multiplier. $6-\sqrt{3}$
$36+\sqrt{108}$
$-\sqrt{108}-3$
Product. 33

After multiplication of the Absolute Numbers, whose Product is 36, the *Surds* are multipli'd into the square of 6, which alternately produce $+\sqrt{108}$ & $-\sqrt{108}$, and in the addition both cancelled being of contrary Signs, and 3 which is − coming of $\sqrt{3}\times\sqrt{3}$, is taken from 36.

Multiplic. $\sqrt{52}+17$
Multiplier. $17-\sqrt{52}$
$\sqrt{15028}+289$
$-15028-52$
Product. 237

Here the Square of 17 multiplied into $\sqrt{52}$, makes in both places $\sqrt{15028}$, the one + and the other —, according to the Signs. And 17×17 gives 289; and $\sqrt{52}\times\sqrt{52}$ produceth 52, cancelling the Character.

Multiplic. $\sqrt{124}-6$
Multiplier. $32+\sqrt{14}$
$\sqrt{126976}-192$
$\sqrt{1736}-\sqrt{504}$
Product. $\sqrt{126976}+\sqrt{1736}-\sqrt{504}-192$

The Work in this Example is like the last before, and the total Product is set at large.

Multiplic. $\sqrt{32}-3$
Multiplier. $\sqrt{8}+2$
$\sqrt{256}-\sqrt{72}$
$\sqrt{128}-6$
Product. $\sqrt{256}+\sqrt{8}-6$

The Multiplication ended in collecting the Total $\sqrt{72}$ is taken from $\sqrt{128}$, and the Remain $\sqrt{8}$ set down.

8) $\sqrt{128}-\sqrt{72}$
16□ 9□
4√ − 3√
1
1□
8
$\sqrt{8}$

Polynomial *multiplied squarely, how changeth a Name.*

From hence it is obvious, that if a *Multinomial* be multiplied into it self, with one of his Signs changed, the Product shall be purged of one Name.

As $3+\sqrt{5}+\sqrt{2}$ multiplied by $3+\sqrt{5}-\sqrt{2}$, the Product shall be $12+\sqrt{180}$.

Example.

Multiplic. $3+\sqrt{5}+\sqrt{2}$
Multiplier. $3+\sqrt{5}-\sqrt{2}$
$9+\sqrt{45}+\sqrt{18}$
$\sqrt{45}+5+\sqrt{10}$
$-\sqrt{18}-\sqrt{10}-2$
$9+\sqrt{180}+5-2$
Product. $12+\sqrt{180}$

45) $\sqrt{45}+\sqrt{45}$
1□ 1□
1√ + 1√
2
4□
45
$\sqrt{180}$

$\sqrt{45}$
4
$\sqrt{180}$

Universals Homogeneal *how multiplied.*

Multiply the Numbers of Universal *Surds* that are Homogeneal, as Compound *Surds*, alternately one into the other, with this difference, that the Sinister Numbers be figurate, according to the Denomination of the Dexter into which they are multiplied; And the particular Roots of all the Dexter Multiplees added to the Product of the two Sinister Numbers, and the Root of the Total shall be the Product in Absolute Numbers; which if thereby cannot be expressed, is to be marked accordingly with its proper Character.

Example.

Example, to multiply the Root Universal of $\sqrt{2}+\sqrt{49}$, by the Universal Root of $\sqrt{3}+\sqrt{36}$; after multiplication of 3 by 2, both 3 and 2 are squared before they are respectively multiplied into 49 and 36, and then 49 and 36 are multiplied together; the Roots of all which Dexter Multiplees are added to 6, and the Total makes 81, which is a Square, and the Root 9 Absolute Numbers is the Product desired.

Multiplicand.

Multiplicand.	√:2+W49		ʒ √
Multiplier.	√:3+W36		√ 2 × 3 ====6
	6+W441		3 . 9 × 49 = 441 | 21
	W144+W1764		
Product.	√:6+W441+W144+W1764		2 . 4 × 36 = 144 | 12
Roots.	6+ 21 + 12 + 42		49 × 36 = 1764 | 42
Total Product.	W81 = 9 Absolute Numbers.		81

If the given Numbers be *Heterogeneal*, then besides the Reduction of the one to the other, as in Compounds before, figurate the Sinister *Surd* according to the Reduction of the Dexter, and then proceed as in other Universals. *Universals Heterogeneal, how multiplied.*

Example, to multiply the Universal Root of W23+wW8, by the Universal Root of W2+wW343. First, 23 is multiplied by 2, for they are *Homogeneal*, that is, both Squares. Then W2 is reduced with wW8 to the √ʒc8, and √ʒc64: And moreover, because 64 the Dexter Number is now of the Zenzicube Denomination, 2 the Sinister Number shall be made a Zenzicube also, which is 64. So shall 64 be multiplied by 64, and the Product with his Character set down, which is √ʒc4096. Likewise W23 and wW343 reduced, make the Dexter Number √ʒc117649; which is to be multiplied by 148035889 the Zenzicube of 23, and the product which is the √ʒc17416274304961 set down. Afterward wW8 and wW343, both Homogeneal Cubes, are multiplied, and the Roots severally taken and added to the Sinister Product 46; the Total is W225, which in Absolute Numbers is 15. *Example.*

Multiplicand.	√:23+wW8
Multiplier.	√: 2+wW343
	46+√ʒc4096
	√ʒc17416274304961+wW2744
Product.	√:46+√ʒc4096+√ʒc17416274304961+wW2744
Roots.	46 + 4 + 161 + 14
Total Product.	W 225 = 15 Absolute Numbers.

If the Universal Root be strictly taken, then the Root of these last Numbers must be taken for the Product; which in the former Example would be 3, the Square Root of 9, in this Example W15.

As the Multiplication of Simple *Surds* takes for the Proof thereof, either Simple Division or Rational Numbers; so doth Multiplication of Compound *Surds*, Particular and Universal, prove it self either by the respective Divisions, *viz.* Particular by Particular, and Universal by Universal, as in the next Chapter is exemplified; or by working with Rational Numbers, as in the Instance following. *Proof of Multiplication of Compound Surds.*

Suppose the Roots, Particular or Universal of W4+wW125 were to be multiplied by themselves, the W4 being 2, and wW125 being 5, added, make the Sum of their Particular Roots 7; which multiplied by it self is 49.

But if wW125 which is 5, be added to 4, the Sum is 9, of which the Square Root is 3 for the Root Universal, and 3 multiplied into it self is 9, for the Product of the Universal Roots; accordingly by the Work thereof are those Numbers produced.

And both if strictly taken must be the Roots of these Roots, to wit, 7 and 3 the Roots of 49 and 9.

W 4+wW125	2+5= 7	√:4+wW125	4+5=9√3
W 4+wW125	2+5= 7	√:4+wW125	4+5=9√3
W16+√ʒc1000000	49	16+√ʒc64000000	9
√ʒc1000000+wW15625		√ʒc64000000+wW15625	
4 + 10 + 10 + 25		16 + 20 + 20 + 25	
49		W81 = 9	

CHAP. X.

Division of Compound Surds.

Compound Surds divided.

Particular.

TO divide *Compound Surds*, let them also as in *Multiplication*, be considered as they are Particular or Universal.

Division of Particular *Compound Surds*, hath sometime the given Divisor Simple, sometime the Dividend Simple, and sometime both Compound: The Varieties of whose Divisions are comprised in the following Cases.

1. Divisor Simple, or an Absolute Number.

Case 1. When the Divisor given is a Simple *Surd*, or Absolute Number, and the Dividend Compound; divide by the Divisor the several Numbers of the Dividend that are of like Denomination; and if any be unlike, reduce the Divisor to the same Denomination, and then divide thereby. And order the Signs + and — as in *Division of Compound Cossicks*; for like Signs give + and unlike —.

Examples of Binomials.

Examples of Binomials.

Dividend. $\sqrt{130}+\sqrt{15}$
Divisor. $\sqrt{5}$ ($\sqrt{26}+\sqrt{3}$ *Quotient.*

Dividend. $\sqrt{56}+\sqrt{24}$
Divisor. $\sqrt{6}$ ($\sqrt{9\frac{1}{3}}+\sqrt{4}$ *Quotient.*

Residuals.

Examples of Residuals.

Dividend. $\sqrt{130}-\sqrt{15}$
Divisor. $\sqrt{5}$ ($\sqrt{26}-\sqrt{3}$ *Quotient.*

Dividend. $\sqrt{56}-\sqrt{24}$
Divisor. $\sqrt{6}$ ($\sqrt{9\frac{1}{3}}-\sqrt{4}$ *Quotient.*

In all these first 4 Examples there appeareth no Difficulty, every Number being divided as Simple *Surds*, and the Quotients + or — according to the likeness or unlikeness of the Divisor's Sign with the Sign of the Numbers of the Dividend.

Mixt.

Examples of Binomials with Residuals.

Dividend. $6+\sqrt{10}-\sqrt[3]{16}$
Divisor. 2 ($3+\sqrt{2\frac{1}{2}}-\sqrt[3]{2}$ *Quotient.*

Dividend. $6-\sqrt{10}+\sqrt[3]{16}$
Divisor. 2 ($3-\sqrt{2\frac{1}{2}}+\sqrt[3]{2}$ *Quotient.*

In both these Examples, after the Absolute Numbers are divided; to divide the Squares, 2 is squared, and to divide the Cubes, 2 is cubed.

2. Dividend Simple, or an Absolute Number.

Case 2. When the Dividend given is a Simple *Surd*, or Absolute Number, and the Divisor a Compound Binomial or Residual, so as the one *Surd* be not equal to the other; then multiply the Divisor, if Binomial, by his Residual; if Residual, by his Binomial. And by the same Number the Divisor is multiplied, multiply the Dividend: this Product divide by the Number remaining of the Multiplication of the Divisor, figurate or not, according to the Denominations of the Dividend so multiplied, ordering the Signs as before.

Examples.

Example, to divide the Absolute Number 49 by $4+\sqrt{9}$. First, the Divisor multiplied by his Residual $4-\sqrt{9}$, produceth 7; which figurated or set as a *Surd*, is $\sqrt{49}$; then the Dividend multiplied by $4-\sqrt{9}$, doth produce $196-\sqrt{21609}$; whereof 196 being Absolute Numbers, divided by 7, the Absolute Number remaining by multiplication of the Divisor, and $\sqrt{21609}$ divided by $\sqrt{49}$, give in the Quotient $28-\sqrt{441}$, as at A.

And if 49 be divided by $4-\sqrt{9}$, the Quotient will be alike in Numbers, only the Sign contrary, *viz.* $28+\sqrt{441}$, as at B.

But if 49 be divided by $\sqrt{9}+4$, the Quotient will be $\overline{\sqrt{441}}+28$, as at C.

And if 49 be divided by $\sqrt{9}-4$, the Quotient will be $\overline{\sqrt{441}}-28$, as at D.

A

	A *Divisor.*	B *Divisor.*	C *Divisor.*	D *Divisor.*
	Binomial. $4+\sqrt{9}$	*Residual.* $4-\sqrt{9}$	*Binomial.* $\sqrt{9}+4$	*Residual.* $\sqrt{9}-4$
	Residual. $4-\sqrt{9}$	*Binomial.* $4+\sqrt{9}$	*Residual.* $\sqrt{9}-4$	*Binomial.* $\sqrt{9}+4$
	$16+\sqrt{144}$	$16-\sqrt{144}$	$9+\sqrt{144}$	$9-\sqrt{144}$
	$-\sqrt{144}-9$	$\sqrt{144}-9$	$-\sqrt{144}-16$	$\sqrt{144}-16$
Product.	$16-9$	$16-9$	$9-16$	$9-16$
Remain.	7	7	-7	-7
Figurated.	$\sqrt{49}$	$\sqrt{49}$	$-\sqrt{49}$	$-\sqrt{49}$
	Dividend.	*Dividend.*	*Dividend.*	*Dividend.*
	49	49	49	49
	$4-\sqrt{9}$	$4+\sqrt{9}$	$\sqrt{9}-4$	$\sqrt{9}+4$
Product.	$196-\sqrt{21609}$	$196+\sqrt{21609}$	$\sqrt{21609}-196$	$\sqrt{21609}+196$

Quotients.

$$\frac{196 \mp \sqrt{21609}}{7 \quad\; \sqrt{49}}\Big(\, 28 \mp \sqrt{441}\begin{cases}A\\B\end{cases} \qquad \frac{\sqrt{21609} \mp 196}{-\sqrt{49} \quad -7}\Big(\, -\sqrt{441} \pm 28\begin{cases}C\\D\end{cases}$$

Another Example, with the Varieties in dividing a Rational Number; the like of which is to be done where the Dividend is a *Surd*. As suppose $\sqrt{81}$ be divided by $3+\sqrt{4}$, or $3-\sqrt{4}$, or $\sqrt{4}+3$, or $\sqrt{4}-3$, the several Quotients appear at E. F. G. H. *Other Examples.*

	E *Divisor.*	F *Divisor.*	G *Divisor.*	H *Divisor.*
	Binomial. $3+\sqrt{4}$	*Residual.* $3-\sqrt{4}$	*Binomial.* $\sqrt{4}+3$	*Residual.* $\sqrt{4}-3$
	Residual. $3-\sqrt{4}$	*Binomial.* $3+\sqrt{4}$	*Residual.* $\sqrt{4}-3$	*Binomial.* $\sqrt{4}+3$
	$9+\sqrt{36}$	$9-\sqrt{36}$	$4+\sqrt{36}$	$4-\sqrt{36}$
	$-\sqrt{36}-4$	$\sqrt{36}-4$	$-\sqrt{36}-9$	$\sqrt{36}-9$
Product.	$9-4$	$9-4$	$4-9$	$4-9$
Remain.	5	5	-5	-5
Figurated.	$\sqrt{25}$	$\sqrt{25}$	$-\sqrt{25}$	$-\sqrt{25}$
	Dividend.	*Dividend.*	*Dividend.*	*Dividend.*
	$\sqrt{81}$	$\sqrt{81}$	$\sqrt{81}$	$\sqrt{81}$
	$3-\sqrt{4}$	$3+\sqrt{4}$	$\sqrt{4}-3$	$\sqrt{4}+3$
Product.	$\sqrt{729}-\sqrt{324}$	$\sqrt{729}+\sqrt{324}$	$\sqrt{324}-\sqrt{729}$	$\sqrt{324}+\sqrt{729}$

Quotients.

$$\frac{\sqrt{729} \mp \sqrt{324}}{\sqrt{25}}\Big(\, \sqrt{29\tfrac{4}{25}} \mp \sqrt{12\tfrac{24}{25}}\begin{cases}E\\F\end{cases} \qquad \frac{\sqrt{324} \mp \sqrt{729}}{-\sqrt{25}}\Big(\, -\sqrt{12\tfrac{24}{25}} \pm \sqrt{29\tfrac{4}{25}}\begin{cases}G\\H\end{cases}$$

Example,

Examples in Cubes.

Example, to divide $\sqrt[3]{27}$, by $4+\sqrt[3]{8}$, or $4-\sqrt[3]{8}$, or $\sqrt[3]{8}+4$, or $\sqrt[3]{8}-4$, the several Quotients are as at I. K. L. M.

	I	K	L	M
Divisor.	*Binom.* $4+\sqrt[3]{8}$	*Resid.* $4-\sqrt[3]{8}$	*Binom.* $\sqrt[3]{8}+4$	*Resid.* $\sqrt[3]{8}-4$
	Resid. $4-\sqrt[3]{8}$	*Binom.* $4+\sqrt[3]{8}$	*Resid.* $\sqrt[3]{8}-4$	*Binom.* $\sqrt[3]{8}+4$
	$16+\sqrt[3]{512}$	$16-\sqrt[3]{512}$	$\sqrt[3]{64}+\sqrt[3]{512}$	$\sqrt[3]{64}-\sqrt[3]{512}$
	$-\sqrt[3]{512}-\sqrt[3]{64}$	$\sqrt[3]{512}-\sqrt[3]{64}$	$-\sqrt[3]{512}-16$	$\sqrt[3]{512}-16$
Product.	$16-4$	$16-4$	$4-16$	$4-16$
Remain.	12	12	-12	-12
Figurat.	$\sqrt[3]{1728}$	$\sqrt[3]{1728}$	$\sqrt[3]{1728}$	$\sqrt[3]{1728}$
Dividend.	$\sqrt[3]{27}$	$\sqrt[3]{27}$	$\sqrt[3]{27}$	$\sqrt[3]{27}$
	$4-\sqrt[3]{8}$	$4+\sqrt[3]{8}$	$\sqrt[3]{8}-4$	$\sqrt[3]{8}+4$
Prod.	$\sqrt[3]{1728}-\sqrt[3]{216}$	$\sqrt[3]{1728}+\sqrt[3]{216}$	$\sqrt[3]{216}-\sqrt[3]{1728}$	$\sqrt[3]{216}+\sqrt[3]{1728}$

Quotients.

$$\frac{\sqrt[3]{1728}\mp\sqrt[3]{216}}{\sqrt[3]{1728}}\Big(\sqrt[3]{1}\mp\sqrt[3]{\tfrac{27}{216}}\begin{cases}\text{I}\\ \text{K}\end{cases}\qquad \frac{\sqrt[3]{216}\mp\sqrt[3]{1728}}{-\sqrt[3]{1728}}\Big(-\sqrt[3]{\tfrac{27}{216}}\pm\sqrt[3]{1}\begin{cases}\text{L}\\ \text{M}\end{cases}$$

3. Divisor Compound Binomial, but one part equal to the other.

Case 3. When the Dividend is Simple or Compound, and the Divisor a Compound Binomial, and one *Surd* is equal to the other; then add them together, and by the Total divide the Dividend, as in the first Case, if the Dividend be Compound, or otherwise as a Simple *Surd*. And if such Residual be given for a Divisor whose Parts are equal, because of the contrary Signs, the Value thereof is 0, and so no Division can be made thereby.

Example.

Example, to divide $\sqrt{60}$ by $\sqrt{5}+\sqrt{5}$, both added make $\sqrt{20}$, by which $\sqrt{60}$ divided maketh the Quotient $\sqrt{3}$. But if the Divisor had been proposed $\sqrt{5}-\sqrt{5}$, it being clear the Divisor is 0, no Division can be made thereby but nugatory.

Divisor. $\sqrt{5}$
$\quad 4$
$\sqrt{20}$

Dividend. $\sqrt{60}$
Divisor. $\sqrt{20}$ $\Big(\sqrt{3}$ *Quotient.*

4. Data both Compound.

Case 4. When both Dividend and Divisor are Compound, and the Divisor a Binomial or Residual; so as the Parts of the Divisor be not equal, as aforesaid: Then proceed as in the second Case before.

Examples.

Examples of Binomials, Residuals, and mixt Surds.

To divide $\sqrt{68}+\sqrt{54}$ by $\sqrt{6}+\sqrt{3}$ Binomial, as at N.
To divide $\sqrt{68}-\sqrt{54}$ by $\sqrt{6}-\sqrt{3}$ Residual, as at O.
To divide $\sqrt{456}-\sqrt{72}$ by $\sqrt{18}+\sqrt{6}$ Mixt, as at P.
To divide $\sqrt{456}+\sqrt{72}$ by $\sqrt{18}-\sqrt{6}$ Mixt, as at Q.

	Divisors. N O	*Dividends.* N O
	$\sqrt{6}\pm\sqrt{3}$	$\sqrt{68}\pm\sqrt{54}$
	$\sqrt{6}\mp\sqrt{3}$	$\sqrt{6}\mp\sqrt{3}$
	$6\pm\sqrt{18}$	$\sqrt{408}\pm\sqrt{324}$
	$\mp\sqrt{18}-3$	$\mp\sqrt{204}-\sqrt{162}$
	$6-3$	$\sqrt{408}\pm\sqrt{324}\mp\sqrt{204}-\sqrt{162}$
	3	
	$\sqrt{9}$	

Quotients.

$$\begin{matrix}\textit{Dividends.}\\ \textit{Divisor.}\end{matrix}\ \frac{\sqrt{408}\pm\sqrt{324}\mp\sqrt{204}-\sqrt{162}}{\sqrt{9}}\Big(\sqrt{45\tfrac{1}{3}}\pm\sqrt{36}\mp\sqrt{22\tfrac{2}{3}}-\sqrt{18}\begin{cases}\text{N}\\ \text{O}\end{cases}$$

Divisors.

Divisors.

$$\begin{array}{l} P\ \sqrt{18} \pm \sqrt{6}\ Q \\ \sqrt{18} \times \sqrt{6} \\ \hline 18 \pm \sqrt{108} \\ \quad \mp \sqrt{108} - 6 \\ \hline 18 - 6 \\ \hline 12 \\ \hline \sqrt{144} \\ \hline \end{array} \qquad \begin{array}{l} P\ \sqrt{456} \mp \sqrt{72}\ Q \\ \sqrt{18} \mp \sqrt{6} \\ \hline \sqrt{8208} \mp \sqrt{1296} \\ \quad \mp \sqrt{2736} + \sqrt{432} \\ \hline \sqrt{8208} \mp \sqrt{1296} \mp \sqrt{2736} + \sqrt{432} \\ \hline \end{array}$$

Quotients.

Dividends. *Divisor.* $\dfrac{\sqrt{8208} \mp \sqrt{1296} \mp \sqrt{2736} + \sqrt{432}}{\sqrt{144}} \Big(\sqrt{57} \mp \sqrt{9} \mp \sqrt{19} + \sqrt{3} \begin{cases} P \\ Q \end{cases}$

Case 5. When the Divisor is a Multinomial, in multiplying him by his Residual, one Part, or all the Parts thereof except one, may be made Residual at pleasure, always provided the Parts made Residual be not equal in Value to the other as aforesaid. And after addition of the Multiplees, rejecting what may be rejected, by reason of contrary Signs, add the Remains of the Total into one Number, if it may be, and proceed as in the second and fourth Cases before in this Chapter. 5. *Divisor a Polynomial.*

Example, to divide 54 Integers by $\sqrt{4} + \sqrt{9} + \sqrt{16}$ Rational Numbers, the Divisor may be multiplied either by $\sqrt{4} + \sqrt{9} - \sqrt{16}$, or $\sqrt{4} - \sqrt{9} + \sqrt{16}$, or $\sqrt{4} - \sqrt{9} - \sqrt{16}$, or $\sqrt{9} - \sqrt{4} - \sqrt{16}$, or by any other Residual to be made, by exchanging the Places of the Numbers in the Divisor. And accordingly the Divisor multiplied by any of them, and the Remains brought into one Number, by this Number I divide the Product of 54, multiplied by the same the Divisor was multiplied. See two of the Varieties at R and S. Example.

Divisor. *Divisor.*

$$\begin{array}{l} R\ \sqrt{4} + \sqrt{9} + \sqrt{16} \\ \sqrt{4} + \sqrt{9} - \sqrt{16} \\ \hline 4 + \sqrt{36} + \sqrt{64} \\ \quad \sqrt{36} + 9 + \sqrt{144} \\ \quad\quad - \sqrt{64} - \sqrt{144} - 16 \\ \hline 4 + \sqrt{144} + 9 - 16 \\ \hline \sqrt{144} - 3 \\ \hline \sqrt{81} \\ \hline \end{array} \qquad \begin{array}{l} S\ \sqrt{4} + \sqrt{9} + \sqrt{16} \\ \sqrt{4} - \sqrt{9} - \sqrt{16} \\ \hline 4 + \sqrt{36} + \sqrt{64} \\ \quad - \sqrt{36} - 9 - \sqrt{144} \\ \quad\quad - \sqrt{64} - \sqrt{144} - 16 \\ \hline 4 - \sqrt{576} - 25 \\ \hline - \sqrt{576} - 21 \\ \hline - \sqrt{2025} \\ \hline \end{array}$$

Dividend. *Dividend.*

$$\begin{array}{l} 54 \\ \sqrt{4} + \sqrt{9} - \sqrt{16} \\ \hline \sqrt{11664} + \sqrt{26244} - \sqrt{46656} \\ \hline \end{array} \qquad \begin{array}{l} 54 \\ \sqrt{4} - \sqrt{9} - \sqrt{16} \\ \hline \sqrt{11664} - \sqrt{26244} - \sqrt{46656} \\ \hline \end{array}$$

Quotient.

$\dfrac{\sqrt{11664} + \sqrt{26244} - \sqrt{46656}}{\sqrt{81}} \Big(\sqrt{144} + \sqrt{324} - \sqrt{576}.$ R

Quotient.

$\dfrac{\sqrt{11664} - \sqrt{26244} - \sqrt{46656}}{-\sqrt{2025}} \Big(-\sqrt{5\tfrac{1539}{2025}} + \sqrt{12\tfrac{1944}{2025}} + \sqrt{23\tfrac{81}{2025}}$ S

Case 6. When both Dividend and Divisor are Multinomials, and the Remains of the Total after Multiplication of the Divisor will not be added into one Number as above-mentioned: then divide your given Numbers after the manner of Compound Cossicks, placing the *Surds* orderly; and by every Number gotten for the Quotient, multiply the Divisor, and substract the Product from the Dividend. 6. *Data both Polynomials.*

Example, to divide $9 + \sqrt{180} + 5 - 2$ by $3 + \sqrt{5} + \sqrt{2}$; at the first Application of the Divisor 3 being gotten for the Quotient, I multiply the Divisor thereby, and the Product is $9 + \sqrt{45} + \sqrt{18}$; which substracted from the Dividend, leaves $\sqrt{45} - \sqrt{18}$. Then by Application of the Divisor is gotten for the Quotient $+\sqrt{5}$, by which the Divisor multiplied, produceth $\sqrt{45} + \sqrt{25} + \sqrt{10}$; Example.

this substracted from the Dividend, leaves —W18—W10. And lastly, applying the Divisor, —W2 is gotten for the Quotient, and the Product of the Divisor multiplied thereby is —W18—W10—W4; which substracted leaves 0 remaining, and the Work stands as at W; the other Paragraphs thereof as at T and V.

Dividend.
W 45—W18 T
Divisor. 3+W 5+W 2) 9+W180+5—2 (3 *Quotient.*
3) 9+W 45+W18 (
9+W45+W18

Dividend.
W45-W18-W10 V
Divisor. 3+W 5+W 2) 9+W180+5—2 (3+W5 *Quotient.*
3+W 5) 9+W 45+W18 (
9+W45+W18 W45+5+W10
W45+5+W10

Dividend. W
W45-W18-W10
Divisor. 3+W5 +W2) 9+W180+5—2 (3+W5—W2 *Quotient.*
3+W5 —W2) 9+W45 +W18 (
9+W45+W18 W45 +5+W10
W45+5+W10 —W18—W10—2
-W18-W10-2

When Names are increased and changed in Division.

Thus Division of particular *Surds* give evidence, that every Compound Divisor, by the Work of the 2*d*, 4*th* and 5*th* Cases, increaseth the Names in the Quotient; the one a Binomial, makes the other a Residual, and the Residual begets a Binomial, although the Dividend be single. But if the Dividend be Compound as well as the Divisor, the Quotient shall be a Multinomial. And great heed is to be taken in right placing the Signs, for that according thereto the Quotient is to be valued; all which is plainly to be observed in the foregoing Examples.

Heed to be took in placing the Signs.

Universals Homogeneal, how divided.

Division of Universal *Surds* Homogeneal, imitate the Division of Particular *Surds*, only before the Quotient is to be prefixed the Universal Sign. And if Operation be made according to the sixth Case; then let the Sinister Number of the Divisor upon every Removal be figurate, as the next Dexter Number he is applied to. And if you proceed according to the fourth Case, then in multiplying the Numbers, the Multiplication proper to Universal *Surds* is to be observed; and besides, if the Divisor be Negative, the Order of the Quotient is inverted; for then the Sinister Number of the Quotient shall be substracted from the Dexter, and the Root of the Remainder taken for the Number desired.

Examples.

Example, to divide √:6+W144+W441+W1764 by √:3+W36, upon application of 3 in the Divisor to 6 in the Dividend, 2 is gotten for the Quotient, by which multiplying the Divisor, the Product is 6+W144: Then 3 is squared, and by applying the Product or Square which is 9, to 441, there is W49 gotten for the Quotient; which multiplying the Divisor, produceth W441+W1764, and substracted, leaves 0 remaining. And the Division according to the sixth Case stands as at X. And if Division were made by √:2+W49, the Numbers should be placed thus, √:6+W441+W144+W1764.

But if nine Absolute Numbers (which is the Root Universal of √:6+W144+W441+W1764) be divided by √:3+W36, according to the 4*th* Case; then multiplying √:3+W36 by his Residual √:3—W36, the Numbers to be used for Divisors will be —27 and —W729. And then √:3—W36 multiplying 9, makes the Dividend 243—W236196, and the Quotient will be —9+W324 as at Y. Then because the Divisor is Negative, 9 shall be taken from the Root of 324, and the square Root of the Remainder, which here will be 3, shall be the Universal Root desired, equivalent, though in other Terms, to the Root of the Quotient at X.

Divisor.

Divisor. *Dividend.* *Quotient.*

√:3+W36) √:6+W144+W441+W1764 (√:2+W49 X

√:2+W49 6+W144

W441+W1764

6+W144

W441+W1764

Divisor. *Dividend.* *Quotient.*

√:3+W36 9

√:3—W36 √:3 — W36

9+W324 243—W236196

—W324—36

9—36

—27

—W729

$\frac{243}{-27}$ — $\frac{W236196}{-W729}$ (√:—9+W324 Y

Nevertheless where the Divisor by this last Sort is Affirmative, the Quotient shall be in the former order.

Example, to divide 30 by √:30+W36, whose Root Universal is 6, or by √:30 —W25, whose Root Universal is 5, the Quotients will accordingly be 5 or 6, and therewith agree the Quotients at Z. z. if their Roots be extracted. *Other Examples.*

Divisor. *Dividend.* *Quotient.*

√:30 +W36 30

30 —W36 30 —— W36

900+W32400 27000—W29160000

—W32400—36

900—36

864

W746496

$\frac{27000}{864}$ — $\frac{W29160000}{W746496}$ (√:31$\frac{1}{4}$—W39$\frac{1}{16}$. Z

Divisor. *Dividend.*

√:30 —W25 30

30 +W25 30 + W25

900—W22500 27000+W20250000

W22500—25

900—25

875

W765625

$\frac{27000}{875}$ + $\frac{W20250000}{W765625}$ (√:30$\frac{6}{7}$+W26$\frac{22}{49}$. z

If the given Numbers be Heterogeneal, then proceeding by the *6th* Case, besides upon Removal of the Divisor to figurate the Sinister Number thereof, according to the Dexter Number he is applied to; if the Quotient of this Division be of higher Denomination than the next Dexter Number of the Divisor, then let this Quotient be depressed thereto by extracting the Root, and this Root with his proper Character shall be set in the Quotient of the first Division. *Universals Heterogeneal, how divided.*

Example. If √:4+√ʒφ4096+W16+√ʒφ4096, the Product of √:2+WW8 by √:2+W4, be divided by the first of the said Factors, then upon the Application of the Divisor 2 to W16 he is to be squared; and the Quotient being a Square, and so not above the Dexter Number of the Divisor, which is of the Cube Denomination, this Quotient shall stand for the Quotient desired. But if Division be made by √:2+W4 the other Factor, then upon application of 2 to √ʒφ4096, after 2 is exalted to the Zenzicube Power, and 4096 divided thereby, the Quotient 64 being a Zenzicube, (and so above the Denomination of the next Dexter *Examples.*

Dexter Number of the Divisor which is a Square) is to be depressed to a Cube by extracting the Square Root thereof.

Divisor. *Dividend.* *Quotient.*
√:2+ww8) √:4+√ʒȼ4096+w16+√ʒȼ4096 (√:2+w4

New Divisor. 2×2=w4) 16 (4

Divisor. *Dividend.* *Quotient.*
√:2+w4) √:4+w16+√ʒȼ4096+√ʒȼ4096 (√:2+ww8

2×ʒȼ=64) 4096 (64√ʒ whereof is 8.

If the Universal Root were strictly to be taken in all these Instances, then at X and Y it would be w1. at Z w5/w6. at z w1 1/5, and in the two last Examples w1.

X: w3/w3 (w1 Y
Z: w5/w6 (w5/w6
z: w6/w5 (w1 1/5
√:2+w4=2 / √:2+ww8=2 (w1

Proof of Division of Compound Surds. As the Proof of Division of Simple *Surds* is by their Simple Multiplication, or by Rational Numbers; so will Division of Compound *Surds* be proved by Compound Multiplication, Particular by Particular, and Universal by Universal respectively; multiplying the Quotient by the Divisor to return the Dividend: And also by working with Rational Numbers.

Among the Examples of Particular Surds in this Chapter at A, 49 was divided by 4+w9, which is 7; so shall the Quotient of the Division be 7; the Quotient being 28—w441, multiplied by the Divisor, shall return 49, and the Square Root of 441, which is 21 taken from 28, shall leave 7.

Among the Examples of Universal *Surds* at X, the Root Universal of √:6+w144+w441+w1764 is 9, and the Root Universal of the Divisor √:3+w36 is 3; so shall the Quotient of that Division be 3, the Quotient at X being √:2+w49 agrees: for if the Square Root of 49 which is 7 be added to 2, it makes 9, whose Square Root is 3. As likewise doth the Quotient at Y, for from the Square Root of 324 which is 18, let 9 be taken, and the Square Root of the 9 remaining is also 3. And if those Quotients be multiplied by the Divisor, the Dividends will respectively be returned, as that at X is to be seen in the foregoing Chapter, the other here follow.

Divisor. 4+w9) *Dividend.* 49 *Quotient.*
4—w9 / 4—w9
16—9 / 196—w21609 (28—w441 (√21
7 / 7 w49 / 4+w9
w49
112—w7056
w7056—w3969 (√63
112—63
49

4+3=7) 49 (7
28—21=7

Universal Root.

Divisor. √:3+w36 / or w9) *Dividend.* √:6+w144+w441+w1764 / or w81 (*Quotient.* √:2+w49 / or w9   w81 or 9 / w9 or 3 (w9 or 3.

Divisor.

Divisor.	*Dividend.*	*Quotient.*	
$\surd{:}3+\text{W}36)$	9		$\text{W}9=3)\,9\,(3$
$3-\text{W}36$	$3-\text{W}36$		$\text{W}18-9=\text{W}9=3$
$9-36$	$243-\text{W}236196\,($	$\surd{:}-9+\text{W}324\,(\,\surd 18$	
-27	$-27-\text{W}729$	$3+\text{W}36$	
$-\text{W}729$		$-27+\text{W}2916$	
		$-\text{W}2916+\text{W}11664\,(\,\surd 108$	
		$-27+108$	
		$\text{W}81=9$	

CHAP. XI.

Of Fractionary Surds.

ALthough little mention of Fractions hath been made before in this Part of *Surds*, yet they oft-times arise upon Division of *Surds*, and sometimes with and without Integers are useful among *Surds*: Wherefore it is necessary to remember, that the Operations proper to common Fractions, mixed with the Operations proper to *Surds*, shall add and substract, multiply and divide them, whether Simple or Compound; so as the particular Rules of their several Elements, being nothing but what hath been before shewed, need no further be spoken to here than to give a few Examples. *Fractionary Surds, how to be ordered.*

Fractionary Surds added and substracted. *Examples of Addition and Substraction.*

Examples. $\left.\begin{array}{r}\text{W}\frac{8}{10}\\ \text{W}\frac{7\frac{1}{4}}{4}\\ \text{WW}\frac{7}{12}\end{array}\right\}$ is added to, and substracted from $\left\{\begin{array}{l}\text{W}\frac{50}{10}\text{ at A and B.}\\ 32\text{ Integers at C and D.}\\ \text{WW}\frac{56}{12}\text{ at E and F.}\end{array}\right.$

A . B	C . D	E . F
$\text{W}\frac{50}{10}+\text{W}\frac{8}{10}$ $-$	$32+\text{W}\frac{72}{4}$ $-$	$\text{WW}\frac{56}{12}+\text{WW}\frac{7}{12}$ $-$
$2)\text{W}50\ (\text{W}25\,.\,5$	$8)\text{W}128\ (\text{W}16\,.\,4$	$7)\text{WW}56\ (\text{WW}8\,.\,2$
$\text{W}\,8\ (\text{W}\,4\,.\,2$	$\text{W}72\ (\text{W}9\,.\,3$	$\text{WW}\,7\ (\text{WW}1\,.\,1$
A. $\text{W}\frac{98}{10}$ $\text{W}49\,.\,7$	C. $\text{W}\frac{392}{4}\,.\,\text{W}49\,.\,7$	E. $\text{WW}\frac{189}{12}\,.\,27\,.\,3$ *Sum.*
B. $\text{W}\frac{18}{10}\,.\,\text{W}9\,.\,3$	D. $\text{W}\frac{8}{4}\,.\,\text{W}1\,.\,1$	F. $\text{WW}\frac{7}{12}\,.\,1\,.\,1$ *Difference.*

Fractionary Surds *multiplied and divided.* *Examples of Multiplication and Division.*

Examples, $\left.\begin{array}{r}\text{W}\frac{5}{9}\ldots\ldots\\ 16\text{ Integers}\\ \text{WW}\frac{2}{9}\ldots\ldots\end{array}\right\}$ is multiplied and divided by $\left\{\begin{array}{l}\text{W}\frac{5}{7}\text{ at G and H.}\\ \text{W}2\frac{1}{4}\text{ at I and K.}\\ \text{WW}1\frac{1}{3}\text{ at L and M.}\end{array}\right.$

G. $\text{W}\frac{5}{9}\times\frac{5}{7}=\text{W}\frac{25}{63}$ I. $\overset{\overset{64}{\text{W}256}}{\frac{16}{1}}\times\text{W}2\frac{\overset{9}{1}}{\underset{1}{4}}=\text{W}\frac{576}{1}$ L. $\text{WW}\frac{2}{9}\times\text{WW}1\frac{\overset{4}{1}}{3}=\text{WW}\frac{8}{27}$ Product.

H. $\text{W}\frac{\overset{1}{5}}{7})\,\text{W}\frac{\overset{1}{5}}{9}\,(\text{W}\frac{7}{9}$ K. $\text{W}2\frac{\overset{9}{1}}{4})\,\overset{\text{W}256}{\frac{16}{1}}\,(\text{W}\frac{1024}{9}$ M. $\text{WW}1\frac{\overset{\overset{2}{4}}{1}}{\underset{1}{3}})\,\text{WW}\frac{\overset{1}{2}}{\underset{3}{9}}\,(\text{WW}\frac{1}{6}$ Quotient.

Proof of the Fractionary Operations.

Common with other Fractions are these Operations of *Fractionary Surds* proved, *Addition* by *Substraction*, *Multiplication* by *Division*, and the contrary. Proper to *Surds*, the truth of their Fractionary Operations may be proved, by taking Rational Numbers, and working therewith as in other *Surds*, as may be easily examined.

CHAP. XII.

Figuration of Surds.

Figurate Surds produced.

TO figurate *Surds*, is to multiply them after the manner of *Surds*; for any *Surd* multiplied by himself, produceth the Square and Product again, by the Root bringeth forth the Cube, *&c.* as other Figural Numbers. But this is proper to *Compound Surds*, for any Simple *Surd* multiplied figuratively, produceth a Rational Number, and so ceaseth to be a *Surd*.

Examples.

As to multiply √3 by √3, produceth √9, which is a Rational Number, and hath 3 for the Root.

To multiply squarely the particular Surds √3+√5, the Square produced is 3+√60+5: but the Universal Root squared, as here appeareth, is √:9+√180+√25.

Simple.		*Particular.*		*Universal.*	
Root.	√3	*Root.*	√3+√5	*Root.*	√:3+√5
	√3		√3+√5		√:3+√5
Square.	√9		3+√15		9+√45
			√15+5		√45+√25
		Square.	3+√60+5	*Square.*	√:9+√180+√25

In extraction of Roots to be noted.

Touching the Extraction of the Root of *Surds* is to be minded,

1. Simple Surds can have no Roots. Rationals have their Roots as before extracted.

1. That Simple *Surds* having no Roots to be expressed by Integers or Fractions exactly, can have no Root extracted: but the Rational Numbers set as *Surds*, have their Roots extracted as figural Numbers before spoken to in the second Part of the second Book.

Examples.

And so √2025 shall be 45, and √√729 shall be 9 Integers; because both 2025 and 729 are Rational Numbers, and not proper *Surds*.

Rational. √2025 (4, 16) |45 Root. *Rational.* √√729. |9 Root.

Gnomon. { 40, 25 }

2. Particular Compounds, some in a sort Irradical, what then to be done.

2. That among particular Compound *Surds*, some are in a sort Irradical, and have their Roots extracted only by altering their Characters. This sort have their Sinister Number, either Absolute or a *Surd*.

When the Sinister Number is Absolute. Examples.

If Absolute, then place before the given Number the Character belonging to the Root to be extracted.

As to extract the Square Root of 10+√5, or the Cube Root thereof, they are set as at A and B.

A: Square 10+√5 (√10+√5 Root. B: Cube 10+√5 (√c10+√5 Root.

When the Sinister is a Surd.

If the Sinister Number be a *Surd*, then multiply the Index of the Root to be extracted, by the Index of the Sinister *Surd*, and the Index amounting shall be the Index of the Root, whose Character is to be prefixed before the given *Surd*.

Examples.

As to extract the Square Root of √10+√5, because 2 and 2 make 4, the Index of squared Squares, the Root shall be √√10+√5; sometime set thus, √.√10+√5, and is as much as to say, the Square Root of the Square Root of 10, and the Square Root of 5.

So to take the Square Root of √√18—2, is √.√√18—2. And the like is to be done for the Cube and Higher Powers.

Square

Square √10+√5 (√√10+√5 Root. Square √√18—2 (√ȝ𝔠18—2 Root.
2x2=4 Index ȝȝ 2x3=6 Index ȝ𝔠.

3. Those Extractions of the Roots of particular Compound *Surds*, that are properly Radical, alter the Numbers in Homogeneals, both Numbers and Characters in Heterogeneals. *3. Particular Compounds Radical, what to be done.*

If the Sinister Number be Absolute, and the Dexter a Square *Surd*, then square the Sinister Number, and substract the Dexter Number from it; take the Square Root of the Difference, which add to the Sinister Number, and also substract it therefrom: half the Sum, and half the Difference, joined with the Sign +, shall be the Binomial Root, and with the Sign — shall be the Residual Root. *When the Sinister is Absolute.*

As to extract the Square Root of 7+√40, the Square of 7 is 49; from which 40 taken, leaves 9, whose Square Root is 3, which added to 7, makes 10; the half is 5, and taken from 7, leaves 4; the half is 2: therefore 2+5 shall be the Binomial Root, and 2—5 the Residual Root, or the contrary. Example.

7 Sinister.	7	7	√2+√5	Binomial Root.
7	3	3	√2—√5	Residual Root.
49 ȝ.	10 Sum.	4 Difference.		of 7+√40
40 Dexter.	5 Half.	2 Half.		or 7—√40
9 Difference.			√5+√2	Binomial Root.
3 Root.			√5—√2	Residual Root.

If the given Numbers be Homogeneal square *Surds*, take the one Square from the other, and the Root of the Difference add to the Root of the greater Square, and also substract from it: half the Sum and half the Difference of these Roots joined with the Sign +, shall be the Binomial Root, and with the Sign — shall be the Residual Root as before. *When the Sinister is a Surd.*

For suppose the Numbers given were √49+√40, or √40+√49: then 40 taken from 49, leaves 9; whose Root 3 added to and substracted from 7, the Root of 49, the Greater, makes the Sum and Difference, and consequently the Halfs, and the Binomial and Residual Roots as before, because the Product of √2+√5, or √5+√2 multiplied squarely, is alike. Example.

49 Greater } Square.	7 Root.	7 Root.	√5±√2 } Roots { Binom.
40 Lesser }	3	3	√2±√5 } { Residual
9 Difference.	10 Sum.	4 Difference.	of √49±√40
3 Root.	5 Half.	2 Half.	or √40±√49

If the given Numbers be Cubes or Higher Powers, or an Absolute Number, and a Cube, or Higher Power, take the Roots of the Numbers, and work as if they were Square Roots till you get the Halfs as before; then advance the Roots of those Halfs into the Powers of the Denominations given, or the Dexter Denomination, if but one. *When the Data are Cubes or Higher Powers.*

As if the Root be desired of 13+√√1728, or √√2197+√√1728, the Cube Root of 2197 is 13, and of 1728 is 12; both Roots squared are 169 and 144, the Difference 25, whose Square Root 5 added to 13, makes 18, the Half thereof is 9, whose Root is 3; the said 5 taken from 13, leaves 8, the half whereof is 4 whose Root is 2: these Roots 2 and 3 advanced to be Cubes, according to the Denominations given, are the desired Numbers. Examples.

√√

ww 2197 | √ 13x13 = ʒ 169
1
Gnomon { 9, 27, ...27

13
5
18 Sum.
9 Half.
3 Root.
27Φ

13
5
8 Differ.
4 Half.
2 Root.
8Φ

ww8 + ww27 Binom. Root.
ww8 — ww27 Resid. Root.
of 13 + ww1728
or 13 — ww1728

ww 1728 | √ 12x12 = ʒ 144
1
Gnomon { 6, 12, ...8
25 Difference.
5 Root.

ww27 + ww8 Binom. Root.
ww27 — ww8 Resid. Root.

When the Sinister is Absolute, and the Dexter an Higher Power.

If the Sinister Number be Absolute, and the Dexter an Higher Power than a Square, different Roots may be had according to the Powers into which the Roots of the Halfs be exalted; for the same Value in the Roots produce Products of like Value, though different in Terms.

Example.

As in the last Example, where the Roots of the Halfs were 3 and 2, let them be both squared, as W9 + W4, or the one squared and the other cubed, as W9 + ww8, or ww27 + W4; any of these shall produce Products alike valuable with the Product of ww27 + ww8, but all different in Terms.

When the Dexter higher Power may have the Index halfed.

Yet if the Higher Power on the Dexter Part of the given Number may have the Index evenly halfed, it is most usual to exalt the Root of one half into the Power answering to half the Index of the Dexter Number, and take the other half for the next Inferior Power thereto.

Example.

As suppose the Binomial and Residual Roots be desired of 29 + √ʒΦ64000000, the Zenzicube Root of 64000000 is 20; which squared is 400, and 29 squared is 841, the Difference is 441, whose square Root is 21; this added to and substracted from 29, makes the Sum 50, the Difference 8, whose Halfs are 25 and 4, their Roots 5 and 2: Of which may be made several Roots, but most usual thus; because the Index of ʒΦ which is 6, may be halfed, 3 the Index of Cube shall be taken for one Root; then shall the other half be a Square, which is next inferior to the Cube.

29 + √ʒΦ64000000 | Φ 8000 | √ 20

ʒ 29 x 29 = 841
20 x 20 = 400
441 | √ 21

29
21
50
25 Half.
5 √

29
21
8
4 Half.
2 √

W4 + ww125 Binomial } Root.
W4 — ww125 Residual } Root.
Or,
W25 + ww8 Binomial } Root.
W25 — ww8 Residual } Root.

4. Universal Surds, how to extract their Roots. When the Sinister is Absolute.

4. The Roots of Universal *Surds* are extracted in all things like those particular Compound *Surds*, properly Radical, last spoken to, till you come to the Halfs: Then for those that are Homogeneal, the Root of half the Sum shall be the one Root desired, and the half Difference the other, this Dexter, and that Sinister. But if the Root of the half Difference be made the Sinister, then the half Sum shall be the Dexter.

Example.

As to extract the Root Universal of 25 + W576, the Square of 576 taken from 625 the Square of 25, leaves 49 whose Root is 7; which added to 25 makes 32, whose half is 16, and the Root thereof 4 for the one Root sought: Then 7 taken from 25 leaves 18, whose half is 9 for the other Root sought. Or if 3 the Root of 9 be the Sinister, then 16 shall be the Dexter.

25+W576 25 25 √:4+W9 } Root { Binomial.
25 7 7 √:4—W9 } Root { Residual.
125 32 Sum. 18 Difference.
50 16 Half. 9 Half.
625 4 Root. 3 Root. √:3+W16 } Root { Binomial.
576 √:3—W16 } Root { Residual.
49 Difference.
7 Root.

If the Root of Universals be sought, that have the Sinister Number given Absolute, and the Dexter an higher Power than a Square: Then proceed as above till the Roots of the Halfs be obtained, and thereby different Roots may be had according to the Powers into which the Roots of the Halfs are advanced. But most usual it is to exalt the Root of one half into the Power answering to half the Index of the Dexter Denomination given, if the same Index may be equally halfed: And take the Root of the other half, with the Character of the next inferior Power thereto, for the other part of the Root desired. *When the Dexter is an higher Power.*

As if the Roots Binomial and Residual be desired of 41+√ʒΦ4096000000, the Zenzicube Root of 4096000000 is 40, which squared is 1600, taken from 1681 the Square of 41 leaves 81, whose Root is 9; which added to and substracted from 41, makes the Sum 50, the Difference 32, the Halfs whereof are 25 and 16, whose square Roots are 5 and 4. Then because the Index of Zenzicube, which is 6, may be equally halfed into 3, the Index of the Cube, either 5 or 4, may be cubed, and the other that is not shall be set with the Character belonging to the Square. *Example.*

ʒΦ Φ √
41+√ʒΦ4096000000|64000|40
ʒ
41×41=1681 41 41
40×40=1600 9 9
√
81|9 50 32
25 Half. 16 Half.
5 √ 4 √

√:4+ww125 } Root { Binomial.
√:4—ww125 } Root { Residual.
Or,
√:5+ww64 } Root { Binomial.
√:5—ww64 } Root { Residual.

Besides the Proof of Extraction of these Roots by the Production of the *Surds*, and their Production by Extraction, Simple by Simple, Particular by Particular, and Universal by Universal reciprocally: The Truth of all may be tried, by taking Rational Numbers, and working with them instead of the *Surds*; nevertheless for brevity sake Examples thereof are omitted here. *Proof of Figuration of Surds.*

Partis quintæ Libri tertii

FINIS.

The Sixth PART of the Third BOOK.

CHAP. I. *Of Species.*

Species the last sort of special Contracts.

SPECIES, as the sixth Sort of Contract Numbers, and Third of those whose Denominations are uncertain, come now to be inspected in the Close of this third Book.

Whence the Name. What Species are. Characters used, and why.

According to the Name, (which with the Latins serveth for the Figure, Form or Shape of any thing) *Species* are Quantities or Magnitudes, denoted by Letters, signifying Numbers, Lines, Lineats, Figures Geometrical, *&c.* And for avoiding the prolix and often rescription of Words, several Marks or Characters are used therewith for Terms Artificial: Some whereof, most customary, are already specified, *Book* 1. *Part* 1. *Chap.* 3. which being remembred, may save their transcribing here.

The same Letter denotes different Species.

Further let it be remembred here, that A while the Question is in, may signify any Number of Pounds, Yards, Ells, Length, Breadth, *&c.* But when the Work is ended, and another Question begun, A may denote another Quantity different from the former. And the like is to be understood of B. C. D. or any other *Species.*

Powers how differenced from plain Numbers.

It is also necessary to observe, That sometime to difference Powers (with whom *Species* also converse) from plain Numbers, the Letters for these are commonly Capital, but for those Small. Likewise given Quantities or Numbers, some will have noted with Consonants, and those sought with Vowels; but this is not essentially necessary, so as by any Distinction the *Data* and *Quæsita* be discerned apart.

Form of the Species to be kept while the Question is working.

Moreover, while a Question is working, the Form of the Letters is strictly to be kept: For if A and E be two given Numbers, then AE, or Æ, shall be the Product, Z the Sum, *&c.* But if *a* and *e* be the given Numbers, then *ae*, or *æ*, shall be the Product, *z* the Sum, *&c.*

The same Species differently called.

One and the same Letter in the beginning and end of a Question may be differently called: As in Extraction of Roots and Equations, hereafter spoken to in the 4*th* Book, A in working the Question is called the Supposititious or Quesitious Root; but when the Root is found, or the Question brought to an Equation, A shall be called the Eductitious or Resolved Root.

Figures prefixed make a Number of Quantities, &c.

As every single Letter signifies a certain Quantity or Magnitude, so by prefixing of Figures to their left Hand, there shall be accordingly made a number of Quantities or Magnitudes after the manner of Cossicks. Wherefore if A signify one Yard, B one House, *&c.* then 10A shall signify 10 Yards, and 19B 19 Houses, *&c.* But if A signify 10 Yards, B 10 Houses; then shall 10A signify 100 Yards, and 19B 190 Houses; and so of others.

Nature of Species. 1. Whole, and they, Homogeneal, or Heterogeneal.

Touching the Nature of *Species*, they are diversly considered.

First, As they are Whole or Integral, Broken or Fracted.

Integral, as A. B. C. *&c.* 4A. 5B. 3C. *&c.* These are also

Homogeneal, as A and A, that is 2A. Or,

Heterogeneal, as A and B, that is A+B.

Broken, & they

Fracted, as $\frac{2}{3}$A. $\frac{A}{B}$. $\frac{3B}{4A}$ or the like. These are also

Commensurable,

Commensurable, as $\frac{BA}{BC}$. $\frac{15BD}{27BD}$ *&c.* Or,

Or Incommensurable.

Incommensurable, as $\frac{BD}{C}$. $\frac{3AB}{4E}$ *&c.*

All

All fracted *Species* may also be divided as Common Fractions, into Proper, Improper, and Equal Fractions; and the Proper into Conjunct and Divided, or Fractions of Fractions, as *Book* 1. *Part* 2. *Chap.* 1. *Fractions also divided as others.*

Examples of Fractions.

Examples of
- Equal, as $\frac{A}{A}$ or $\frac{B}{B}$. *&c.* which is always an Unit.
- Improper, as $\frac{BD+C}{D}$, *&c.* when the Numerator is the greater *Species*.
- Proper $\left\{\begin{array}{l}\text{Conjunct, as } \frac{C}{CA} \text{ and } \frac{B}{CA}, \text{ \&c.} \\ \text{Divided, as } \frac{C}{CA} \text{ of } \frac{B}{CA}, \text{ \&c.}\end{array}\right\}$ when the Denominator is the greater.

2. *Species* both Integral and Fracted, are considered as they are Simple or Compound. *2. Simple, and they*

Simple, are such as have some one single *Species* and no more,
Whether
Integral, as 6A. 8B. 9C. *&c.* Or, *Whole, or*
Fracted, as $\frac{1}{2}$A. $\frac{2}{3}$B. $\frac{3}{4}$C. *&c.* These, as was noted in Cossicks before, may be represented as Compound, by placing the *Species* to the Numerator, and leaving the Denominator solitary, for $\frac{3}{4}$A is all one with $\frac{3A}{4}$. *Broken.*

Compound, consists of several *Species*. And these are also *Compound, and they*
Integral, conjoined with the Signs + or —; or both, As
Binomials, A+E . B+C . B+D. *&c.* *Whole, as Binomials.*
Residuals, A—E . B—C . B—D. *&c.* *Residuals.*
Polynomials, A+E—B . A—E+B. *&c.* Or, *Polynomials.*
Fracted, compound in *Species*, or Signs, or both; as *Broken, as*
Dual, compound in *Species* $\frac{A}{B}$. $\frac{2B}{3D}$. *&c.* *Dual.*

Plural, compound in Signs $\frac{A+B+E}{D+C}$. $\frac{A-B-E}{D-C}$. *&c.* *Plural.*

Mixt, compound in both $\frac{A+B-E}{D-C}$. $\frac{A-B+E}{D+C}$. *&c.* *Mixt.*

3. *Species*, both Simple and Compound, are considered as they are Plain or Figurate. *3. Plain or*

Plain, as A. B. C. or any other *Species* Integral or Fracted, below the Power of any Number.

Figurate, as Aq. Bq. or Ac. Bc. or any other Figural *Species*. *Figurate.*
These are also divided into Rational or Irrational. *These are Rational or*
Rational, are such as denote some figural rooted Number, or Quantity of figural rooted Numbers: The former resembling the figural rooted Numbers handled in *Book* 2. *Part* 2. The latter Cossicks in the 4th Part of this third Book: Those without Numbers annexed; these with Numbers prefixed to their left Hand.

As Aq. Ac. Aqq. *&c.* Bq. Bc. Bqq. *&c.*
2Aq. 4Ac. 5Aqq. *&c.* 3Bq. 4Bc. 6Bqq. *&c.*
$\Bigg\}$ Figural and Cossical *Species*.

Irrational *Species*, as Surds, treated of before, have no Roots to be expressed by Absolute Numbers. *Irrational.*

As $\sqrt{q}5$. $\sqrt{c}7$. *&c.*
$\sqrt{q}B$. $\sqrt{c}BA$. *&c.*
$\Bigg\}$ Surd and Irrational *Species*.

Both Rational and Irrational figurate *Species* are capable of like Divisions with Cossicks and Surds into Whole and Broken, and either sort into Simple and Compound. Compound again into Binomial, Residual and Polynomial, Homogeneal and Heterogeneal, Symmetral and Asymmetral; and their Fractions into Single, *Both admit like Divisions as Cossicks and Surds respectively.*

Dual

Dual and Plural: And all are ſometime intermixt, and aſſume Society one with another, both Plain and Figural; of all which Examples will but take up room.

Notes of Figurate Species.

Touching Figurate *Species*, let be further noted:

1. Species for all Powers compoſed of the two Prime.

Firſt, That the *Species* for all the Powers, are compoſed of the *Species* for the two Prime Powers, *viz.* the Square and Cube. The *Species* for the Square or Quadrat being q, and for the Cube c: Of theſe are all the others compounded according to the Addition of their Indices. So ſhall the 4*th* Power be noted by

Examples.

qq. the 5*th* by qc. the 6*th* by cc. *&c.* The proper *Species* for Root is l, ſignifying *Latus*, latin for *a Side*; yet the Character √ is uſed as before for Root, and √: for Root Univerſal, and other Characters for Surds occaſionally.

Indices.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	*&c.*
Species.	l.	q.	c.	qq.	qc.	cc.	qqc.	qcc.	ccc.	qqcc.	qccc.	cccc.	*&c.*
Powers in Numbers.	2.	4.	8.	16.	32.	64.	128.	256.	512.	1024.	2048.	4096.	*&c.*
Coſſical Characters.	℞.	ʒ.	φ.	ʒʒ.	ſs.	ʒφ.	Bſs.	ʒʒʒ.	φφ.	ʒſs.	Cſs.	ʒʒφ.	*&c.*
Example in the Root A.	A.	Aq.	Ac.	Aqq.	Aqc.	Acc.	Aqqc.	Aqcc.	Accc.	Aqqcc.	Aqccc.	Acccc.	*&c.*

For A multiplied into A, or A×A, or AA, is Aq. So A×A×A, or AAA, or AqA, is Ac; and the like is to be underſtood of others.

2. Magnitudes Parodical.

Secondly, All Magnitudes under the Power propoſed, are called *Parodical to the Power*; as A. Aq. Ac. Aqq. are Parodical to Aqc.

3. Coefficient, what.

Thirdly, If the Parodical Degree have a known Magnitude joined therewith, it is called the *Coefficient*, as in Aq—AB, where B is the Coefficient, and A joined to it, is the Parodical Degree under Aq.

Signs uſed as before.

The Signs + and —, with or without Aſteriſk, are uſed in all things, as before in *Coſſicks* and *Surds*.

Examples of ſome Species, and what underſtood thereby.

By theſe previous Directions it will not be difficult to underſtand both the Name and Nature of any given *Species*. And ſuppoſing ſome Abſolute Number for the Prime *Species* in any Operation, the Value of the reſt will be ſeen as in a *Speculum*. For if A and E ſignify two Numbers, and you appoint A the Greater, and E the Leſſer; as let A be 3, and E 2, then ſhall

Species.			*Value.*
Z. or A+E,	ſignify the Sum which is	(by Addition)	3+2= 5
X. or A—E,	Difference	(by Subſtraction)	3−2= 1
Æ. AE. or A×E	Product.	(by Multiplication)	3×2= 6
$\frac{R}{S}$.	Ratio, or Subratio	(by Diviſion)	2) 3 (= $\frac{3}{2}$
Aq.	Greater Square		3×3= 9
Eq.	Leſſer Square		2×2= 4
Ac.	Greater Cube		3×3×3=27
Ec.	Leſſer Cube		2×2×2= 8
Z.	Sum of the Squares		9+4=13
X.	Difference of the Squares		9−4= 5
Z·	Sum of the Cubes		27+8=35
X·	Difference of the Cubes		27−8=19
AqE.	Greater Square multiplied by the leſſer Number		9×2=18
EqA.	Leſſer Square multiplied by the greater Number		4×3=12
AqEq.	Greater Square multiplied by the Leſſer		9×4=36
AcEc.	Greater Cube multiplied by the Leſſer		27×8=216
$\frac{Aq}{Eq}$.	Greater Square divided by the Leſſer		4) 9 (=2¼
$\frac{Ac}{Ec}$.	Greater Cube divided by the Leſſer		8) 27 (=3⅜

Species.

Species.		Value.
$ZA+Aq$.	Sum of the two Numbers multiplied by the Greater, and added to the greater Square	$5\times3+9=24$
$ZA-Aq$.	Sum of the two Numbers multiplied by the Greater, and made less by the Square of the Greater	$5\times3-9=6$
$ZA+Eq$.	Sum of the two Numbers multiplied by the Greater, and added to the lesser Square	$5\times3+4=19$
$ZA-Eq$.	Sum of the two Numbers multiplied by the Greater, and made less by the Square of the Lesser	$5\times3-4=11$
$ZE+Aq$.	Sum of the two Numbers multiplied by the Lesser, and added to the greater Square	$5\times2+9=19$
$ZE-Aq$.	Sum of the two Numbers multiplied by the Lesser, and made less by the greater Square	$5\times2-9=1$
$ZE+Eq$.	Sum of the two Numbers multiplied by the Lesser, and added to the lesser Square	$5\times2+4=14$
$ZE-Eq$.	Sum of the two Numbers multiplied by the Lesser, and made less by the lesser Square	$5\times2-4=6$
$\frac{SqAq}{Rq}$.	Square of the lesser Proportional multiplied by the Square of the greater Number, and the Product divided by the Square of the greater Proportional	$\frac{4\times9}{9}=\frac{36}{9}=4$
$\frac{RqEq}{Sq}$.	Square of the greater Proportional multiplied by the Square of the lesser Number, and the Product divided by the Square of the lesser Proportional	$\frac{9\times4}{4}=\frac{36}{4}=9$
$Zq+Eq-2ZE$ or $Zq-2ZE+Eq$	Sum of the 2 Numbers multiplied by the Lesser, and the Product doubled, is to be substracted from the Sum of the 2 Numbers squared and added to the lesser Square	$25+4-20$ or $25-20+4$ $=9$
$Zq\overset{*}{-}2ZE+Eq$.	Double Sum of the 2 Numbers multiplied by the Lesser, and added to the lesser Square, substracted from the Sum of the 2 Numbers squared	$25\overset{*}{-}20+4=1$
$\sqrt{Z.-Aq}$.	Square of the greater Number substracted from the Sum of the Squares of the 2 Numbers, and the square Root of the Remain is to be taken	$\sqrt{13-9}=\sqrt{4}-2$
$\frac{\sqrt{Z.Rq-RqAq}}{Aq}$.	Sum of the Squares multiplied by the Square of the greater Proportional, made less by the same Square, multiplied into the Square of the greater Number. The Remain is to be divided by the Square of the greater Number, & the square Root of the Quot. to be taken.	$\frac{\sqrt{13\times9-9\times9}}{9}=\frac{36}{9}\left(\sqrt{4}=2\right.$
$\frac{Z}{2}+\frac{\sqrt{Zq-4\text{Æ}}}{4}$ or $\frac{Z}{2}+\frac{\sqrt{Zq-4P}}{4}$	Half the Sum of the 2 Numbers added to the square Root of the Sum squared, made less by 4 times, the Rectangle or Product divided by 4 (for P is sometime the Species for Product as well as Periphery).	$\frac{5}{2}+\frac{\sqrt{25-24}}{4}$ that is $2\frac{1}{2}+\sqrt{\frac{1}{4}}=3$

Species.		*Value.*
$\sqrt{:}\frac{Z.}{2} - \frac{W Z.q - 4Pq}{4}$	The Universal square Root of half the Sum of the Squares, made less by the square Root of the Sum of the Squares squared and lessened by 4 Rectangles squared and divided by 4.	$\sqrt{:}\frac{13}{2} - \frac{W 169 - 144}{4}$ or $\sqrt{:}\frac{13}{2} - W\frac{25}{4}$ or $\sqrt{:}\frac{13}{2} - \frac{5}{2} = \frac{8}{2} = W4 = 2$
$E = \frac{Zq - X.}{2Z}$	The lesser Number is equal to the Quotient of the Sum of both Numbers squared, made less by the Difference of the Squares, and the Remain divided by the doubled Sum ——	$2 = \frac{25-5}{10}$ or $2 = \frac{20}{10} = 2$
$A = \frac{X. + Xq}{2X}$.	The greater Number is equal to the Difference of the Squares, and the Difference of the Numbers squared & divided by the same Difference doubled.	$3 = \frac{5+1}{2}$ or $3 = \frac{6}{2} = 3$
$2Aq - 2XA = Z. - Xq$	The Square of the greater Number doubled, made less by the greater, multiplied by the Difference doubled, is equal to the Sum of the Squares made less by the Difference squared.	$9+9-6 = 13-1$ or $18-6 = 12$

Thus with wonderful Variety, and unimaginable Celerity, may *Species* be written and beheld, of both which this is but a Drop; yet may serve as an Introduction, as well to their Knowledg and true Position, as for a Basis to the Resolution of several Propositions deducible therefrom; which is easily discernable by the different *Species*, whereby one and the same Number in value may be expressed. For if the Sum of two Numbers be Z, and the two Numbers A+E, then shall A+E be equal to Z and Z to them. So in like manner shall other *Species* of different Forms be equal in value, of which more in the next Book among *Equations*.

The same Number in value expressed by different Species.

CHAP. II. *Addition of Integral and Rational Species.*

Addition of Integral and Rational Species.

THE *Addition of Integral and Rational Species*, whether Sole or Mixt, with Integers, Simple or Compound, may be comprised under the four Cases following.

1. Homogeneal, and of like Signs.

Case 1. If the *Species* be Homogeneal, and of like Signs, then add the Number of the *Species* as if they were Integers, and annex to the Total the same Signs.

Examples.

As A added to A, shall be 2A; and —A added to —A, shall be —2A. The like is to be observed in the rest of the Examples following.

Addends	A	5A	—A—B	A+B	3C—E
	A	A	—2A	A+B	2C—E
Total.	2A	6A	—3A—B	2A+2B	5C—2E

2. Homogeneal, and of unlike Signs.

Case 2. If the *Species* be Homogeneal and of unlike Signs, then substract the lesser Number from the Greater, and to the Difference subscribe the Sign of that *Species* wherein the Excess lieth.

Examples.

As to add 2A to —3A, the 2 taken from 3, leaves 1; which is here to be Negative, because 3 which exceeded 2 was Negative. See further in the following Examples.

Addends.	A	5A	3B—C	2D—E	A+3C	
	—A	—3A	—5B	—D+E	—2A—2C	
Total.	0A	2A	—2B—C	D	—A+C	Remaining.

Case

Case 3. If the *Species* be Heterogeneal, then conjoin the *Species* to be added with their proper Signs. *3. Heterogeneal.*

As A added to B, ſhall be A+B; but −A added to B, ſhall be B—A. This is further clear in the Examples enſuing. *Examples.*

Addends.	A E	4B 5D	—A —4B	A+E B	A—B —E	Ac+A Aq
Total.	A+E	4B+5D	—A—4B	A+B+E	A—B—E	Ac+Aq+A

Case 4. If the *Species* be mixt Homogeneal and Heterogeneal, with Signs like and unlike; then as the Caſe is, ſo ſhall the Addition be. *4. Mixt.*

As A+B added to A—B, the *Species* B being of contrary Signs by the ſecond Caſe, are to be ſubſtracted one from the other, and ſo o will remain. But A added to A, makes the Total 2A by the firſt *Case*. *Examples.*

Alſo A+B added to A—C, the *Species* B and C being Heterogeneal, are by the third *Case* to be conjoined with their proper Signs, and A added to A by the firſt *Case*, makes the Total 2A+B—C.

More Examples of this kind follow.

Addends.	—B+C C—D	A+D A—5D	3B—A 5C+B	A—B+C —A—B	A+B—C A—B+2C
Total.	2C—B—D	2A—4D	4B+5C—A	C—2B	2A+C

As in other Numbers, ſo in *Species*, *Substraction* will prove the Truth of *Addition*, as in the next Chapter appeareth. *Proof of Addition of Int. and Rat. Species.*

Moreover, ſuppoſing the *Species* to be Abſolute Numbers, compare the Addition of the one with the other, and the Truth of the Work will appear thereby. As in the laſt Example where the Total is 2A+C, ſuppoſing A 10, and C 6, then ſhall the Total be 26, and ſo ſhall anſwer the Value of the *Species* to be added when the Quantities — are taken from them with +.

Suppoſing A=10. B=5. C=6. Then ſhall

Species	A+B—C	=10+5—6	=9 Value.
	A—B+2C	=10—5+12	=17
	2A+C	20+6	26

CHAP. III. *Substraction of Integral and Rational Species.*

INtegral and Rational *Species* are ſubſtracted under like Caſes with *Addition*. *Substraction of Int. and Rat. Species.*

Case 1. If the *Species* be Homogeneal and of like Signs, then abate the Quantities to be ſubſtracted from the other; and to the Difference prefix the Common Sign, except the Greater be the *Subtrahend*; then prefix the contrary Sign to the Difference. *1. Homogeneal, and of like Signs.*

As to take 2A from 3A, there reſts 1A. But to take 3A from 2A, there will remain —1A, becauſe the greater Quantity is the Subtrahend. More of like ſort the following Examples ſhew. *Examples.*

From	A	3A	A	E	3A+5B	3C—E
Subſtract	A	A	4A	2E	4A+B	2C—E
Remaineth	oA	2A	—3A	—E	4B—A	C

Case 2. If the *Species* be Homogeneal and of unlike Signs, then add the Quantities together, and to the Total prefix the Sign of the *Species* from which Subſtraction is made. And if any odd *Species* have none to fellow him, annex him to the Remain with his own Sign if in the upper Number; but with the contrary Sign if in the *Subtrahend*. *2. Homogeneal, and of unlike Signs.*

As to take 2A from —3A, the Remain ſhall be —5A. But —2A taken from 3A, ſhall leave remaining 5A. The like may be obſerved in the Examples following. *Examples.*

From

From	A	−3A	3B—C	−5B	2D—E
Subſtract	−A	5A	−5B	3B—C	−D+E
Remaineth	2A	−8A	8B—C	−8B+C	3D—2E

3. Heterogeneal. *Caſe* 3. If the *Species* be Heterogeneal, then conjoin the *Species* to be ſubſtracted with the contrary Signs.

Examples. As to take E from A, makes the Remain A—E; but —E taken from A, makes the Remain A+E. See further in the following Examples.

From	4B	—A	A+E	A	A	Aq
Subſtract	5D	—4B	B	B—C	B+C	A
Remaineth	4B—5D	—A+4B	A+E—B	A—B+C	A—B—C	Aq—A

4. Mixt. *Caſe* 4. If the *Species* be mixt Homogeneal and Heterogeneal, with Signs like and unlike; then as the Caſe is, ſo ſhall the Subſtraction be.

Examples. As to take C—D from —B+C, the *Species* C being Homogeneal, and of like Signs, the one taken from the other, by the firſt *Caſe*, leaves the remaining Quantity cleared of both. And —D ſubſtracted from —B being Heterogenal, makes the Remain —B+D by the third *Caſe*.

Alſo A+D ſubſtracted from A—5D, the *Species* A in both Homogeneal, and of like Signs, leaves o remaining of that Quantity by the firſt *Caſe*. But +D taken from —5D, leaves —6D by the ſecond *Caſe*. The like is to be obſerved in the Examples enſuing.

From	2A—4D	4B+5C—A	C—2B	2A+C	3Aq—3BA+CD
Subſtract	A—5D	5C+B	−A−B	A—B+2C	CD—Dq—Aq
Remaineth	A+D	3B—A	A−B+C	A+B—C	4Aq—3BA+Dq

Proof of Subſtraction of Int. and Rat. Species. The Quantities from which Subſtraction is made in four of theſe laſt Examples, being the Total of the Additions in the fourth *Caſe* of the foregoing Chapter: And the Remains here being one of the Addends there, and the Subtrahends here the other; ſufficiently ſhew the Proof of Addition by Subſtraction, and Subſtraction by Addition, without farther Example.

Alſo ſuppoſing the *Species* to be Abſolute Numbers, compare the Subſtraction of the one with the other; and by their exact Agreement in Value of the Remains, will the Truth of the Work be made manifeſt.

As in the laſt Example ſave one, where the Remain is A+B—C, ſuppoſing A10, B5, and C6, then ſhall the Remain be 9, that is 15 lacking 6, and ſo accordingly will remain, when the Value of the *Species* to be ſubſtracted is taken from the other.

Suppoſing A = 10. B = 5. C = 6. Then ſhall

2A + C	= 20 + 6	= 26.
A−B+2C	= 10—5+12	= 17
A+B—C	10+5—6	9

CHAP. IV. *Multiplication of Integral and Rational Species.*

Multiplication of Int. and Rat. Species. FOUR *Caſes* comprehend all needful to the *Multiplication of Integral and Rational Species*, whether Homogeneal or Heterogeneal, Simple or Compound, with or without Integers; in all which, as before in *Coſſicks* and *Surds*, like Signs produce +, and unlike —.

1. Simple and Homogeneal. *Caſe* 1. If the *Species* be Simple and Homogeneal, then to the right Hand of one of the given Quantities adjoin q, which ſignifieth a Quadrate or Square; becauſe any Number multiplied by himſelf, produceth the Square thereof.

As

As to multiply A by A. B by B, *&c.* they are set thus. Examples.

Multiplicands.	A	B	C	*&c.*	Roots.
Multipliers.	A	B	C		
Products.	Aq	Bq	Cq		Squares.

Case 2. If the *Species* be Simple and Heterogeneal; then set one of the given *Species* besides the other, in a Right Line, from the Right Hand to the Left. *2. Simple and Heterogeneal.*

As to multiply A by B, the Product shall be AB or BA; and sometimes set with the Sign of Multiplication between them thus, A×B or B×A. Other Examples of this sort follow. Examples.

Multiplicands.	B	A	D	——B	*&c.*
Multipliers.	C	E	A	A	
Products.	BC	AE	AD	—AB	

Case 3. If the *Species* have any Numbers joined with them, or annexed to them by the Signs + or —: then multiply the Numbers as Integers, and the *Species* as *Species.* *3. Species with Numbers.*

Examples.

Multiplicands.	3E	4A	2B+1
Multipliers.	2E	3E	A
Products.	6Eq	12Æ	2BA+A

Case 4. If the *Species* or one of them be Compound, or one of the Factors be some multiplied *Species*; then multiply every Quantity in the Multiplicand, by every Quantity in the Multiplier, as before in Compound Cossicks, Integers as Integers, and Species as Species. And place the Multiplees orderly from the Left Hand forwards to the Right, whether the Signs be more or less. And when the *Species* are multiplied by themselves, or any other Power by the Root, or by some higher Power, whereby figural Numbers arise; let the Character of the Magnitude or Power arising by the Addition of their Indices, be added to the Product. *4. Compound Species.*

Examples.

Multiplicands.	AE	A+E	A—E	A+E—I
Multipliers.	AE	B	B	Z
Products.	AqEq	BA+BE	BA—BE	ZA+ZE—ZI

Multiplicands.	A+BE	A—B	A+E	A+E
Multipliers.	B	BA	A+E	A—E
Products.	BA+BqE	BAq-BqA	Aq+AE AE+Eq	Aq+AE —AE—Eq
			Aq+2AE+Eq	Aq— Eq

Multiplicands.	3A—2E	5A+CD
Multipliers.	4B—C	3BA—2CD
	12BA—8BE —3CA+2CE	15BAq+3BACD —10DCA—2CqDq
Products.	12BA—8BE—3CA+2CE	15BAq+3BACD—10DCA-2CqDq

Species, as other Numbers, prove the Truth of their Multiplications by Division, as will appear in the next Chapter. And by supposing the *Species* to be Absolute Numbers, the Product of their Multiplication will agree with the Product of their *Species* multiplied, if the Work be right. *Proof of Multiplication of Int. & Rat. Species.*

As in the last Example, if A be 3, B 4, C 5, D 6, then shall 5A be 15, BA 12, and 3BA 36, CD 30, and 2CD 60. So will the Multiplicand being 5A+CD be 15+30 or 45. And the Multiplier being 3BA—2CD, will be 36—60, that is —24; by which 45 multiplied, makes the Product —1080; and so is the Value of the other Product at the supposed Rate aforesaid, when the *Species* with + are taken from those with —.

Supposing A=3. B=4. C=5. D=6. Then shall

5A + CD ============	5×3 + 5×6 == 15 + 30 == 45
3BA — 2CD ============	3×4×3 — 2×5×6 = 36 — 60 = −24
15BAq + 3BACD ===========	15×4×9 + 3×4×3×5×6 = 540 + 1080
— 10DCA — 2CqDq =======	— 10×6×5×3 — 2×25×36 = −900 − 1800
15BAq + 3BACD — 10DCA — 2CqDq	540 + 1080 — 900 — 1800

15BAq	=== 540	— 10DCA	= 900		45
3BACD	== 1080	— 2CqDq	= 1800		—24
	1620				180
			— 2700	=	90
					—1080

CHAP. V. *Division of Integral and Rational Species.*

Division of Int. & Rat. Species. TO divide *Integral and Rational Species*, consider the six *Cases* following; in all which, as in *Cossicks*, *Surds*, and other Contract Numbers, like Signs shall give + and unlike —. And where Integers are adjoined, divide them as Integers are divided.

1. Simple. *Case* 1. If any Simple *Species* be to divide the same Simple *Species*, then set in the Quotient an Unit; but if Integers be annexed to the given *Species*, divide Integers by Integers, as if there were no *Species*.

Examples. As to divide A by A, or B by B, the Quotients shall be 1. But 6A divided by 2A, shall give 3 in the Quotient. And 8B by —2B, shall give —4B.

Divisor A) *Dividend* A (1 *Quotient*, or thus *Dividend* / *Divisor* $\frac{A}{A}$ (1A *Quotient.*

Divisor B) *Dividend* B (1 *Quotient*, or thus *Dividend* / *Divisor* $\frac{B}{B}$ (1B *Quotient.*

Dividend / *Divisor* $\frac{6A}{2A}$ (3A *Quotient.* *Dividend* / *Divisor* $\frac{8B}{-2B}$ (−4B *Quotient.*

2. Simple Species of the Divisor figurate in the Dividend. *Case* 2. If the Simple *Species* of the Divisor be figurate in the Dividend; then place the Divisor in the Quotient with such a Note of Abatement in the Power of the *Species*, as the Index of the Divisor being substracted from the Index of the Dividend will leave.

Examples. As if Aq be divided by A, the Quotient shall be A only; because A being the Root, whose Index 1 taken from 2, the Index of Aq, the Square leaves 1 the Index of the Root.

Dividend / *Divisor* $\frac{Aq}{A}$ (A *Quotient.* *Indices* 2—1=1 *Dividend* / *Divisor* $\frac{4Ac}{2A}$ (2Aq *Quotient.* *Indices* 3—1=2

Dividend / *Divisor* $\frac{AqEq}{AE}$ (AE *Quotient.* *Indices* 2—1=1 *Dividend* / *Divisor* $\frac{Aqqc}{Aqq}$ (Ac *Quotient.* *Indices* 7—4=3

3. Simple Species of the Divisor specified in the Dividend. *Case* 3. If the Simple *Species* of the Divisor be specified in the Dividend, then after the *Species* of like Form are cancelled or substracted, the one from the other, let the Residue be set in the Quotient.

Examples. As if BA be divided by A, the *Species* A being in both, only B is placed in the Quotient. So 10AE divided by 5A, makes the Quotient 2E. Other Examples of like sort are set with them as followeth.

Dividends / *Divisors* $\frac{BA}{A}$ (B $\frac{10AE}{5A}$ (2E $\frac{BA+A}{A}$ (B+1 *Quotients.*

Dividends / *Divisors* $\frac{BA-BE}{B}$ (A—E $\frac{BA+BE}{B}$ (A+E *Quotients.*

Case

Case 4. If the *Species* of both Dividend and Divisor be Compound, then as before in the third *Case*, substract like *Species*; and for every two of the remaining *Species* in the Dividend, from which Substraction is made, let one be set in the Quotient. 4. *Compound Species.*

As to divide BA—BE by A—E, substracting A—E from the Dividend, there is left only B—B, for which B only is put in the Quotient. See further in the following Examples. Examples.

Dividends $\frac{BA-BE}{A-E}\Big(B \qquad \frac{BA+CA}{B+C}\Big(A$ *Quotients.*
Divisors

Dividend $\frac{BA-BE-CA+CE}{B-C}\Big(A-E$ *Quotient.*
Divisor

Case 5. If the *Species* be Heterogeneal, then place the Divisor under the Dividend in form of a Fraction. 5. *Heterogeneal.*

As to divide B by A, being set thus $\frac{B}{A}$ they are left as not otherwise dividable. The Examples following are of like sort. Examples.

Dividends $\frac{-BC}{E}\Big(-\frac{BC}{E} \qquad \frac{-BC}{-A}\Big(\frac{BC}{A} \qquad \frac{B+C}{A-E}\Big(\frac{B+C}{A-E}$ *Quotients.*
Divisors

Case 6. If the *Species* given to be divided be mixt, then accordingly let their Division be by mixture of the Cases under which they fall. 6. *Mixt.*

As to divide BAc by Aq, by the second *Case*, Aq dividing Ac, shall give A in the Quotient; and because B hath nothing substracted from him, he shall be set in the Quotient by the third *Case*. Examples.

So BA—BE divided by A, maketh the Quotient B—$\frac{BE}{A}$; that is, by the third *Case* is B left for the Quotient, and by the fifth *Case* $\frac{BE}{A}$ because they are Heterogeneals.

Dividends $\frac{BAc}{Aq}\Big(BA \qquad \frac{BA-BE}{A}\Big(B-\frac{BE}{A}$ *Quotients.*
Divisors

The Quotient of every Division in *Species* multiplied by the Divisor, returning the Dividend; and the Product of every Multiplication in *Species* divided by one of the Factors, giving the other in the Quotient, is a sufficient Testimony, that the proof of either is reciprocally by each other. *Proof of Division of Int. and Rat. Species.*

Likewise by supposing the *Species* to be divided Absolute Numbers, and dividing them as such, the Quotient of this Division will be the Value of the *Species* in the Quotient of their Division, if the Work be right.

As in the last Example; Suppose A 6, B 2, E 4, then shall BA be 12, and BE be 8, and the Dividend 12—8 to be divided by 6; so shall the Quotient be 2—$\frac{8}{6}$, agreeing with B—$\frac{BE}{A}$.

Supposing A=6. B=2. E=4. Then shall

$$\frac{BA-BE}{A} = \frac{2\times6-2\times4}{6} = \frac{12-8}{6} = 2-\tfrac{8}{6} = B-\frac{BE}{A}.$$

CHAP. VI. *Reduction of Fracted and Rational Species.*

INtegral and Rational *Species* discussed, the next that shew themselves are their Fractions. These in the first Chapter of *Species*, were divided and subdivided into several sorts, and so need no farther Description here; wherefore I proceed to *Reduction*. *Reduction of Fract. and Rat. Species.*

Fracted and Rational *Species* are to be reduced either into their least Terms, or into like Denominators when they will admit thereof.

The first sort is called *Abbreviation*, as in Common Fractions; and if the Numbers be commensurable in Single Fractions, or the *Species* in Dual, or both Num- *To their least Terms.*

bers

bers and *Species* in Plural Fractions, or either of them: then dividing them by the Common Divisor, they will be brought to their least Terms.

Examples. As $\frac{6}{8}$ A will be reduced to $\frac{3}{4}$ A, by the Common Divisor 2; and $\frac{6}{9}$ B, by the Common Divisor 3, will be reduced to $\frac{2}{3}$ B.

And $\frac{BA}{BC}$ will be reduced to $\frac{A}{C}$ by the Common Divisor B. And $\frac{BCD}{BDA}$ by the Common Divisor BD will be reduced to $\frac{C}{A}$.

And $\frac{6BA}{8BC}+\frac{DE}{DC}$ will be reduced in Numbers to $\frac{3BA}{4BC}+\frac{DE}{DC}$ and in *Species* to $\frac{3A}{4C}+\frac{E}{C}$ by the Common Divisors B and D.

Abbreviated.

Plain { Simple Fracted *Species* 2) $\frac{6}{8}$A ($\frac{3}{4}$A 3) $\frac{6}{9}$B ($\frac{2}{3}$B.

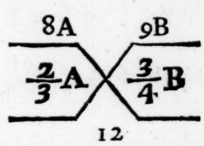
Compound { Dual Fracted *Species* B) $\frac{BA}{BC}$ ($\frac{A}{C}$ BD) $\frac{BCD}{BDA}$ ($\frac{C}{A}$

Plural Fracted Species 2 B+D) $\frac{6BA}{8BC}+\frac{DE}{DC}$ ($\frac{3A}{4C}+\frac{E}{C}$

To one Denominator. The second sort of Reduction of Fracted and Rational *Species*, to bring different Denominators into one comprehendeth,

1. *Simple.* *First*, To reduce simple Fracted *Species*, in which proceed with the Numbers, as in Common Fractions, without altering the *Species*.

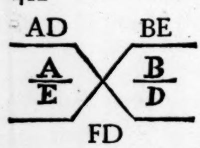
Example. And so $\frac{2}{3}$ A, and $\frac{3}{4}$ B, shall be reduced to $\frac{8A \quad 9B}{12}$

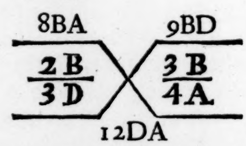


2. *Dual.* *2ly*, To reduce Dual Fracted Species into like Denominators, multiply the Numbers as Common Fractions, and the *Species* as *Species*.

Examples. And so $\frac{A}{E}$ and $\frac{B}{D}$ shall be reduced to $\frac{AD \quad BE}{ED}$

And $\frac{2B}{3D}$ and $\frac{3B}{4A}$ shall be reduced to $\frac{8BA \quad 9BD}{12DA}$

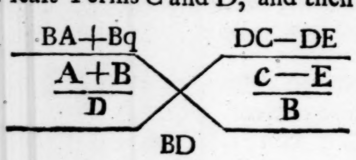


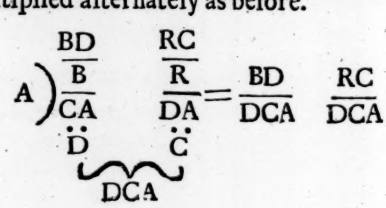


3. *Plural.* *3ly*, To reduce Plural Fracted *Species* into one Denomination, is alike to the Reduction of Dual, *mutatis mutandis*. And if in either of these the Denominations given are Commensurable, reduce them to their least Terms.

Examples. As to reduce $\frac{A+B}{D}$ and $\frac{C-E}{B}$ into one Denomination, multiplying alternately D into C—E, and B into A+B, the new Numerators are gotten, and D into B is the common Denominator. So are the new Fractions $\frac{BA+Bq}{BD}$ and $\frac{DC-DE}{BD}$.

But if $\frac{B}{CA}$ and $\frac{R}{DA}$ be given to be reduced to one Denomination; then because their Denominators are commensurable by A, they are first reduced to their least Terms C and D, and then multiplied alternately as before.





4*ly*, To reduce many proper Fractions of *Species* into one Denomination, is like the Method uſed in vulgar Fractions, by multiplying the Denominators one into another for the common Denominator; and every fraction's Numerator into the other Denominators except his own. *4. Many proper Fractions.*

And ſo $\frac{B}{D}$ and $\frac{C}{A}$ and $\frac{E}{H}$ reduced to one Denomination, are $\frac{BAH}{DAH}$ & $\frac{CDH}{DAH}$ & $\frac{EDA}{DAH}$ Example.

5*ly*, To reduce Fractions of Fractions, is to multiply after the manner of *Species*, Numerator by Numerator, and Denominator by Denominator. *5. Fractions of Fractions.*

As $\frac{B}{D}$ of $\frac{E}{A}$ reduced, ſhall be $\frac{BE}{DA}$ by multiplying B into E, and D into A. Examples.

And $\frac{A}{C}$ of $\frac{3Aq}{2B}$ reduced, ſhall be $\frac{3Ac}{2BC}$ by multiplying A into 3 Aq, and C into 2B.

6*ly*, To reduce Integral *Species* and Fracted, into improper Fractions, multiply the Integral *Species* by the Denominator of the Fraction, and to the Product add the Numerator. *6. Mixt into Improper.*

As $B\frac{C}{D}$ and $D\frac{C}{A}$ reduced into improper Fractions, ſhall be $\frac{BD+C}{D}$ and $\frac{DA+C}{A}$ Examples.

by multiplying B into D in the one, and D into A in the other, and adjoining to the Products C, with the Sign of Addition $+$.

So $B-C\frac{R+D}{S}$ reduced, ſhall be $\frac{B3-CS+R+D}{S}$ the improper Fraction.

7*ly*, To reduce improper Fracted *Species* back into Integers, or an Integral and Fracted Species, divide the Numerator by the Denominator after the manner of *Species*. *7. Improper into Mixt.*

As $\frac{BD+C}{D}$ divided by D, makes the Quotient B an Integral *Species*, and the remaining C is ſet over D as a Fraction thus, $B\frac{C}{D}$. Example.

8*ly*, To reduce an Integral *Species* into ſome deſired Denomination, multiply the whole *Species* by the given Denominator: And any whole *Species* may be ſet as a Fraction, by placing 1 under the *Species*. *8. Species turned to a Denomination deſired.*

As if B be deſired as a Fraction, whoſe Denominator ſhall be A, then is B to be multiplied into A, and ſet thus $\frac{BA}{A}$. Examples.

So B+C into D, ſhall be $\frac{BD+CD}{D}$.

Integer ſet as a Fraction.

And B, or B+C ſet as Fractions, ſhall be $\frac{B}{1}$ and $\frac{B+C}{1}$.

Hence it appeareth, that as by this laſt-mentioned Reduction, whole *Species* may be ſet as Fractions: So by the firſt ſort of Reduction may ſome fracted *Species* be turned into Integral, being abbreviated into their leaſt Terms. *Some Fractions by Abbreviation brought to Integers.*

As $\frac{BA}{B}$ may be abbreviated into A the Integer.

And $\frac{4Aq}{2A}$ into 2A. And $\frac{3Aq}{6A}$ into $\frac{A}{2}$.

Reduction of Fracted and Rational Species, as well as other Reductions, may clearly be diſcerned to prove one part thereof, by the other part reciprocal thereto. And beſides by ſuppoſing the Fracted *Species* abſolute Numbers or common Fractions, every Part of Reduction may be proved with ſufficient Demonſtration. *Proof of Reduction of Fract. and Rat. Species.*

As in the laſt Example, if A be ſuppoſed 3, then ſhall 6A be 18, and Aq ſhall be 9, and 3 Aq 27; which abbreviated or divided by 18, ſhall be $1\frac{1}{2}$ equal to $\frac{A}{2}$.

Suppoſing $A=3$. Then ſhall

$$\frac{3Aq}{6A} \; \begin{matrix} = 3\times 3\times 3 \\ = 6\times 3 \end{matrix} \; = \frac{27}{18}\left(\frac{3}{2} = \frac{A}{2}\right.$$

CHAP. VII. *Addition of Fracted and Rational Species.*

Addition of Fract. and Rat. Species.

THE several Cases and Varieties in Addition of Common Fractions, *Book* 1. *Part* 2. *Chap*. 3. might here be run over again: But three Cases contain sufficient for the *Addition of Fracted and Rational Species*, all the rest differing materially only in Examples.

1. Like Denominators without Numbers.

Case 1. If the Fractions to be added be without Numbers annexed, and of the same Denomination; then add the Numerators as *Species* are added, that is, by conjoining them with the Signs proper thereto, if the Numerators be unlike; or if alike, by adding the Number of them together, their Signs also being alike; but when unlike, take their Difference, and subscribe the common Denominator.

Examples.

As to add $\frac{B}{D}$ to $\frac{A}{D}$ the Total shall be $\frac{B+A}{D}$ by joining B to A, because they are Heterogeneal by the Sign of Addition.

But $\frac{B}{D}$ added to $\frac{B}{D}$ shall make the Sum $\frac{2B}{D}$; because both Numerators are Homogeneal, they are added as Integral *Species*.

So Z the Integer added to the Fraction $\frac{E}{B}$, makes the Total $Z\frac{E}{B}$ or $\frac{ZB+E}{B}$.

And $\frac{3D}{B}$ added to $\frac{-D}{B}$, makes the Sum $\frac{2D}{B}$, because the Signs are contrary; when $-D$ is taken from $+3D$, the Remain is $+2D$.

Other Examples.

Addends.	*Total.*	*Addends.*	*Total.*
$\frac{BA}{DE}$ and $\frac{CD}{DE}$ added, are	$\frac{BA+CD}{DE}$.	$\frac{A+E}{B}+\frac{D-C}{B}=$	$\frac{A+E+D-C}{B}$.

2. Unlike Denominators without Numbers.

Case 2. If the Fractions to be added be without Numbers, and of different Denominations, then first reduce them as fracted *Species* are reduced, and afterwards add their Numerators as before.

Examples.

As $\frac{B}{A}$ added to $\frac{B}{C}$ makes, first by Reduction $\frac{BC}{AC}$ and $\frac{BA}{AC}$, and then joined by the Sign of Addition $\frac{BC+BA}{AC}$. So $\frac{B}{D}$ added to $\frac{D}{A}$ make the Total $\frac{BA+Dq}{DA}$.

The former Example.

$$\begin{array}{ccc} BC & + & BA \\ \frac{B}{A} & \times & \frac{B}{C} \\ & AC & \end{array} = \frac{BC+BA}{AC}.$$

The latter Example.

$$\begin{array}{ccc} BA & + & Dq \\ \frac{B}{D} & \times & \frac{D}{A} \\ & DA & \end{array} = \frac{BA+Dq}{DA}.$$

Likewise $\frac{B}{D}$ the proper Fraction added to $\frac{ZB+E}{B}$ the improper Fraction, makes the Total $\frac{ZBD+ED+Bq}{BD}$.

Other Examples.

Addends.	*Total.*	*Addends.*	*Total.*
$\frac{BA}{DE}+\frac{CD}{BE}=$	$\frac{BqA+CDq}{BDE}$.	$\frac{A+B}{E}+\frac{D-C}{B}=$	$\frac{AB+Bq+DE-CE}{EB}$

Thus,

$$E)\begin{array}{ccc} BqA & & CDq \\ \frac{BA}{DE} & + & \frac{CD}{BE} \\ \ddot{B} & & \ddot{D} \end{array} \quad \underbrace{B \quad D}_{BDE}$$

Thus,

$$\begin{array}{ccc} AB+Bq & + & DE-CE \\ \frac{A+B}{E} & \times & \frac{D-C}{B} \\ & EB & \end{array}$$

Addends.

$$\begin{array}{ccc} \textit{Addends.} & & \textit{Total.} \\ \frac{B+C}{Dq+A}+\frac{DE+1}{A-C} & = & \frac{BA+CA-BC-Cq+DcE+Dq+ADE+A}{DAq+Aq-DqC-AC.} \end{array}$$

Case 3. If the Fractions have Numbers annexed to them, then order the Numbers as common Fractions, and the *Species* as *Species*. 3. *With Numbers.*

As to add $\frac{2}{5}A$ to $\frac{1}{5}A$, the Sum ſhall be $\frac{3}{5}A$.

So 2B and $\frac{1}{2}A$ added, make the Total $2B+\frac{1}{2}A$, or $\frac{4B+A}{2}$. Examples.

Other Examples.

$$\begin{array}{cc} \textit{Addends.} & \textit{Total.} \\ \frac{2}{3}A+\frac{3}{4}B = & \frac{8A+9B}{12}. \end{array} \qquad \frac{2B}{3D}+\frac{3B}{4A}=\frac{8BA+9BD}{12DA}$$

Thus, Thus,

$$\begin{array}{ccc} 8A & + & 9B \\ \frac{2}{3}A & \times & \frac{3}{4}B \\ & 12 & \end{array} \qquad \begin{array}{ccc} 8BA & & 9BD \\ \frac{2B}{3D} & \times & \frac{3B}{4A} \\ & 12DA & \end{array}$$

Here are fitly to be inſerted ſuch Propoſitions as require to add a Part or Parts of a given Number or Magnitude to the ſame Number or Magnitude, or to any other of his Parts. In both which the Deſire is thus obtained; Firſt by the Reduction of Fractions of Fractions get the Part or Parts to be added, and then by Addition add them as the Caſe requires.

Examples of the firſt Sort. *Part of a Number added thereto.*

As $\frac{1}{2}$ of $\frac{3A}{5}$ added to $\frac{3A}{5}$, maketh the Total $\frac{9A}{10}$.

And $\frac{2}{5}$ of $\frac{3A+D}{E}$ added to $\frac{3A+D}{E}$, maketh the Total $\frac{21A+7D}{5E}$.

For in the firſt of theſe $\frac{1}{2}$ of $\frac{3A}{5}$ is found by Reduction to be $\frac{3A}{10}$, which added to $\frac{3A}{5}$ as Common Fractions are added, make the Total as above.

And in the latter Example $\frac{2}{5}$ of $\frac{3A+D}{E}$ is found by Reduction to be $\frac{6A+2D}{5E}$, which by Addition to $\frac{3A+D}{E}$, makes the Total as before.

$$\frac{3A}{\frac{1}{2}\text{ of }\frac{3A}{5}}\Big/10 \qquad \begin{array}{c} 9A \\ \hline 3A \quad 6A \\ 5)\frac{3A}{10}+\frac{3A}{5} \\ 1 \quad\; 2 \\ 10 \end{array} \qquad \frac{6A+2D}{\frac{2}{5}\text{ of }\frac{3A+D}{E}}\Big/5E \qquad \begin{array}{c} 21A+7D \\ \hline \quad\quad 15A+5D \\ \frac{6A+2D}{5E}+\frac{3A+D}{E} \\ \hline 5E \end{array}$$

Examples of the ſecond Sort. *Part of a Number added to other Parts thereof.*

As $\frac{1}{3}$ of 5B added to $\frac{2}{5}$ of 5B, makes the Total $\frac{11B}{3}$.

And $\frac{1}{2}$ and $\frac{1}{3}$ of $\frac{15A-12}{8}$ added, make the Total $\frac{25A-20}{16}$.

For in the firſt of theſe $\frac{1}{3}$ of 5B is $\frac{5B}{3}$, and $\frac{2}{5}$ of 5B is $\frac{2B}{1}$; both which added make $\frac{11B}{3}$ as before.

And in the latter Example $\frac{1}{2}$ of the *Data* is $\frac{15A-12}{16}$, and $\frac{1}{3}$ is $\frac{5A-4}{8}$, which added together make the Total as above.

5B

$$\begin{array}{c} 5B \\ \hline \frac{1}{3}\text{ of }\frac{5B}{1} \\ \hline 3 \end{array} \qquad \begin{array}{c} 11B \\ \hline \frac{5B}{3}+\frac{6B}{\frac{2B}{1}} \\ \hline 3 \end{array} \qquad \begin{array}{c} 15A-12 \\ \hline \frac{1}{2}\text{ of }\frac{15A-12}{8} \\ \hline 16 \end{array} \qquad \begin{array}{c} 25A-20 \\ \hline 8)\underbrace{\dfrac{15A-12}{\underset{1}{16}}+\dfrac{10A-8}{\dfrac{5A-4}{\underset{2}{8}}}}_{16} \end{array}$$

$$\begin{array}{c} 2B \\ \hline 1 \\ \frac{2}{3}\text{ of }\frac{5B}{1} \\ 1 \\ \hline 1 \end{array} \qquad \begin{array}{c} 5A-4 \\ \hline 5A-4 \\ \frac{1}{3}\text{ of }\frac{15A-12}{8} \\ 1 \\ \hline 8 \end{array}$$

Proof of Addition of Fract. & Rat. Species. *Addition of Fracted and Rational Species* is to be proved both by Substraction, as in the next Chapter, and by converting the Fracted *Species* into Absolute Numbers or Common Fractions; with the Addition whereof the Addition of these Fracted *Species* will exactly agree.

As in the last Example of the third *Case* above, suppose A2, B3, D4, then shall 2B be 6, and 3D 12. So $\frac{2B}{3D}$ is $\frac{6}{12}$ or $\frac{1}{2}$. And 3B shall be 9, and 4A 8. So $\frac{3B}{4A}$ is $\frac{9}{8}$. And $\frac{1}{2}$ and $\frac{9}{8}$ added together, make $\frac{13}{8}$ or $1\frac{5}{8}$ equal to the Total $\frac{8BA+9BD}{12DA}$

Supposing A=2. B=3. D=4. Then shall

$$\frac{2B=6}{3D=12}+\frac{3B=9}{4A=8} \qquad \begin{array}{l} 8BA=8\times3\times2=48 \\ 9BD=9\times3\times4=108 \\ \hline 12DA=12\times4\times2=96 \end{array}\Big\}=\frac{156}{96}\Big(1\frac{60}{96}\text{ or }1\frac{5}{8}$$

$$2)\underbrace{\frac{4}{\underset{4}{\frac{1}{2}}}+\frac{9}{\underset{1}{\frac{9}{8}}}}_{8}=\frac{13}{8}\Big(1\frac{5}{8}$$

CHAP. VIII. *Substraction of Fracted and Rational Species.*

Substraction of Fract. and Rat. Species. As in *Addition*, so here, sufficient for the *Substraction of Fracted and Rational Species*, is contained under the three *Cases* following. The other *Cases* mentioned in Substracting of Common Fractions, *Book* 1. *Part* 2. *Chap*. 4. being more occasional than essentially necessary.

1. Like Denominators without Numbers. *Case* 1. If the Fractions to be substracted be without Numbers annexed, and of like Denomination, then after the manner of *Species* substract the Numerator of the Subtrahend from the other Numerator set to the left Hand of the Subtrahend; that is, by connexing them with the Sign of Substraction, if the Numerators be unlike and their Signs alike: But if the Numerators be alike, by taking the one from the other, or adding with contrary Signs both Numerators as before in Cossicks, and under such Sum or Difference subscribe the Common Denominator.

Examples. As to take $\frac{D}{C}$ from $\frac{B}{C}$, the Remain is $\frac{B-D}{C}$ conjoining B to D, because they are Heterogeneal by the Sign of Substraction.

But $\frac{B}{D}$ taken from $\frac{3B}{D}$, shall leave remaining $\frac{2B}{D}$, because both Numerators are Homogeneal, the one is substracted from the other as Integral *Species*.

So $\frac{E}{B}$ the proper Fraction taken from $\frac{ZB+E}{B}$ the improper Fraction, the Remain will be Z.

And $\frac{-D}{B}$ taken from $\frac{2D}{B}$, leaves remaining $\frac{3D}{B}$; for the Signs being contrary, −D shall be added to 2D, which makes 3D as before in Cossicks.

Other Examples.

Subtrahend. *Remain.*

$\frac{AB}{D}$ substracted from $\frac{BC}{D}$, shall leave $\frac{BC-AB}{D}$

Subtrahend. *Remain.*

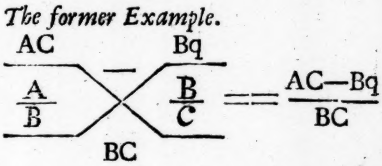
$\frac{A-E}{B}$ substracted from $\frac{D-C}{B}$, shall leave $\frac{D-C-A+E}{B}$.

Case 2. If the Fractions to be substracted be without Numbers, and of different Denominations; then first reduce them as Fracted *Species* are reduced, and then substract the Numerator of the Subtrahend from the other as before. *2. Unlike Denominators without Numbers.*

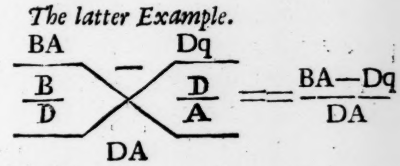
As from $\frac{A}{B}$ substract $\frac{B}{C}$, the Rednction makes them $\frac{AC}{BC}$ and $\frac{Bq}{BC}$; then by Substraction the Remain is $\frac{AC-Bq}{BC}$. Examples.

And from $\frac{B}{D}$ take $\frac{D}{A}$, the Remain shall be $\frac{BA-Dq}{DA}$.

The former Example.



The latter Example.



Other Examples.

Subtrahend. *Remain.*

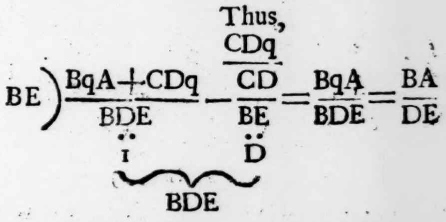
$\frac{BA}{DE}-\frac{CD}{BE}=\frac{BqA-CDq}{BDE}$.

Subtrahend. *Remain.*

$\frac{BqA+CDq}{BDE}-\frac{CD}{BE}=\frac{BA}{DE}$.

Thus,

Thus,



In which last Example is to be noted, that in the Reduction the Common Divisor BE takes away all the Denominator of the Subtrahend; and so nothing being left to be brought to the Denominator of the other Number, an Unit is there placed and not a Cipher, lest it should be taken for a *Species*. After the Reduction the Numbers stand thus.

$\frac{BqA+CDq-CDq}{BDE}$; where because CDq is both Affirmative and Negative, in the Numerators they are both to be cancelled, and the Remain $\frac{BqA}{BDE}$ abbreviated to $\frac{BA}{DE}$, because Bq in the Numerator is figurate; which divided by B, leaves B only to A for the Numerator.

Case 3. If the Fractions have Numbers annexed to them, then order the Numbers as Common Fractions, and the *Species* as *Species*. *3. With Numbers.*

As from $\frac{2}{7}$ A take $\frac{1}{7}$A, and the Remain shall be $\frac{1}{7}$A. Examples.

But to take $\frac{3}{4}$A from $\frac{13}{12}$A, the Fractions must first be reduced to one Denomination; and then from $\frac{13}{12}$A let $\frac{9}{12}$A be taken, and the Remain will be $\frac{4}{12}$A or $\frac{1}{3}$ A.

So 2B taken from $\frac{4B+A}{2}$ leaves $\frac{A}{2}$ or $\frac{1}{2}$A.

 Other

Other Examples.

Subtrahend. *Remain.*

$\frac{2}{3}A - \frac{3}{4}B = \frac{8A-9B}{12}$.

Thus,

$$\begin{array}{ccc} 8A & & 9B \\ \frac{2}{3}A & - & \frac{3}{4}B \\ & 12 & \end{array}$$

Subtrahend. *Remain.*

$\frac{8A+9B}{12} - \frac{3}{4}B = \frac{2}{3}A$.

Thus,

$$4)\frac{8A+9B}{12} - \frac{\overset{9B}{3}B}{4} = \frac{8A}{12} = \frac{2}{3}A.$$

$$\underbrace{1 \quad 3}_{12}$$

In this place may be fitly inſerted ſuch Propoſitions as require to take a Part or Parts of a Number or Magnitude given, from the ſame Number or Magnitude, or from any other of his Parts. In both which the Queſited is gotten thus: Firſt by Reduction of Fractions of Fractions, get the Part or Parts to be ſubſtracted, and then by Subſtraction, as the Caſe may require, you will have the Remain.

Part of a Number taken therefrom.

Example of the firſt Sort.

As to take $\frac{1}{3}$ and $\frac{1}{5}$ of $\frac{B+8A}{C}$ from the ſame, the Remain will be $\frac{7B+56A}{15C}$. For $\frac{1}{3}$ and $\frac{1}{5}$ being together $\frac{8}{15}$, are $\frac{8B+64A}{15C}$; which ſubſtracted from the *Data*, leave the Remain as before.

$$\begin{array}{c} 8 \\ \hline \frac{5}{\frac{1}{3}} + \frac{3}{\frac{1}{5}} \\ \hline 15 \end{array} \qquad \begin{array}{c} 8B+64A \\ \hline \frac{8}{15} \text{ of } \frac{B+8A}{C} \\ \hline 15C \end{array} \qquad \begin{array}{c} 7B-56A \\ \hline 15B+120A \\ \frac{B+8A}{C} - \frac{8B+64A}{15C} \\ \hline 15C \end{array}$$

Parts of a Number taken from other Parts thereof.

Example of the ſecond Sort.

As to take $\frac{3}{4}$ of $3B+C$ from $\frac{4}{5}$ thereof, the Remain is $\frac{3B+C}{20}$.

For $\frac{3}{4}$ of $3B+C$ is $\frac{9B+3C}{4}$, and $\frac{4}{5}$ is $\frac{12B+4C}{5}$, and the former taken from this latter, leaves the Remain as aforeſaid.

$$\begin{array}{c} 9B+3C \\ \hline \frac{3}{4} \text{ of } \frac{3B+C}{1} \\ \hline 4 \end{array} \qquad \begin{array}{c} 12B+4C \\ \hline \frac{4}{5} \text{ of } \frac{3B+C}{1} \\ \hline 5 \end{array} \qquad \begin{array}{c} 3B+C \\ \hline 48B+16C \quad\quad 45B+15C \\ \frac{12B+4C}{5} - \frac{9B+3C}{4} \\ \hline 20 \end{array}$$

Proof of Subſtraction of Fract. and Rat. Species.

Foraſmuch as in the laſt Example here in the third *Caſe*, and alſo in the laſt Example of the ſecond *Caſe* above, the Numbers from which the Subſtraction is made, are the Totals of their Additions in the former Chapter; and the Subtrahends in both, are one of the Addends there: It is enough to ſhew the Reciprocal Proof of Addition by Subſtraction, and Subſtraction by Addition in theſe Fracted *Species*.

Subſtraction alſo, as Addition before, may be proved by ſuppoſing the *Species* to be Numbers Abſolute; becauſe after Subſtraction made with them, the Remain will be equally valuable with the Fracted *Species* remaining.

As in the laſt mentioned Example, if A be ſuppoſed 2, and B 3, then ſhall 8A be 16, and B 27. And 8A+9B, that is 16+27, ſhall be 43; which divided by 12, ſhall be $3\frac{7}{12}$: From which $\frac{3}{4}$B taken which is $\frac{9}{4}$, the Remain is $\frac{16}{12}$, that is 8A to be divided by 12, or $\frac{2}{3}$A, as the *Species* above ſhewed.

Suppoſing A=2. B=3. Then ſhall

Subtrahend. *Remain.*

$\left.\begin{array}{l} 8A=16 \\ 9B=27 \end{array}\right\} \frac{43}{12} - \frac{3}{4}B = \frac{9}{4}$ $\qquad \frac{2}{3}A = \frac{4}{3} = 1\frac{1}{3}$

16

16 Or thus, $16+27-27=16$

27

$4)\frac{43}{12}-\frac{9}{4}=\frac{16}{12}=1\frac{1}{3}$

$\underbrace{1 \quad 3}_{12}$

$4)\frac{8A+9B}{12}-\frac{\frac{9B}{3}}{4}B=8A$

$\underbrace{1 \quad 3}_{12}=\frac{}{12}$

CHAP. IX. *Multiplication of Fracted and Rational Species.*

SPECIES Fracted and Rational, mixed with Integral *Species*, or purely Fracted, are multiplied as Common Fractions, *Book* 1. *Part* 2. *Chap.* 5. by comparing the Heterologal Terms, and multiplying the Homologal. And therefore but two *Cases* are necessary, in both which the Signs are to be ordered as in Cossical Fractions. *Multiplication of Fract. and Rat. Species.*

Case 1. If the Heterologal Terms need no Reduction, then multiply Numerator by Numerator, and Denominator by Denominator, Numbers as Numbers, and *Species* as *Species*. 1. *Heterologal Terms not reducible.*

As $\frac{B}{D}$ multiplied into $\frac{C}{A}$, makes the Product $\frac{BC}{DA}$. Examples.

And $\frac{2B}{3D}$ multiplied by $\frac{2C}{3A}$, makes the Product $\frac{4BC}{9DA}$.

So Z the Integral *Species* multiplied by the Fraction $\frac{A}{B}$ makes the Product $\frac{ZA}{B}$, where I set or suppose an Unit for the Denominator to Z, as if it were a Fraction.

Other Examples.

Mds.		*Mrs.*		*Products.*	*Mds.*		*Mrs.*		*Products.*
$\frac{A}{B}$	$\times$	$\frac{ZA}{C}$	$=$	$\frac{ZAq}{BC}$.	B	$\times$	$\frac{DA}{C}$	$=$	$\frac{BDA}{C}$.
$\frac{3B}{4C}$	$\times$	$\frac{1}{5}A$	$=$	$\frac{3BA}{20C}$.	$\frac{2B+C}{D-E}$	$\times$	$\frac{3A+O}{P}$	$=$	$\frac{6BA+3CA+2BO+CO}{DP-EP}$.

Case 2. If the Heterologal Terms need Reduction, then reduce them according to the Nature of the Fraction, Numbers as Numbers, and *Species* as *Species*; And after Reduction into their least Terms, multiply the Homologal Terms as before. 2. *Heterologal Terms reducible.*

As $\frac{B}{A}$ multiplied into $\frac{A}{C}$ produceth $\frac{B}{C}$, because A in the Numerator of the one, and Denominator of the other, being alike, are both set aside. Examples.

And $\frac{2B}{3D}$ multiplied by $\frac{2C}{5B}$ produceth $\frac{4C}{15D}$, the Heterologal B in both Fractions being useless in the Product.

So B the Integral *Species* multiplied into the Fraction $\frac{A}{B}$ makes the Product A; for A is equal to $\frac{BA}{B}$.

Other Examples.

Mds.		*Mrs.*		*Reductions.*				*Products.*
$B\frac{C}{A}$	$\times$	$\frac{C}{D}$	$=$	$\frac{BA+C}{A}$	$\times$	$\frac{C}{D}$	$=$	$\frac{BAC+Cq}{AD}$.
$\frac{BA}{RE}$	$\times$	$\frac{DE}{CA}$	$=$	$\frac{A)\overset{B}{BA}}{E)\underset{R}{RE}}$	$\times$	$\frac{\overset{D}{DE}}{\underset{C}{CA}}$	$=$	$\frac{BD}{RC}$.
$\frac{2B}{3C}$	$\times$	$\frac{3}{4}A$	$=$	$\frac{2)\overset{1}{2B}}{3)\underset{1}{3C}}$	$\times$	$\overset{1}{\underset{1}{\tfrac{3}{4}}}A$	$=$	$\frac{1BA}{2C}$.

Hence

Heterolog. Terms equal.

Hence as in Multiplication of Common Fractions, if the Heterologal Terms either way are equal, the other Terms ſhall ſtand for the Product. And if they are alike both ways, then ſhall the Product be an Unit or equal Fraction. For

Examples.

$\frac{2B}{3C} \times \frac{3C}{5B} = \frac{2BC}{5CB}$, and omitting the *Species* to $\frac{2}{5}$ the Number 3 in both being ſet aſide. And $\frac{2B}{3C} \times \frac{3C}{2B} = \frac{6BC}{6CB}$, that is 1.

To find a Part or Parts of a Number.

Alſo here properly may be inſerted a Propoſition to find a Part or Parts of a given Number or Magnitude, which is no more than to multiply the ſame by the Part or Parts, after the manner of Fractions.

Examples of both Sorts.

Examples.

As to know what $\frac{3}{4}$ of 3B is, the Product ſhews it $\frac{9B}{4}$.

And if $\frac{1}{3}$ and $\frac{1}{4}$ of $\frac{B+8A}{C}$ be demanded, the Multiplication produceth $\frac{7B+56A}{12C}$ for Anſwer. For $\frac{1}{3}$ and $\frac{1}{4}$ added, make $\frac{7}{12}$, which multiplying the *Data*, makes the Product as laſt above-mentioned.

Proof of Multiplication of Fract. and Rat. Species.

Multiplication of Fracted and Rational Species is to be proved, both by Diviſion as in the next Chapter; and by turning the Fracted *Species* into Common Fractions or Abſolute Numbers, with the Multiplication whereof will agree the Multiplication of theſe Fracted *Species*.

For ſuppoſe in the Example above-mentioned (where $\frac{2B}{3C} \times \frac{3C}{5B}$ produce $\frac{2BC}{5CB}$) B be 2, and C 3, then ſhall 2B be 4, and 3C 9. So will $\frac{2B}{3C}$ be $\frac{4}{9}$ and $\frac{3C}{5B}$ will be $\frac{9}{10}$, and the Product by Abbreviation $\frac{2}{5}$, as is the Value of the Product $\frac{2BC}{5CB}$ by that Suppoſition.

Suppoſing B = 2. C = 3. Then ſhall

$$\frac{2B = 4}{3C = 9} \times \frac{3C = 9}{5B = 10} = \frac{2BC = 2\times2\times3}{5CB = 5\times3\times2} = \frac{12}{30} \text{ or } \frac{2}{5}.$$

CHAP. X. *Division of Fracted and Rational Species.*

Division of Fract. and Rat. Species.

Contrary to Multiplication *Species* Fracted and Rational, Pure or Mixt with Integral *Species*, are divided by comparing the Homologal Terms, and multiplying the Heterologal as in Common Fractions, *Book* 1. *Part* 2. *Chap.* 6. And ſo but two Caſes are here neceſſary, and in both of them the Signs are to be ordered as in Coſſical Fractions.

1. Homologal Terms not reducible.

Caſe 1. When the Homologal Terms cannot be reduced lower, then multiply the Numerator of the Dividend by the Denominator of the Diviſor, to produce the Numerator of the Quotient: And the Denominator of the Dividend by the Numerator of the Diviſor, to produce the Denominator of the Quotient; Numbers as Numbers, and *Species* as *Species*.

Examples.

As $\frac{B}{D}$ divided by $\frac{C}{A}$ gives in the Quotient $\frac{BA}{DC}$.

And $\frac{2B}{3D}$ divided by $\frac{3C}{4A}$, gives in the Quotient $\frac{8BA}{9DC}$.

So $\frac{Aq}{D}$ divided by the Integral *Species* B, makes the Quotient $\frac{Aq}{BD}$. Suppoſing or ſetting an Unit for Denominator to B.

Other Examples.

Diviſors.	*Dividends.*	*Quotients.*	*Diviſors.*	*Dividends.*	*Quotients.*
$\frac{Bc}{C}$)	$\frac{Ec}{A}$	($\frac{EcC}{BcA}$	$\frac{A}{D}$)	$\frac{BC}{1}$	($\frac{BCD}{A}$
$\frac{DA}{C}$)	$\frac{B}{1}$	($\frac{BC}{DA}$	$\frac{3B}{4C}$)	$\frac{1}{5}A$	($\frac{4AC}{15B}$

Caſe

Case 2. When the Homologal Terms may be reduced, then, according to the Nature of the Fractions, reduce them, Numbers as Numbers, and *Species* as *Species*: And after Reduction into their least Terms, multiply the Heterologal Terms as before. 2. *Homologal Terms reducible.*

As $\frac{A}{C}$ dividing $\frac{B}{C}$ giveth in the Quotient $\frac{B}{A}$, because C the Denominator of both the given Fractions is set aside as useless. Examples.

And $\frac{4B}{15C}$ divided by $\frac{7B}{11D}$ brings in the Quotient $\frac{44D}{105C}$, the Homologal B in both being understood as cancelled.

So BC the Integral *Species* divided by $\frac{B}{A}$, makes the Quotient CA.

Other Examples.

Divisors. Dividends. Reductions. Quotients.

$$\frac{BA}{RT}\Big)\ \frac{CA}{DT} = \frac{A}{T}\Big)\frac{\overset{B}{BA}}{\underset{R}{RT}}\Big)\frac{\overset{C}{CA}}{\underset{D}{DT}}\Big(\frac{CR}{BD}.$$

$$\frac{BA}{CR}\Big)\ \frac{A}{R} = \frac{A}{R}\Big)\frac{\overset{B}{BA}}{\underset{C}{CR}}\Big)\frac{\overset{1}{A}}{\underset{1}{R}}\Big(\frac{C}{B}.$$

$$\tfrac{3}{4}A\Big)\ \tfrac{1}{2}\frac{BA}{C} = \frac{A}{2}\Big)\frac{\overset{1}{\tfrac{3}{4}A}}{\underset{2}{}}\Big)\tfrac{1}{2}\frac{\overset{B}{BA}}{\underset{1}{C}}\Big(\frac{2B}{3C}.$$

Hence, as in Division of Common Fractions, if the Homologal Terms be either way equal, the other Terms shall be taken for the Quotient: And if they are alike both ways, the Quotient shall be an Unit or equal Fraction. *Homologal Terms equal.*

For $\frac{2B}{3C}\Big)\frac{2B}{5D}\Big(\frac{3C}{5D}$ the Number 2 being set aside with the *Species* B, because alike in both Divisor and Dividend. And $\frac{2B}{3D}\Big)\frac{1C}{3D}\Big(\frac{1C}{2B}$, because 3D in both Denominators are equal, the other *Species* make up the Quotient. Example.

And $\frac{2B}{3D}\Big)\frac{2B}{3D}\Big($ 1 is the Quotient, the Terms and *Species* in both Numerators and Denominators being alike.

Sometimes it is the easiest way to reduce the Numbers given, part before Division and part after; which happens if one or both be figurate. *Best sometime to reduce the Data, part before and part after Division.*

As in dividing $\frac{ZAq}{BC}$ by $\frac{ZA}{C}$ and $\frac{BAC+Cq}{AD}$ by $\frac{C}{D}$ in the first Quotient $\frac{Aq}{BA}$ is reduced to $\frac{A}{B}$, and the other $\frac{BA+C}{A}$ to $B\frac{C}{A}$. Examples.

Divisors. Dividends. Reductions. Quotients Reduced.

$$\frac{ZA}{C}\Big)\ \frac{ZAq}{BC} = \frac{Z}{C}\Big)\frac{ZA}{\underset{1}{C}}\Big)\frac{ZAq}{\underset{B}{BC}}\Big(\frac{Aq}{BA}\Big(\frac{A}{B}.$$

$$\frac{C}{D}\Big)\frac{BAC+Cq}{AD} = \frac{C}{D}\Big)\frac{C}{D}\Big)\frac{BAC+Cq}{AD}\Big(\frac{BA+C}{A}\Big(B\frac{C}{A}.$$

Here may be inserted a Proposition, to find the principal Number or Magnitude, if any of his Parts be given; which is done by dividing the same Number or Magnitude by the Part or Parts, after the manner of Fractions. *To find a Number by any of its Parts.*

Examples of both Sorts. Examples.

As to know what is the principal Number whereof B is the $\frac{4}{5}$?

Answer, $\frac{5B}{4}$.

And if $\frac{B+8A}{C}$ be $\frac{2}{3}$ of another Number which is desired, it will be found to be $\frac{3B+24A}{2C}$. For by dividing B by $\frac{4}{5}$, and $\frac{B+8A}{C}$ by $\frac{2}{3}$, their Quotients are as before.

Proof of Division of Fract. & Rat. Species.

Seeing the Quotients of these Divisions multiplied by their Divisors will return the Dividends, and the Dividends in both the Examples above are the Products of their Multiplications in the former Chapter, it is sufficient to shew the Alternate Proof of Multiplication by Division, and Division by Multiplication in these Fracted *Species*.

Division also, as Multiplication before, may be proved by supposing the *Species* to be Numbers Absolute: For after Division made thereby, the Quotient will be equally valuable with the Quotient of the divided Fractions.

As in the Example above, supposing A 2. B 3. C 4. D 5. then will the Divisor $\frac{C}{D}$ be $\frac{4}{5}$; and the Dividend $\frac{BAC+Cq}{AD}$ be $\frac{40}{10}$ or $\frac{4}{1}$, that is $\frac{3\times2\times4+4\times4}{2\times5}$; and the Quotient of that Division 5 Integers equal to $B\frac{C}{A}$ that is $3\frac{4}{2}$.

Supposing A=2. B=3. C=4. D=5. Then shall

$$\frac{C=4}{D=5}\Bigg)\frac{\overset{24+16}{BAC+Cq}=40}{AD==10}\Bigg(B\frac{C}{A}=3\frac{4}{2} \qquad \frac{4)\overset{1}{4}}{5)\underset{1}{5}}\Bigg)\frac{\overset{10}{40}}{\underset{2}{10}}\Bigg(\frac{10}{2}\Big(5$$

CHAP. XI. *Figuration of Rational Species.*

Figurate Rational Species produced.

HOW to add, substract, multiply and divide whole and broken *Species*, as well Plain as Figurate, or Rational, intermixt one with another, is already dispatcht in the foregoing Chapters; wherein because the Cossical or Rational *Species* are subject to like Orders and Directions, and receive like Resolutions with plain *Species*, little need be added here save something of their Figuration.

Common Way.

Touching the Production of Rational *Species*, as others before them, so they are produced by Multiplication: For any *Species* multiplied into it self, produceth the Square; and that multiplied by the Root, produceth the Cube, &c. And whether the Root be Simple or Compound, or if one Power be multiplied by another; by Addition of their Indices, the due Index of the Product is had, as was touched in their Multiplication before, and fully seen in Cossicks in the 4th Part of this Book, and Figural Numbers in the second Part of the second Book.

Examples.

Examples
Of Simple Rational Species *produced.*

Species	A A	Aq A	Ac A	2Ac 3A	Aqc Aq	Aqc Aqq	multiplied.
	Aq	Ac	Aqq	6Aqq	Aqqc	Accc	
Indices	1 1	2 1	3 1	3 1	5 2	5 4	added.
	2	3	4	4	7	9	

Examples

Examples
Of Compound Rational Species *produced.*

√ A +E
A +E
Aq+AE
AE+Eq
q Aq+2AE+Eq
A+E
Ac+2AqE+AEq
AqE+2AEq+Ec
c Ac+3AqE+3AEq+Ec
&c.

√ A—E
A—E
Aq—AE
—AE+Eq
q Aq—2AE+Eq
A—E
Ac—2AqE+AEq
— AqE+2AEq—Ec
c Ac—3AqE+3AEq—Ec

√ 2A +3E
2A +3E
4Aq+6AE
6AE+9Eq
q 4Aq+12AE+9Eq
2A+3E
8Ac+24AqE+18AEq
12AqE+36AEq+27Ec
c 8Ac+36AqE+54AEq+27Ec.
&c.

√ A +B+C
A +B+C
Aq+BA+CA
BA+Bq+BC
CA+BC+Cq.
q Aq+2BA+Bq+2CA+2BC+Cq.

Besides this ordinary way of Production, the Power of any Binomial *Species* may speedily be had, thus: Set down all the Parodical Degrees of both *Species* under the Power to the Root, and then place the highest Powers of each in opposition, and let the rest be coupled contrary, the highest of one sort to the lowest of the other, and to them prefix the Numbers proper to the Power to be produced, mentioned in the Table for Extraction of Roots, *Book* 2. *Part* 2. *Chap.* 3. which said Numbers are sometime called *Unciæ*. *Power of a Binomial otherwise produced.* *Unciæ what.*

As if the 7*th* Power of A+E (that is the second Sursolid) be sought. *Example.*

Parodical Degrees.		*Rightly placed*		*with the Unciæ.*
		Aqqc		Aqqc
A	E	Acc	E	7AccE
Aq	Eq	Aqc	Eq	21AqcEq
Ac	Ec	Aqq	Ec	35AqqEc
Aqq	Eqq	Ac	Eqq	35AcEqq
Aqc	Eqc	Aq	Eqc	21AqEqc
Acc	Ecc	A	Ecc	7AEcc
Aqqc	Eqqc		Eqqc	Eqqc

The Parodical Degrees with the *Unciæ* to the 10*th* Power, stand as in the Table following, which may be enlarged at pleasure. *A Table of the Parodical Degrees, with the Unciæ to the 10th Power.*

1	2	3	4	5	6	7	8	9	10
A E	Aq 2AE Eq	Ac 3AqE 3AEq Ec	Aqq 4AcE 6AqEq 4AEc Eqq	Aqc 5AqqE 10AcEq 10AqEc 5AEqq Eqc	Acc 6AqcE 15AqqEq 20AcEc 15AqEqq 6AEqc Ecc	Aqqc 7AccE 21AqcEq 35AqqEc 35AcEqq 21AqEqc 7AEcc Eqqc	Aqcc 8AqqcE 28AccEq 56AqcEc 70AqqEqq 56AcEqc 28AqEcc 8AEqqc Eqcc	Accc 9AqccE 36AqqcEq 84AccEc 126AqcEqq 126AqqEqc 84AcEcc 36AqEqqc 9AEqcc Eccc	Aqqcc 10AcccE 45AqccEq 120AqqcEc 210AccEqq 252AqcEqc 210AqqEcc 120AcEqqc 45AqEqcc 10AEccc Eqqcc

By this Table we have a farther Demonstration of the Truth of that Theorem of *Euclid* touching the Square; and the other of *Ramus* touching the Cube, mentioned before, *Book* 2. *Part* 2. *Chap.* 2. *Sect.* 6, 7. *Theorem of Euclid demonstrated.*

As

As in producing the Square and Cube of 845; firſt A ſhall be ſet for 8, and E for 4; then 84 ſhall be A, and 5 ſhall be E; the reſt of the Work, with the *Species* annexed, follow.

Square.

8	4	5		
64			Aq	
6	4		2AE	*Gnomon.*
	16		Eq	
70	56		Aq	
	84	0	2AE	*Gnomon.*
		25	Eq	
71	40	25		

Cube.

8	4	5		
512			Ac	
76	8		3AqE	
3	84		3AEq	*Gnomon.*
	64		Ec	
592	704		Ac	
10	584	0	3AqE	
	63	00	3AEq	*Gnomon.*
		125	Ec	
603	351	125		

What the Diagonals, Complements Unciæ and Gnomon.

The two extream Powers of every kind in this Table, are ſometime called *Diagonals*, and the intermediate Species *Complements*. The Numbers or *Unciæ* affixed, ſhew the number of Complements to be taken in the Conſtitution of the Power; and all the Complements with the leſſer Power make up the *Gnomon*.

Mr. Oughtred*'s Obſervations on the Table.*

Mr. *Oughtred* in the 17*th* Chapter of his *Clavis*, makes farther inſpection into the Table, and obſerves:

1. What Species Affirmative, and what Negative, &c.

1. That all the *Species* of the Powers of a Binomial Root are Affirmative, and thoſe Parts ſimply taken without Unity, are in continual Proportion: But all the *Species* of the Powers of a Reſidual Root, are alternately Negative.

Example in Binomials.

As Powers.	*Binomial.*		*Parts.*		*Ratio.*
Q:	A + E	is	Aq+AE+Eq	∺	$\frac{9}{6}(1\frac{1}{2}$ $\frac{6}{4}(1\frac{1}{2}$
			9 . 6 . 4		
C:	3 + 2		Ac+AqE+AEq+Ec	∺	$\frac{27}{18}(1\frac{1}{2}$ $\frac{18}{12}(1\frac{1}{2}$ $\frac{12}{8}(1\frac{1}{2}$
			27 . 18 . 12 . 8		
QQ:			Aqq+AcE+AqEq+AEc+Eqq	∺	$\frac{81}{54}(1\frac{1}{2}$ $\frac{54}{36}(1\frac{1}{2}$, &c.
			81 . 54 . 36 . 24 . 16		

Examples in Reſiduals.

Powers.	*Reſidual.*	*Parts.*
Q:		Aq —2AE +Eq.
C:	A—E	is Ac —3AqE+3AEq—Ec.
QQ:		Aqq—4AcE+6AqEq—4AEc+Eqq.
		&c.

2. Difference of Names, what.

2. That the difference of the Names of every Binomial or Reſidual, is the Homogeneal Power of the Difference of the Names of the Root.

Example.

Scilicet, Ac+3AEq$\overset{*}{—}$3AqE+Ec, or Ac+3AEq—3AqE—Ec, is the Cube of A—E.

3. Difference of Squares, &c. what.

3. That the Difference of the Squares of the Names of every Binomial or Reſidual, is the Homogeneal Power of the difference of the Squares of the Names of the Root.

Example.

Scilicet, Q: Ac+3AEq$\overset{*}{—}$Q:3AqE+Ec, is C:Aq—Eq.

4. Species aggregate in Names, &c. what.

4. That if the *Species* aggregate in Names alternately be Affirmative and Negative, the Sum of the Squares of the Names is the Homogeneal Power of the Sum of the Squares of the Names of the Root.

Example.

Scilicet, Q: Ac—3AEq$\overset{*}{+}$ Q:3AqE—Ec, is C:Aq+Eq.

5. Intermediate Species, Powers of the Means.

5. That all the intermediate *Species* of every Order, are alſo Powers of the mean Proportionals between A and E.

Example.

Scilicet, between Ac and Ec, there are two mean Proportionals AqE and AEq, which are alſo Cubes of M and N, (the four continual Proportionals being A.M.N.E): Wherefore A.√cAqE.√cAEq.E. are continual Proportionals. For AqE=AMN=Mc. And AEq=MNE=Nc.

Invention of Means ariſe hence.

And hence ariſeth the Invention of any mean Proportionals between A and E; of which further is to be ſeen in the next Book. As if five mean Proportionals be ſought, then ſhall the Power be the ſixth Quantity or cc; the Index whereof exceeds

exceeds by an Unit the Number of the Means sought, and shall be A.√ccAqcE. √ccAqqEq.√ccAcEc.√ccAqEqq.√ccAEqc.E. ∸

6. That every mean *Species* in every kind is made of the two Powers of the Names of the Root, the Indices whereof together, are equal to the Index of the same kind; but the mean Distances of the same *Species* from their Extreams, shall be equal to the Indices of the Alternate Factors, and answer to them in the Common Angle. *6. Mean Species made of what.*

Scilicet. AqEc in the 5*th* Power, is made of Aq multiplied into Ec, which answer in the common Angle; and is from Aqc the Third, and Eqc the Second. Example.

And hence came the former speedy and easy Way of producing the Power of any Binomial by the Parodical Degrees. The Example given by Mr. *Oughtred* there, is in the 5*th* Power of the Binomial Root A+√qÆ, of which the Parodical Degrees may be thus placed with their Indices. *Hence came the production of Figurals in the Pages before.*

Example.

	4	3	2	1			Aqc
	Aqq .	Ac .	Aq .	A		5	√qÆAqcc
5					√qÆqc	10	ÆAc
Aqc					5	10	√qÆcAqq
	√qÆ .	Æ .	√qÆc .	Æq		5	ÆqA
	1	2	3	4			√qÆqc

This explained in Numbers, taking for the Root A+E, 82 of which the Segments being 80+2 or 8. 2. the Rectangle or Æ is 16, the square Root or √qÆ is 4. the Numbers that answer to the Parodical Degrees, and make up the whole Power of 84, *viz.* 4182119424, are as followeth. *Explained in Numbers.*

A .8	√qÆ .4 .		Aqc	32768
Aq .64	Æ .16 .	5	√qÆAqcc	81920
Ac .512	√qÆc .64 .	10	ÆAc	81920
Aqq.4096	Æq .256 .	10	√qÆcAqq	40960
Aqc.32768	√qÆqc.1024.	5	ÆqA	10240
			√qÆqc	1024
				4182119424

7. That if any *Species* be multiplied in Æ, the Magnitude produced shall be the mean *Species* collateral in the following alternate Order, and the same in Number from their Extreams. *7. What produced by a Species, × in Æ.*

As Ac×Æ, is AqqE, which is the first from Aqc, and the fourth from Eqc. Examples.

So AcE×Æ, is AqqEq, which is the second from Acc, and the fourth from Ecc.

8. That if any *Species* be multiplied by A—E or X, the Magnitude produced shall be the Difference between the two *Species* next each of the following Order. *8. What the Product of A—E × into a Species.*

As AcX =Aqq —AcE. AqEX=AcE—AqEq. Examples.

AEqX=AqEq—AEc. EcX =AEc— Eqq.

Wherefore if all the *Species* of every Order be multiplied by X, the Difference of the two extream Powers of the next upper Order will be produced.

As of Ac+AqE+AEq+Ec multiplied into X, will be made Aqq—Eqq.

9. That in the Orders of unequal Indices (l. c. qc. &c.) the Sum of the two extream Powers; but in the Orders of equal Indices, (q. qq. cc. &c.) the Difference of them, made of A+E multiplied into the singular *Species* of the lesser Order precedent, is alternately Affirmative and Negative. *9. Sum of unequal difference of the equal Indices × into a Species, what produced.*

As Ac+Ec, made of Aq—AE+Eq, multiplied into A+E. Examples.

Also Aqq—Eqq, made of Ac—AqE+AEq—Ec, multiplied into A+E.

10. That if the same Magnitude be multiplied into two contrary Magnitudes, the Magnitudes made of them shall be also contrary. *10. Magnitude in contrary, what they produce.*

As Aq−2AE+Eq multiplied into A−E, shall be made Ac−3AqE+3AEq−Ec. Examples.

But the same multiplied into −A+E, shall be made −Ac+3AqE−3AEq+Ec.

11. That the *Unciæ* or Numbers prefixt to the *Species*, are numerary Figures: For all under A and E are Roots; all under Aq and Eq are Triangulars; all under Ac and Ec are Pyramidals; all under Aqq and Eqq are Triangle-Triangulars; all under Aqc and Eqc are Triangle-Pyramidals; all under Acc and Ecc are Pyramidal-Pyramidals, *&c.* 11. Unciæ *what they are, and how called.*

12. That if in any *Species* the number of the negative Sides be unequal, that *Species* shall be Negative. 12. *Unequal Sides Negative, so the Species.*

Examples. As Q: A+B—C=Aq+2AB+Bq—2BC+Cq—2AC.
C: A+B—C=Ac+3AqB+3ABq+Bc—3BqC+3BCq—Cc—3CAq+3CqA—6ABC.

Root.	*Square.*		*Cube.*		
A+B—C	Aq	2AB	Ac	Bc	—Cc
	Bq	—2BC	3AqB		3ABq
	Cq	—2CA	—3BqC		3BCq
			3CqA		—3CAq
				—6ABC	

Roots of Rational Species extracted. Thus much may suffice for the Genesis. Now for the Analysis of Rational *Species* or Extraction of their Roots, wherein nothing of difficulty occurs, the Extraction of Roots in other Numbers being learned, and especially Cossicks, between which and these Rational *Species* there is great Concinity: For Figurate *Species* Simple, as Simple Cossicks, and compound Figurate *Species* Binomial and Residual, as compound Cossicks, have their Roots extracted. Yet *Species* of a Polynomial Root differ in the pointing or pricking of the Number; for whereas in all other Extractions the figural Numbers were pricked according to their Quantities, in *Species* respect is to be had both to the Quantities and to the Number of distinct *Species* in the Root: For the Square that hath three *Species* in the Root, shall leave two Magnitudes unprickt between the right-hand Prick and the next, and but one Magnitude unprickt between the two next left-hand Pricks. But if the Root have four *Species* in it, then between the Dexter and next Sinister Prick, shall three Magnitudes be left unprickt, and 2 between the two next Pricks, and but one between that and the uttermost Sinister Prick. Likewise in pricking Cubes, if the Root consist of three *Species*, there shall be five Magnitudes left unprickt, between the Dexter and next Sinister Prick, and between that and the utmost Sinister Prick but two: And if the Root have four *Species* in it, then are nine Magnitudes left unprickt between the first two right-hand Pricks, five between the two next, and two between those next the left-hand, &c.

Figurate Species of a Polynomial Root pricked different from others.

Examples in the Square. As to extract the Root of Aq+2AE+Eq, because the Root is a Binomial, only one *Species* is left unprickt as others: And the Root of Aq which is A, is placed in the Quotient; this doubled, that is 2A, is Divisor, which gets E in the Quotient: Then multiplying the Divisor thereby, and adjoining the Square of E, makes the *Gnomon* to be substracted.

Square Aq+2AE+Eq |A+E Root.
Aq
Divisor 2A
E
2AE+Eq *Gnomon.*

But to extract the Square Root of Aq+2BA+Bq+2CA+2BC+Cq, because the Root is a Polynomial of three *Species* Aq. Bq. & Cq. are the pricked *Species*. And after A+B is gotten in the Quotient, as by the Work above, then 2A+2B is Divisor, which gets C in the Quotient; and the Divisor multiplied thereby, and the Square of C adjoined, makes the second *Gnomon.*

Square Aq+2BA+Bq+2CA+2BC+Cq |A+B+C. Root.
Aq
Divisor 2A
B
2BA+Bq *Gnomon.*
Divisor 2A+2B
C
2CA+2BC+Cq *Gnomon.*

Examples in the Cube. So to extract the Cube Root of Ac+3AqE+3AEq+Ec, the Root being a Binomial, the *Species* unprickt are as others: And the Root of Ac which is A, is placed in the Quotient: This squared and tripled is Divisor, which gets E in the Quotient, whereby the Divisor multiplied, and the triple of A by the Square of E and the Cube of E added together, make up the *Gnomon* to be substracted.

Cube

$$\begin{array}{lll} \text{Cube} & Ac+3AqE+AEq+Ec & |\,\underline{A+E}\ \text{Root.} \\ & Ac & \\ & \text{Divisor}\ \ 3Aq \quad\ 3A & \\ & \qquad\quad\ E \qquad Eq & \\ & \overline{3AqE+3AEq+Ec}\ \textit{Gnomon.} & \end{array}$$

But to extract the Cube Root of $Ac+3AqB+3ABq+Bc+3CAq+6ABC+3BqC+3ACq+3BCq+Cc$, because the Root is a Polynomial of three *Species*, according to the Instructions aforesaid, the Number is pricked: And after $A+B$ is gotten in the Quotient, as by the Work of the other Cube last mentioned; then $A+B$ squared and tripled, makes the Divisor $3Aq+6AB+3Bq$, whereby C is gotten for the Quotient; this multiplying the Divisor, and the Square of C by the triple of $A+B$, and added to the Cube of C, makes up the second *Gnomon* to be substracted.

$$\begin{array}{ll} & \textit{Cube.} \qquad\qquad\qquad\qquad \textit{Root.} \\ & Ac+3AqB+3ABq+Bc+3CAq+6ABC+3BqC+3ACq+3BCq+Cc\ |\,\underline{A+B}+C \\ & Ac \\ \text{Divisor} & 3Aq \quad 3A \\ & \ \ B \quad\ \ Bq \\ \textit{Gnomon} & \overline{3AqB+3ABq+Bc} \\ & \qquad\qquad \text{Divisor}\ \ 3Aq+6AB+3Bq \quad 3A+3B \\ & \qquad\qquad\qquad\qquad\quad C \qquad\qquad\quad Cq \\ & \qquad\qquad \textit{Gnomon}\ \ \overline{3AqC+6ABC+3BqC+3ACq+3BCq+Cc.} \end{array}$$

Besides this real Difference in pointing the Numbers, there is another seeming Difference in extraction of the Roots of Rational *Species* and others in the Divisor of all Powers above the Square: For whereas in all other Extractions, the Power next inferior to the given Number, whose Root is to be extracted, multiplied by the Index of the given Power, is counted the Divisor; As in the Square the Root multiplied by 2, the Index of the Square; in the Cube the Square multiplied by 3, the Index of the Cube; in the squared Square the Cube multiplied by 4, the Index of the squared Square, &c. In *Species* the Divisor is usually reckoned to consist of all the Parodical Degrees under the Power whose Root you are extracting, with the *Unciæ* annexed to those Degrees. *Divisors used in Extraction, how different from others.*

As in the Square 2A, in the Cube 3Aq and 3A, in the squared Square 4Ac and 6Aq and 4A, &c. Nevertheless, inquiry for the next Quotient, Figure or *Species*, is made only by the left-hand *Species* thereof, which is the same with the Divisor in other Extractions: And the other Parts of the Divisor are set down, the better to direct the Operator to place the Parodical Degrees of the new-gotten Quotient, Figure or *Species* thereby, for Multiplication of them one into another to make up the *Gnomon*. Examples.

$$\begin{array}{lll} \textit{Square.} & \textit{Cube.} & \textit{Squared Square.} \\ \quad Aq & \quad Ac & \quad Aqq \\ \text{Divisor}\ |\ 2A:E \\ \qquad\qquad\ \ Eq \end{array}\Big\}\textit{Gnomon.} \quad \begin{array}{l} \text{Divi-} \\ \text{sor} \end{array}\left\{\begin{array}{l} 3Aq:E \\ 3A\ :Eq \\ \qquad Ec \end{array}\right\}\textit{Gnomon.} \quad \begin{array}{l} \text{Divi-} \\ \text{sor} \end{array}\left\{\begin{array}{l} 4Ac:E \\ 6Aq:Eq \\ 4A\ :Ec \\ \qquad Eqq \end{array}\right\}\textit{Gnomon.}$$

Thus that most curious Analyst Mr. *Oughtred* practises, not only in all his Resolution of Affected Equations, but even in extracting the Roots of plain Figural Numbers, to which as more demonstrative he adjoineth the *Species*, after the manner in the two Examples following, of the Square and Cube of 845; in which latter the first Divisor is 1944, which by the common Way is but 192: And the next Divisor, which ordinarily would be but 21168, is 211932. In the Square there ariseth no Difference in the Divisor, because the Quantity is next the Root. *Practice of Mr. Oughtred.*

Square

Examples.

Square 784 / 714025 Root. (845

64			Aq	
x	6		2A *Divisor.*	
6	4		2AE	*Gnomon.*
	16		Eq	
6	56			
	x6	8	2A *Divisor.*	
	84	0	2AE	*Gnomon.*
		25	Eq	
	84	25		

Cube 10 / 91647 / 603351125 Root. (845

512			Ac	
x9	2		3Aq	*Divisor.*
	24		3A	
x9	44			
76	8		3AqE	*Gnomon.*
3	84		3AEq	
	64		Ec	
80	704			
2	x16	8	3Aq	*Divisor.*
	2	52	3A	
2	x19	32		
10	584	0	3AqE	*Gnomon.*
	63	00	3AEq	
		125	Ec	
10	647	125		

Proof of Figuration of Rational Species.

Production of Rational *Species* and Extraction of their Roots, are mutual Proofs of the Truth of each other's Operations, as in other Figural Numbers. Besides which, if any Scruple arise in either, trial may be made of both, by taking Absolute Numbers, and working therewith. As instead of other Examples, the last of the Analysis of the Square and Cube of 845, with the Examples of their Production before in this Chapter, are sufficient Evidence.

CHAP. XII. *Reduction of Irrational Species.*

Reduction of Irrational Species.

IN pursuit of *Species*, I am now come to *Irrationals*, which in their Operations and Resolutions follow *Surds*, as the Rational *Species Cossicks*. And having been described before, *Chap.* 1. of *Species*, I proceed forthwith to their Reduction.

Irrational Species are either to be reduced to their least Terms, or from different Denominations into one.

To their least Terms.

To abbreviate or lessen the Terms of an *Irrational Species*, is when the Denomination is Compound, and the Number annexed hath a Root that may be expressed by part of that Compound Denomination; then extract the Root of the Number, and alter the *Species* accordingly.

Examples.

As √qq25 and √cc81, the Indices 4 and 6 reduced to their least Terms, are 2 and 3, by the common Divisor 2 the Index of the Square: If therefore the Square Root of 25 and 81 be taken, the Surd *Species* shall be reduced to √q5 and √c9.

	2) 4	6	2	3	
√qq25 and √cc81	qq	cc	q	c	√q5 and √c9
	25	81	5	9	

So √cc27 may be reduced to √q3, and discharged of c.
And √qqcc32 discharged of qcc, may be reduced to √q2.

To one Denominator.

To reduce *Irrational Species* of different Denominations into one, comprehends,

With an Absolute Number.

First, To reduce an Absolute Number into the Denomination of a Surd or *Irrational Species*; and this is done by multiplying the Number according to the Power of the *Species*.

Examples.

As 2 and √qB reduced, is √q4 and √qB.
So 2 and √cB reduced, is √c8 and √cB.

Secondly,

Secondly, To reduce two *Irrational Species* of uncompounded Indices into one Denomination; And this is effected by multiplying each Quantity as his Alternate Power or *Species* doth shew. *With uncompounded Indices.*

As √qB & √cC reduced, is √ccBc & √ccCq *Example.*

√q		√c	√q
B	X	C	√c
Bc		Cq	√cc

Thirdly, To reduce two *Irrational Species* of compounded Indices into one Denomination. And this is performed by dividing the Indices by the common Divisor, and then by the least Terms thereof multiplying alternately both the given Indices for the new Index, and the Numbers into the Powers of these least Terms. *With compounded Indices.*

	Examples in Numbers.			*Species.*		
Surds to be reduced	√qq10 and √cc7			√qAq and √qqBq		
Common Divisor	2) 4qq		6cc	2) 2q		4qq
	2 q	X	3c	1√	X	2q
Least Terms.	3		2	2		1
Surds reduced	√cccc1000		√cccc49	√qqAqq		√qqBq

Examples.

As before in Reduction of Surds one part of Reduction, which lessened the Terms, was observed to prove the other, which exalted the lessened Surds into the Powers from whence they were abated; and reciprocally that part of Reduction which increaseth their Terms and Denominations, by extracting the Roots and abating the Characters: So here. *Proof of Reduction of Irrational Species.*

Also all sorts of Reduction of *Irrational Species* may be proved, by taking in their stead Rational Numbers, and working with them after the manner of Surds, as in the Chapter of *Reduction of Surds* is so fully exemplified that Example here need not.

CHAP. XIII. *Addition of Simple Irrational Species.*

Irrational Species agreeing in their Simple Elements with Surds, before explicated in the 5*th* Part of this third Book, the fewer Examples, and a more brief Repetition of the Rules may serve turn here. *Addition of Irrational Species Simple.*

Addition of the Simple may be included in four *Cases*.

Case 1. If the *Species* and Numbers (if any be annexed) be commensurable, then order the Numbers, whether Integers or Fractions, as Simple Surds before, and the *Species* in like manner; that is, figurate the Sum of their Roots according to their Quantities, and multiply that Quantity by the Common Divisor. *1. Commensurable.*

As to add WRPq to WRSq. The Common Divisor is R. The Squares Pq & Sq. The Roots P & S; which squared and multiplied into R, makes the Total WRPq +2PRS+RSq, which is all one in equality with WRPq+WRSq. *Examples.*

		Addends.					
Common Divisor	WR	WRPq WRSq	Squares	Pq Sq	Roots	P. S.	
Total		WRPq+2PRS+RSq					

P+S Root.
P+S

Pq+PS
PS+Sq

Pq+2PS+Sq Square.
WR Com. Divisor.

WRPq+2PRS+RSq Total.

Another Example with Numbers.

	Addends.		
W3	W27Aq	9Aq	3A.
	W 3Bq	1Bq	1B.
Total	W27Aq+18BA+3Bq		

3A +B
3A +B

9Aq+3BA
3BA+Bq

9Aq+6BA+Bq
W3

W27Aq+18BA+3Bq=W27Aq+W3Bq.

 Case

2. Incommensurable. *Case* 2. If the *Species* or Numbers annexed, or both be incommensurable, then join them together with +.

Examples. As √qB and √qC added, are √qB+√qC.

So √c5B added to √c10C, make √c5B+√c10C.

3. Heterogeneal. *Case* 3. If the Denominations of the Quantities be Heterogeneal, they may be reduced, and then added or conjoined as before.

Example. As to add the √qB & √cC, they are reduced as in the precedent Chapter to the Denomination of cc, and then conjoined with the Sign of Addition thus; √ccBc +√ccCq.

4. Different Signs. *Case* 4. If the Signs be different, conjoin the given Irrationals with the Sign —.

Example. As to add √qB with —√qA, the Total shall be √qB—√qA.

Irrational added to it self. Here also, as in Simple Surds, may be observed, to add any *Irrational Species* to it self, is to multiply the Squares by 4, the Cubes by 8, *&c.*

Examples. As √qB+√qB, is √q4B. And √cD+√cD, is √c8D.

Proof of Addition of Simp. Irrat. Species. *Addition of Simple Irrational Species*, is to be proved by their Substraction in the next Chapter, and by taking Rational *Species* or Numbers instead of Irrational. As in the last Example, supposing the √cD be 2, then shall D be 8, and 8D 64, whose Cube Root will be 4 equal to 2 added to it self.

CHAP. XIV. *Substraction of Simple Irrational Species.*

Substraction of Irrational Species Simple. AS Addition, so the Substraction of these Irrationals may be included in four *Cases*.

1. Commensurable. *Case* 1. If the *Species* and Numbers (if any be annexed) be commensurable, then (whether Integers or Fractions) order the Numbers as in Substraction of Simple Surds before, and the *Species* likewise; that is, figurate the Difference of their Roots according to the given Quantities, and multiply that Quantity by the Common Divisor.

Examples. As to take √RSq from √RPq, the Common Divisor is R, the Squares Pq and Sq, the Roots P and S, the Difference P—S; which squared and multiplied into R, makes the Remain √RPq—2PRS+RSq, which is equal to √RPq—√RSq.

Subtrahend.

√RPq—√RSq Squares (Pq Roots P
Common Divisor √R (Sq S

Remain √RPq—2PRS+RSq

P—S
P—S
Pq-PS
—PS+Sq
Pq—2PS+Sq
√R
√RPq—2PRS+RSq

Another Example with Numbers.

Subtrahend.

√27Aq—√3Bq (9Aq . 3A.
Common Divisor √3 (1Bq . 1B.

Remain √27Aq-18BA+3Bq

3A—B
3A—B
9Aq—3BA
—3BA+Bq
9Aq—6BA+Bq
√3
√27Aq—√3Bq=√27Aq—18BA+3Bq

2. Incommensurable. *Case* 2. If the *Species* or Numbers annexed, or both, be Incommensurable, then join them together with —.

Examples. As to take √qC from √qB, the Remain shall be √qB—√qC.

So √c9B taken from √c10C, shall make the Remain √c10C—√c9B.

3. Heterogeneal. *Case* 3. If the given Denominations of the Quantities be Heterogeneal, they may be reduced, and then substracted or conjoined as before.

Example. As to take √qB from √cC, being reduced as before in *Chap.* 12. *Of Reduction of Irrational Species*, to the Denomination of cc, they are then conjoined with the Sign of Substraction thus, √ccCq—√ccBc.

Case

Case 4. If the Signs be different, the Irrationals given are to be added; And the Sign of the Remain shall be contrary to the Sign of the Subtrahend, as before in Simple Surds, if they be commensurable; but if incommensurable, conjoin them by the Sign of Addition $+$. *4. Different Signs.*

As to take $-\sqrt{q}3BD$ from $\sqrt{q}12A$, because the Signs are unlike, the one $+$ and the other $-$ they are added, and the Remain shall be $\sqrt{q}12A+\sqrt{q}3BD$. *Examples.*

And $-\sqrt{q}C$ taken from $\sqrt{q}B$, and $\sqrt{q}C$ taken from $-\sqrt{q}B$, leave their Remains $\sqrt{q}B+\sqrt{q}C$, and $-\sqrt{q}B+\sqrt{q}C$.

Here also, as in Substraction of Simple Surds, may be observed, that to take any Irrational from it self, leaves nothing remaining; but to take half any Irrational *Species*, is to divide the Squares by 4, the Cubes by 8, &c. *Irrational taken from it self, or the Half taken.*

As to take $\sqrt{q}B$ from $\sqrt{q}B$, the Remain is Nought. *Examples.*

But to take half the $\sqrt{q}B$ is $\frac{\sqrt{q}B}{4}$. And half the $\sqrt{c}B$ is $\frac{\sqrt{c}B}{8}$

Substraction of Simple Irrational *Species* is proved by their Addition: And by taking Rational *Species* or Numbers, instead of those Irrational, and working therewith: As in the last Example, if half the $\sqrt{c}B$ be $\frac{\sqrt{c}B}{8}$, then by the Addition thereof to it self, it shall be $\frac{\sqrt{c}8B}{8}$, and by clearing the Number of 8, it shall be $\sqrt{c}B$ as before. *Proof of Substraction of Sim. Irrat. Species.*

Also suppose B were 64, the $\sqrt{c}$ thereof shall be 4; which halfed, shall be 2 equal to the $\sqrt{c}8$.

CHAP. XV. *Multiplication of Simple Irrational Species.*

To the Multiplication of these Irrationals, two Cases will serve.

Case 1. If the *Species* be Homogeneal in their figurate Denominations, then multiply Numbers as Numbers, and *Species* as *Species*, Integers as Integers, and Fractions as Fractions. *Multiplication of Irrational Species Simple. 1. Homogeneal.*

As $\sqrt{c}12B$ multiplied by $\sqrt{c}5C$, produceth $\sqrt{c}60BC$. *Examples.*

Factors.	*Product.*	*Factors.*	*Product.*
$\sqrt{q}A\times\sqrt{q}B$	$=\sqrt{q}BA$	$\sqrt{qq}12B\times\sqrt{qq}D$	$=\sqrt{qq}12BD$.

Case 2. If the *Species* be Heterogeneal in their figurate Denominations; then after Reduction to one Denomination, multiply them as before. *2. Heterogeneal.*

As $\sqrt{q}A$ to be multiplied into $\sqrt{c}B$, is reduced to $\sqrt{cc}Ac$, and $\sqrt{cc}Bq$; and then multiplied to $\sqrt{cc}AcBq$. *Example.*

Reduction.

$$\begin{array}{ccccc} A & & B & \sqrt{q} \\ \sqrt{q} & \times & \sqrt{c} & \sqrt{c} \\ \hline Bq & & Ac & \sqrt{cc} \end{array}$$

Multiplication.

$$\begin{array}{l} \sqrt{cc}\ \begin{array}{c} Ac \\ Bq \end{array} \\ \hline \sqrt{cc}AcBq \end{array}$$

Here, as in Multiplication of Simple Surds, these Consectaries take place. *Consectaries.*

1. To multiply any *Irrational Species*, is to increase him by the Power of a Root Homogeneal. *1. How to double, triple, &c.*

Irrational.	*Doubled.*	*Tripled.*	*Quadrupled.*
	$\sqrt{q}B$	$\sqrt{q}B$	$\sqrt{q}B$
Squares.	4	9	16
	$\sqrt{q}4B$	$\sqrt{q}9B$	$\sqrt{q}16B$
	$\sqrt{c}D$	$\sqrt{c}D$	$\sqrt{c}D$
Cubes.	8	27	64
	$\sqrt{c}8D$	$\sqrt{c}27D$	$\sqrt{c}64D$

Examples.

2. To multiply some Irrationals, produce Rationals, and the Product, as occasion is, may be cleared of the Surd Character. *2. Rationals produced.*

Irrationals.	*Square.*	*Cube.*
	$\sqrt{q}18B$	$\sqrt{c}250C$
	$\sqrt{q}\ 2E$	$\sqrt{c}\ 4D$
Rationals.	$\sqrt{q}36BE=6BE$.	$\sqrt{c}1000CD=10CD$.

Examples.

3. To

3. What to cancel the Character.

3. To multiply the Side of any Power according to the Exigency of his Kind, is but to blot out or cancel the Note of the Side, and leave the *Species* absolute.

Examples.

As $\surd qB \times \surd qB = B$. And $\surd c12D \times \surd c12D \times \surd c12D = 12D$.

4. What produced by the Side of a Power, &c.

4. To multiply the Side of a Power, whose Index is a Compound Number, according to the Exigency of one of the Kinds compounding; the Side of either Kind may be prefixed to the special Number alone.

Examples.

As the Square of the $\surd ccB$, is $\surd cB$; and the Cube of the $\surd ccB$, is $\surd qB$: For $\surd cc$ is $\surd$ of 2×3.

5. Homogeneal Figurals multiplied, what produced.

5. To multiply a figural Number by an Homogeneal figural Number, the Product shall be a figural Number of the same Kind, whose Side or Root shall be equal to the Product of the Sides of the Number multiplied.

Examples.

As $Aq \times Eq = AqEq$. And $Ac \times Ec = AcEc$.

q. $25 \times 16 = 400$ c $27 \times 8 = 216$

$\surd$. $5 \times 4 = 20$ $\surd$ $3 \times 2 = 6$

6. Sides of such multiplied, what produced.

6. To multiply the Sides of Homogeneal *Species* Irrational, begetteth Sides of Irrational *Species* Homogeneal.

Examples.

As $\surd qA \times \surd qE = \surd qAE$. And $\surd cB \times \surd cC = \surd cBC$.

Proof of Multiplication of Sim. Irrat. Species.

Multiplication of Simple Irrational *Species*, is to be proved by their Division in the next Chapter; and by taking Rational *Species* or Numbers, and working therewith instead of Irrational. As in the last Example, if B be 27, and C 8, the Product will be 216; the Cube Root whereof is 6, equal to 3 the Root of 27, multiplied into 2 the Root of 8.

CHAP. XVI. *Division of Simple Irrational Species.*

Division of Irrational Species Simple.

AS Multiplication, so the Division of these Irrationals may be included in two Cases.

1. Homogeneal.

Case 1. If the *Species* be Homogeneal in their figurate Denominations, then divide the Dividend by the Divisor, Numbers as Numbers, and *Species* as *Species*, Integers as Integers, and Fractions as Fractions.

Examples.

As $\surd qBA$ divided by $\surd qB$, giveth in the Quotient $\surd qA$.

And $\surd c60BC$ divided by $\surd c12B$, giveth in the Quotient $\surd c5C$.

Dividend. *Dividend.*

Divisor $\surd qB)\ \surd qBA\ (\surd qA$ Quotient. $\surd c12B)\ \surd c60BC\ (\surd c5C$.

2. Heterogeneal.

Case 2. If the *Species* be Heterogeneal in their figurate Denominations, then first reduce them into an Homogeniety, and afterward divide them as before.

Example.

As $\surd ccAcBq$ divided by the $\surd qA$, the Divisor is first reduced to $\surd ccAc$; and then Division being made, the Quotient is $\surd ccBq$, which may be depressed to $\surd cB$.

Reduction. *Division.*

$$\begin{array}{lcl} & \surd ccAc & & \surd ccAcBq \\ 2) & \surd qA & \times & \surd ccAcBq \\ & \surd & & c \\ \hline & \surd ccAc & & \surd ccAcBq \end{array}$$

$\surd ccAc)\ \surd ccAcBq\ (\surd ccBq$

Consectaries.

Here, as in Division of Simple Surds, these Consectaries are fitly inserted.

1. To take half, &c.

1. To divide any Irrational *Species*, is to diminish him by the Power of a Root Homogeneal.

Examples.

Irrational.	*The Half.*	*The third Part.*	*The fourth Part.*
Squares.	$\frac{\surd q12B}{4}(\ \surd q3B$	$\frac{\surd q27B}{9}(\ \surd q3B$	$\frac{\surd q48B}{16}(\ \surd q3B$
Cubes.	$\frac{\surd c24B}{8}(\ \surd c3B$	$\frac{\surd c81B}{27}(\ \surd c3B$	$\frac{\surd c192B}{64}(\ \surd c3B$

2. Quotient sometime Rational.

2. To divide some Irrationals, brings forth Rationals in the Quotient. And the Quotients, as occasion is, may be cleared of the Surd Character.

Square

Examples.

Square.

Irrational. $\frac{\surd q27BE}{\surd q3B}\Big(\surd q9E = 3E$ *Rational.*

Cube.

Irrational. $\frac{\surd c24DC}{\surd c\,3\,C}\Big(\surd c8D = 2D$ *Rational.*

3. To divide any *Irrational Species* by himself, giveth in the Quotient one *Irrational Species* or Unit. *3. Irrational dividing himself gives 1.*

Examples in *Squares.* $\frac{\surd qB}{\surd qB}\Big(1$, or B. *Cubes.* $\frac{\surd c9B}{\surd c9B}\Big(1$, or B. *Examples.*

4. To divide a Power whose Index is compounded, by one Side of the Compounding Powers, shall give the Quotient higher or lower according to the Dividing Power. *4. Quotient of a Power, &c. divided by the Side.*

As $\surd qB)\ \surd ccB\ (\surd cB$. And $\surd cB)\ \surd ccB\ (\surd qB$. *Examples.*

5. To divide a Figural Number by another Homogeneal figural Number, the Quotient shall be a figural Number of the same Kind, whose Side is equal to the Quotient of the greater Side divided by the Lesser. *5. Homogeneal Figurals divided, what the Quotient.*

As $\surd q\frac{AqEq}{Bq}\Big(\frac{AE}{B}$. *Example.*

6. To divide the Sides of Homogeneal Irrational *Species*, begetteth Sides of Irrational *Species* Homogeneal. *6. Sides of such divided, what begotten.*

Ergo, $\surd qA)\ \surd qAE\ (\surd qE$. *Example.*

Division of Simple Irrational Species, is proved by their Multiplication; and by taking Rational *Species* or Numbers instead of Irrational, and working therewith. As in the last Example, and several others of this Chapter made exemplary in Multiplication, bring here in the Quotient one of the Factors there. *Proof of Division of Simple Irrat. Species.*

Also suppose A 25 and E 9, then $\surd qA$ which is 5 multiplied into $\surd qE$ that is 3, will be 15 for $\surd qAE$: And this divided by $\surd qA$ that is 5, giveth 3 in the Quotient equal to $\surd qE$.

CHAP. XVII. *Addition of Compound Irrational Species.*

Addition of Irrational *Species* that are Compound, includes the Addition of Particulars and Universals. *Addition of Comp. Irrat. Species. Particulars.*

Particulars, like or unlike, are added as the Simple, and joined together into one Total with the respective Signs.

Examples.	*Binomials.*	*Residuals.*	*Mixt.*
Addends.	$Z+\surd qBC$	$\surd qB-3D$	$\surd cRq+6B$
	$R+\surd qDE$	$\surd qB-2C$	$\surd cRq-6B$
Totals.	$Z+R+\surd qBC+\surd qDE$	$\surd q4B-3D-2C$	$\surd c8Rq$

Examples.

Another Example.

Addends $\begin{cases} \frac{Z}{2} \ * + \surd q\frac{Zq-4P}{4} \\ \frac{Z}{2} \ * - \surd q\frac{Zq-4P}{4} \end{cases}$

Total. Z

Here the plain and fracted Sinister *Species* being Z and Z, that is 2Z to be divided by 2, makes Z only to be left for the Total. All the dexter Part of the Addends being of contrary Signs, *viz.* one + and the other −, and so Equal, are taken away.

Universals are added as other Compound Irrationals, with respect had to their Signs, and the Sign Universal $\surd$: prefixed, as was largely discoursed in the Addition of Universal Surds before. *Universals.*

Examples.

Examples.	*Binomials.*	*Residuals.*	*Mixt.*
Addends.	$\sqrt{:}\ Zq+4Pq$	$\sqrt{:}\ Zq-4Pq$	$\sqrt{:}\ Zq+4Pq$
	$\sqrt{:}\ Zq+4Pq$	$\sqrt{:}\ Zq-4Pq$	$\sqrt{:}\ Zq-4Pq$
Totals.	$\sqrt{:}4Zq+16Pq$	$\sqrt{:}4Zq-16Pq$	$\sqrt{:}2Zq+\sqrt{q}4Zqq-64Pqq$

The two first of these Examples are wrought according to the 7*th* Direction in Addition of Universal Surds, *Chap.* 7. *Book* 3. *Part* 5. but the last according to the Second Form of Addition of Square Surds, *Chap.* 3. *Book* 3. *Part* 5.

Other Examples explained in Numbers.

Other Examples in Species *and* Numbers *corresponding.*

$$\text{Supposing}\begin{cases} A\,3.\ E\,2.\ Z\,5.\ P\,6. \\ Aq\,9.\ Eq\,4.\ Z\,13.\ X\,5. \end{cases}$$

$$\begin{array}{l} \sqrt{:}\ Z+\sqrt{q}\ Zq-4Pq \\ \sqrt{:}\ Z+\sqrt{q}\ Zq-4Pq \\ \hline \sqrt{:}4Z+\sqrt{q}16Zq-64Pq \end{array} \quad \begin{array}{l} \sqrt{:}13+\sqrt{q}\ 169-144 \\ \sqrt{:}13+\sqrt{q}\ 169-144 \\ \hline \sqrt{:}52+\sqrt{q}2704-2304 \\ \quad 20 \quad -\frac{2304}{400}\left(\begin{array}{l}\sqrt{} \\ 20\end{array}\right. \\ \sqrt{q}72\ \textit{Value} \end{array} \left.\right\} \text{or} \begin{cases}\sqrt{:}13+\sqrt{q}25 \\ \sqrt{:}13+\sqrt{q}25\end{cases} \text{or} \begin{cases}\sqrt{q}18 \\ \sqrt{q}18\end{cases} \ \overline{\sqrt{q}72}$$

$$\begin{array}{l} \sqrt{:}\ \frac{Z}{2}\overset{*}{+}\sqrt{q}\frac{Zq-4Pq}{4} \\ \sqrt{:}\ \frac{Z}{2}+\sqrt{q}\frac{Zq-4Pq}{4} \\ \hline \sqrt{:}\frac{4Z}{2}+\sqrt{q}\frac{16Zq-64Pq}{4} \\ \text{or } \sqrt{:}2Z+\sqrt{q}4Zq-16Pq \end{array} \quad \begin{array}{l} \sqrt{:}\frac{13}{2}\overset{*}{+}\sqrt{q}\frac{169-144}{4} \\ \sqrt{:}\frac{13}{2}+\sqrt{q}\frac{169-144}{4} \\ \hline \sqrt{:}\frac{52}{2}+\sqrt{q}\frac{2704-2304}{4} \\ \hline \sqrt{:}26+\sqrt{q}\ 676-576 \\ \quad 10 \quad -\frac{576}{100}\left(\begin{array}{l}\sqrt{} \\ 10\end{array}\right. \\ \sqrt{q}36 \\ \quad (6\ \textit{Value}. \end{array} \left.\right\} \text{or} \begin{cases}\sqrt{:}\frac{13}{2}+\sqrt{q}\frac{25}{4} \\ \sqrt{:}\frac{13}{2}+\sqrt{q}\frac{25}{4}\end{cases} \text{or} \begin{cases}\sqrt{q}\frac{18}{2}=3 \\ \sqrt{q}\frac{18}{2}=3\end{cases} \ \overline{6}$$

$$\begin{array}{l} \sqrt{:}\ Z-\sqrt{q}\ Zq-4Pq \\ \sqrt{:}\ Z-\sqrt{q}\ Zq-4Pq \\ \hline \sqrt{:}4Z-\sqrt{q}16Zq-64Pq \end{array} \quad \begin{array}{l} \sqrt{:}\ 13-\sqrt{q}\ 169-144 \\ \sqrt{:}\ 13-\sqrt{q}\ 169-144 \\ \hline \sqrt{:}\ 52-\sqrt{q}2704-2304 \\ \quad -20 \quad -\frac{2304}{400}\left(\begin{array}{l}\sqrt{} \\ 20\end{array}\right. \\ \sqrt{q}\ 32 \\ \quad (\textit{Value}. \end{array} \left.\right\} \text{or} \begin{cases}\sqrt{:}13-\sqrt{q}25 \\ \sqrt{:}13-\sqrt{q}25\end{cases} \text{or} \begin{cases}\sqrt{q}8 \\ \sqrt{q}8\end{cases} \ \overline{\sqrt{q}32}$$

$$\begin{array}{l} \sqrt{:}\ \frac{Z}{2}\overset{*}{-}\sqrt{q}\frac{Zq-4Pq}{4} \\ \sqrt{:}\ \frac{Z}{2}-\sqrt{q}\frac{Zq-4Pq}{4} \\ \hline \sqrt{:}\frac{4Z}{2}-\sqrt{q}\frac{16Zq-64Pq}{4} \\ \text{Or } \sqrt{:}2Z-\sqrt{q}4Zq-16Pq \end{array} \quad \begin{array}{l} \sqrt{:}\frac{13}{2}\overset{*}{-}\sqrt{q}\frac{169-144}{4} \\ \sqrt{:}\frac{13}{2}-\sqrt{q}\frac{169-144}{4} \\ \hline \sqrt{:}\frac{52}{2}-\sqrt{q}\frac{2704-2304}{4} \\ \hline \sqrt{:}26-\sqrt{q}676-576 \\ \quad -10 \quad -\frac{576}{100}\left(\begin{array}{l}\sqrt{} \\ 10\end{array}\right. \\ \sqrt{q}16 \\ \quad (4\ \textit{Value}. \end{array} \left.\right\} \text{or} \begin{cases}\sqrt{:}\frac{13}{2}-\sqrt{q}\frac{25}{4} \\ \sqrt{:}\frac{13}{2}-\sqrt{q}\frac{25}{4}\end{cases} \text{or} \begin{cases}\sqrt{q}\frac{8}{2}=2 \\ \sqrt{q}\frac{8}{2}=2\end{cases} \ \overline{4}$$

All these four last Examples are wrought according to the 7*th* Direction in Addition of Universal Surds, *Chap.* 7. *Book* 3. *Part* 5.

√:Z

Other Examples explained in Numbers.

$$\begin{array}{l} \surd{:}\,Z + \surd q\,Zq - 4Pq \\ \surd{:}\,Z - \surd q\,Zq - 4Pq \\ \hline \surd{:}\,2Z + 4P \end{array} \qquad \left.\begin{array}{l} \surd{:}\,13 + \surd q\,169 - 144 \\ \surd{:}\,13 - \surd q\,169 - 144 \\ \hline \surd{:}\,26 + 24 \end{array}\right\} \text{or} \left\{\begin{array}{l} \surd{:}\,13 + \surd q\,25 \\ \surd{:}\,13 - \surd q\,25 \end{array}\right\} \text{or} \left\{\begin{array}{l} \surd q\,18 \\ \surd q\,8 \\ \hline \surd q\,50 \end{array}\right.$$

$$\begin{array}{l} \surd{:}\,\frac{Z}{2} \overset{*}{+} \surd q\,\frac{Zq - 4Pq}{4} \\ \surd{:}\,\frac{Z}{2} - \surd q\,\frac{Zq - 4Pq}{4} \\ \hline \surd{:}\,Z + 2P \end{array} \qquad \left.\begin{array}{l} \surd{:}\,\frac{13}{2} \overset{*}{+} \surd q\,\frac{169 - 144}{4} \\ \surd{:}\,\frac{13}{2} - \surd q\,\frac{169 - 144}{4} \\ \hline \surd{:}\,13 + 12 \end{array}\right\} \text{or} \left\{\begin{array}{l} \surd{:}\,\frac{13}{2} + \surd q\,\frac{25}{4} \\ \surd{:}\,\frac{13}{2} - \surd q\,\frac{25}{4} \end{array}\right\} \text{or} \left\{\begin{array}{l} \surd q\,\frac{18}{2} = 3 \\ \surd q\,\frac{8}{2} = 2 \\ \hline \qquad 5 \end{array}\right.$$

The Work of these two last Examples is like the second Form of Addition of square Surds, *Chap.* 3. *Book* 3. *Part* 5. For first the Surds are added, which in the upper Example make $2Z$, in the lower $\frac{2Z}{2}$, that is Z being divided; then they are multiplied, the upper Product is Zq the Sinister part, and $-Zq + 4Pq$ the Dexter part, the lower Product is $\frac{Zq}{4} - \frac{Zq + 4Pq}{4}$: for according to the third Consectary in Multiplication of Simple Surds, and also of Simple Irrational *Species*, the Surds multiplied by themselves have their Characters cancelled: So as $\surd q\,Zq - 4Pq$ multiplied by it self, shall be $Zq - 4Pq$; but in the Examples above it is $Zq + 4Pq$, because the Signs of the given Numbers were $+$ and $-$. The Products thus gotten, the Parts found with $+$ and $-$ of like Denomination are set aside; so as of the upper Product is left only $4Pq$, of the lower $\frac{4Pq}{4}$, which divided is Pq. The Roots of these are $2P$ and P; which multiplied by 2, the Index of the Square, make $4P$ to be added to the upper $2Z$, and $2P$ to the lower Z.

Another Example explained with Numbers.

$$\begin{array}{l} \surd{:}\surd\frac{Xq + 4Pq}{4} \overset{*}{+} \frac{X}{2} \\ \surd{:}\surd\frac{Xq + 4Pq}{4} - \frac{X}{2} \\ \hline \surd{:}\surd\,Xq + 4Pq + 2P \end{array} \qquad \left.\begin{array}{l} \surd{:}\,\frac{25 + 144}{4} \overset{*}{+} \frac{5}{2} \\ \surd{:}\,\frac{25 + 144}{4} - \frac{5}{2} \\ \hline \surd{:}\,25 + 144 + 12 \\ \frac{144}{169}\left(\begin{array}{l}\surd \\ 13 + 12\end{array}\right. \end{array}\right\} \text{or} \left\{\begin{array}{l} \surd q\,\frac{169}{4} + \frac{5}{2} \\ \surd q\,\frac{169}{4} - \frac{5}{2} \end{array}\right\} \text{or} \left\{\begin{array}{l} \surd q\,\frac{18}{2}\ (3 \\ \surd q\,\frac{8}{2}\ (2 \\ \hline \quad 5 \end{array}\right.$$

The Total of this Example is misprinted in *Moore*, being there $\surd Xqq + Pq + 2P$; which cannot be, for the Addition is $\frac{4Xq + 16Pq}{4}$, and by Division to clear the Fraction, is $Xq + 4Pq$; the Multiplication is thus, $\frac{Xq + 4Pq}{4} - \frac{Xq}{4}$, and by cancelling the contrary *Species*, is left only $\frac{4Pq}{4}$; which by Division is Pq, whose Root is P; this multiplied by 2, is $2P$ to be added to the first Addition, as I have set it.

Proof of Addition of Compound Irrat. Species.

Addition of Compound Irrational Species is proved by Substraction, or by Rational Numbers, like Addition of Compound Surds before; the Work of Particulars by Particulars, and Universals by Universals: And many of these Examples have their Rational Numbers besides them to save farther Instance.

CHAP. XVIII. *Substraction of Compound Irrational Species.*

Substraction of Comp. Irrat. Species. Particulars.

As Addition before, so *Substraction of Compound Irrational Species*, includes the Substraction both of Particulars and Universals.

Particulars, as the Simple Irrationals take like *Species* from like, and to the Remain their proper Sign is subscribed unless the greater be taken from the lesser, for then the Sign is to be changed: And if the Signs be contrary, though the

Species

Species be alike, then the Sign of the Sum is to be contrary to the Sign of the Subtrahend. Also *Species* unlike and Asymmetral as before, are substracted, by conjoining them with —.

Examples.

Examples.	*Binomials.*	*Residuals.*	*Mixt.*
Greater Numbers.	$Z+\sqrt{q}SR$	$\sqrt{q}B-8$	$\sqrt{c}Rq+4D$
Subtrahends.	$R+\sqrt{q}AE$	$\sqrt{q}B-4$	$\sqrt{c}Rq-4D$
Remains.	$Z-R+\sqrt{q}SR-\sqrt{q}AE$	4	$8D$

Another Example.

Greater Number. $\frac{Z}{2} \overset{*}{+} \sqrt{q}\frac{Zq-4P}{4}$

Subtrahend. $\frac{Z}{2} \overset{*}{-} \sqrt{q}\frac{Zq-4P}{4}$

Remain. $\sqrt{q}\frac{4Zq-16P}{4}$

Or $\sqrt{q}Zq-4P$

In this Example the sinister Parts of the *Data* being both alike and equal, *viz.* $\frac{Z}{2}$, are set aside; the Residue being of contrary Signs, are added together; and afterward Division by 4 being made to discharge the Fraction, the Remain is $\sqrt{q}Zq-4P$.

Universals.

Universals, like other Compound Irrationals, are substracted with respect to their Signs, and the Sign Universal prefixed, as before discoursed in Surds.

Examples.

Examples.	*Binomials.*	*Residuals.*	*Mixt.*
Greater Numbers.	$\sqrt{:}4Zq+16Pq$	$\sqrt{:}4Zq-16Pq$	$\sqrt{:}2Zq+\sqrt{q}4Zqq-64Pqq$
Subtrahends.	$\sqrt{:}\ Zq+4Pq$	$\sqrt{:}\ Zq-4Pq$	$\sqrt{:}\ Zq-4Pq$
Remains.	$\sqrt{:}\ Zq+4Pq$	$\sqrt{:}\ Zq-4Pq$	$\sqrt{:}\ Zq+4Pq$

The first two of these Examples are wrought according to the *7th* Direction in Substraction of Universal Surds, *Chap.* 8. *Book* 3. *Part* 5. but the last according to the second Form of Substraction of Square Surds, *Chap.* 4. *Book* 3. *Part* 5.

Other Examples explained in Numbers.

Other Examples in Species and Numbers corresponding.

Supposing $\begin{cases} A\,3. & E\,2. & Z\,5. & P\,6. \\ Aq\,9. & Eq\,4. & Z\,13. & X\,5. \end{cases}$

$$\sqrt{:}\frac{Z}{2}\overset{*}{+}\sqrt{q}\frac{Zq-4Pq}{4} \qquad \sqrt{:}\frac{13}{2}\overset{*}{+}\sqrt{q}\frac{169-144}{4}$$
$$\sqrt{:}\frac{Z}{2}-\sqrt{q}\frac{Zq-4Pq}{4} \qquad \sqrt{:}\frac{13}{2}-\sqrt{q}\frac{169-144}{4}$$
$$\text{or}\begin{cases}\sqrt{:}\frac{13}{2}+\sqrt{q}\frac{25}{4}\\ \sqrt{:}\frac{13}{2}-\sqrt{q}\frac{25}{4}\end{cases}\text{or}\begin{cases}\sqrt{q}\frac{18}{2}=3\\ \sqrt{q}\frac{8}{2}=2\end{cases}$$
$$\sqrt{:}\,Z-2P \qquad \sqrt{:}13-12 \qquad 1$$

$$\sqrt{:}\sqrt{\frac{Xq+4Pq}{4}}\overset{*}{+}\frac{X}{2} \qquad \sqrt{:}\frac{25+144}{4}\overset{*}{+}\frac{5}{2}$$
$$\sqrt{:}\sqrt{\frac{Xq+4Pq}{4}}-\frac{X}{2} \qquad \sqrt{:}\frac{25+144}{4}-\frac{5}{2}$$
$$\text{or}\begin{cases}\sqrt{:}\frac{169}{4}+\frac{5}{2}\\ \sqrt{:}\frac{169}{4}-\frac{5}{2}\end{cases}\text{or}\begin{cases}\sqrt{q}\frac{18}{2}=3\\ \sqrt{q}\frac{8}{2}=2\end{cases}$$
$$\sqrt{:}\sqrt{X}q+4Pq-2P \qquad \sqrt{:}25+144-12 \qquad 1$$

$$\frac{144}{169}\ \sqrt{(13-12}$$

The Work of these two last Examples may be seen in the precedent Chapter of their Addition; between which and their Substraction, all the Difference is to connex the multiplied Root 2P, to the Sum of the Addition of the Sinister part of the *Data*, with the Sign of Substraction —.

Another Example explained with Numbers.

$$\sqrt{:}\frac{2Zq-4Pq}{4}\overset{*}{+}\sqrt{q}\frac{4Zqq-16PqZq}{16} \qquad \sqrt{:}\frac{338-144}{4}\overset{*}{+}\sqrt{q}\frac{114244-97344}{16}$$
$$\sqrt{:}\frac{2Zq-4Pq}{4}-\sqrt{q}\frac{4Zqq-16PqZq}{16} \qquad \sqrt{:}\frac{338-144}{4}-\sqrt{q}\frac{114244-97344}{16}$$
$$\sqrt{:}\ Zq-4Pq \qquad \sqrt{:}169-144$$
$$1 \qquad \sqrt{q}25=5$$

The

The Work of this last Example in *Moore* is thus to be understood: The Addition at first being $\frac{4Z.q - 8Pq}{4}$, by Division is brought to $Z.q - 2Pq$. The Multiplication is $\frac{4Z.qq - 16Z.qPq + 16Pqq}{16} - \frac{4Z.qq + 16PqZ.q}{16}$; and abating 16 the Denominator, because alike in both Parts, they are reduced to Integers, and by cancelling the contrary *Species*, resteth only $\frac{16Pqq}{16}$, and by Division Pqq; whose Root Pq multiplied by 2, the square Index is 2Pq, which is to be adjoined to the first Addition: So will the Remain be $\sqrt{:}Z.q - 2Pq - 2Pq$, that is $\sqrt{:}Z.q - 4Pq$. But if the Total of those *Data's* had been required, it would have been $\sqrt{:}Z.q - 2Pq + 2Pq$, that is $\sqrt{q}Z.q$; and by Reduction only Z, and in Numbers thus:

$$\begin{array}{r} 338 \\ -144 \\ \hline 194 \end{array} \qquad \begin{array}{r} 114244 \\ -97344 \\ \hline 16900 \end{array} \sqrt{(}\, \frac{130}{4} \qquad 16(\,4$$

$$\frac{194}{4} + \frac{130}{4} = \frac{324}{4} \;\sqrt{(}\,\frac{18}{2}(\; 9$$

$$\frac{194}{4} - \frac{130}{4} = \frac{64}{4} \;(\,\frac{8}{2}(\; 4$$

13 Total.

5 Difference.

Substraction of Compound Irrational Species, Particulars and Universals respectively, is proved as Substraction of Compound Surds, both by Addition and by Rational Numbers. Enough of which may be seen by the Examples of this Chapter compared with those in the former of Addition. *Proof of Substraction of Comp. Irrat. Species.*

CHAP. XIX. *Multiplication of Compound Irrational Species.*

IN the Multiplication of these Compound Irrationals, Particulars agree with the Multiplication of particular Compound Surds; and Universals here with Universals there. *Multiplication of Comp. Irrat. Species.*

Particulars therefore are like to be multiplied with like, or else reduced thereto, and the Signs + and — ordered as in the Multiplication of Compound Cossicks. *Particulars.*

Examples.

Examples.	*Binomials.*	*Residuals.*	*Mixt.*
Multiplicands.	$\sqrt{q}B + \sqrt{q}C$	$\sqrt{q}D - \sqrt{q}B$	$\sqrt{c}K + \sqrt{c}P$
Multipliers.	$\sqrt{q}B + \sqrt{q}D$	$\sqrt{q}B - \sqrt{q}D$	$\sqrt{c}K - \sqrt{c}P$
	$B + \sqrt{q}BC$	$\sqrt{q}BD - B$	$\sqrt{c}Kq + \sqrt{c}KP$
	$\sqrt{q}BD + \sqrt{q}DC$	$-D + \sqrt{q}BD$	$-\sqrt{c}KP - \sqrt{c}Pq$
Products.	$B + \sqrt{q}BC + \sqrt{q}BD + \sqrt{q}DC$	$\sqrt{q}4BD - B - D$	$\sqrt{c}Kq - \sqrt{c}Pq$

Another Example out of Moore *explained with Numbers.* *Another Example explained with Numbers.*

Supposing A 3. E 2. Z 5. P 6.

Multiplicand. $\frac{Z}{2} * + \sqrt{q}\frac{Zq - 4P}{4}$

Multiplier. $\frac{Z}{2} * + \sqrt{q}\frac{Zq - 4P}{4}$

$$\frac{Zq}{4} * + \sqrt{q}\frac{Zqq - 4ZqP}{16}$$

$$\sqrt{q}\frac{Zqq - 4ZqP}{16} + * \frac{Zq - 4P}{4}$$

Product. $\frac{2Zq - 4P}{4} * + \sqrt{q}\frac{4Zqq - 16ZqP}{16}$

By breaking the Line into two Segments, the two Squares thereof are $\frac{Zq}{4} + \frac{Zq - 4P}{4}$, that is, $\frac{2Zq - 4P}{4}$ each of the two Rectangles is $\sqrt{q}\frac{Zqq - 4ZqP}{16}$, which added, after the manner of Surds, by their multiplication with 4, make the Product as in the ordinary way of Multiplication here set down.

The Explanation by Numbers.

$$\begin{array}{ll} \text{Multiplic.} & \tfrac{5}{2}\overset{*}{+}\sqrt{q}\dfrac{25-24}{4} \\ \text{Multiplier.} & \tfrac{5}{2}\overset{*}{+}\sqrt{q}\dfrac{25-24}{4} \end{array}\Bigg\}\text{ or }\left\{\begin{array}{l}\tfrac{5}{2}+\sqrt{q}\tfrac{1}{4} \\ \tfrac{5}{2}+\sqrt{q}\tfrac{1}{4}\end{array}\right\}\text{ or }\left\{\begin{array}{l}\tfrac{6}{2} \\ \tfrac{6}{2}\end{array}\right(\begin{array}{l}3 \\ \underline{3} \\ \underline{9}\ \text{Product.}\end{array}$$

$$\tfrac{25}{4}\overset{*}{+}\sqrt{q}\frac{625-600}{16}$$

$$\sqrt{q}\frac{625-600}{16}\overset{*}{+}\frac{25-24}{4}$$

$$\text{Product.}\quad \left.\frac{50-24}{4}\overset{*}{+}\sqrt{q}\frac{2500-2400}{16}\right\}\text{ or }\left\{\frac{26}{4}+\sqrt{q}\frac{100}{16}\right\}\text{ or }\left\{\frac{26}{4}+\frac{10}{4}\right\}\text{ or }\left\{\frac{36}{4}\right(9$$

Universals. Universals Homogeneal, as Surds before, are to be multiplied one into the other alternately, the Sinister Numbers and *Species* being Figurate, according to the Denomination of the Dexter into which they are multiplied; and Heterogeneals according to the Reduction.

Examples in Homogeneals.

$$\begin{array}{lll} \textit{Examples.} & \textit{Binomials.} & \textit{Residuals.} \\ \text{Multiplicands.} & \sqrt{:}B+\sqrt{q}H & \sqrt{:}D-\sqrt{q}I \\ \text{Multipliers.} & \sqrt{:}D+\sqrt{q}K & \sqrt{:}B-\sqrt{q}I \\ & BD+\sqrt{q}DqH & DB-\sqrt{q}BqI \\ & \quad\sqrt{q}BqK+\sqrt{q}HK & \quad-\sqrt{q}DqI+\sqrt{q}Iq \\ \text{Products.} & \sqrt{:}BD+\sqrt{q}DqH+\sqrt{q}BqK+\sqrt{q}HK & \sqrt{:}DB-\sqrt{q}BqI-\sqrt{q}DqI+\sqrt{q}Iq \end{array}$$

$$\begin{array}{ll} & \textit{Mixt.} \\ \text{Multiplicand.} & \sqrt{:}D+\sqrt{q}P \\ \text{Multiplier.} & \sqrt{:}B-\sqrt{q}R \\ & BD+\sqrt{q}BqP \\ & \quad-\sqrt{q}DqR-\sqrt{q}PR \\ \text{Product.} & \sqrt{:}BD+\sqrt{q}BqP-\sqrt{q}DqR-\sqrt{q}PR. \end{array}$$

Examples in Heterogeneals, Binomial and Residual.

Examples in Heterogeneals.

$$\begin{array}{lll} \text{Multiplicands.} & \sqrt{:}B+\sqrt{c}D & \sqrt{:}B-\sqrt{c}D \\ \text{Multipliers.} & \sqrt{:}B+\sqrt{c}D & \sqrt{:}B-\sqrt{c}D \\ & Bq+\sqrt{cc}DqBcc & Bq-\sqrt{cc}DqBcc \\ & \quad\sqrt{cc}DqBcc+\sqrt{c}Dq & \quad-\sqrt{cc}DqBcc+\sqrt{c}Dq \\ \text{Products.} & \sqrt{:}Bq+\sqrt{cc}64DqBcc+\sqrt{c}Dq & \sqrt{:}Bq-\sqrt{cc}64DqBcc+\sqrt{c}Dq \end{array}$$

The following Examples by the former Supposition in this Chapter, are explained by Numbers.

Examples explained by Numbers.

$$\sqrt{:}\,\frac{Z}{2} \ast + \sqrt{q}\,\frac{Zq-4Pq}{4} \qquad \sqrt{:}\,\frac{13}{2} \ast + \sqrt{q}\,\frac{169 \quad 144}{4}$$

$$\sqrt{:}\,\frac{Z}{2} - \sqrt{q}\,\frac{Zq-4Pq}{4} \qquad \sqrt{:}\,\frac{13}{2} - \sqrt{q}\,\frac{169-144}{4}$$

$$\frac{Zq}{4} + \sqrt{q}\,\frac{Zqq-4ZqPq}{16} \qquad \frac{169}{4} + \sqrt{q}\,\frac{28561-24336}{16}$$

$$-\sqrt{q}\,\frac{Zqq-4ZqPq}{16} - \frac{Zq-4Pq}{4} \qquad -\sqrt{q}\,\frac{28561-24336}{16} - \frac{169-144}{4}$$

$$\sqrt{q}\,\frac{4Pq}{4} \text{ or } \sqrt{q}Pq, \text{ or } P \qquad \sqrt{q}\,\frac{144}{4}\left(\frac{12}{2}\sqrt{}\right.\left(6\right.$$

$$\sqrt{:}\,\frac{Z}{2} \ast + \sqrt{q}\,\frac{Zq-4Pq}{4} \qquad \sqrt{:}\,\frac{13}{2} \ast + \sqrt{q}\,\frac{169-144}{4}$$

$$\sqrt{:}\,\frac{Z}{2} + \sqrt{q}\,\frac{Zq-4Pq}{4} \qquad \sqrt{:}\,\frac{13}{2} + \sqrt{q}\,\frac{169-144}{4}$$

$$\frac{Zq}{4} + \sqrt{q}\,\frac{Zqq-4ZqPq}{16} \qquad \frac{169}{4} + \sqrt{q}\,\frac{28561-24336}{16}$$

$$\sqrt{q}\,\frac{Zqq-4ZqPq}{16} + \frac{Zq-4Pq}{4} \qquad \sqrt{q}\,\frac{28561-24336}{16} + \frac{169-144}{4}$$

$$\sqrt{:}\,\frac{2Zq-4Pq}{4} \ast + \sqrt{q}\,\frac{4Zqq-16ZqPq}{16} \qquad \sqrt{:}\,\frac{338-144}{4} \ast + \sqrt{q}\,\frac{114244-97344}{16}$$

$$\begin{array}{r} 338 \\ -144 \\ \hline 194 \\ \hline 4 \end{array} \qquad \begin{array}{r} 114244 \\ -97344 \\ \hline 16900 \\ \hline 16 \end{array}\left(\frac{\sqrt{}\;130}{4}\right.$$

$$\frac{194}{4} + \frac{130}{4} = \frac{324}{4}\left(\sqrt{}\,\frac{18}{2}\right.\left(9\right.\left(3\sqrt{}\right.$$

$$\frac{194}{4} + \frac{130}{4} = \frac{324}{4}\left(\frac{18}{2}\right.\left(9\right.\left(3\sqrt{}\right.$$

$\overline{\;9\;}$ Product.

Multiplication of Compound Irrational Species, both Particulars and Universals, like Multiplication of Compound Surds before, will be proved by Division and by Rational Numbers, as already hath been sufficiently seen. *Proof of Multiplication of Com. Irrat. Species.*

CHAP. XX. *Division of Compound Irrational Species.*

THE *Division of Compound Irrationals*, takes in Particulars and Universals. *Division of Com. Irrat. Species. Particulars.*
Particulars are to be divided by a Mixture of Division of *Species* and Compound Surds, variegated as the *Case* requires.

Case 1. When the Divisor is Simple, and the Dividend Compound. *1. Divisor Simple.*

	Divisor.	*Dividend.*	*Quotient.*
Binomial.	$\sqrt{q}B)$	$\sqrt{q}BS+\sqrt{q}BD$	$(\sqrt{q}S+\sqrt{q}D$
Residual.	$\sqrt{q}C)$	$\sqrt{q}BC-\sqrt{q}BD$	$(\sqrt{q}B-\sqrt{q}\frac{BD}{C}$
Mixt.	$3B)$	$6B+\sqrt{q}54BC-\sqrt{c}81BAD$	$(2B+\sqrt{q}6C-\sqrt{c}3AD$

Examples.

These Examples are like the first in Division of Compound Surds.

Case 2. When the Divisor is Compound, and the Dividend Simple. *2. Dividend Simple.*

	Divisor.	*Dividend.*	*Quotient.*
Binomial.	$\sqrt{q}B+\sqrt{q}C)$	$\sqrt{q}D$	$\left(\frac{\sqrt{q}D}{\sqrt{q}B+\sqrt{q}C}\right.$
Residual.	$\sqrt{q}B-\sqrt{q}C)$	$\sqrt{q}B$	$\left(\frac{\sqrt{q}B}{\sqrt{q}B-\sqrt{q}C}\right.$
Mixt.	$\sqrt{q}B+\sqrt{q}C-\sqrt{q}D)$	$\sqrt{q}C$	$\left(\frac{\sqrt{q}C}{\sqrt{q}B+\sqrt{q}C-\sqrt{q}D}\right.$

Examples.

The

The Quotients of these Examples are set as Fractions, like those in the *5th Case* of Division of Integral and Rational *Species*; for to divide *Compound Irrational Species*, it is requisite the *Species* be Homogeneal, as well as the Figurate Quantities to be divided.

3. Divisor reduced to a Simple. *Case* 3. When the Divisor is Compound, but both Parts being equal, may be reduced to a Simple as the Dividend is.

Examples.

	Divisor.	*Reduced.*	*Dividend.*	*Quotient.*
Binomial.	$\surd qB+\surd qB$)	$\surd q4B$)	$\surd q8Bq$	($\surd q2B$
Residual.	$\surd q4B-\surd qB$)	$\surd qB$)	$\surd qBc$	($\surd qBq$
Mixt.	$\surd q4B-\surd q4B+D$)	D)	$\surd qBD$	($\surd qB$

The Work of these Examples is according to the third *Case* of Division of Compound Surds.

4. Data Compound. *Case* 4. When both Divisor and Dividend are Compound.

Examples.

$$\begin{array}{lll} & \textit{Divisor.} & \textit{Dividend.} \qquad\qquad \textit{Quotient.} \\ \text{Binomial.} & \surd qB+\surd qD) & \surd qBq+\surd qBC+\surd qBD+\surd qDC\ (\surd qB+\surd qC \\ & \surd qB+\surd qC & \surd qBq+\surd qBD \\ & \overline{\surd qBq+\surd qBD} & \qquad \surd qBC++\surd qDC \\ & \quad \surd qBC+\surd qDC & \end{array}$$

$$\begin{array}{lll} & & \surd qBD \\ \text{Residual.} & \surd qB-\surd qD) & \surd q4BD-B-D\ (\surd qD-\surd qB \\ & \surd qD-\surd qB & \surd qBD--D \\ & \overline{\surd qBD-D} & \surd qBD-B \\ & \quad -B+\surd qBD & \end{array}$$

$$\begin{array}{lll} & & \quad -\surd cKP \\ \text{Mixt.} & \surd cK+\surd cP) & \surd cKq-\surd cPq\ (\surd cK-\surd cP \\ & \surd cK-\surd cP & \surd cKq+\surd cKP \\ & \overline{\surd cKq+\surd cKP} & \quad -\surd cKP-\surd cPq \\ & \quad -\surd cKP-\surd cPq & \end{array}$$

These Examples are wrought according to the *6th Case* of Division of Compound Surds.

5. Heterogeneal. *Case* 5. When the Quantities are Heterogeneal, reduce them to like Denominations before Division.

Examples.

$$\begin{array}{llcl} & \textit{Divisor.} & \textit{Dividend.} & \textit{Quotient.} \\ \text{Binomial.} & \surd qB+\surd qD\ \Big) & \begin{array}{c} B+\surd q4BD+D \\ \text{Reduced.} \\ \surd qBq+\surd q4BD+\surd qDq \end{array} & \Big(\ \surd qB+\surd qD \end{array}$$

$$\begin{array}{llcl} & \textit{Divisor.} & \textit{Dividend.} & \textit{Quotient.} \\ \text{Residual.} & \begin{array}{c} \surd qB-\surd qD \\ \text{Reduced.} \\ \surd ccBc-\surd ccDc \end{array}\Big) & \begin{array}{c} \surd ccBc-\surd ccDc \\ \text{Reduced.} \\ \surd ccBcc-\surd cc64BcDc+\surd ccDcc \end{array} & \Big(\ \surd ccBc-\surd ccDc \end{array}$$

In these two Examples the Reduction agrees with that of Surds and Irrational *Species*, and the Division according to the second *Case* of Division of the Simple Irrationals in both, is performed after the Reduction, as the next precedent *Case* of this Chapter.

6. Fractionary. *Case* 6. When the *Species* given are Fractionary, the Work is mixed after the manner of Fractions and Surds both.

Example.

$$\begin{array}{ccc} \textit{Divisor.} & \textit{Dividend.} & \textit{Quotient.} \\ \frac{Z}{2}\overset{*}{+}\surd q\frac{Zq-4P}{4}\Big) & \frac{2Zq-4P}{4}\overset{*}{+}\surd q\frac{4Zqq-16ZqP}{16} & \Big(\frac{4Zq-8P}{4Z}\overset{*}{+}\surd q\frac{16Zqq-64ZqP}{16Zq-64P} \end{array}$$

Explained in Numbers.

$$\frac{5}{2}\overset{*}{+}\surd q\frac{25-24}{4}\Big)\frac{50-24}{4}\overset{*}{+}\surd q\frac{2500-2400}{16}\Big(\frac{100-48}{20}\overset{*}{+}\surd q\frac{10000-9600}{400-384}$$

The Quotients being depressed by Reduction in *Species*, may be brought to $\frac{Z}{2}\overset{*}{+}\surd q\frac{Zq-4P}{4}$, and in Numbers to $\frac{100-48}{20}\overset{*}{+}\surd q\frac{400}{16}$, or $\frac{52}{20}+\frac{20}{4}$. But as in Multiplication of this Dividend in the foregoing Chapter, the Denominator 2 of the

the Sinister part, was led into the Numerator of the Dexter part, contrary to the manner of other Fractionary Multiplication. So here this Denominator 4 is to be doubled and made Numerator to 20; which $\frac{8}{20}$ added to $\frac{52}{20}$, make $\frac{60}{20}$, that is 3 Integers for the Value of the Root of the Quotient. And this is worthy to be noted in other Examples of like Nature in Particular Irrationals, when as noted by the Asterisque any Quantity Compound is affirmed or denied of a Simple, or a Simple of a Compound. But in others the Division of Fracted Irrationals agreeth with the Division of Fractionary Surds and *Species* mixt: And will appear plainer in this Example, if there be made two Divisions thereof, as in effect there is, *viz.* the Sinister part of Dividend and Divisor for one, and the Dexter part for the other.

Universals. Homogeneals.

Division of Universal *Species* Homogeneal, is like the Division of Universal Homogeneal Surds; and according to the *4th Case* of Division of *Species* in this Chapter, only before the Quotient prefix the Sign Universal, and upon every Removal of the Divisor figurate the Sinister Number thereof according to the Dexter Number to which he is applied; and in multiplying the Divisor by the Quotienary *Species*, the Multiplication must be also proper to Universals.

Example.

$$\begin{array}{lll}
\textit{Divisor.} & \textit{Dividend.} & \textit{Quotient.} \\
\surd{:}B+\surd qH) & \surd{:}BD+\surd qDqH+\surd qBqK+\surd qHK & (\surd{:}D+\surd qK \\
\underline{D+\surd qK} & BD+\surd qDqH \quad BqK+\surd qHK & \\
BD+\surd qDqH & & \\
\quad BqK+\surd qHK & &
\end{array}$$

Heterogeneals.

In Heterogeneals, besides Figuration of the Sinister part of the Divisor as aforesaid, if the Quotienary *Species* gotten on the second Removal of the Divisor be of higher Denomination than the next Dexter Number of the Divisor, this Quotienary *Species* ought to be depressed in Quantity by extracting the Root and abating the Index, and this shall be the true Quotienary Number.

Example.

$$\begin{array}{lll}
\textit{Divisor.} & \textit{Dividend.} & \textit{Quotient.} \\
 & DqBcc & \\
\surd{:}B+\surd cD) & \surd{:}Bq+\surd cc64DqBcc+\surd cDq & (\surd{:}B+\surd cD \\
\underline{B+\surd cD} & Bq+\surd ccDqBcc & \\
Bq+\surd ccDqBcc & DqBcc+\surd cDq & \\
\quad \surd ccDqBcc+\surd cDq & &
\end{array}$$

Here besides figurating the divising B to Bcc, when applied to DqBcc, because Dq will be gotten thereby for the Quotient, which is a Power above D the next Number of the Divisor, therefore the Root D shall be set in the Quotient with the Cube Index.

Another Example.

$$\surd{:}B+\surd qD)\ \surd{:}Bq+\surd qBqD+\surd ccBccCq+\surd cCqDc\ (\surd{:}B+\surd cC$$

Explained in Numbers.

Example explained by Numbers.

$$\surd{:}2+\mathcal{W}4)\ \surd{:}4+\mathcal{W}16+\surd\mathfrak{z}\,\phi 4096+\surd\mathfrak{z}\,\phi 4096\ (\surd{:}2+\mathcal{W}8.$$

Proof of Division of Comp. Irrat. Species.

The Proof of Division of these Compound Irrationals, is by their Multiplications, or by Rational Numbers, that is to say, Particular by Particular, and Universal by Universal, as is to be seen before in Division of Surds, and may further be beheld in these two last Examples, where the Dividends are returned, by multiplying the Divisors into the Quotients respectively, and so may others be examined.

$$\begin{array}{lll}
\textit{Species.} & \textit{Numbers supposed.} & \\
\surd{:}B+\surd cD & \surd{:}2+\mathcal{W}8 & 2+2=4 \\
\underline{\surd{:}B+\surd cD} & \underline{\surd{:}2+\mathcal{W}8} & \underline{2+2=4} \\
Bq+\surd ccBccDq & 4+\surd\mathfrak{z}\,\phi 4096 & \mathcal{W}16 \\
\quad \underline{\surd ccBccDq+\surd cDq} & \quad \underline{\surd\mathfrak{z}\,\phi 4096+\mathcal{W}64} & \\
\underline{\surd{:}Bq+\surd cc64BccDq+\surd cDq} & \underline{\surd{:}4+\surd\mathfrak{z}\,\phi 262144+\mathcal{W}64} & \\
 & \underline{4 \quad + \quad 8 \quad + \quad 4} & \\
 & \underline{\mathcal{W}16 = 4} &
\end{array}$$

Species.	*Numbers supposed.*	
√:B+√qD	√:2+w4	2+2=4
√:B+√cC	√:2+ww8	2+2=4
Bq+√qBqD	4+w16	w16
√ccBccCq+√ccDcCq	√3Φ4096+√3Φ4096	
√:Bq+√qBqD+√ccBccCq+√ccDcCq.	4+w16+√3Φ4096+√3Φ4096	
	4+4+4+4	
	w16=4	

CHAP. XXI. *Figuration of Irrational Species.*

Figurate Irrational Species produced.

As was observed before in Figuration of Surds; so here it may be remembred that any *Species* simply Irrational, multiplied figurately, produceth a Rational: But Compound Irrational *Species* may be squared, cubed, *&c.* by multiplying them figurately as other Figural Numbers are multiplied; but the *Unciæ* to the Parodical Degrees here, will be the Squares of them in Rational *Species.* See the Examples that follow.

	Simple.		*Particular.*	*Universal.*
Root	√qB	Root	√qB+√qD	√:B+√qD
	√qB		√qB+√qD	√:B+√qD
Square	√qBq		B+√qBD	Bq+√qBqD
	√qB		√qBD+D	√qBqD+√qDq
Cube	√qBc	Square	B+√q4BD+D	√:Bq+√q4BqD+√qDq
			√qB+√qD	√:B+√qD
			√qBc+√q4BqD+√qBDq	Bc+√q4BqqD+√qBqDq
			√qBqD+√q4BDq+√qDc	√qBqqD+√q4BqDq+√qDc
		Cube	√qBc+√q9BqD+√q9BDq+√qDc.	√:Bc+√q9BqqD+√q9BqDq+√qDc

Explained in Numbers, supposing B 4. *and* D 9.

	Particular.		*Universal.*	
Root	w4+w9	2+3=5 z	√: 4+w9	4+3=7z
	w4+w9	2+3=5	√: 4+w9	4+3=7
	4+w36	25ʒ	16+w144	49√7
	w36+9	5	w144+w81	
Square	4+w144+9	125Φ	√:16+w576+w81	
	4+12+9=25ʒ		16+24+9=49√7	
	w4+w9		√:4+w9	
	w64+w576+w324		64+w9216+w1296	
	w144+w1296+w729		w2304+w5184+w729	
Cube	w64+w1296+w2916+w729		√:64+w20736+w11664+w729	
	8+36+54+27=125Φ		√:64+144+108+27=343√7	

Roots of Irrational Species extracted.

Extraction of the Roots of Irrational *Species* is performed like the Extraction of Surd Roots, at large discussed *Chap.* 12. of the precedent part of this third Book, with a mixture of the Extraction of Roots in Cossicks: So as a Retrospection thither may save a large Repetition of the Rules here in the different *Cases* occurrent.

1. *Simple.*

Case 1. If the Surd be Simple, he hath no Root to be otherwise expressed, than by prefixing the proper Character before the *Species*.

Example.

As to extract the Square Root of B, or the Cube Root of B+D, they are set thus:

√qB √cB+D

Eut

But any Rational *Species* set as an Irrational, may have the Root extracted.

As the Square Root of $\sqrt{q}Bq$ is B.
And the Cube Root of $\sqrt{c}Bc$ is B.

Case 2. If any particular Compound *Species* Irradical in the *Species*, have the Sinister *Species* absolute, then prefix before the same the proper Character for the Root to be extracted. 2. *Particular Irradical.*

As to extract the Square Root of $B+\sqrt{q}D$, then it is to be set thus: Example.

$\sqrt{q}B+\sqrt{q}D.$

But if the Sinister *Species* be figurate, then multiply the Index thereof by the Index of the Root to be extracted.

As to extract the Square Root of $\sqrt{q}B+\sqrt{q}D$, it shall be $\sqrt{qq}B+\sqrt{q}D$.

Case 3. If the Compound *Species* be not Irradical, then are they formally Rational or Irrational: And if they be formally Rational, the Root may be extracted after the manner of Cossicks, keeping the Addition and Substraction of the Multiplees as in Surds. But because Operations in *Species* keep the Prime and Original *Species* for the most part throughout the Work, and in all formal Figuration only increase their Quantities; therefore it is easy to see the Root in any Figural Quantity in *Species* of never so high a Power. 3. *Particular Radical if formally Rational.*

As in the former Examples in this Chapter of

$\sqrt{q}Bc+\sqrt{q}9BqD+\sqrt{q}9BDq+\sqrt{q}Dc.$

$\sqrt{:}\ Bc+\sqrt{q}9BqqD+\sqrt{q}9BqDq+\sqrt{q}Dc.$ Example.

there being no other *Species* used save the Notes of Quantities B and D, it is easy to discern, the Root did consist of B and D.

Case 4. If the Particular or Universal Compound *Species* be not formally Rational, then figurate the Sinister Part as the Dexter, and from thence take the Dexter, add the Square Root of the Difference to the Sinister, and substract it therefrom, half the Sum and half the Difference joined with + shall be the Binomial Root, and with — shall be the Residual Root. 4. *Particular or Universal, if not formally Rational.*

As to extract the square Root of $\frac{2Zq-4P}{4} \overset{*}{+} \sqrt{q}\frac{4Zqq-16ZqP}{16}$ Examples.

Sinister Part squared is $\frac{4Zqq-16ZqP+16P}{16}$. From whence if the

Dexter Part $\frac{4Zq4-16ZqP}{16}$ be substracted, then will the

Remain be 16P, whose Root is 4P; and this Root when

Added to the Sinister part, makes the Sum $\frac{2Zq-8P}{4}$

Substracted therefrom, makes the Difference $\frac{2Zq}{4}$

Half the Sum.	*Half the Difference.*
$\frac{Zq-4P}{4}$	$\frac{Zq}{4}$ or $\frac{Z}{2}$

Binomial Root $\frac{Z}{2}+\sqrt{q}\frac{Zq-4P}{4}$

Residual Root $\frac{Z}{2}-\sqrt{q}\frac{Zq-4P}{4}$

So

So to extract the Root of $\surd\!:\frac{2Z,q-4Pq}{4} \overset{*}{+} \surd q\frac{4Z,qq-16Z,qPq}{16}$

$\frac{4Z,qq-16Z,qPq+16Pqq}{16}$ Sinister part squared.

$\frac{4Z,qq-16Z,qPq}{16}$ Dexter part substracted.

16Pqq Remain. 4Pq Root.

$\frac{2Z,q-8Pq}{4}$ Sum. $\frac{2Z,q}{4}$ Difference.

$\frac{Z,q-4Pq}{4}$ Half Sum. $\frac{Z,q}{4}$ or $\frac{Z}{2}$ Half Difference.

Root $\frac{Z}{2}+\surd q\frac{Z,q-4Pq}{4}$ Binomial. $\frac{Z}{2}-\surd q\frac{Z,q-4Pq}{4}$ Residual.

Proof of Extraction of Roots of Irrat. Species. These Extractions are to be proved by Production of their Figurals, as those Productions by these Extractions. And also by Rational Numbers, working with them instead of the Irrationals; as by the last Example explained in Numbers will appear.

Species $\surd\!:\frac{2Z,q-4Pq}{4} \overset{*}{+} \surd q\frac{4Z,qq-16Z,qPq}{16}$

Numbers $\surd\!:\frac{338-144}{4} \overset{*}{+} \surd q\frac{114244-97344}{16}$

Sinister Part squared $\frac{114244-97344+20736}{16}$

Dexter Part substracted $\frac{114244-97344}{16}$

Remain 20736 | Root 144

Sum $\frac{338-288}{4}$ Half $\frac{169-144}{4}$

Difference $\frac{338}{4}$ Half $\frac{169}{4}$ or $\frac{13}{2}$

Binomial Root $\surd\!:\frac{13}{2}+\surd q\frac{169-144}{4}$

Residual Root $\surd\!:\frac{13}{2}-\surd q\frac{169-144}{4}$

$$\begin{array}{rrr}
338- & 144 & \\
338- & 144 & \\
\hline
2704- & 1152+ & 576 \\
1014 & 432 & 576 \\
1014 & 432 & 144 \\
\hline
114244- & 48672+ & \\
 & -48672+ & 20736 \\
\hline
114244- & 97344+ & 20736 \\
\end{array}$$

And thus much may suffice in this place to be spoken of *Species*, and in them of the Simple Elements of both Abstract and Contract Numbers.

Partis sextæ & Libri tertii

FINIS.

ARITHMETICK.

The Fourth BOOK.

CONCERNING

Numbers Proportional, Abstract and Contract.

In Four PARTS.

WHEREIN

Ratio's	are	Deciphered.
Proportions disjunct		Dissected.
Proportions continued		Computed.
Aequations		Enodated.

And their *Comparative* ELEMENTS.

CHAP. I.

Of RATIO'S.

HAVING thus in the three former Books run through the *Simple Elements* of *Numbers*, in their proper Nature, *viz. Abstract*, and generally and specially *Contract*: I now come to review them in their common Nature, as they are Relational, and uncover their *Comparative Elements*; which shew the Comparison of Numbers among themselves, the Description whereof takes up this Chapter, and the Computation the rest of this Book. *Numbers in their common Nature as Relational, and their Comparative Elements, treated of in this Fourth Book.*

Description gives us an Account of the several Species or Kinds of Comparison. Description, *what.*

Computation is an Account of the several Operations belonging to these Species. Computation, *what.*

Comparison of Numbers, shews what Likeness or Relation there is between the Numbers compared; and is twofold, *Ratio*, or *Proportion*. Comparison *twofold.*

Ratio is a Comparison of Terms, and is, when the Relation or Conference is extended, but to two Numbers or Magnitudes only, as 21 to 12, or 3 to 6, or any such-like: This is sometimes promiscuously called *Proportion*. Ratio, *what. How sometime called.*

Proportion is properly a Comparison of *Ratio's*, and is when the Conference reacheth unto many Numbers: This is sometimes called *Proportionality*, and often *Analogy*. Proportion, *what, and how called.*

Ratio twofold. Every Relation between two Numbers, is either *Ratio* of Equality or Inequality.

Ratio of Equality. *Ratio* of Equality, is when one Number is compared to himself, as 5 to 5, always agreeing in Unity.

Ratio of Inequality. *Ratio* of Inequality, is when one Number is compared to another different from him: Wherein also there is a double Conference, *viz.* *Ratio* of the Greater, and *Ratio* of the Lesser Inequality.

This twofold. *Greater Inequality also twofold.* *Ratio* of the Greater Inequality, is when the greater Number is compared to the Lesser; as 6 to 3, or 5 to 2, *&c.* And this is of two Sorts, either Simple and Prime, or Compound and Conjunct.

Prime, and this, Prime, or Simple *Ratio* of the greater Inequality, is of two sorts.

1. *Containeth the Lesser once and a Part.* The first is, when the greater Number containeth the Lesser more than once, and not twice, but once and a Part more, called generally by the Latin Name, *Superparticularis*, or a Part more: For that one Part of the lesser Number, is the just Difference or Excess betwixt it and the Greater. Specially, and for distinction-sake, each receiveth his Name from that Part it containeth: As if it contain the Half more, then it is called *Sesquialtera*, as 3 to 2.

A farther view of such *Ratio's*.

Examples.

Sesquialtera ———	as 3 to 2.	6 to 4, &c.	$1\frac{1}{2}$	an Half more.
Sesquitertia ———	as 4 to 3.	8 to 6, &c.	$1\frac{1}{3}$	a Third more.
Sesquiquarta ———	as 5 to 4.	10 to 8, &c.	$1\frac{1}{4}$	a Fourth more.
Sesquiquinta ———	as 6 to 5.	12 to 10, &c.	$1\frac{1}{5}$	a Fifth more.
Sesquisexta ———	as 7 to 6.	14 to 12, &c.	$1\frac{1}{6}$	a Sixth more.
Sesquiseptima ———	as 8 to 7.	16 to 14, &c.	$1\frac{1}{7}$	a Seventh more.
Sesquioctava ———	as 9 to 8.	18 to 16, &c.	$1\frac{1}{8}$	an Eighth more.
Sesquinona ———	as 10 to 9.	20 to 18, &c.	$1\frac{1}{9}$	a Ninth more.
Sesquidecima ———	as 11 to 10.	22 to 20, &c.	$1\frac{1}{10}$	a Tenth more.
Sesquiundecima ———	as 12 to 11.	24 to 22, &c.	$1\frac{1}{11}$	an Eleventh more.
&c.				

2. *Containeth the lesser once, and some Parts.* The second Simple sort is, when the Difference is 2, 3, or more Parts of the whole; this is generally called *Superpartiens*, or Parts more, intimating above one Part; but specially every one hath his proper Name, according to his Content, as the former had. For if 5 be compared to 3, then is it called *Superbipartiens-tertias*, because it containeth the Whole, and $\frac{2}{3}$ of the Whole. And so may other Names be given infinitely. Some Examples whereof follow.

Examples.

Superbipartiens.	*Tertias* ———	as 5 to 3.	10 to 6, &c.	$1\frac{2}{3}$	Two Parts more.
	Quintas ———	as 7 to 5.	14 to 10, &c.	$1\frac{2}{5}$	
	Septimas ———	as 9 to 7.	18 to 14, &c.	$1\frac{2}{7}$	
	Nonas ———	as 11 to 9.	22 to 18, &c.	$1\frac{2}{9}$	
	Undecimas ———	as 13 to 11.	26 to 22, &c.	$1\frac{2}{11}$	
	Decimas tertias --	as 15 to 13.	30 to 26, &c.	$1\frac{2}{13}$	
	&c.				
Supertripartiens.	*Quartas* ———	as 7 to 4.	14 to 8, &c.	$1\frac{3}{4}$	Three Parts more.
	Quintas ———	as8 to 5.	16 to 10, &c.	$1\frac{3}{5}$	
	Septimas ———	as 10 to 7.	20 to 14, &c.	$1\frac{3}{7}$	
	Octavas ———	as 11 to 8.	22 to 16, &c.	$1\frac{3}{8}$	
	Decimas ———	as 13 to 10.	26 to 20, &c.	$1\frac{3}{10}$	
	Undecimas ———	as 14 to 11.	28 to 22, &c.	$1\frac{3}{11}$	
	Decimas tertias --	as 16 to 13.	32 to 26, &c.	$1\frac{3}{13}$	
	&c.				
Superquadrupartiens.	*Quintas* ———	as 9 to 5.	18 to 10, &c.	$1\frac{4}{5}$	Four Parts more.
	Septimas ———	as 11 to 7.	22 to 14, &c.	$1\frac{4}{7}$	
	Nonas ———	as 13 to 9.	26 to 18, &c.	$1\frac{4}{9}$	
	Undecimas ———	as 15 to 11.	30 to 22, &c.	$1\frac{4}{11}$	
	Decimas tertias --	as 17 to 13.	34 to 26, &c.	$1\frac{4}{13}$	
	Decimas quintas -	as 19 to 15.	38 to 30, &c.	$1\frac{4}{15}$	
	&c.				

Superquintupartiens.	*Sextas*	as 11 to 6.	22 to 12, &c.	$1\frac{5}{6}$	Five Parts more.
	Septimas	as 12 to 7.	24 to 14, &c.	$1\frac{5}{7}$	
	Octavas	as 13 to 8.	26 to 16, &c.	$1\frac{5}{8}$	
	Nonas	as 14 to 9.	28 to 18, &c.	$1\frac{5}{9}$	
	Undecimas	as 16 to 11.	32 to 22, &c.	$1\frac{5}{11}$	
	Duodecimas	as 17 to 12.	34 to 24, &c.	$1\frac{5}{12}$	
	&c.				

Here is to noted, that those *Ratio's* which some have set to fill up the Complement of the other, as *Superbipartiens-Secundas*, *Quartas*, *Sextas*, *Octavas*, &c. and *Supertripartiens-Secundas*, *Tertias*, *Sextas*, *Nonas*, &c. are all Irregular; for properly there are no such *Ratio's*; but those are *Ratio's* that fall under some other Name agreeable thereto: As 6 to 4, called by some *Superbipartiens-Quartas*, is indeed *Sesquialtera*. So 10 to 6, called by some *Superquadrupartiens-Sextas*, is properly no other than *Superbipartiens-Tertias*. The like is to be understood of many others: For as in Common Fractions, it is most regular to set them in their least Terms; so in *Ratio's*: wherefore 10 to 6 shall be accounted $1\frac{2}{3}$, not as $1\frac{4}{6}$.

Compound, otherwise called *Conjunct Ratio's* of the greater Inequality, are of three Kinds. *Compound, and this*

The first is, when the greater Number containeth the Lesser, divers times generally called *Multiplex*, or *Manifold*; and particularly named, according to the Times that the lesser Number is contain'd in the Greater: So that if it contain it twice, then it is called *Dupla*, or *Double*, as 2 to 1, *&c.* A farther Account of their Names follows. *1. Containeth the Lesser many times.*

Examples.

Dupla	as 2 to 1.	4 to 2, &c.	$\frac{2}{1}$	*Duple*, or Double.
Tripla	as 3 to 1.	6 to 2, &c.	$\frac{3}{1}$	*Triple*, or Threefold.
Quadrupla	as 4 to 1.	8 to 2, &c.	$\frac{4}{1}$	*Quadruple*, or Fourfold.
Quintupla	as 5 to 1.	10 to 2, &c.	$\frac{5}{1}$	*Quintuple*, or Fivefold.
&c.				

The second Sort of *Ratio's* of the greater Inequality compound, is named *Multiplex-Superparticularis*; which importeth, that the greater Number containeth the lesser many times, and a Part more, as 5 to 2, which contains 2 twice and a half more, and therefore is called *Dupla-Sesquialtera*. These Kinds of *Ratio's* may be diversly divided, as into *Double*, *Triple*, *Quadruple*, &c. and every of them into their several Subdivisions. *2. Containeth the Lesser many times, and a Part.*

As for Instance.

Examples.

Duplex	*Sesquialtera*	as 5 to 2.	10 to 4, &c.	$2\frac{1}{2}$
	Sesquitertia	as 7 to 3.	14 to 6, &c.	$2\frac{1}{3}$
	Sesquiquarta	as 9 to 4.	18 to 8, &c.	$2\frac{1}{4}$
	&c.			
Triplex	*Sesquialtera*	as 7 to 2.	14 to 4, &c.	$3\frac{1}{2}$
	Sesquitertia	as 10 to 3.	20 to 6, &c.	$3\frac{1}{3}$
	Sesquiquarta	as 13 to 4.	26 to 8, &c.	$3\frac{1}{4}$
	&c.			
Quadruplex	*Sesquialtera*	as 9 to 2.	18 to 4, &c.	$4\frac{1}{2}$
	Sesquitertia	as 13 to 3.	26 to 6, &c.	$4\frac{1}{3}$
	Sesquiquarta	as 17 to 4.	34 to 8, &c.	$4\frac{1}{4}$
&c.	&c.			

The third Sort is called *Multiplex-Superpartiens*, and implieth, that the greater Number containeth the Lesser divers times, and some Parts thereof besides: And are likewise distinguished into *Double*, *Triple*, &c. Some of these, with their Subdivisions, appear in the following Examples. *3. Containeth the Lesser many times, and some Parts.*

Dupla

Examples.

Dupla	*Superbipartiens*	*Tertias*	as 8 to 3, &c.	$2\frac{2}{3}$
		Quintas	as 12 to 5, &c.	$2\frac{2}{5}$
		Septimas	as 16 to 7, &c.	$2\frac{2}{7}$
		&c.		
	Supertripartiens	*Quartas*	as 11 to 4, &c.	$2\frac{3}{4}$
		Quintas	as 13 to 5, &c.	$2\frac{3}{5}$
		Septimas	as 17 to 7, &c.	$2\frac{3}{7}$
		&c.		
	Superquadripartiens	*Quintas*	as 14 to 5, &c.	$2\frac{4}{5}$
		Septimas	as 18 to 7, &c.	$2\frac{4}{7}$
		Nonas	as 22 to 9, &c.	$2\frac{4}{9}$
		&c.		
Tripla	*Superbipartiens*	*Tertias*	as 11 to 3, &c.	$3\frac{2}{3}$
		Quintas	as 17 to 5, &c.	$3\frac{2}{5}$
		&c.		
	Supertripartiens	*Quartas*	as 15 to 4, &c.	$3\frac{3}{4}$
		Quintas	as 18 to 5, &c.	$3\frac{3}{5}$
		&c.		
	Superquadripartiens	*Quintas*	as 19 to 5, &c.	$3\frac{4}{5}$
		Septimas	as 25 to 7, &c.	$3\frac{4}{7}$
		&c.		
Quadrupla	*Superbipartiens*	*Tertias*	as 14 to 3, &c.	$4\frac{2}{3}$
		Quintas	as 22 to 5, &c.	$4\frac{2}{5}$
		&c.		
	Supertripartiens	*Quartas*	as 19 to 4, &c.	$4\frac{3}{4}$
		Quintas	as 23 to 5, &c.	$4\frac{3}{5}$
		&c.		
	Superquadripartiens	*Quintas*	as 24 to 5, &c.	$4\frac{4}{5}$
		Septimas	as 32 to 7, &c.	$4\frac{4}{7}$
&c.		&c.		

Lesser Inequality twofold. *Ratio* of the lesser Inequality, is when the lesser Number is conferred to the Greater, as 3 to 5, or 2 to 6, *&c.*

Containing a Part. These *Ratio's* are also divided into two Sorts, *viz.* either such as contain a Part of the Number only, as one Third, one Fourth, *&c.* as 1 to 3, and 4 to 16, *&c.*

Containing some Parts. Or such as contain many Parts of the greater Number, as Three Quarters, Four Fifths, *&c.* as 3 to 4, and 4 to 5, *&c.*

How differenced in the Names from the other. Both these kinds of *Ratio's* of the lesser Inequality, have the same Names which the *Ratio's* of the greater Inequality had; save that to the beginning of every Name *Sub* is to be adjoined, as *Subdupla*, *Subtripla*, *Subsesquialtera*, &c. Examples of both Sorts follow.

Examples of the first Sort.

Examples of both.

Subdupla	as 1 to 2.	2 to 4, &c.	$\frac{1}{2}$	one Part.
Subtripla	as 1 to 3.	2 to 6, &c.	$\frac{1}{3}$	
Subquadrupla	as 1 to 4.	2 to 8, &c.	$\frac{1}{4}$	
&c.				

Examples of the second Sort.

Subsesquialtera	as 2 to 3.	4 to 6, &c.	$\frac{2}{3}$	many Parts.
Subsesquitertia	as 3 to 4.	6 to 8, &c.	$\frac{3}{4}$	
Subsesquiquarta	as 4 to 5.	8 to 10, &c.	$\frac{4}{5}$	

Ratio's properly conversing with Abstract Numbers, appear in the Front of Relational Numbers, and so are the Subject of the first Part of this 4*th* Book.

Proportion dissected into Simple and Compound. Proportion, as was said before, is referred to more than two Numbers, and therein may be a Conference of the former several *Ratio's* in their several Terms: But as conversing both with Abstract and Contract Numbers, and so best befitting Arithmetick, the usual Dissection of all Analogy, is into Simple and Compound Proportion.

Simple,

Simple, becauſe the Numbers ſo compared, need make no uſe of the Signs + or —. This is alſo divided into Diſcontinual and Continual Proportion. *Simple into Diſcontinual and Continual.*

Diſcontinual, called alſo *Disjunct Proportion*, is when there is an Equality of the Difference or *Ratio* between ſome of the Terms given; but not current through all, either ſort ſeldom exceeding four Terms. *Diſcontinual twofold.*

This kind of Proportion is double, either Arithmetical or Geometrical.

Arithmetical Diſcontinual Proportion is, when the Equality of the Difference is not continued alike throughout all the Terms, but the Difference of the firſt and ſecond Numbers is ſomewhere diſtracted. As 4 . 7 : 5 . 8 . where 3, the Difference between 4 and 7, is diſcontinued between 7 and 5. *Arithmetical, what.*

Hence aroſe thoſe Proportions called *Muſical*, which Mr. *Blundevil*, and ſome others, make a third Species: And as Mr. *Oughtred*, in Chap. 6. of his *Clavis* tells us is, when in 4 Numbers; As the Firſt is to the Fourth, ſo is the Difference of the Firſt and Second, to the Difference of the Third and Fourth. As 5. 8. 12. 30. are Muſical Proportions, becauſe 5. 30 : : 8 — 5. 30 — 12 : : 3. 18. Alſo in Species, A, M, N, E, let A. E : : M — A. E — N. Wherefore A E — A N = M E — A E in theſe Terms duly ordered, the Rule ſhall be $\frac{AN}{2A—M} = E$ and $\frac{EM}{2E—N} = A$. That is, If the Product of the Firſt and Third be divided by the Exceſs of the Firſt doubled above the Second, the Quotient ſhall be a Fourth in Muſical Proportion. *Hence ariſe Muſical Proportions. What they are. How to make them.*

Geometrical Diſcontinual Proportion is, when the *Ratio* is diſtracted: As 5. 15 : : 6. 18. where the *Subtriple Ratio* between 5 and 15, agreeth not with the *Ratio Duplaſeſquialtera* between 15 and 6. *Geometrical Proportion, what.*

This ſort of Proportion is either Plain or Figurate. *This twofold.*

Plain or Simple, becauſe the Number found thereby, agreeth in the ſimple Nature of the *Data*. *Plain.*

Such as theſe, by the 12, 13, 14, 15, and 16 Definitions of the 5*th* Book of *Euclid*, may be divided into *Alternate*, *Inverſe*, *Compounded*, *Divided*, and *Converſe*; but as moſt ſutable to *Arithmetick*, they may be divided into *Direct* or *Indirect*. *How ſorted by* Euclid. *How here.*

Direct is, when the Term by which the Queſtion is made, (which is the Third) by how much it is greater than the Firſt, by ſo much it requireth a Fourth Number greater than the Second: And by how much it is leſſer, by ſo much it requireth a Leſſer. On this is founded the *Golden Rule Direct*: For of 3, Numbers given, if the Second multiply the Third, and that Product be divided by the Firſt, the Quotient ſhall be a Fourth Proportional to the three given Numbers. *Direct what, and what founded thereon.*

Example when the Greater requireth a Greater. If 7 give 28, (being *quadrupled)* What ſhall 9 give? *facit* 36; which is in proportion to 28, as 9 is to 7. *Examples.*

Example when the Leſſer requireth a Leſſer. If 6 give 4, (being diminiſhed by $\frac{1}{3}$) what ſhall 3 give? *facit* 2; for 2 is in proportion to 4, as 3 is to 6.

Indirect, called alſo *Reciprocal* or *Reverſed Proportion*, is, when the Term by which the Queſtion is made, by how much it is greater than the Firſt, by ſo much it requireth a Fourth Number leſſer than the Second: And by how much it is leſſer, by ſo much it requireth a Greater. And on this is bottomed the *Backward Rule of Three*, or *Golden Rule Reverſed*: For of three Numbers given, if the Firſt multiply the Second, and that Product be divided by the Third, the Quotient ſhall be a Fourth, proportional to the three given Numbers. *Indirect what, how called, and what founded thereon.*

Example when the Greater requireth a Leſſer. If 4 give 10, then ſhall 8 give 5; becauſe as 8 is double to 4, ſo the Double of 5 is 10. *Examples.*

Example when the Leſſer requireth a Greater. If 4 give 10, then ſhall 2 give 20; for 20 is double to 10, as 4 is to 2.

Figurate Proportions here, are not to be taken ſo much for any Simple *Ratio* or Proportion that is between Figural Numbers *Homogeneal* or *Heterogeneal*, or any of their Complements or Parodical Degrees mentioned before *in Species*; nor yet only for the Proportional Figural Numbers, found out by the *Direct* or *Indirect* Analogy aforeſaid; but the Proportions uſed about Geometrical Figures; and together with theſe, ſuch as are diſcovered by the Operations proper to the Figural Numbers themſelves, or depending thereupon. And theſe receive Names according to the *Index* of the Figural Numbers they deal with; as if Squares, then are they called *Doubled Proportions*; if with Cubes, *Tripled Proportions*; if with Squared-Squares, *Quadrupled Proportions*, &c. Figurate Proportions *how to be underſtood here.*

All these Plain and Figurate-Geometrical-Discontinual Proportions, with their farther Subdivisions, and the Issues and Operations to them properly belonging, fill up the Second Part of this *4th* Book, in the Computation thereof.

The other Sort of Proportions before-mentioned, are *Continual.*

Continual *Proportion twofold.* *Continual*, otherwise called *Conjunct Proportion*, is, when three or more Numbers bear like Proportion in their Progression: So as well the Second Number may be referred to the Third, as the First to the Second. And the same Difference, or *Ratio*, shall be between the two last Terms, as between the two First.

This kind of Proportion is also both *Arithmetical* and *Geometrical.*

Arithmetial, *what.* *Arithmetical* Continual Proportion is, when between every two Numbers, or Terms, the Difference or Excess is equal, as 2, 4, 6, 8, &c. where the Excess 2 is continued throughout all the Terms, *&c.*

Geometrical, *what.* *Geometrical* Continual Proportion is, when between every two Numbers, or Terms, the *Ratio* is equal, as 4, 8, 16, 32, &c. where the *Ratio* 2 is continued throughout all the Terms.

Both called Progression. These two Sorts deal with plain Numbers especially, yet the latter in a manner figurates the first Terms; both are called *Progression*, and have their useful Operations computed in the Third Part of this *4th* Book.

Compound *Proportion how otherwise called.* *Compound* Proportions hold Community especially with Contract Numbers; make use of the Signs + and —, and are those called *Æquations*; that is, Numbers equal to others. This may be called *Proportion of Equality.* And of these there are two principal Sorts.

Æquations of two sorts. First, *Pure*, when one Number is compared as equal to another.

Secondly, *Mixt*, (commonly called *Affected*) when one or divers Numbers are compared in Equality to divers others.

Examples of Pure.

Pure { Example in *Geodædicals* – 1 *s.* = 12 *d.*
Example in *Cossicks* — 2ɣ = 6ℨ. Root 3.

Examples of Affected.

Examples in *Abstract* Numbers.

Mixt {
4 = 2 + 2 4 = 6 — 2
4 + 2 = 3 + 3 4 — 2 = 3 — 1

Examples in *Contract* Numbers.

1 ϕ = 2ɣ + 3ℨ. 1 ϕ = 4 ɣ — 3 ℨ
1ϕ + 1ɣ = 9ℨ + 9 N 1 ϕ — 1 ɣ = 7 ℨ — 3 N Root 3.
1ϕ + 1ɣ = 13ℨ – 3N 1 ϕ — 1 ɣ = 5 ℨ + 8 N

These, as the most profound Part of *Arithmetick*, occupy the last Part of this Fourth Book, and close up the whole Survey of this Numbering Art.

Notes of Ratio's. Touching *Ratio's* is here further to be noted.

1. Antecedent *&* Consequent. *1st*, As in *Fractions* there is *Numerator* and *Denominator*: So in *Ratio's* there are two Terms; the first whereof is called the *Antecedent*, and the second the *Consequent.*

2. Ratio's, *how expressed.* *2dly*, Though *Ratio's* are set commonly one before another, as in the Instances before-mentioned; yet for better conveniency in working, they are also set one over another like Fractions. And by some, to distinguish them from Fractions, instead of the intervening Line, two Pricks are set; and so the *Ratio Sesquialtera* is thus expressed $\frac{3}{2}$.

3. Ratio's *are Commensurable, or Incommensurable.* *3dly*, *Ratio's*, as common Fractions before, are either Commensurable, or Incommensurable. For if the two Terms compared have any common Part, that will equally divide them both: then they are Commensurable, as 12 to 21; because 3, a Part of them, is a Common Divisor to both. But on the contrary, if the Numbers have no such Part for a Common Divisor; they are Incommensurable, as 18 to 25: for 25 can evenly be divided by no Number but 5; and 18 cannot be divided equally thereby.

4. Ratio *and Proportion promiscuously used.* *4thly*, The words *Ratio* and *Proportion* may be found promiscuously used one for the other in good Authors; which the curious cannot stumble at, since they agree in the Genus, for *Ratio* is a Single Proportion, and Proportion but Plural *Ratio's.*

5. *Conjunct* Ratio's *have their Names so.* *5thly*, In Conjunct *Ratio's*, the Names are Conjunct; as $\frac{3}{1}$ *Triple-Sesquiquarta.*

6thly, The Difference between two Numbers, is the Distance of a Number from a Number, and is found by *Substraction:* But the *Ratio* is the containing of a Number in a Number, and is found by *Division*, and therefore to be understood differently. 6. *Difference and* Ratio *how they differ.*

7thly, The same Difference between Terms may fall out, and the *Ratio* divers. As 6 to 3, the *Ratio* is double, and the Difference 3: But 12 to 9, the *Ratio* is *Sesquitertia*, yet the Difference 3 as before. 7. *Terms alike in Difference, not in* Ratio.

8thly, If one Number shall multiply two Numbers, the Products shall be proportional. And if one Number shall divide two Numbers, the Quotients also shall be proportional. As if 4 multiply 7 and 9, the Products 28 and 36 will be alike proportional: And if 28 and 36 be divided by 4, their Quotients 7 and 9 will be alike proportional. 8. *A Number multiply or divide two others, the Products and Quotients equal.*

$$\text{Multiplier } 4\left\{\begin{matrix}7\,.\,28\\9\,.\,36\end{matrix}\right. \qquad \text{Divisor } 4\Big)\begin{matrix}28\,(7\\36\,(9\end{matrix} \quad \frac{9}{7}\Big(1\tfrac{2}{7} \quad \frac{36}{28}\Big(1\tfrac{2}{7}$$

Example in Species.

$$A\times\left\{\begin{matrix}B\,.\,BA\\C\,.\,CA\end{matrix}\right. \qquad A\Big)\begin{matrix}BA\,.\,(B\\CA\,.\,(C\end{matrix}$$

9thly, Mr. *Oughtred* in his *Clavis*, Chap. 6. adds; If the Consequents of two *Ratio's* are equal, they are as the Antecedents: but if the Antecedents are equal, they are reciprocal as the Consequents. 9. *What follows if the Terms are equal.*

$$\text{As } \frac{7}{1}\cdot\frac{9}{1} :: 7\,.\,9 \qquad \text{And } \frac{1}{7}\cdot\frac{1}{9} :: 7\,.\,9$$

10thly, The *Ratio* of the Antecedent to the Consequent, is compounded either of the *Ratio* of the Antecedent to the Third, and of the Third to the Consequent; or of the *Ratio* of the Third to the Consequent, and of the Antecedent to the Third. 10. Ratio *of the Terms, of what compounded.*

$$\text{As } 7\,.\,9 :: \times\left\{\begin{matrix}7\,A\\A\,9\end{matrix}\right. \qquad \text{Also } 7\,.\,9 :: \times\left\{\begin{matrix}A\,9\\7\,A\end{matrix}\right.$$

CHAP. II.

Reduction of RATIO'S.

THE Description of *Relational Numbers* passed in the former Chapter, *Computation* comes next on the Stage. *Computation of Relational Numbers.*

Ratio's as they are single Proportions, so they challenge the Simple Elements of Numbers to their Accompt; and as Contract Numbers have their Ortive Numeration before their Original. Ratio's *have their Simple Elements.*

The Ortive Numeration of *Ratio's* consists in *Reduction.*

Reduction of *Ratio's*, is to reduce them to their least Terms, or to like Antecedents or Consequents. The first is to be performed as they are of the greater or lesser Inequality. Reduction *of* Ratio's, *of two sorts.*

Ratio's of the greater Inequality are to be divided as Integers; and if any thing remain abbreviated as common Fractions with the Divisor to the least Terms: And then if Occasion be, this Quotient, and the least Terms of the Remain and Divisor so abbreviated, may be reduced into the Form of an improper Fraction. *To reduce* Ratio's *of the greater Inequality to their least Terms.*

As 80 to 32, after Division 16 remaineth; which abbreviated with 32, makes $\frac{1}{2}$ this, and the Quotient 2 is $2\frac{1}{2}$; or reduced like an improper Fraction is $\frac{5}{2}$ and sheweth the *Ratio* of 80 to 32 in its least Terms, is *Dupla-Sesquialtera.* *Example.*

Ratio's of the lesser Inequality, are to be abbreviated as common Fractions to their least Terms. As 32 to 80, when abbreviated, is $\frac{2}{5}$ *Subdupla-Sesquialtera.* *To reduce* Ratio's *of the Lesser Inequality, &c.* *Example.*

Of

To reduce Ratio's to like Consequents. *Example.*

Of the other sort of *Reduction*, that to reduce *Ratio's* to like Consequents, is like Reduction of Common Fractions to like Denominators. As $\frac{1}{20}$ and $\frac{1}{30}$ reduced to like Consequents, is $\frac{30}{600}$ and $\frac{20}{600}$, and abbreviated is $\frac{3}{60}$ and $\frac{2}{60}$.

To reduce Ratio's to like Antecedents. *Example.*

If *Ratio's* are to be reduced to like Antecedents; then, contrary to the other, their Antecedents are to be multiplied one into another for the common Antecedent; and then crosswise every Antecedent is to be multiplied into the other Consequent, except his own. As to reduce $\frac{20}{1}$ and $\frac{30}{1}$ to one Antecedent, 20 is to be multiplied into 30 for the common Antecedent; and then crosswise, 20 into 1, and 30 into 1. So is this Reduction $\frac{600}{30}$ and $\frac{600}{20}$, and by Abbreviation $\frac{60}{3}$ and $\frac{60}{2}$.

Proof of Reduction of Ratio's.

Reduction of *Ratio's* is proved one part by another, after the manner of other *Reductions*.

CHAP. III.

Addition of RATIO'S.

Genesis *and* Analysis *of* Ratio's. Continuation *and* Diminution, *what.*

RATIO'S have their Original Numeration in their *Genesis* and *Analysis*; and their *Genesis* Prime in *Addition*, (called sometime *Continuation*) Compound in *Multiplication*: their *Analysis* Prime in *Substraction*, (sometime called *Diminution*) Compound in *Division* as others before them; but being Comparative herein they differ, for the Operations of the prime Parts of their Numeration are as the Compound Parts of others, and their Compound Parts as Figurals: Wherefore *Addition* of *Ratio's*, in all respects, is performed as Multiplication of Fractions. And so two *Cases* are sufficient.

Addition *of* Ratio's *in two Cases.*

1\. Heterological *Terms not reducible.* *Example.*

Case 1. Where the *Heterologal* Terms need no Reduction, multiply Antecedent by Antecedent, for a new Antecedent: And in like manner Consequent by Consequent, for a new Consequent of the Total.

As if I would add $\frac{2}{3}$ to $\frac{2}{1}$, that is the *Ratio* of 2 to 3, which is *Sub-Sesquialter* to the *Ratio* 2 to 1, which is double: I multiply 2 by 2, and 4 is the new Antecedent of the Total, and 3 by 1. So is 3 the new Consequent, and the Total *Ratio* is *Sesquitertia*.

Antecedents.

Addends $\frac{2}{3} + \frac{2}{1} = \frac{4}{3}$ Total.

Consequents.

2\. Heterologal *Terms reducible.*

Case 2. Where the *Heterologal* Terms, or either of them may be reduced, reduce them like Fractions as low as you can; and then multiply the reduced Terms as above.

Example.

As to add $\frac{4}{3}$ to $\frac{3}{2}$, they may be reduced to $\frac{2}{1}$ and $\frac{1}{1}$, and then their *Addition* will make the Total $\frac{2}{1}$.

Addends $\begin{array}{c}2\\4\\3\\1\end{array} + \begin{array}{c}1\\3\\2\\1\end{array} = \frac{2}{1}$ Total $\frac{12}{6} \mid \frac{2}{1}$

Musical Proportions, how gotten.

Hereby it appeareth that the *Ratio's* commonly called *Harmonica*, or *Harmoniaca Ratio*; and sometime Musical Proportions, (which are such as are to be found in Musical Consorts) though accompted by some a distinct Species from the *Ratio's* before described; yet are nothing else but several *Ratio's* of the former Sorts, only they have other Names, as appeareth in the speculative Part of Musick. For the *Diapente* is *Sesquialter*. *Diatesseron*, *Sesquitertia*, *Diapason*, *Dupla*. *Diapason*

pason with the *Diapente*, *Tripla*; and the *Tone*, *Sesquioctava*. So that the *Diapason* is made of the *Diatesseron* added to the *Diapente*: And the *Diapente* is made of the *Diatesseron* added to the *Tone*.

Examples.

	Diapason.	*Diatesseron.*		*Diapente.*		*Diapason,*	or	*Dupla.*
Ant.	2	4	+	3	added make	12	=====	2
Cons.	1	3		2		6		1

	Diapente.	*Diatesseron.*		*Tone.*		*Diapente,*	or	*Sesquialter.*
Ant.	3	4	+	9	added make	36	=====	3
Cons.	2	3		8		24		2

If many *Ratio's* are to be added together, proceed as in Reduction of Fractions of Fractions; that is, multiply all their Antecedents one into another, and likewise all their Consequents. And if the Terms be reduced before Multiplication, the Total will be in its least Terms. *Many* Ratio's *added.*

Example.

Antecedents. *Reduced.*

Addends $\frac{3}{2} + \frac{4}{3} + \frac{2}{1} = \frac{24}{6}$ Total. $\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} + \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{3}}} + \frac{2}{1} = \frac{4}{1}$

Consequents.

In *Addition* of *Ratio's* may be observed; *Observations.*

1*st*, That *Addition* of *Ratio's*, shews how far the *Ratio's* added are distant from the *Ratio* of *Equality*. For *Dupla* and *Subdupla* added, shall make the *Ratio* of *Equality*. 1. *What* Addition *of* Ratio's *shews.*

As $\frac{2}{1} + \frac{1}{2} = \frac{2}{2}$ or $\frac{1}{1}$ So $\frac{3}{5} + \frac{4}{1} = \frac{12}{5}$ Therefore the Complements of $\frac{3}{5} + \frac{4}{1}$ to Unity, which are $\frac{5}{3} + \frac{1}{4} = \frac{5}{12}$ & $\frac{5}{12} + \frac{12}{5}$ make the *Ratio* equal. *Examples.*

Otherwise if out of each particular *Ratio* I add or substract the Remains of the Complement, and make comparison between the Remains, it will likewise so appear. For 3 is less than 5 by 2, and more than 1 by 2; after the Substraction there rests 1. Likewise in the *Ratio* of the Complement so working there will also remain 1, which is in the *Ratio* of *Equality* with the other 1.

$$\begin{array}{c} 12 \\ \hline \frac{3}{5} + \frac{4}{1} \\ \hline 5 \end{array} \quad \begin{array}{c} 5 \\ \hline \frac{5}{3} + \frac{1}{4} \\ \hline 12 \end{array} \quad \begin{array}{c} 3 \\ 4 \\ 5 \\ 1 \\ \hline 13 \end{array} \quad \begin{array}{c} 5 \\ 1 \\ 3 \\ 4 \\ \hline 13 \end{array} \quad \begin{array}{c} 12 \\ 5 \\ \hline 17 \end{array} \quad \begin{array}{c} 5 \\ 12 \\ \hline 17 \end{array} \quad \begin{array}{c} 60 \\ \hline \frac{12}{5} + \frac{5}{12} \\ \hline 60 \end{array}$$

Also $-1\frac{2}{5} + 3 = +1\frac{2}{5} - 3$. And $-2 + 3 = +2 - 3$
$+1 = -1$

2*dly*, That Equal *Ratio's* added to Equal *Ratio's*, make the Total Equal. 2. *When the Total will be Equal.*

As $\frac{2}{2} + \frac{3}{3} = \frac{6}{6}$ or $\frac{1}{1}$. *Example.*

3*dly*, That Equal *Ratio's* added to Inequal *Ratio's*, render the Total in the same *Ratio* of Inequality the Inequal *Ratio* was before *Addition*. 3. *When the Total is Inequal.*

As $\frac{3}{3} + \frac{3}{2} = \frac{9}{6}$ And $\frac{3}{3} + \frac{2}{3} = \frac{6}{9}$ *Examples.*

Ratio $0 + 1\frac{1}{2} = 1\frac{1}{2}$ *Ratio* $0 + \frac{2}{3} = \frac{2}{3}$

Questions Resolved by Addition *of* Ratio's.

Questions.

1. The *Ratio* of a Penny to a Farthing is as 4 to 1. The *Ratio* of a Shilling to a Penny is as 12 to 1. What is the *Ratio* of a Shilling to a Farthing? 1. Ratio *of a Shilling to a Farthing.*

Answ. As 48 to 1, for so is the Total of the added *Ratio's*.

$$\frac{4}{1} + \frac{12}{1} = \frac{48}{1}$$

2. *Of two Horses Draught.* 2. An Horse draws a Weight, but can draw three times as much; and if another Horse be joined to him, How much will both Horses draw betwixt them, when the first draws as much as he can, and the other but three quarters as much as he can?

Answ. Twice as much and a quarter more, as the *Addition* of the given *Ratio's* shews.

$$\frac{3}{1} + \frac{3}{4} = \frac{9}{4} \ (2\tfrac{1}{4}$$

Proof of Addition of Ratio's. *Addition* of *Ratio's* is to be proved by *Substraction*, as in the next Chapter is made plain.

CHAP. IV.

Substraction of RATIO'S.

Substraction of Ratio's in two Cases. RATIO'S are substracted as Fractions are divided. So two Cases are sufficient for their *Substraction*.

1. *Homologal Terms not reducible.* *Case* 1. When the *Homologal* Terms need no Reduction, multiply crosswise the Antecedent of the *Ratio*, out of which *Substraction* is to be made by the Consequent of the *Ratio* to be substracted, and the Product shall be the remaining Antecedent. And likewise the Consequent of the *Ratio*, from which *Substraction* is to be made by the Antecedent of the *Ratio* to be substracted, and you shall have the Consequent of the Remain.

Examples. As the *Ratio* of $\frac{4}{3}$ substracted from the *Ratio* of $\frac{3}{2}$, the Remain will be $\frac{9}{8}$. And if the *Ratio* of $\frac{3}{2}$ be taken from the *Ratio* of $\frac{4}{3}$, there will remain $\frac{8}{9}$.

$\frac{3}{2} - \frac{4}{3} = \frac{9}{8}$ Remain. $\frac{4}{3} - \frac{3}{2} = \frac{8}{9}$ Remain.

2. *Homologal Terms reducible.* *Case* 2. Where the *Homologal* Terms, or either of them, may be reduced, reduce them like Fractions as low as you can; and then multiply the reduced Terms as above.

Example. As to take the *Ratio* of $\frac{4}{3}$ from $\frac{2}{1}$, because 4 and 2 will abbreviate, they are reduced to 2 and 1: And being multiplied, the Remain shall be in its least Terms $\frac{3}{2}$, which otherwise will be $\frac{6}{4}$.

$$\frac{\overset{1}{2}}{1} - \frac{\overset{2}{4}}{3} = \frac{3}{2}$$

Substraction of many from one, or one from many. If many *Ratio's* were to be substracted from one, or one *Ratio* from many, then first add together the *Ratio's* exceeding one, and afterward make *Substraction* as above.

Example. As to substract $\frac{2}{1}$ from $\frac{4}{3}$ and $\frac{3}{2}$, I first add $\frac{4}{3}$ and $\frac{3}{2}$ and the Total is $\frac{12}{6}$, or by Reduction $\frac{2}{1}$; From which if $\frac{2}{1}$ be taken, there remaineth the *Ratio* of Equality $\frac{2}{2}$ or $\frac{1}{1}$.

Addition. *Substraction.* Without *Reduction.*

$$\frac{\overset{2}{4}}{\underset{1}{3}} + \frac{\overset{1}{3}}{\underset{1}{2}} = \frac{2}{1} \qquad \frac{\overset{1}{2}}{1} - \frac{\overset{1}{2}}{1} = \frac{1}{1} \qquad \frac{4}{3} + \frac{3}{2} = \frac{12}{6} - \frac{2}{1} = \frac{12}{12} \text{ or } \frac{1}{1}$$

Substraction of many from many. If many *Ratio's* are to be substracted from many, proceed in like manner, adding together the *Ratio's* from which *Substraction* is to be made, and also the *Ratio's* to be substracted: And then make *Substraction* as above.

Example. As $\frac{4}{3}$ and $\frac{3}{2}$ to be substracted from $\frac{2}{1}$ and $\frac{3}{4}$, the Remain will be $\frac{3}{4}$.

For $\frac{4}{3} + \frac{3}{2} = \frac{2}{1}$ And $\frac{2}{1} + \frac{3}{4} = \frac{3}{2}$ And $\frac{3}{2} - \frac{2}{1} = \frac{3}{4}$

Some-

Sometime more to puzzle a young Practitioner than otherwiſe, many *Ratio's* are given to be ſubſtracted from many, annexed with the Signs + and —, the which though improper, becauſe, as was noted before, thoſe Signs properly belong to *Æquations*; yet there is more Difficulty apparent than real therein. For it is but to add what is to be added, and ſubſtract what is to be ſubſtracted of the *Data*. And for a final Reſolution, work with the New *Ratio's*. *If Signs be uſed, how to proceed.*

As if $2:1 + 3:2 - 1:3$ were to be ſubſtracted from $4:3 + 5:6 - 4:5$; then firſt I add $2:1 + 3:2$, and from the Total $3:1$ I ſubſtract $1:3$ and there reſteth $9:1$; Alſo $4:3 + 5:6$ added, make $10:9$, from which $4:5$ taken, leaves $25:18$. Laſtly, From thence I withdraw the *Ratio* $9:1$, and the ultimate Remain is $25:162$. *Example.*

$$2:4:3 + 5:6:3 = 5:10:9 - 2:4:5 = 25:18 \qquad 1:2:1 + 3:2:1 = 3:1 - 1:3 = 9:1$$

And $25:18 - 9:1 = 25:162$ Ultimate Remain.

In *Subſtraction* of *Ratio's* may be obſerved: *Obſervations.*

1ſt, That Equal *Ratio's*, ſubſtracted from Equal *Ratio's*, leave the Remains Equal. *1. When the Remain will be equal.*

For $2:2 - 3:3 = 6:6$ *Example.*

2dly, That if Equal *Ratio's* be ſubſtracted from Inequal *Ratio's*, the Remains will be left in the ſame *Ratio* of Inequality the Inequal *Ratio* was before *Subſtraction* made: And is all one as if the *Ratio's* were added. *2. When the Remain will be inequal.*

As $1:3:2 - 1:3:3 = 3:2$ *Examples.*

Ratio $1\frac{1}{2} - 0 = 1\frac{1}{2}$

And $2:3:1 - 3:3:1 = 2:3$

Ratio $2:3 - 0 = 2:3$

3dly, Inequal *Ratio's* ſubſtracted from Equal, leave the Remain of the ſame Inequal Terms the ſubſtracted *Ratio* was, but alters the ſame ſo, that if before *Subſtraction* the *Ratio* were of the Greater Inequality, now it ſhall be of the Leſſer; and if before of the Leſſer, now of the contrary. *3. Inequal from Equal, what the Remain.*

As $1:3:3 - 1:3:2 = 2:3$ *Examples.*

Ratio $0 - 1\frac{1}{2} = 2:3$

And $3:3:1 - 2:3:1 = 3:2$

Ratio $0 - 2:3 = 1\frac{1}{2}$

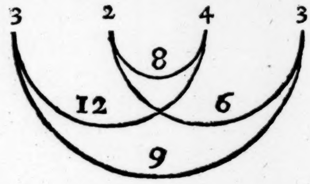
4thly, *Subſtraction* ſhews which is the greateſt *Ratio* of two given; for after the *Subſtraction*, if the Remain be of the greater Inequality, then was the *Ratio* from which *Subſtraction* was made greater than the ſubſtracted *Ratio*: But if the Remain be of the leſſer Inequality, then underſtand the contrary. Both theſe are ſo evident in the laſt Examples, that no farther Demonſtration thereof is needful. *4. What Subſtraction of Ratio's ſhews.*

5thly, *Addition* and *Subſtraction* of *Ratio's* ſerves to find out new Proportional *Ratio's*; for the new *Ratio's* found by both, are reciprocally proportional. *5. What Addition and Subſtraction of Ratio's ſerves for.*

As if $4:3$ be added to $3:2$ without Reduction, the Total will be $12:6$. And if $3:2$ be ſubſtracted from $4:3$, the Remain will be $8:9$. Then it is evident that 12 to 8 is as 3 to 2; and 12 to 9, is as 4 to 3: Alſo 8 to 6, is as 4 to 3; and 9 to 6, is as 3 to 2: So that 8 and 9 have proportion with the other reciprocally, as by the Scheme following may appear: After which Form the Antient Writers of Muſick were wont to expreſs both the *Addition* and *Subſtraction* of *Ratio's*, as Mr. *Oughtred* teſtifieth, Chap. 10. of his *Clavis*. *Examples.*

3

The Antient Form of expressing Addition *and* Substraction *of* Ratio's.



$\genfrac{}{}{0pt}{}{3}{2}+\genfrac{}{}{0pt}{}{4}{3}=\genfrac{}{}{0pt}{}{12}{6}$ *Ergo* $\genfrac{}{}{0pt}{}{12}{8}+\genfrac{}{}{0pt}{}{8}{6}=\genfrac{}{}{0pt}{}{96}{48}$ or $\genfrac{}{}{0pt}{}{12}{6}$ Also $\genfrac{}{}{0pt}{}{9}{6}+\genfrac{}{}{0pt}{}{12}{9}=\genfrac{}{}{0pt}{}{108}{54}$ or $\genfrac{}{}{0pt}{}{12}{6}$

$\genfrac{}{}{0pt}{}{4}{3}-\genfrac{}{}{0pt}{}{3}{2}=\genfrac{}{}{0pt}{}{8}{9}$ *Ergo* $\genfrac{}{}{0pt}{}{8}{12}-\genfrac{}{}{0pt}{}{9}{12}=\genfrac{}{}{0pt}{}{96}{108}$ or $\genfrac{}{}{0pt}{}{8}{9}$ Also $\genfrac{}{}{0pt}{}{9}{6}-\genfrac{}{}{0pt}{}{8}{6}=\genfrac{}{}{0pt}{}{54}{48}$ or $\genfrac{}{}{0pt}{}{9}{8}$

Questions.

Questions resolved by Substraction *of* Ratio's.

1. Ratio *of a Pound to a Shilling.*

1. The *Ratio* of a Pound to a Penny, is as 240 to 1. The *Ratio* of a Shilling to a Penny, is as 12 to 1. What is the *Ratio* of a Pound to a Shilling?

Answ. As 20 to 1, for so is the Remain left after *Substraction.*

$$\begin{smallmatrix}20\\240\\1\end{smallmatrix}-\begin{smallmatrix}1\\12\\1\end{smallmatrix}=\genfrac{}{}{0pt}{}{20}{1} \qquad \frac{240}{12}(20$$

2. *Of the Draught of one Horse.*

2. Suppose two Horses together draw twice as much, and a quarter part more at one time as at another when the Ways are bad: One of which Horses alone can draw a Weight of no greater *Ratio* than 3 to 4. What *Ratio* shall the Weight drawn by the other Horse bear?

Answ. 3 to 1: for if the *Ratio* $\genfrac{}{}{0pt}{}{3}{4}$ be substracted from the *Ratio* $2\frac{1}{4}$, the Remain will be $\genfrac{}{}{0pt}{}{3}{1}$.

$$\begin{smallmatrix}3\\9\\4\\1\end{smallmatrix}-\begin{smallmatrix}1\\3\\4\\1\end{smallmatrix}=\genfrac{}{}{0pt}{}{3}{1} \qquad \frac{36}{12}(3$$

Proof of Substraction *of* Ratio's.

Substraction and *Addition* of *Ratio's* are Reciprocal Proofs of each other, as *Multiplication* and *Division* of Fractions, and need no more than these two Questions, being of a contrary Nature to those in *Addition*, compared one with another to make it plain: For the Remain of any *Substraction* of *Ratio's*, added to the *Subtrahend*, after the manner of *Ratio's*, will return the Number from which *Substraction* was made, that is, the Total of their *Addition.*

CHAP. V.

Multiplication of RATIO'S.

Multiplication *of* Ratio's.

THE prime Parts of *Numeration* of *Ratio's*, in their *Addition* and *Substraction*, have been now seen to be like the Compound Parts of *Numeration* in other Numbers. And as before observed, their Compounds Parts of *Numeration*, viz. *Multiplication* and *Division*, are like Figuration of other Numbers.

Rule.

To multiply therefore any *Ratio*, is to multiply the Antecedent and Consequent so often into themselves respectively, as there be Units in the Multiplier.

Examples.

As to double $\genfrac{}{}{0pt}{}{3}{2}$, the *Ratio Sesquialter*, or multiply it by 2, is to multiply 3 by 3, and 2 by 2: So is the Product $\genfrac{}{}{0pt}{}{9}{4}$

And to triple the *Ratio Sesquitertia*, I either set down $\genfrac{}{}{0pt}{}{4}{3}$ three times, and multiply the Antecedents one into another, and so likewise the Consequents: Or multiply 4 cubically, and 3 likewise: Whereby there is produced $\genfrac{}{}{0pt}{}{64}{27}$.

Factors.	*Product.*	*Factors.*	*Product.*
$\genfrac{}{}{0pt}{}{3}{2}\times\genfrac{}{}{0pt}{}{3}{2}$	$=\genfrac{}{}{0pt}{}{9}{4}$	$\genfrac{}{}{0pt}{}{4}{3}\times\genfrac{}{}{0pt}{}{4}{3}\times\genfrac{}{}{0pt}{}{4}{3}$	$=\genfrac{}{}{0pt}{}{64}{27}$

In

In like manner as to multiply any *Ratio* by 2, is to square the same; and to multiply a *Ratio* by 3, is to cube it. So to multiply by any other Number, is to figurate the Terms of the *Ratio* accordingly.

Multiplication figurates the Terms.

Quest. Ptolomy in his *Almagest*, proposes the Diameter of the Sun to the Diameter of the Earth, to be as 11 to 2. And by *Prop.* 18. of *Euclid*'s 12*th* Book, Spheres or Globes have triple Proportion to their Diameters. How much then shall the Sun be bigger than the Earth?

Question of the Bigness of the Sun.

Answ. 166 times and $\frac{3}{8}$; for the *Ratio* of $\frac{11}{2}$ tripled that is cubed, produceth $\frac{1331}{8}$, which reduced by *Division*, gives $166\frac{3}{8}$ as before.

Answer.

$$\frac{11}{2} \times \frac{11}{2} \times \frac{11}{2} = \frac{1331}{8} \left(166\tfrac{3}{8} \right.$$

Multiplication of *Ratio's* is to be proved by their *Division*, as in the next Chapter is to be seen.

Proof of Multiplication *of* Ratio's.

CHAP. VI.

Division of RATIO'S.

AS *Multiplication* figurates the Terms of the *Ratio*; so on the contrary, *Division* of *Ratio's* is to extract from each Term of the *Ratio* given to be divided, a Root of the second, third, or fourth Quantity, *&c.* according to the Units contained in the Divisor. As to divide a *Ratio* by 2, is to extract the Square Root thereof: And to divide by 3, is to extract the Cube Root, *&c.*

Division of Ratio's *is as Extraction of Roots.*

Thus $\frac{9}{4}$ divided by 2, gives in the Quotient $\frac{3}{2}$

And $\frac{27}{8}$ divided by 3, gives in the Quotient $\frac{3}{2}$

But $\frac{256}{81}$ if divided by 4, makes the Quotient $\frac{4}{3}$

Examples.

Dividend.

Divisor $\frac{4}{1}$) $\frac{256}{81}$ ($\frac{4}{3}$ Quotient

$$\begin{array}{c|c|c} 256 & 16 & 4 \\ 81 & 9 & 3 \end{array}$$

Quest. If the Body of the Sun, according to *Ptolomy*, be bigger than the Globe of the Earth 166 times and $\frac{3}{8}$; then what *Ratio* is there between their Diameters?

Question of the Diameters of the Sun and the Earth.

Answ. $\frac{11}{2}$: for $166\frac{3}{8}$ reduced, and the *Ratio* divided by 3, that is, the Cube Root of each Term taken, the Quotient or Root is $\frac{11}{2}$ as aforesaid.

Answer.

Reduced.	Divided.	
$166\frac{3}{8} = \frac{1331}{8}$	$\frac{3}{1}$) $\frac{1331}{8}$ ($\frac{11}{2}$	$\begin{array}{c\|c} 1331 & 11 \\ 8 & 2 \end{array}$

Division and *Multiplication* of *Ratio's*, being as Production of Figural Numbers, and Extraction of Roots; it must needs be that the one shall be Proof of the other reciprocally as they were, and hath been largely discoursed before in Figural Numbers. And the last Question being but the Reverse of the Question before in *Multiplication*, will clearly evince without farther Testimony.

Proof of Division *of* Ratio's.

Partis primæ Libri quarti

FINIS.

The Second PART of the Fourth BOOK.

CHAP. I.

Of PROPORTIONS Disjunct.

Proportions, their Computations and Comparative Elements.

NEXT to the Computation of *Ratio's*, come the Computation of *Proportions*, and their Comparative Elements, to be viewed.

Among *Proportions*, the first that present themselves are Simple, and of them those called *Disjunct*.

Disjunct, where described.

Disjunct *Proportions*, in the first Chapter of this 4*th* Book, were generally described to be *Arithmetical* and *Geometrical*.

Those Arithmetical of little use. Geometrical excel.

Those *Arithmetical* (yet of little use in *Arithmetick*) were before remembred to give being to some Musical *Proportions*, and have somewhat mentioned of them hereafter in this Chapter, with others *Geometrical*; but give place to these, as being of much more excellent Use, not only in *Arithmetick*, but in several other Arts and Sciences.

Wherein their Comparative Elements consist.

The Comparative Elements of Geometrical Disjunct *Proportions*, consist in the Invention of new Proportional Numbers, Plain or Figural, according to the *Data* by which they are found.

What they are, vide antea. Comparative Elements twofold.

What Plain and Figural *Proportions*, Disjunct and Geometrical, are, the first Chapter of this 4*th* Book hath told us. And that these plain *Proportions* are Direct and Indirect; both which, with their Comparative Elements, may be subdivided into *Primitive* and *Derivative*.

Primitive, what.

Primitive give three Numbers to find out a Fourth or new Proportional by a single Operation, and need nothing before Operation save a due and orderly Disposition of the *Data*.

How divided, and where handled.

Primitives are again subdivided

Into { Common, in the *Rule of Three* { *Direct*, Chap. 2. / *Indirect*, Chap. 3. }; Peculiar, in *Practice*, Chap. 4. }

Derivative of four sorts.

Derivative before Operation, need some or other of the Simple Elements of Numbers to fit them for Resolution.

Or, 2. give more than three Numbers.

Or, 3. deal with some particular Subject.

Or, 4. have some peculiar Operation requisite to their Resolution.

Where handled.

Derivatives {
- of the first Sort, are Specificks *Direct* and *Indirect*, — Chap. 5.
- of the second Sort, The Rule of 5 Numbers { *Direct*, — Chap. 6. / *Indirect*, — Chap. 7. }
- of the third Sort {
  - *Fellowship*, — Chap. 8.
  - *Alligation*, — Chap. 9.
  - *Barter* and *Exchange*, — Chap. 10.
  - *Loss* and *Gain*, — Chap. 11.
  - *Æquation* of Paiment, — Chap. 12.
  - *Factorship*, — Chap. 13. }
- of the fourth Sort, *Falshood* or *Position*, — Chap. 14. }

Figural Proportions, where handled.

Figural Disjunct Proportions follow them, { *Doubled*, — Chap. 15. / *Tripled*, &c. — Chap. 16. }

Concerning

Concerning Disjunct Proportions in general, take this farther Account.

Notes on Disjunct Proportions.

1. The *Data*, or Numbers given, are called *Terms*; of which the first is set to the left Hand, and the Residue in order towards the Right.

1. Terms what; and how set.

2. The least and greatest Terms are called *the Extreams*, and the others *the Means*, or *middle Proportionals*. Also the Sinister Term is called the *Antecedent*, and the Second his *Consequent*. So likewise the Third and Fourth.

2. Extreams and Means, what.

3. In Arithmetical *Disjunct Proportions*, the Aggregate of the Extreams shall be equal to the Aggregate of the Means.

3. Arithmetical Disjunct Proportions, the Total of the Means and Extreams equal. Example.

As in 4 . 7 : 5 . 8, the Sum of 4 and 8, is equal to the Sum of 7 and 5.

4 · 7 : 5 · 8

12

12

4. If of three Terms in Arithmetical *Disjunct Proportion*, a fourth be sought; from the Second added to the Third, the First shall be substracted.

4. Three Terms thereof given, to find a fourth. Example.

As in the former Example, if 4 . 7 . 5 be given; then from 12, the Sum of 7 and 5, shall 4 be abated; so will the Remain be 8.

For $7 + 5 - 4 = 8$.

5. Numbers or Magnitudes in Geometrical *Disjunct Proportion*, have the Aggregate of the greatest and least Terms, greater than the Aggregate of the Residue: but the Product of the Extreams, equal to the Product of the Means.

5. Geomet. Disjunct Proportions, the Product of the Extreams and Means equal. Example.

As 5 . 15 :: 6 . 18, the Total of 5 and 18 is 23, greater than 21, the Total of 15, and 6; but the Product of 5 by 18, which is 90, is equal to the Product of 15 by 6.

Added.	Multiplied.
5 . 15 :: 6 . 18	5 . 15 :: 6 . 18
21	90
23	90

Example in Species.

R . S :: Z . A   R . S :: Z . A

Ergo, R + A shall be greater than S + Z. But $SZ = RA$, and by Consequence $\frac{ZS}{R} = A$ in Numbers $\frac{90}{5} = 18$.

From hence sprang the *Rule of Three*, spoken to in the next Chapter.

Whence the Rule of Three.

6. Four Numbers or Magnitudes in Disjunct Geometrical *Proportion* Direct, shall also be proportional; if they be Alternate, Inversed, Compounded, Parted, Conversed and Mixt.

6. Four in Geom. Disjunct Proportion, how otherwise Proportional. Alternate.

As admit 7 get 28, and accordingly 9, 36, then alternately shall 7 . 9 :: 28 . 36; that is, Antecedent to Antecedent, and Consequent to Consequent; and there the *Ratio* of 7 to 9, is alike to that of 28 to 36.

Inversed.

And Inversed 28 . 7 :: 36 . 9, the *Quadruple Ratio* of 28 to 7, is the same with 36 to 9. Here the Consequent is taken as the Antecedent, and so compared to the Antecedent as the Consequent.

Compounded.

Also Compounded; as 7 and 28 which is 35 is to 28, so shall 9 and 36 that is 45 to 36. This is when the Antecedent and Consequent, together as one, are compared to the Consequent.

Parted.

Again, being divided 7 — 28 . 28 :: 9 — 36 . 36. And if withdrawing the first Term from the Second, the Residue be compared the Second, yet the *Ratio* shall be alike to the *Ratio* of the Remainder of the third Term substracted from the Fourth, compared to the Fourth. For divided *Ratio* is, when the Excess wherein

wherein the Antecedent exceedeth the Consequent, is compared to the Consequent, as $21 . 28 :: 27 . 36$; where as well 27 to 36, as 21 to 28, are in the *Ratio Subsesquitertia*.

Conversed. Likewise, if conversed, that is, when the Antecedent is compared to the Excess, wherein the Antecedent exceeds the Consequent, 7 to 35, which is the Sum of the first and second Terms, are in a like *Ratio* as 9 to 45, the Sum of the third and fourth Terms.

Mixt. Moreover, Mixt, as $7 + 28 . 7 - 28 :: 9 + 36 . 9 - 36$; which in effect, is as $35 . \overline{21} :: 45 . \overline{27}$, both agreeing in the *Ratio Superbipartiens-Tertias*.

Examples.

An Example in Species explained, with other Numbers.

Data	$A . \alpha :: B . \beta$	$4 . 16 :: 6 . 24$	
Alternate	$A . B :: \alpha . \beta$	$4 . 6 :: 16 . 24$	*Subsesquialter.*
Inversed	$\alpha . A :: \beta . B$	$16 . 4 :: 24 . 6$	*Quadruple.*
Compounded	$A + \alpha . \alpha :: B + \beta . \beta$	$4 + 16 . 16 :: 6 + 24 . 24$	*Sesquiquarta.*
	$A + B . B :: \alpha + \beta . \beta$	$4 + 6 . 6 :: 16 + 24 . 24$	*Superbipartiens-Tertias.*
Divided	$A - \alpha . \alpha :: B - \beta . \beta$	$4 - 16 . 16 :: 6 - 24 . 24$	*Subsesquitertia.*
	$A - B . B :: \alpha - \beta . \beta$	$4 - 6 . 6 :: 16 - 24 . 24$	*Subtriple.*
Conversed	$A . A \pm \alpha :: B . B \pm \beta$	$4 . 4 \pm 16 :: 6 . 6 \pm 24$	*Subquintuple. Subtriple.*
	$A . A \pm B :: \alpha . \alpha \pm \beta$	$4 . 4 \pm 6 :: 16 . 16 \pm 24$	*Subduplasesquial. Subdupla.*
Mixt	$A + \alpha . A - \alpha :: B + \beta . B - \beta$	$4 + 16 . 4 - 16 :: 6 + 24 . 6 - 24$	*Superbipartiens-Tertias.*
	$A + B . A - B :: \alpha + \beta . \alpha - \beta$	$4 + 6 . 4 - 6 :: 16 + 24 . 16 - 24$	*Quintuple.*

7. Numbers Proportional, the Consequents.

7. Certain Numbers or Magnitudes proportional, it shall be, That as one Antecedent to his Consequent; so the Sum of the Antecedents to the Sum of the Consequents.

Examples.

Example in Numbers.

As $4 . 16 :: 6 . 24 :: 3 . 12 :: 2 . 8$; It shall be then,
That $4 . 16 :: 4 + 6 + 3 + 2 . 16 + 24 + 12 + 8$.

Example in Species.

As $A . a :: B . b :: C . c :: D . d$; Therefore it shall be,
That $A . a :: A + B + C + D . a + b + c + d$.

8. Antecedents equal the Consequents.

8. When the Antecedents of many *Proportions* are equal, it shall be; That as one Antecedent, to the Sum of his Consequents: So another Antecedent to the Sum of his Consequents.

Example in Numbers and Species.

Examples.

As $\begin{matrix} 4 . 16 :: 6 . 24 \\ A . B :: a . b \end{matrix}$ And $\begin{matrix} 4 . 12 :: 6 . 18 \\ A . C :: a . c \end{matrix}$ And $\begin{matrix} 4 . 10 :: 6 . 15 \\ A . D :: a . d \end{matrix}$

Ergo $\begin{matrix} 4 . 16 + 12 + 10 :: 6 . 24 + 18 + 15 \\ A . B + C + D :: a . b + c + d \end{matrix}$ That is, $4 . 38 :: 6 . 57$.

9. Four in Arith. Disjunct Proportion added to, or taken from others the Consequents.

9. If 4 Numbers or Magnitudes in Arithmetical *Proportion Disjunct*, be added to, or substracted from 4 others alike Proportional, the Totals and Remains will be Proportionals.

Example.

	$3 . 5$	$9 . 11$	Excess 2
	$2 . 4$	$5 . 7$	Excess 2
Totals	$5 . 9$	$14 . 18$	4
Remains	$1 . 1$	$4 . 4$	0

10. Four in Geo. Disjunct Proportion, multiply or divide others the Consequents.

10. If four Numbers, or Magnitudes, in Disjunct Geometrical Proportion, be multiplied or divided by four others respectively Proportional, The Products and Quotients shall be accordingly Proportional: And the *Ratio* in the Products will likewise be multiplied, but in the Quotients divided.

7 . 28

	7 .	28 ::	9 .	36	Ratio	4	*Example.*
	8 .	32 ::	9 .	36	Ratio	4	
Products	56 .	896 ::	81 .	1296		16	
Quotients	$1\frac{1}{7}$.	$1\frac{1}{7}$ ::	1 .	1		1	

CHAP. II.

The Direct Rule of Three.

IN the Front of the Comparative Elements of Disjunct Geometrical Proportions stands *The Rule of Three*, so called, because three Numbers are given to find out a Fourth Proportional Number; sometime *The Rule of Proportion*, because the Likeness or Agreement of some Numbers within themselves, or each to other, is thereby declared: But most frequently it is called *The Golden Rule* for its Excellency, the Conclusions wrought and obtained thereby being so many, and so profitable and useful, as exceed all Credence in the Unskilful. And to difference this from *The Double Rule of Three*, spoken to hereafter, in the 6th and 7th Chapters, this is called *The Simple Golden Rule*.

Direct Rule of Three, *why so called.* *Why called the* Rule of Proportion. *Why the* Golden Rule. *Why the* Simple Golden Rule.

This *Rule of Three*, as in the Chapter before noted, is *Direct* and *Indirect*: to the Performance and perfect Understanding of *The Direct Rule of Three*,

Rule of Three, Direct *and* Indirect.

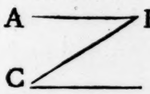
Some things are { *Preparatory.* / *Operatory.* / *Probationary.* }

The *Preparatory* Part, is the Right Disposition and Consideration of the *Data*, or Numbers given. And this is to be had in the Precepts following.

Preparatory *to the Direct.*

1. Place the three given Numbers, as A . B . C, in one or other of the three following Varieties: but the last to the right Hand is of late as brief and best, most in use; that is, to divide the first Term from the Second by a Point, the Second from the Third by four Points, and the Third from the Fourth when found by another single Point. So shall it be read, as A is to B; so is C to D, signifying the Fourth new Proportional when found. And the *Data* so standing, the Number standing in the Place of A, shall be called the first Number or Term; and the Number placed as B, shall be the Second; and the Number in the Place of C, the Third.

1. *How to place the Numbers.*

Common Way.	Old Way.	Late Way.
1 2 3 A — B — C —		A . B :: C.

2. Let the Number upon which the Question propounded depends, be always set in the third Place, or C, and called the third Number.
3. That Number of the *Data* which is of one Denomination and Nature with the Third, must be set in the Place of A, for the first Number.
4. The right Places found for two of the three given Numbers, of necessity the Number of another Nature or Denomination, left unplaced, shall be set in the second Place instead of B; and so will all the Three be rightly placed.
5. When the Denominations seem to be doubled, as Yards-long, Yards-broad, Pounds-principal, or Pounds-profit, &c. the latter Denomination is to be respected in placing the Numbers.
6. The fourth Number, when found, shall be of like Denomination with the Second.

2. *Where to set that on which the Question depends.* 3. *Which must be first.* 4. *Which the second Number.* 5. *Denominations doubled, how to place them.* 6. *Fourth of what Denomination.*

The *Operatory* Part consists in the { Invention of the *Divisor.* / Resolution of the *Question.* }

Operatory *in the Direct.*

Ppppp To

To find the Divisor.

To find the Divisor after the *Data* are rightly placed, consider whether the fourth Number, which is the Number quesited, must be greater or lesser than the Second. For if greater, then the least of the two Extreams (which are the first and third Numbers) shall be Divisor: But if less, then the greatest of those two Extreams is to be Divisor.

To resolve the Question.

To resolve the Question, when the first Number is found to be Divisor, the Rule is called the *Direct Rule of Three*, and is to multiply the second Number by the Third, and divide the Product by the First. The Quotient of this Division shall be the Answer to the Question, and the fourth Proportional: For such Proportion as the third Number beareth to the First, and is greater or less, such Proportion shall this fourth Number bear to the Second.

Example.

Example. What shall 18 ℔. of Spice [Yards of Cloth, Bushels of Wheat, *&c.*] cost me, when 3 ℔. [Yards, Bushels, *&c.*] of the same Commodity costeth me 5 *s.*?

The Question thus propounded, I see, by the second Precept, 18 ℔. must be the third Number, because the Question depends thereon; and by the third Precept 3 ℔. shall be the first Number, because it is alike denominate to 18, the third Number. So of necessity 5 *s.* which is the odd denominate Number, shall be set in the second Place, according to the *4th* Precept; and the Numbers thus rightly placed, stand as here set.

℔.	*s.*	℔.		℔. *s.* ℔.
As 3 .	5 ::	18.		Commonly if 3—5—18?

And if wrote at length shall be, If 3 ℔. cost me 5 *s.* what shall 18 ℔?

Then to find the Divisor, consulting with Reason, it is evident, that 18 ℔. shall cost me more than 3 ℔. Wherefore enquiring for a greater Number than 5 *s.* the least Extream, which here is 3, shall be Divisor. And so 5 and 18 must be multiplied together, and the Product 90 divided by 3, the fourth Number and Resolution is obtained, which is found to be 30, and they to be Shillings, denominate as 5 *s.* by the *6th* Precept.

℔. *s.* ℔. *s.*
As 3 . 5 :: 18 . 30

$$\frac{5}{90} \qquad \frac{90}{3}\Big(30\,s.$$

Proof of the Direct Rule of three, by

The *Probationary* Part is either to { Reverse the Question, Or, Multiply the Extreams and Means.

Reversing the Question.

To reverse the Question: Let the third Number of the one Question, be the First of the other; the former *4th* the Second of the next Work; and the first of the former, the Third of the Latter; and after Operation, the Fourth of this Latter will be the former second; and so prove both, if there be no Error in the Operations.

Multiplying the Extreams and Means.

To multiply the Extreams and Means respectively, according to the Fifth of the foregoing Chapter, the Products being equal, will also prove the Truth of any Question resolved by the *Rule of Three.*

For in the Instance before, If the Question be stated thus;

Example.

If 18 ℔. cost 30 *s.* what shall 3 ℔. cost? Seeing a lesser Number than 30 is looked for, 18 the greater of the two Extreams shall be Divisor; which shall divide 90, the Product of 30, into 3. So will 5, the Quotient, be equal to the former second Number, and prove the former Work true.

Also if the Extreams and Means be multiplied in either Operation, they will produce in both Operations 90.

℔. *s.* ℔. *s.*
As 18 . 30 :: 3 . 5

$$18\Big)\frac{3}{90}\Big(5$$

3 . 5 :: 18 . 30
90
90

18 . 30 :: 3 . 5
90
90

And

And the *Sexcuple Ratio* between 18 and 3, is equal to the *Ratio* of 30 to 5.

Four Numbers in Proportion, how diversly transposed.

Neverthelefs though this above be the common Courfe in reverfing the Queftion, it may be noted further, That where four Numbers are proportional, their Order may be fo tranfpofed, that each of thofe Terms may be laft in Proportion. As, firft, 1 . 2 :: 3 . 6. Second, 3 . 6 :: 1 . 2. Third, 2 . 1 :: 6 . 3. Fourth, 6 . 3 :: 2 . 1. So as every Proportion doth implicitly contain four Orders, two defcending, and two afcending; by one of which Orders, if of four Proportionals any three be given, the other Unknown may be found out.

In Refolution of all Queftions proper, Multiplication and Divifion to be ufed.

The Benefit thereof.

That nothing may be wanting to compleat the Neceffaries of this Chapter, let it be obferved, that to the Refolution of every Queftion propounded, the Multiplication and Divifion ufed be proper to the Nature of the Numbers given: And this will fave a burdenfome Charge of the Memory with many Rules; which is always to be avoided, where one Rule and Method of proceeding is fufficient. As to refolve a Queftion propounded in Fractions, or other Contract Numbers, it is but to multiply and divide after the manner of Fractions, or other Contract Numbers proper to the *Data, mutatis mutandis:* Whereas moft Arithmetical Writers deliver the Rule in Fractions, thus, Multiply the Numerator of the firft Fraction, by the Denominator of the fecond Fraction, and that Product again by the Denominator of the third Fraction, and this Product fhall be Divifor. Again, multiply the Denominator of the firft Fraction, by the Numerator of the Second, and that Product again by the Numerator of the Third for the Dividend. Much more long and tedious to the Memory, than to multiply the Second by the Third, and divide by the Firft, as Fractions are to be multiplied and divided.

Example in Fractions.

Examples in Fractions of the Price of $\frac{1}{2}$ an Ell.

If $\frac{3}{4}$ of one Ell coft me $\frac{4}{5}$ of a Pound *Sterling*, What fhall $\frac{1}{2}$ of an Ell coft? Here $\frac{3}{4}$ being Divifor, $\frac{4}{5}$ and $\frac{1}{2}$ are multiplied together; and the Product $\frac{2}{5}$ divided by $\frac{3}{4}$, giveth in the Quotient $\frac{8}{15}$ of a Pound, or 10 *s.* 8 *d.*

	Ell.	*l.*	*Ell.*	*l.*	
As	$\frac{3}{4}$.	$\frac{4}{5}$ ::	$\frac{1}{2}$.	$\frac{8}{15}$	For $\overset{2}{\cancel{\frac{4}{5}}} \times \underset{1}{\cancel{\frac{1}{2}}} = \frac{2}{5}$ And $\frac{3}{4}) \frac{2}{5} (\frac{8}{15}$

The other Way used by several, is thus.

$$\begin{array}{lcccc}
 & \textit{Ell.} & \textit{l.} & \textit{Ell.} & \textit{l.} \\
\text{As} & \frac{3}{4} \cdot & \frac{4}{5} :: & \frac{1}{2} \cdot & \frac{8}{15}
\end{array}
\qquad
\begin{array}{r} 3 \\ \underline{5} \\ 15 \\ \underline{2} \\ \text{Divifor } \underline{30} \end{array}
\qquad
\begin{array}{r} 4 \\ \underline{4} \\ 16 \\ \underline{1} \\ \text{Dividend } \underline{16} \end{array}
\qquad
\frac{16}{30}\Big|\frac{8}{15}
\qquad
\text{For}\begin{cases} 4\times4\times1=16 \\ 3\times5\times2=30 \end{cases}$$

Examples in other Contract Numbers, with their proper Multiplications and Divifions.

Decimals.

Decimals.

Of the Price of 640 Ells.

If $158\frac{1}{4}$ Ells coft 79 *l.* 2 *s.* 6 *d.* What will 640 Ells coft at that Rate? The firft and fecond Numbers turned into Decimals, are 158,25; and 79,125. Operation being made as aforefaid, the Refolution is 320 *l.* and the whole Work is as here followeth.

$$\begin{array}{ccccccc}
 & \textit{Ells.} & & \textit{l.} & & \textit{Ells.} & \textit{l.} \\
\text{As} & 158{,}25 & . & 79{,}125 & :: & 640 & . \; 320{,}0
\end{array}$$

$$\begin{array}{rr}
(3) & \underline{640} \\
\underline{(2)} & 3165\,000 \\
\text{Index } (1) \text{ of the} & \underline{47475\,0} \\
\text{Quotient.} & \underline{50640{,}000}
\end{array}
\qquad
\begin{array}{r}
3165 \\
50640000 \\
158255 \\
1582
\end{array}
\Bigg(\begin{array}{l} \textit{l.} \\ 320{,}0 \end{array}$$

Aftronomicals.

Aftronomicals.

Of the Digits eclipfed.

If the Diameter of the Moon be fuppofed 33′ 28″, and the deficient Scruples at a Lunar Eclipfe are found to be 29′ 5″, What are the Digits eclipfed? Aftronomers allowing 12 Digits for the whole, 12 fhall be the fecond Number, by the proper

proper multiplication whereof into the third Number, after Division made there is found 10 Digits, 25′ 41″, and almost 50‴.

$$\text{As } 33' ,\ 28'' .\ \overset{\text{Digits.}}{12} :: 29' ,\ 5'' .\ \overset{\text{Digits.}}{10} ,\ 25' ,\ 41'', \&c.$$

```
  29′ . 5″                  23 | 27
  12°                   24 . 20 . 20 | 52
 ―――――――――――        ―――――――――――――――――――――――――――――
  348 . 60          ) 5° , 49′ , 00″ , 00‴ , 00⁗, &c. ( 10° , 25′ , 41″
 ―――――――――――        ―――――――――――――――――――――――――――――
  5° . 49′ . 00″      5 . 34 . 40
 ―――――――――――            13 . 56 . 40
                            22 . 52 . 08
```

Logarithms.

Logarithms. *Of Interest-Money.*

If at Simple Interest, 6 *s.* be the Gain of 5 *l.* Principal Money, What in the same space of Time, and at the same Rate, shall 100 *l.* gain? Because Addition of Logarithms is equivalent to Multiplication, and Substraction to Division; The Logarithms of the second and third Numbers added, and the Log. of 5 *l.* the first Number substracted, the Log. for Resolution will be 2,07918,12461, which is the Log. for 120 *s.* or 6 *l.*

	l.	*s.*	*l.*	*s.*	*l.*
As	5 .	6 ::	100 .	120 .	or 6

Or, 0,69897,00043. 0,77815,12504 : 2,00000,00000. 2,07918,12461.

Log. of 6.	0,77815,12504		
Log. of 100.	2,00000,00000		
Sum	2,77815,12504	Product.	Log. of 600.
Log. of 5.	0,69897,00043		
Difference	2,07918.12461	Quotient.	Log. of 120.

Cossicks.

Cossicks. *Of the Equality of 30 . φ.*

If 16ʒ be equal to 2ʃs, what shall 30φ (coming from the same Root) be equal to? Answer, by the following Operation to 3ʒφ and ¼.

As 16ʒ . 2ʃs :: 30φ . 3¼ ʒφ.

For 2ʃs × 30φ = 60ʒʒʒ. And 16ʒ) 60ʒʒʒ (3¼ʒφ.
Indices 5+3 = 8 Indices 8−2 = 6

The like for Compound *Cossicks* whole and broken.

Surds.

Surds. *Of the Likeness of $5+\sqrt{4}$.*

If $3+\sqrt{16}$ be alike to $4+\sqrt{9}$, What shall $5+\sqrt{4}$ be like to? By the Operation following, (performed according to the Nature of the *Data*) the Answer appears to be $\sqrt{25}+\sqrt{4}$.

As $3+\sqrt{16}$. $4+\sqrt{9}$ :: $5+\sqrt{4}$. $\sqrt{25}+\sqrt{4}$.

$$\begin{array}{l|l}
3-\sqrt{16} & 5+\sqrt{4} \\
\hline
9+\sqrt{144} & 20+\sqrt{225} \\
\quad -\sqrt{144}-16 & \quad \sqrt{64}+\sqrt{36} \\
\hline
9-16 & 20+\sqrt{225}+\sqrt{64}+\sqrt{36} \\
\hline
-7 & 3-\sqrt{16} \\
\hline
-\sqrt{49} & 60+\sqrt{2025}+\sqrt{576}+\sqrt{324} \\
 & -\sqrt{6400}-\sqrt{3600}-\sqrt{1024}-\sqrt{576} \\
\hline
 & -\sqrt{1225}-\sqrt{196}
\end{array}$$

$$-\sqrt{49})-\sqrt{1225}-\sqrt{196}(\sqrt{25}+\sqrt{4}$$

The like for *Simple Surds* as well as *Compound*, both Integers and Fractionary.

Species.

Species.

If 8 AB require 56 BC, What shall 7 AD require? Operation being made according to the Simple Elements of Species, the Resolution for Answer will be 49 CD. *Species. Of 7 AD, what required.*

As 8AB . 56BC :: 7AD . 49CD.
For 56BC x 7AD=392BCAD. And 8AB)392BCAD(49CD.

The like for other *Species*, Simple, Compound, Integral, Fracted, Rational or Irrational, *mutatis mutandis.*

And as the Process in the Work of these Contract Numbers: So the Proof of the Work, when done, needs no new Institution. For the Reverse of the Question, or the Multiplication of the Extreams and Means, shall prove them all. And over and above, as the Simple Elements of Contract Numbers had their singularity of Proof in reducing them to other Numbers: So the Comparative Elements of Contract Numbers may be proved, by taking other Numbers in their stead, and comparing the Resolutions together: As in the last Example thus proved is evident. *Proof of Contract like others.*

Example.

Reverse of the Question.	As 7AD . 49CD	:: 8AB . 56BC.
Extreams in the one,	56BC	49CD
Means in the other,	7AD	8AB
	392BCAD ==	392CDAB

Reduction into other Numbers, supposing A=1.B=2.C=3.D=4.
Then 8AB . 56BC :: 7AD . 49CD . shall be
As 16 . 336 :: 28 . 588 .
For 336 x 28=9408. And 16)9408(588.

CHAP. III.

The Indirect Rule of Three.

THE next of the Comparative Elements of *Disjunct Proportions Geometrical*, is, *The Indirect Rule of Three*, called severally by Authors, as, *The Reversed*, *Conversed*, *Inversed*, *Reciprocal*, and *Backer Rule*; though improperly, except the First and two Last. Indirect Rule of Three. *How called.*

In this *Indirect Rule of Three*, all the Preparatory and Probationary Parts, and the finding of the Divisor, is exactly the same with that in the precedent Chapter of *The Direct Rule of Three*, the only Difference is in the Resolution. *What herein like to the* Direct.

To resolve the Question propounded; the third Number being found to be Divisor, as it always will in Indirect Proportions, the Rule is, Multiply the first and second Numbers together, and divide the Product by the Third. The Quotient of this Division shall be the 4*th* Proportional Number, and answer the Question. For the Proportion between the third and first Numbers, is the same between the Second and Fourth. *The third Number always Divisor.*

Example. If at 4*s.* the Price of a Bushel of Wheat, the Penny white Loaf must weigh 9 Ounces *Troy*, What shall it weigh at 6 *s.* a Bushel? *Question of the Weight of Bread.*

The Numbers right placed, according to the Precepts of the former Chapter, to find the Divisor, I consider, the dearer the Corn, the lesser the Loaf: So the Number quesited being less than the Second, the greater of the two Extreams, which is 6, the third Number must be Divisor. Therefore 4 multiplied into 9, and the Product 36 divided by 6, giveth 6 Ounces for the Number desired.

	s.	*Ounces.*		*s.*	*Ounces.*	*Ounces.*
As	4 .	9	::	6 .	6	
		4				36 (6
		36				6

Proof

Proof of the Indirect Rule of Three.

Proof as in the *Rule of Three Direct*, by

Reverse of the Question. Ratio.

	s.	*Ounces.*		*s.*	*Ounc.*	*Extreams.*		*Means.*
As	6	. 6	::	4	. 9	4 . 9	::	6 . 6
		6				36	==	36

$4)\overline{36}(9$ Ounces.

$2)\frac{6}{4}(\frac{3}{2}$ $3)\frac{9}{6}(\frac{3}{2}$ abbreviated.

$4)6(1\frac{1}{2}$ $6)9(1\frac{1}{2}$ divided.

Proper Multiplication and Division here to be used. The Benefit thereof.

Further as necessary to this Chapter is to be remembred, for the Reason before rendred in the *Direct Rule of Three*, That whensoever any Question resolvable by the *Indirect Rule of Three*, is propounded in Fractions, or other Contract Numbers, the Multiplication and Division proper to the *Data* be used. It being much shorter to remember, to multiply the first and second Numbers after the manner of Fractions, and divide the Product by the Third as Fractions are divided, than to multiply the Numerator of the first Fraction, by the Numerator of the second Fraction; and that Product again by the Denominator of the Third for the Dividend: And again, to multiply the Denominator of the First by the Denominator of the Second; and that Product again by the Numerator of the Third for the Divisor, as some deliver the Rule.

Example in Fractions.

Examples in Fractions. Of keeping Money lent.

If I lend $\frac{3}{4}$ of a Pound to a Friend for 7 Months. And when I come to borrow of him, he can spare me but $\frac{5}{12}$ of a Pound: How long may I keep from paiment thereof again, to equalize the Interest of his Loan to me?

Answ. 12 Months and $\frac{3}{5}$ of a Month.

l. *M.* *l.* *M.*

As $\frac{2}{4} . \frac{7}{1} :: \frac{5}{12} . 12\frac{3}{5}$ For $\frac{3}{4} \times \frac{7}{1} = \frac{21}{4}$ And $\frac{5}{12})\frac{21}{4}(\frac{63}{5}(12\frac{3}{5}$

(3 and 1 set under 12 and 4.)

The other Way used by several.

l. *M.* *l.* *M.*

As $\frac{3}{4} . \frac{7}{1} :: \frac{5}{12} . 12\frac{3}{5}$

3	4
7	1
21	4
12	5
42	
21	
Dividend 252	Divisor 20

(1
25(2(12$\frac{3}{5}$
20

Example in Decimals.

Decimals. Of the Weight of Bread.

Suppose at 5 *s.* 9 *d.* the Bushel of Wheat, the Penny white Loaf weigh 9 Ounces and 6 Penny Weights; What shall the same weigh, when Wheat is risen to 6 *s.* 6 *d.* the Bushel?

Answ. 8 Ounces, 4$\frac{1}{2}$ Penny Weights, and somewhat more.

	s.		*Ounces.*		*s.*		*Ounces.*
As	0,2875	.	0,775	::	0,325	.	0,6855

	,775		1(25
(4)	14375	(7)	71870
(3)	20125	(3)	,2228125(,6855
(7)	20125	(4)	,325
	,2228125		,325
			,325
			,325

Example

Example in Logarithms.

Logarithms. Of paying Money lent.

In case I lend a Friend 320 *l.* for 20 Months: And when I want, he courteously lendeth me 400 *l.* When shall I pay him again, not to trespass on his Kindness?

Answ. At 16 Months end.

	l.		*Mon.*		*l.*		*Mon.*
As	320	.	20	::	400	.	16

Or, 2,50514,99783 . 1,30102,99957 :: 2,60205,99913 . 1,20411,99827

Log. of 320 . 2,50514,99783
Log. of 20 . 1,30102,99957
Sum 3,80617,99740 Product. Log. of 6400.
Log. of 400 . 2,60205,99913
Difference 1,20411,99827 Quotient. Log. of 16.

Example in Species.

Species. Of Workmen to build a Wall.

If 15 Pioneers, with the help of 3 Boys, can dig a Trench, and build a Wall in 12 Days; and Expedition requires the same to be done in 9 Days: How many Men and Boys are there needful?

Answ. 20 Pioneers and 4 Boys.

As 12 A . 15 B + 3 C :: 9 A . 20 B + 4 C.
For 12 A × 15 B + 3 C = 180 BA + 36 CA.
And 9 A) 180 BA + 36 CA (20 B + 4 C.

Proof of these and others.

The Proof of these, and such others, are as before, by reversing the Question, or multiplying the Extreams and Means; or, moreover, by taking other Numbers instead of these Contracts, and working therewith. All which by the last Example is here instanced.

Example.

Reverse of the Question. As 9 A . 20 B + 4 C :: 12 A . 15 B + 3 C.

Extreams $\left\{\begin{array}{c} 20B + 4C \\ 9A \\ \hline 180BA + 36CA \end{array}\right.$ Means $\left\{\begin{array}{c} 15B + 3C \\ 12A \\ \hline 180BA + 36CA \end{array}\right.$

		Days.		*Labourers.*		*Days.*		*Labourers.*
Omitting the Species.	As	12	.	18	::	9	.	24.
	12 × 18 = 216					9) 216 (24		

CHAP. IV.

PRACTICE.

Practice.

THE common Comparative Elements, in the *Direct* and *Indirect Rule of Three visited*, the other Part of the Primitives which is Peculiar, comes to be patent, and this is *Practice*.

What, and why so called.

Practice is so called, from the frequent Use and general Practice thereof, and is a Compendium or Breviat of the brief Rules and most expeditious Method of resolving the Propositions resolvable by the *Rule of Three*, after the Common Way. So as if any Process therein be shorter than other, it falls under consideration here.

Heads of Practice.

The practical shortning the common Work of the *Rule of Three*, may be comprised under one of these three Heads.

1. When

1. When the first and third Numbers will abbreviate.

1. When the first and third Numbers may be abbreviated, then reduce them to their least Terms, and by those Terms proceed, as in the *Rule of Three*, to the Resolution of the Question.

Example in the Direct Rule of Three.

Examples. Of the Price of Rods of Wall.

If 30 Rods of Wall making cost 8 *s.* What shall 48 Rods cost? Because 30 and 48 will abbreviate, they may be reduced to 5 and 8 for the first and third Numbers, and the Resolution thereby gotten will be 12 *s.* $\frac{4}{5}$, as by the Common Way.

$6)\frac{30}{48}(\frac{5}{6}$

	Rods.	*s.*	*Rods.*	*s.*
As	5 .	8 ::	8 .	$12\frac{4}{5}$

8

$5)\ 64\ (12\frac{4}{5}$

Common Way.

As 30 . 8 :: 48 . $12\frac{4}{5}$

8

$30)\ 384\ (12\,s.\ \frac{4}{5}$

Example in the Indirect Rule of Three.

Of Labourers to finish a Piece of Work.

If 18 Labourers in 12 Days can finish a Piece of Work; In how many Days shall 24 Labourers finish the same Work? Here 18 and 24, the first and third Numbers, will be reduced to their least Terms, 3 and 4; by which the Resolution will be 9 Days, as by the *Common Way.*

$6)\frac{18}{24}(\frac{3}{4}$

	Lab.	*Days.*	*Lab.*	*Days.*
As	3 .	12 ::	4 .	9

3

$4)\ 36\ (9$

Common Way.

As 18 . 12 :: 24 . 9

18

96

12

$24)\ 216\ (9$

2. When the common Work may be shortned.

2. Though the first and third Numbers will not abbreviate, yet may the common Work of the *Rule of Three* be shortned, if in the *Direct Rule* the third Number be divided by the First; and in the *Indirect Rule*, the first Number be divided by the Third, and the Quotient multiplied by the Second.

Example in the Direct Rule of Three.

Examples. Of Gain by Trade.

If 750 *l.* Stock in merchandising gain 360 *l.* What shall 1250 *l.* Stock gain?

Answ. 600 *l.* For 1250 divided by 750, gives in the Quotient $1\frac{2}{3}$, which multiplying 360, makes the Product 600.

	l. Stock.	*l. Gain.*	*l. Stock.*	*l. Gain.*
As	750 .	360 ::	1250 .	600

$1\frac{2}{3}$

360

120

120

600 *l.*

$\frac{50}{125|0}(1\frac{2}{3}$, $75|0$

Common Way.

As 750 . 360 :: 1250 . 600

360

75000

3750

$750)\ 450000\ (600$

Example in the Indirect Rule of Three.

Of Length, &c. to make an Acre of Land.

If 8 Perches broad require 20 Perches long to make an Acre of Land: What length shall a Piece of Land have that is 2 Perches broad to make an Acre?

Answ. 80 Perches: For 8 divided by 2, gives 4 in the Quotient; which multiplying 20, makes the Product 80.

	Perches-broad.	*Perches-long.*	*Perches-broad.*	*Perches-long.*
As	8 .	20 ::	2 .	80

$\frac{8}{2}(4$ $\quad$ $\frac{4}{80}$ Perches long.

Common Way.

As 8 . 20 :: 2 . 80

8

$2)\ 160\ (80$

The

The first and third Numbers in these two last Examples, abbreviated as above-mentioned, the Work will shew it self thus.

As 750 . 360 :: 1250 . 600   As 8 . 20 :: 2 . 80

$250)\frac{750}{1250}(\frac{3}{5}$ $3)\frac{5}{1800}(600$ $2)\frac{8}{2}(\frac{4}{1}$ $\frac{4}{80}$

3. Because one doth neither multiply nor divide. If an Unit among *Geodeticals* be one of the three given Numbers, and the First and Third be of like Denominations, and so need no Reduction as some do, which will be seen among Specificks in the next Chapter; then the Unit is set by, and Operation being made with the other two Numbers of the *Data*, the Work may be shorter than that commonly by the *Rule of Three*, especially when the Unit happens to stand in the first Place, as in the ensuing Process of this Chapter will be proved: Which Operations are all that with some Authors pass by the Name of *Practice*.

3. When an Unit is one of the Data.

The ensuing Operations of *Practice* under this third Head, will be comprehended under these ten Cases following.

This sort of Practice seen in ten Cases.

Case 1. If the given Price of one Ell, Yard, Pound-weight, *&c.* be any certain Number of Shillings, then multiply the given Quantity of Ells, Yards, Pounds, *&c.* whose Price is desired, by half the Number of Shillings which one shall cost: And from the Product cut off by a Dash of the Pen the right-hand Figure for Primes, (every Unit whereof is in value 2 *s.*) the Residue is Pounds. If a Cipher be in the Dexter Place cut off, it signifies nothing.

1. If the Price be any Number of Shillings.

Example 1. If one Ell cost 4 *s.* what shall 401 Ells cost at that Price?

Answ. 80 *l.* 4 *s.* For multiplying 401 by 2, which is the half of 4 the Price given; the Product is 802, of which the Right-hand Figure 2, cut off for Primes, is 4 *s.*

Example in Numbers. Even and Odd.

		Common Way.
	Ells.	*Ell.* *s.* *Ells.* *l.* *s.*
At 4 *s.* *per* Ell, what costs	401 ?	As 1 . 4 :: 401 . 80 : 4
	2	4
½ of 4 *s.* is 2.	*l.* 80|2 Primes.	160|4
		l. 80:4 *s.*

Example 2. If one Ell cost 19 *s.* what will 300 Ells cost?

Answ. 285 *l.* For after Multiplication by 9½, half the Price of 1 Ell, the total Product is 2850; from which the Cipher as insignificant cut off, the Residue is 285.

Odd and Even.

		Common Way.
	Ells.	*Ell.* *s.* *Ells.* *l.*
At 19 *s.* *per* Ell, what costs	300 ?	As 1 . 19 :: 300 . 285
	9½	19
½ of 19 *s.* is 9½	2700	2700
	150	300
	l. 285|0 Primes.	570|0
		l. 285:0 *s.*

Example 3. If 1 Yard cost 15 *s.* what shall 343 Yards cost?

Answ. 257 *l.* 5 *s.* For multiplying 343 by 7½, the half Price of one Yard, there comes to be cut off 2½ Primes, which is 5 *s.*

Both odd.

		Common Way.
	Yards.	*Yard.* *s.* *Yards.* *l.* *s.*
At 15 *s.* *per* Yard, what costs	343 ?	As 1 . 15 :: 343 . 257 : 5.
	7½	15
½ of 15 *s.* is 7½	2401	1715
	171½	343
	l. 257|2½ Prim.	514|5
		l. 257:5 *s.*

2. If the Price be an Aliquot Part of a Pound or Shilling.

Case 2. If the given Price of one Yard, Ell, Pound-weight, *&c.* be any *Aliquot*, or even Part of a Pound or Shilling: Then divide the Quantity of Yards, Ells, *&c.* whose Price is desired by that *Aliquot* Part; and the Quotient shall be the Resolution in Pounds or Shillings, according to the Part divided by. And if any thing remain upon the Division, every Unit thereof is in Value so much as that Part of a Pound or Shilling which was the Divisor.

The Collection of the Aliquot *Parts of a Pound and Shilling, commonly called* Practice Tables, *here follow.*

Practice *Tables.*

Aliquot Parts of a Pound.

Parts	*Value* s. d.
1	20.0
2	10.0
3	6.8
4	5.0
5	4.0
6	3.4
8	2.6
10	2.0
12	1.8
15	1.4
16	1.3

Parts	*Value* s. d.
20	1.0
24	0.10
30	0.8
40	0.6
48	0.5
60	0.4
80	0.3
120	0.2
240	0.1
480	$0.0\frac{1}{2}$
960	$0.0\frac{1}{4}$

Aliq. Parts of a Shilling.

Parts	*Value* d.
1	12
2	6
3	4
4	3
6	2
8	$1\frac{1}{2}$
12	1
16	$0\frac{3}{4}$
24	$0\frac{1}{2}$
48	$0\frac{1}{4}$

The Tables explained.

The Tables are easy to be understood, the Value of every Aliquot Part standing directly against the same on the same Line: As the third Part of a Pound is 6 *s.* 8 *d.* the third Part of a Shilling 4 *d.* the like of all the rest.

How made.

The Value of every Part is found, by dividing the Shillings in a Pound by the Parts under 20: The Pence in a Pound by the Parts under 240, *&c.* The like for the Parts of a Shilling, or the lowest Denomination, 960 Farthings in a Pound, and 48 Farthings in a Shilling by the Part desired; and so is the Quotient Farthings, which may be reduced into Pence and Shillings by *Geodetical* Reduction.

Their Use.

Operation by the Parts of a Pound.

Examples of the first Table.

Example 1. If one Yard cost 6 *s.* 8 *d.* what shall 348 Yards cost?

Answ. 116 *l.* For 348 divided by 3, or the third part thereof taken, because 6 *s.* 8 *d.* is $\frac{1}{3}$ of a Pound, the Quotient will be 116.

At 6 *s.* 8 *d.* *per* Yard, what costs 348 *Yards*?

$\frac{1}{3}$ *l.* 3) 348 (116 *l.*

Common Way.

As *Yard.* 1 . *d.* 80 :: *Yards.* 348 . *l.* 116

80

27840

12 (232|0

l. 116

Example 2. If one pound Weight cost 1 *s.* 4 *d.* what shall 8976 Pounds cost?

Answ. 598 *l.* 8 *s.* For dividing 8976 by 15, because 1 *s.* 4 *d.* is the fifteenth part of a Pound: the Quotient is 598, and 6 remaining is $\frac{6}{15}$; which, if $\frac{1}{15}$ be 1 *s.* 4 *d.* is 8 *s.* that is, 6 Shillings and 6 Groats.

At

Common Way.

At 1 s. 4 d. per ℔, what costs 8976 ℔?

As 1 ℔. 16 d. :: 8976 ℔. : 598 l. : 8 s.

$\frac{1}{15}$ l.

1		
142	l.	s.
8976	598	. 8
15		
15		
15		

16

53856

8976

143616 (1196|8

12 l.598:8 s.

Operation by the Parts of a Shilling.

Example 1. If 1 ℔ cost 3 q. what shall 4048 ℔. cost? *Examples of the Second.*

Answ. 12 l. 13 s. For dividing 4048 by 16, which part of a Shilling 3 q. is, the Quotient is 253 s. and by Reduction 12 l. 13 s.

Common Way.

At 3 q. per ℔, what costs 4048 ℔?

As 1 ℔. 3 q. :: 4048 ℔. 12 l. : 13 s.

$\frac{1}{16}$ s.

8

4048 (25|3

16 l.12:13 s.

16

16

3 6

12144 (3036 (25|3

4 12 l.12:13 s.

Example 2. If one Pound cost 2 q. what shall 1227 ℔. cost?

Answ. 2 l. 11 s. 1 $\frac{1}{2}$ d. For 2 q. being $\frac{1}{24}$ of a Shilling, and dividing 1227, the Quotient shall be 51 s. and the 3 which remain 3 odd Half-pence.

Common Way.

At 2 q. per ℔. what costs 1227 ℔?

As 1 ℔. 2 q. :: 1227 ℔. 2 l. : 11 s. : 1 $\frac{1}{2}$ d.

$\frac{1}{24}$ s.

(3

1227 (51 s.

24 l. s. d.

24 2:11:1 $\frac{1}{2}$

2

2454

(29 (1 d. (1 s.

2454 (613 (51 (1 (2 l.

4 12 20

The price of 1 ℔. in this last Example, 2 q. being also an Aliquot part of a Pound, (as oftentimes happeneth in the given Price) Operation may be made by the Parts of a Pound. *Price oft-times to be found in both Tables.*

As at 2 q. per ℔. what costs 1227 ℔?

$\frac{1}{480}$ l.

26

122|7 (2 l.

480

2(3

267 (11 $\frac{1}{8}$ s.

24

Case 3. If the given Price of one Yard, Ell, &c. be not any even Part of a Pound or Shilling: Then break the given Price into any Aliquot Parts, and work as before with every of those Parts, and add their Quotients together, with the Remains if any be. Or for the Shillings, work as in the first Case; and for the Pence as in the second Case. *3. If the Price be no Aliquot Part of a Pound or Shilling.*

Example 1. If one hundred Weight cost 13 s. 7 d. what shall 245 hundred Weight cost? *Example 1.*

Answ. 166 l. 7 s. 11 d. as appeareth by both Works so plain, that nothing is needful for Explication.

By

By the first and second Cases.

```
                               C.
At 13s.7d. per C. what costs 245 ?
                              6½
 ½ of 13 s. is 6½          ------
 1/40 l.  0 : 6 d.          1470
 1/240 l. 0 : 1             122½
         -------         l. 159|2½ Primes.
          13 : 7          ------
                         159 : 5 s. d.
              24(5
              ----(    6 : 2 : 6
              40
              24(5
              ----(    1 : 0 : 5
              240      ----------
                       166 : 7 : 11
```

By the second Case.

```
                              C.
At 13 s. 7 d. per C. what costs 245 ?
      s.   d.           (1
 1/2 l. 10 : 0       245   ( l.   s.  d.
                     ---   (122 : 10 : 0
                      2
 1/6 l.  3 : 4       24(5
                     ----  (  40 : 16 : 8
                      6
 1/80 l. 0 : 3       24(5
        -------      ----  (   3 : 01 : 3
         13 : 7       80      ------------
                              166 : 7 : 11
```

Example 2. *Example* 2. If 1 C. cost 11 s. 8 d. what shall 146 C. cost? *Answ.* 85 l. 3 s. 4 d.

By the first and second Cases.

```
                               C.
At 11s.8d. per C. what costs 146 ?
                              5½
 ½ of 11 s. is 5½          ------
 1/30 l.  0 : 8d.            730
         -------              73
          11 : 8            80|3 Primes.
                          ------
                         l. 80 : 6s. d.
              (2
              24(6   (  4 : 17 : 4
              ----      ----------
              30        85 : 3 : 4
```

By the second Case.

```
                                  C.
At 11 s. 8 d. per C. what costs 146 ?
      s.  d.         2(2
 1/3 l. 6 : 8        146  ( l.   s.  d.
                     ---  (48 : 13 : 4
                      3
 1/4 l. 5 : 0        2(2
       -------       146  ( 36 : 10 : 0
        11 : 8       ---    -----------
                      4     85 : 3 : 0
```

4. Operation by the Parts of 24.

Case 4. If the Aliquot Parts of 24 be taken for the Price given, and Operation made thereby as before by the Aliquot Parts of 20, under the second Case: A brief Resolution of the Question will be had in the Quotient of Division by these Parts; which Quotient shall be Pounds also, because 24 hath but the Cipher cut off from 240, the Pence in a Pound; and therefore the Dexter Figure or Cipher of the Quotient shall be cut off for Primes: And if any thing remain upon the Division, it is always less than one Prime, or 2 s. every Unit being in Value so much as the Divisor contained of 24.

The Aliquot Parts of 24, are known by the single Numbers in this Table standing against them, the uneven Parts having some 2, and some 3 Numbers against them.

A Table of the Parts of 24.

Pence.	*Parts.*		*Pence.*	*Parts.*
1	24		13	2.24
2	12		14	3.4
3	8		15	2.8
4	6		16	3.3
5	12.8		17	3.4.8
6	4		18	2.4
7	8.6		19	2.8.6
8	3		20	2.3
9	4.8		21	2.4.8
10	4.6		22	2.4.6
11	3.8		23	2.3.8
12	2		24	1

The

The Numbers under the Title *Parts* are for Divisors, when such a Number of Pence as stands against them happens to be the given Price of one Ell, Yard, Pound, &c. As if 4 *d.* be the Price given, then shall 6 be Divisor: But if 5 *d.* be the given Price, which is no even Part of 24, then 12 and 8 shall be Divisors, 12 because it is the Part answering to 2 *d.* and 8, because the Part that answers to 3 *d.* which 2 and 3 make 5, and both the Quotients must be added together, as those under the third Case. *The Table explained.*

Operation by the Parts of 24.

Example 1. If 1 ℔. cost 8 *d.* what will 447 ℔. cost? *Example 1.*

Answ. 14 *l.* 18 *s.* For 447 divided by 3, because 8 is the third Part of 24, the Quotient is 149; from which 9 cut off for the Primes, which is 18 *s.* the rest is Pounds.

At 8 *d.* *per* ℔. what cost 447 ℔.?

$\frac{1}{3}$ of 24 — $\frac{447}{3}$ (14|9 Primes.

l. 14 : 18 *s.*

By the second Case.

At 8 *d.* *per* ℔. what costs 447 ℔.?

$\frac{1}{3}$ *l.* — (2 — $\frac{44(7}{30}$ (*l.* 14 : *s.* 18

Example 2. If 1 ℔. cost 9 *d.* what will 4638 ℔. cost? *Example 2.*

Answ. 173 *l.* 18 *s.* 6 *d.* For dividing 4638 by 4 and 8, the Divisors for 9, and adding the Quotients together, the Total is 173 *l.* 9$\frac{1}{4}$ Primes, or 18 *s.* 6 *d.*

At 9 *d.* *per* ℔. what costs 4638 ℔.?

$\frac{1}{4}$ of 24 is 6 *d.*

$\frac{1}{8}$ of 24 is 3 *d.*

9 *d.*

2(2 — $\frac{4638}{4}$ (1159$\frac{1}{2}$

7(6 — $\frac{4638}{8}$ (579$\frac{3}{4}$

l. 173|9$\frac{1}{4}$ Pri.

l. 173 : 18 : 6 *d.*

By the second Case.

At 9 *d.* *per* ℔. what costs 4638 ℔.?

$\frac{1}{40}$ *l.* 6 *d.*

$\frac{1}{80}$ *l.* 3 *d.*

9 *d.*

2 — $\frac{46(38}{40}$ (*l.* 115 : *s.* 19 : *d.* 0

(7 — $\frac{463(8}{80}$ (57 : 19 : 6

173 : 18 : 6

Case 5. If the Price given be above 24 *d.* or 2 *s.* and yet the same may be evenly divided by some Aliquot Part of 24: then see how many such Parts there is in the given Price, and after Division by that Aliquot Part, multiply the Quotient by the other, and cut off the dexter Figure or Cipher as before. *5. If the Price be above 2 s.*

Example 1. If 1 Ell cost 3 *s.* 6 *d.* what shall 400 Ells cost? *Example 1.*

Answ. 70 *l.* For 6 *d.* is an Aliquot Part of 24, and $\frac{1}{4}$ thereof, and in 3 *s.* 6 *d.* there are 7 Six-pences: Therefore 400 divided by 4, gives 100, which must be multiplied by 7; and from that Product 700, a Cipher cut off leaves 70.

At 3 *s.* 6 *d.* *per* Ell, what costs 400 *Ells*?

$\frac{1}{4}$ of 24 is 6 *d.*

7

3 : 6

$\frac{400}{4}$ (100

7

l. 70|0 Primes.

By the second Case.

At 3 *s.* 6 *d.* *per* Ell, what costs 400 *Ells*?

$\frac{1}{8}$ *l.* 2 *s.* : 6 *d.*

$\frac{1}{20}$ 1 : 0

3 : 6

$\frac{400}{8}$ (50 *l.*

$\frac{400}{20}$ (20

70 *l.*

Example 2. If 1 Ell cost 2 *s.* 8 *d.* what shall 500 Ells cost? *Example 2.*

Answ. 66 *l.* 13 *s.* 4 *d.* For 4 *d.* is an even Part of 24, and in 2 *s.* 8 *d.* are 8 Groats: Therefore dividing by 6, because 4 is the sixth Part of 24, and multiplying the Quotient by 8, there is 66 *l.* 6$\frac{2}{3}$ Primes, or 13 *s.* 4 *d.*

Ells.
At 2 *s.* 8 *d.* *per* Ell, what costs 500?

$\frac{1}{6}$ of 24 is 4 *d.*
8
2:8

2(2
500 ($83\frac{1}{3}$
6 8
664
$2\frac{2}{3}$
l. 66,$6\frac{2}{3}$ Primes.
l. 66:13:4*d.*

By the second Case.
Ells.
At 2 *s.* 8 *d.* *per* Ell, what costs 500?

$\frac{1}{8}$ *l.* 2 : 6 (*s. d.*)
$\frac{1}{120}$ 2
2 : 8

2(4
500 (62 : 10 : 0 (*l. s. d.*)
8
2
500 | 0 (4 : 03 : 4
120 | 0
66 : 13 : 4

6. *If a Fraction be in the* Data.

Case 6. If besides the whole Quantity given, there be some small Part annexed, as $\frac{1}{4}$ or $\frac{1}{2}$, or such-like: After Operation for the whole Quantity, the Price of that Part is to be added.

Example. If one Yard cost 3 *s.* 4 *d.* what shall 300$\frac{1}{2}$ Yards cost?

Answ. 50 *l.* 1 *s.* 8 *d.* Where after 300 is divided by 6, because 3 *s.* 4 *d.* is $\frac{1}{6}$ of a Pound, 1 *s.* 8 *d.* for the half Yard is added to the Quotient.

Yards.
At 3 *s.* 4 *d.* *per* Yard, what costs 300$\frac{1}{2}$?

$\frac{1}{6}$ *l.*

300 (50 *l.*
6 1 : 8 (*s. d.*) for the $\frac{1}{2}$ Yard.
50 1 : 8

7. *If the Price exceed Pounds.*

Case 7. If the Price given exceed Pounds, the Pounds are to be multiplied by the Quantity, and the rest of the Price wrought out by some or other of the foregoing Cases: And sometime Geodetical Multiplication is used as the shorter Way.

Example. If one Hundred Weight cost 3 *l.* 8 *s.* 4 *d.* what shall 7 C?

Answ. 23 *l.* 18 *s.* 4 *d.* For so by Multiplication of 3 *l.* 8 *s.* 4 *d.* into 7, it will appear.

By Geodetical Multiplication.
C.
At 3 *l.* 8 *s.* 4 *d.* *per Cent.* what costs 7?

7
21:56:28
23:18: 4

Otherwise.
C.
At 3 *l.* 8 *s.* 4 *d.* *per* C. what costs 7?

3 × 7
$\frac{1}{2}$ of 8 is 4.
$\frac{1}{60}$ *l.* 4 *d.*

7 7
3 4
l. 21 2|8 Primes.
$\frac{7}{60}$ 2 : 16
2:4
23 : 18:4

8. *If the Price of an hundred Weight,* &c.

Case 8. If the Price of an hundred Weight be given, and it be required to know what some certain Number of Pounds will cost, which are not the equal Half or Quarters of the hundred Weight, and so easily reckoned without the Pen: then those Pounds may be broken into Aliquot Parts of 112, (the Weight of one Hundred) and Operation made thereby, like as in the other Cases before.

A Table of the Aliquot Parts of 112.

The Aliquot Parts of 112.

Pounds.	*Parts.*		*Pounds.*	*Parts.*
1	112		14	8
2	56		16	7
4	28		28	4
7	16		56	2
8	14		112	1

Example.

Example. If 1 C. cost 16 *l.* what will 35 ℔. cost? *Example of the Use of the Table.*

Answ. 5 *l.* For breaking 35 into Aliquot Parts of 112, that is, 28 and 7; and dividing 16 by 4, which stands against 28, and by 16 which stands against 7, among the Aliquot Parts, and adding the Quotients together, the Total will be 5 *l.*

```
                         ℔.                 Common Way.
At 16 l. per C. what costs 35?            ℔.    l.    l.   l.
 1/4 }          { 28       16 ( 4 l.    As 112 . 16 :: 35 . 5
 1/16} of 112   {  7       --                   35
                  ---       4                   --
                  35       16 ( 1               80             560 ( 5 l.
                           --                   48             ---
                           16                   --             112
                           ------              560
                             5 l.
```

Case 9. If the Price of one Pound be given, and it be required to know what the hundred Weight will cost; or the contrary by the Price of the Hundred to know the Price of 1 ℔. the Rule called the *Billingsgate Rule*, is made use of, (being so easily remembred, that every common Costermonger at *Billingsgate* can presently tell what it comes to) which is thus: For every Farthing in one Pound, take so many Groats, and double the Number of Shillings, and this shall be the Price of the hundred Weight. So that every Farthing in the Pound increases the hundred Weight so many times seven Groats. *9. The Price of 1 Pound given, to know the Price of the Hundred. Billingsgate-Rule.*

Example. If 1 ℔. cost 2 *d.* $\frac{1}{2}$, what shall 1 C. cost? *Example.*

Answ. 1 *l.* 3 *s.* 4 *d.* For in 2 *d.* $\frac{1}{2}$ there being 10 Farthings, the Hundred shall cost 10 Groats, and twice 10 Shillings, which is 23 *s.* 4 *d.*

Case 10. If an Unit stand in the third Place of the *Data*, and the first Number be an Article, then, according to the Ciphers in the Article cut off (by a Perpendicular Line from the highest Geodetical Denomination in the second Number) so many Figures, and reduce the rest by common Reduction, and the Numbers that exceed that Line to the Left-hand, shall be the Resolution. *10. If 1 be in the third place.*

Example 1. If 10 Pieces of Cloth cost 42 *l.* 5 *s.* 5 *d.* what doth one Piece thereof cost? *Example 1.*

Answ. 4 *l.* 4 *s.* 6 *d.* $\frac{1}{2}$. Here I cut off 2, of the 42 *l.* because of the Cipher in 10; and that 2 *l.* with the 5 *s.* reduced, makes 45 *s.* of which 4 exceeds the Line; the other 5 *s.* with 5 *d.* is turned into 65 *d.* of which 6 exceeds the Line; and 5 left, the half of 10, is for the Half-penny; or multiplied by 4, is 20, of which the 2 exceeds the Line, and are Farthings.

```
                                                  Common Way.
Pieces.  l. s. d. Piece. l. s. d.        P.  l.   s.  d.   P.  l. s. d.
As 10  . 4|2:5:5 :: 1 . 4:4:6½           As 10 . 42 : 5 : 5 :: 1 . 4:4:6½
          |20                                   20
      ----|--                                  ---
     s. 4 |5                                   845
          |12                                   12
      ----|--                                  ----
     d. 6 |5                                   1690
          |4                                   845.5             5(6 d.
      ----|--                            10 ) 1014|5 ( 1014 ( 8,4 s.
     q. 2 |0                                               --   ----------
     -------                                               12   l. 4:4:6½
```

Example 2. If 100 ℔. of Spice cost 28 *l.* 10 *s.* 6 *d.* what doth one Pound thereof cost? *Example 2.*

Answ. 5 *s.* 8 *d.* 1 *q.* $\frac{21}{25}$. Here, because 100 hath 2 Ciphers, 28 is wholly cut off. So the Price of one ℔. shall not reach to one *l.* The Work in the rest is as the last above.

As

```
     lb.  l.   s.  d.   lb. s.  d. q.
As 100 . 28 : 10 : 6 :: 1 . 5 : 8 : 1 21/25.
         | 20
 s. 5    | 70
         | 12
 d. 8    | 46
         | 4
 q. 1    | 84/100, or 21/25.
```

Common Way.

```
     lb.   l.   s.   d.    lb.  s.  d. q.
As 100 . 28 : 10 : 6 :: 1 . 5 : 8 : 1 21/25
           20
          570
           12
         1140
         5706
         6846
            4
100) 273|84 ( 3(1q
               273 ( 6(8d ( 5 s.
                4     12
```

Proof of Practice. The Proof of all Proceedings in *Practice*, is by the common Work of the *Rule of Three*, and one Variety by another, as by the Examples before is sufficiently clear.

Variety of Proof. Some prove the Operations of *Practice* by a contrary Question, thus; Mark the Number given for the Price of one Ell, Yard, Pound, *&c.* in the first Question, and how much it wanteth of 20 *s.* and by that which it wanteth, work with the first given Quantity stated into a second Question. And if both Resolutions added together be equal to the Quantity, the Work of both is right.

As if 8 *s.* buy 1 Ell, then will 100 Ells cost 40 *l.* on the contrary, because 8 *s.* wanteth 12 *s.* of 20. if the Price of 100 Ells be enquired at 12 *s.* *per* Ell, the Resolution will be 60 *l.* Now because 40 and 60 make up 100, both Works are found right.

```
                          Ells.                              Ells.
At 8 s. per Ell, what costs 100?     At 12 s. per Ell, what costs 100?
                             4                                   6
                        l. 40|0           Proof.            l. 60|0
                                    40 + 60 = 100.
```

CHAP. V.

SPECIFICKS.

Specificks. THE last Chapter ended with those Comparative Elements called *Primitive*; therefore this must begin with those called *Derivative*, because they borrow their Being from the former.

Why so called. Foremost, in the order of *Derivatives*, stand *Specificks*; so called, because something is specifical, in or about the Resolution of the Questions propounded, either by reason of the contract Numbers mixed in the *Data*, or the different Denominations of some sorts of Contract Numbers in one or more of the 3 given Numbers; *The Sorts thereof.* or the right Preparation of the *Data*, or the intermixture of Questions or Proportions in the Proposition, and that both in the *Direct* and *Indirect Rule of Three*.

Of these Diversities, the two first Sorts are specifical in respect of the Nature of the Numbers given, *viz.* the first in Fractions and Geodeticals; the second in Geodeticals and Astronomicals: But the two last Sorts are Specifical in respect of the Nature of the Question propounded.

1. If Fractions and Geodeticals be in the Data. 1. When Fractions and Geodeticals are mixed in the *Data*, place an Unit under the Geodeticals; and if any Fraction be mixed with an Integer, reduce the same into an Improper Fraction: then for Resolution of the Question, proceed as in the *Rule of Three*, multiplying and dividing after the manner of Fractions.

Operation by the Direct Rule of Three.

Examples in the Direct Rule of Three. *Example* 1. If 2 Ells cost $\frac{2}{3}$ *l.* what shall $1\frac{3}{4}$ Ell cost?

Answ.

Answ. $\frac{7}{12}$ *l.* or 11 *s.* 8 *d.* Here $1\frac{3}{4}$ is to be reduced to $\frac{7}{4}$, and multiplied into $\frac{2}{3}$, as Fractions whose Product is to be divided by $\frac{2}{1}$, or 2 the first Number. *Of the Price of $1\frac{3}{4}$ Ell.*

	Ells.	*l.*		*Ells.*	*l.*
As	2 .	$\frac{2}{3}$	::	$1\frac{3}{4}$.	$\frac{7}{12}$

For $\frac{2}{3} \times \frac{7}{4} = \frac{7}{6}$ And $\frac{2}{1}$) $\frac{7}{6}$ ($\frac{7}{12}$ *l.*

Example 2. If $1\frac{1}{2}$ Ell cost $1\frac{1}{4}$ *l.* what shall $3\frac{3}{4}$ Ells cost? *Of the Price of $3\frac{3}{4}$ Ells.*

Answ. $3\frac{1}{8}$ *l.* or 3 *l.* 2 *s.* 6 *d.* The *Data* reduced into improper Fractions, make $\frac{3}{2} . \frac{5}{4} . \frac{15}{4}$; the rest of the Work needs nothing to explain it.

	$\frac{3}{Ell.}$	$\frac{5}{l.}$		$\frac{15}{Ells.}$	*l.*
As	$1\frac{1}{2}$.	$1\frac{1}{4}$	::	$3\frac{3}{4}$.	$3\frac{1}{8}$

For $\frac{5}{4} \times \frac{15}{4} = \frac{75}{16}$ And $\frac{3}{2}$) $\frac{75}{16}$ ($\frac{25}{8}$ or $3\frac{1}{8}$ *l.*

Operation by the Indirect Rule of Three. *Examples in the Indirect Rule of Three.*

Example 1. If at 4 *s.* the Bushel, the Penny White-loaf must weigh 9 Ounces *Troy*: what must it weigh at 5 *s.* $\frac{1}{2}$ *per* Bushel?

Answ. $6\frac{6}{11}$ Ounces. Here $5\frac{1}{2}$ reduced into an Improper Fraction, is $\frac{11}{2}$, under the other Numbers an Unit is placed, and the Operation is as the Indirect Rule of Three in Fractions. *Of the Weight of Bread.*

	s.	*Ounc.*		$\frac{11}{s.}$	*Ounc.*
As	$\frac{4}{1}$.	$\frac{9}{1}$	::	$5\frac{1}{2}$.	$6\frac{6}{11}$

For $\frac{4}{1} \times \frac{9}{1} = \frac{36}{1}$ And $\frac{11}{2}$) $\frac{36}{1}$ ($\frac{72}{11}$, or $6\frac{6}{11}$. *Ounces.*

Example 2. If $6\frac{6}{11}$ Ounces *Troy* be the Weight of a Loaf, when Wheat is sold for 5 *s.* 6 *d.* the Bushel: what shall the same Loaf weigh when Wheat is sold for 12 *s.* the Bushel? *Another of the Weight of Bread.*

Answ. 3 Ounces. Here $6\frac{6}{11}$ is reduced to $\frac{72}{11}$, and $5\frac{1}{2}$ to $\frac{11}{2}$; and under 12 is placed an Unit; and the Multiplication and Division is as before.

	$\frac{11}{s.}$	$\frac{72}{Oun.}$		*s.*	*Ounces.*
As	$5\frac{1}{2}$.	$6\frac{6}{11}$	::	$\frac{12}{1}$.	3

For $\frac{11}{2} \times \frac{72}{11} = \frac{36}{1}$ And $\frac{12}{1}$) $\frac{36}{1}$ ($\frac{3}{1}$, or 3. *Ounces.*

2. When the *Data* are Geodeticals or Astronomicals of different Denominations, either the Multiplication and Division used for Resolution of the Questions propounded in such Numbers, must be proper according to the Nature of the Numbers; or else they must be either reduced Geodetically, by common Reduction, into the lowest Denomination, or turned into Decimals. *2. If the Data be of divers Denominations. These reduced or turned into Decimals.*

Touching Reduction of the given Numbers, observe:

§. 1. If the second Number be reduced, he must be reduced into the lowest Denomination of them given, lower than which he need not be brought. As if the second Number be Pounds, Shillings, and Pence, it must be brought all into Pence: And if Pounds, Shillings, Pence, and Farthings, then all must be reduced into Farthings, *&c.* *Their common Reduction is to be noted in four things.* 1.

§. 2. If the second Number be reduced, then the fourth Number, when found, shall be of like Denomination to the Second when reduced, whether Shillings, Pence, Farthings, *&c.* 2.

3. §. 3. If the second Number multiplied into the Third, in the *Direct Rule of Three*, or multiplied into the First in the *Indirect Rule of Three*, be too little to be divided by the Divisor; then to avoid the Work in Fractions, the second Number may be reduced from a gross Denomination into a smaller or subtiler, as from Pounds to Shillings, *&c.* And likewise the fourth Number, when found, often-times admits of Reduction from a smaller Denomination into a Grosser.

4. §. 4. If the first and third Numbers be of plural Denominations, or single Numbers but of different Denominations, as they must be reduced into the lowest of the given Denominations; so they both must be reduced to one and the same Denomination. As if the one be Pounds and Shillings, and the other Pounds, Shillings and Pence, or Shillings and Pence; both Numbers must be reduced into Pence, *&c.*

These things observed, the rest of the Work differs not from that in the *Rule of Three* Direct or Indirect for *Geodeticals*, or Direct only in *Astronomicals*, they being all Direct Proportions.

Examples in Geodeticals. *What* 3 l. *will buy.*

Operation in Geodeticals, *by the* Direct Rule of Three.

Example. If 4 *l.* 14 *s.* 4 *d.* buy 2 C. 12 ℔. 6 ℥. what shall 3 *l.* buy?

Answ. 1 C. 38 ℔. 5 ℥ $\frac{145}{283}$. Here the first and third Numbers reduced into Pence, are 1132, and 720. The second Number brought into Ounces, is 3782. And after multiplication of 720 into 3782, and division of the Product by 1132, the Quotient is 2405$\frac{145}{283}$ Ounces, according to the Reduction of the second Number, and may be brought into the grosser Denominations of Pounds and Hundreds. And if the first and third Numbers after Reduction were abbreviated, the Multiplication and Division would be shorter.

```
   l.   s.  d.    C.  ℔.  ℥.     l.      C.  ℔.  ℥.
As 4 : 14 : 4 .   2 : 12 : 6 :: 3 .      1 : 38 : 5 145/283.
   20              112           20
   94 s.           236 ℔         60 s.
   12               16           12
   188             1416          120
   944             2366          60
   1132 d.         3782 ℥.       720 d.
                    720
                  75640
                 26474
                 2723040

        6|5
      4592|8                            ℔.
   2723040|0 (  8  ℥ (  |38 ( C.
   1132       2405|5     150    1
    1132       16        112
     1132       16
      1132       16
```

Operation in Geodeticals *by the* Indirect Rule of Three.

What Stock to raise a Profit proposed.

Example. If Stock to the Value of 10 *l.* 13 *s.* 3 *d.* raise a considerable Profit in 12 Months: what Stock shall raise the same Profit in 16 Months?

Answ. 7 *l.* 19 *s.* 11 *d.* $\frac{1}{4}$. Here the second Number only needs Reduction into Pence, or may be turned into a Decimal. And because the first and third Numbers will abbreviate, may be wrought as in *Practice*.

```
   Mon.  l.  s.  d.  Mon. l.  s.  d.         Mon.  l.  s.  d.   Mon. l.  s.  d.
As 12 . 10: 13 : 3 :: 16 . 7 : 19 : 11 1/4   As 12 . 10: 13 : 3 :: 16 . 7 : 19 : 11 1/4
   3    20            4                         3 . 10,6625    ::  4 . 7,9968 1/4
        213                                            3
        12              2   d.                  4)31,9875(7,9968 1/4
        426      3 3(1  7|11                             |  20
        2133     7677 ( 1919 ( 15|9                 s. 19|9375
        2559       4     12                              |  12
          3                    l.7:19:11 1/4            1|8750
        7677                                            9|375
                                                    d. 11|2500
                                                         |   4
                                                    q.  1|0000
```

Operation

Operation in Astronomicals.

Example. Suppose the Æquation of ♂ be desired, and his Anomaly given be 0 Sign, 18 Deg. 12′. 30″. and the Difference of Æquations found by the Astronomical Tables, between the 18*th* and 19*th* Degree, be 9′ : 36″ decreasing, the Question then will stand thus: *Example in Astronomicals. Of the Æquation of Mars.*

If 1 Degree, or 60′, give 9′ 36″, what shall 12′ 30″?

And the Proportional Part gained will be 2′, to be substracted from the given Anomaly, as the following Operations make plain.

By Geodetical Reduction.

```
     '    '  "    '  "   '
As 60 . 9 : 36 :: 12 : 30 . 2
   60    60          60
 3600   576         750
                    576
                   4500
                  5250
                 3750
3600)432000(120"(2'
              60
```

By the Sexagenary Table.

```
  Deg.  '  "     '   "   '
As 1 . 9 : 36 :: 12 : 30 . 2
      12 : 30
       1 . 55 . 12'''
           4 . 48 . 00''''
       2 . 00 . 00 . 00
```

By Decimals.

```
    '    '        '       '
As 60 . 9,6 :: 12,5 . 2
                 9,6
                 750
               1125
60)120,00(2',00
```

	s.	d.	′	″
Anomaly of ♂	0	18	12	30
Proportional Part			2	00
Æquation of ♂	0	18	10	30 desired.

3. When the *Data* are not the same three Numbers Resolution is to be had by, but these are included in the Question, and according to the State thereof, by a due preparation of the *Data*, those more covert Numbers are discovered, through help of some or other of the Simple Elements of Numbers which they call to their Aid. *3. If the Data must be prepared.*

In this third Sort of *Specificks*, diligent consideration must be had of the State of the Question, and Nature of the Number quesited thereby to find the three Numbers to work by in the *Rule of Three*, since no Rule can be given to reach all Cases; but sometime one, sometime another, and sometime more than one of the Simple Elements of Numbers are needful to prepare the *Data*. So as much depends on the Ingenuity of the Operator, as *Ptolomy* once said, *Abs te & à Scientia.* *State of the Question to be diligently heeded, and the reason.*

After the Numbers fit for Resolution are obtained, which must be first done, (seeing without the true *Data* no Question can be truly resolved) let the Numbers, however proposed in the Question, (sometime only to try the Skill of the Resolver) be duly disposed, according to the Precepts before given in the *Rule of Three*, and then operate by the Direct or Indirect Rule as the Case requires; for all Operation by a wrong Rule will render the Result false. *Numbers to be duly disposed, however propounded.*

Addition needful. Addition useful.

Example 1. Two Posts, (suppose *A* and *B*) depart one from another, one directly Eastward, and the other directly Westward: *A* travelleth 18 Miles a Day, and *B* 30: How far are they distant the third Day after their Departure? *Q. Of the Distance of two Posts.*

Answ. 144 Miles. Here 18 and 30, the Travel of both in one Day, are to be added, and the Work with the Total committed to the *Direct Rule of Three*, thus. *Answer.*

			Day.	Miles.	Days.	Miles.
A . 18		As	1 .	48 ::	3 .	144.
B . 30				3		
48				144 Miles.		

Q. *Of the lining of two Cloaks.*

Example 2. If one Cloak may be lined with $4\frac{1}{2}$ Yards of Stuff that is yard broad: what Plush will line two of those Cloaks when the Plush is but $\frac{3}{4}$ of a yard broad, and one Cloak will be lined with $\frac{1}{2}$ Yard less than the other?

Answer.

Answ. $11\frac{1}{3}$ Yards of Plush. Here the breadth of both Cloaks is to be added, and the Work with the Total committed to the *Indirect Rule of Three*, thus.

		Yard-broad.	Yards long.		Yards broad.	Yards long.
1. Cloak $4\frac{1}{2}$	As	$\frac{1}{1}$.	$8\frac{1}{2}$	::	$\frac{3}{4}$.	$11\frac{1}{3}$
2. Cloak 4						
$8\frac{1}{2}$						

$$\frac{3}{4}\Big)\frac{17}{2}\Big(\frac{34}{3}\Big(11\frac{1}{3} \text{ Yards long.}$$

Substraction useful.

Substraction needful.

Q. *Of the filling of a Cistern.*

Example 1. A Conduit-Pipe rnnning into a Cistern, holding 250 Gallons, poureth in every Hour 24 Gallons; but the Cistern by another Pipe running out thereof, emptieth of it self in every Hour 16 Gallons: in what time shall the Cistern be filled, both the Pipes running?

Answ.

Answ. $31\frac{1}{4}$ Hours. Here the Gallons emptied by the one Pipe, must be substracted from the Gallons filled by the other, that is, 16 from 24; and the Work with the Remain committed to the *Direct Rule of Three*, thus.

		Gallons.	Hour.		Gallons.	Hours.
Greater Pipe 24	As	8 .	1	::	250 .	$31\frac{1}{4}$
Lesser Pipe 16						
8						

$$\frac{250}{8}\Big(31\frac{1}{4} \text{ Hours.}$$

Q. *Of the length, &c. to $\frac{1}{2}$ Acre.*

Example 2. If 4 Perches in breadth, when 40 is the length of a Piece of Land, make 1 Acre: what length must there be to make one half-Acre, when the breadth is 6 Perches?

Answ.

Answ. $13\frac{1}{3}$ Perches in length. Here, because the Enquiry is but for half an Acre, and 4 was a Proportion of breadth given for a whole Acre; therefore half the 4 shall be taken away, and the Work with the Remain committed to the *Indirect Rule of Three*: Thus,

		Perches broad.	Perches long.		Perches broad.	Perches long.
1. Acre 4	As	2 .	40	::	6 .	$13\frac{1}{3}$
$\frac{1}{2}$. Acre 2			2			
2						

$$6\Big)80\Big(13\frac{1}{3} \text{ Perches long.}$$

Multiplication useful.

Multiplication needful.

Q. *Of the Powder spent by seven Guns.*

Example 1. Suppose 5 Guns being often discharged, spend 60 Barrels of Powder: what will 7 Guns which spend three times as much as the other at every shot?

Answ.

Answ. 252 Barrels. Here 60 is multiplied by 3, and the Work with the Product is committed to the *Direct Rule of Three*: Thus,

		Guns.	Barrels.		Guns.	Barrels.
Barrels 60						
Increased 3	As	5 .	180	::	7 .	252
180			7			

$$5\Big)1260\Big(252 \text{ Barrels.}$$

Example

Example 2. If one have right to pasture on a Common 100 Sheep 40 Days, and he pastureth there four times as many 12 Days: hath he transgressed or not? *Q. Of Pasturage of Sheep.*

Answ. Yes, by the space of two Days. Here 4 is to multiply 100, and the Work with the Product to be committed to the *Indirect Rule of Three*, by which it is resolved he ought to have pastured 400 Sheep but 10 Days. Answer.

Sheep 100
Increased 4
400

	Sheep.		*Days.*		*Sheep.*		*Days.*
As	100	.	40	::	400	.	10

100
400) 4000 (10 Days.

Division needful. Division useful.

Example 1. Four Guests at Table drank 16 *d.* in Wine: how many Guests that drink but half so much as the former will 18 Pennyworth of Wine serve? *Q. Of Wine drunk.*

Answ. 9 Guests. Here 16 is to be halfed, or divided by 2, and the Work with the Quotient committed to the *Direct Rule of Three*: Thus, Answer.

Wine 16
Halfed 2 (8

	d.		*Guests.*		*d.*		*Guests.*
As	8	.	4	::	18	.	9

4
8) 72 (9 Guests.

Example 2. If a Parcel of Hay will maintain 90 Head of Cattel 10 Weeks, and $\frac{1}{3}$ of the Cattel be put out to keeping: how long will the Hay maintain the Rest? *Q. Of Hay to feed Cattel.*

Answ. 15 Weeks. Here 90 is divided by 3, and the Quotient, or the third Part of 90 taken from thence, the Residue is committed to the *Indirect Rule of Three*: Thus, Answer.

Cattel 90
Divided 3 (30

90—30=60

	Cattel.		*Weeks.*		*Cattel.*		*Weeks.*
As	90	.	10	::	60	.	15

90
60) 900 (15 Weeks.

Division and Addition needful. Div. *and* Add. *used.*

Example. Certain Reapers in 5 Days can reap 24 Acres of Corn: in how many Days can they, with each Man his Servant, every of which doth half as much Work as his Master, reap 144 Acres? *Q. Of Reapers.*

Answ. In 20 Days. Here the half of 24 being added thereto, the Work is committed to the *Direct Rule of Three*: Thus, Answer.

Acres 24
Halfed 2 (12

24
12
36

	Acres.		*Days.*		*Acres.*		*Days.*
As	36	.	5	::	144	.	20

5
36) 720 (20 Days.

Substraction and Multiplication needful. Sub. *and* Mult. *used.*

Example. A having stolen certain Goods, fleeth 40 Miles a Day: *B* setting out 4 Days after him, pursueth 50 Miles a Day: in how many Days may *B* overtake *A*? *Q. Of pursuing a Thief.*

Answ. In 16 Days. Here the Difference between the Journey of *A* and *B* in one Day first taken, and the Day's Journey of *A*, multiplied by the number of Days *B* set out after *A*, the Work is committed to the *Direct Rule of Three*: Thus, Answer.

B . 50	A . 40		Miles. Day.	Miles. Days.
A . 40	4		As 10 . 1 ::	160 . 16
Differ. 10 Miles.	160			

In like manner Questions may be composed, wherein *Multiplication* and *Addition*, or *Division* and *Substraction*, &c. may be needful: but these are sufficient for Example here.

4. If Questions be intermixt.

4. When there is an Intermixture of Questions or Proportions in the Proposition, the one will be *express*, and the other *implied*.

Expresly.

§. 1. When the several Questions are expressed in the Proposition, then proceed according to the State of the Question in the Resolution of either; first with one, and then with the other.

Q. Of the Miles travel to overtake a Thief.

Example 1. Suppose in the last Proposition it had been demanded, not only in how many Days *B* should overtake *A*, but also after how many Miles travel? Then after the first Work, as above, had resolved *B* to overtake *A* in 16 Days, another Question should be committed to the *Direct Rule of Three*, for the Resolution of this latter Query: Thus,

If *B* travel in 1 Day 50 Miles; how far shall he travel in 16 Days?

Answer. *Answ.* 800 Miles.

	Day.	Miles.	Days.	Miles.
As	1 .	50 ::	16 .	800
			50	
			800 Miles.	

Q. Of a Castle besieged.

Example 2. A Castle besieged hath Victuals enough for 1200 Men 7 Months; but the Captain finding the Siege is like to be long, and that fewer Men will defend it, he would disband some of his Men, and lengthen out his Provision to 12 Months: how many Men shall he retain, and how many disband?

Answer. *Answ.* By the *Indirect Rule of Three*, 700 Men are to be retained; which found, that Number is to be taken from the 1200 propounded, and the Residue, that is 500, to be disbanded.

	Months.	Men.	Months.	Men.
As	7 .	1200 ::	12 .	700
		7		
	12)	8400 (	700 Men retained.	
			500 Men disbanded.	
			1200	

Implicitly, with a Mixture of Ratio's, &c.

§. 2. When in the Proposition there is such an Intermixture of *Ratio's* or Proportions, that as necessary to the Resolution of the Demand, either the Elements proper to *Ratio's* must be used, or more than one Operation of the *Rule of Three*, though perhaps but three Numbers given; and in such Operations the Quotient of the one Work is to be added to, or substracted from some or other of the *Data*, or the contrary, before Resolution can be had by the other: which differs from those Questions falling under the *Rule of five Numbers*, treated of in the two next Chapters, because they may be resolved at one Operation; yet they always give five Numbers; and if they be resolved by two Works, the Quotient of the first is taken whole without Alteration for the next Work, which is not so in these *Specificks*.

Wherein these differ from those that fall under the Rule of five Numbers.

Data, 3 Numb. and Add. used.

Data, 3 *Numbers and Addition needful.*

Q. Of two doing a piece of Work.

Example 1. *A* and *B* are hired to do a Piece of Work, which *A* can do alone in 30 Days, and *B* in 20 Days: in how many Days can they do it together?

Answer. *Answ.* In 12 Days. If I work by *Ratio's*, always where the *Ratio* is manifold, or the contrary, that is, to an Unit, it matters not which of the Terms be made Antecedent: For if the Unit be made Consequent, the *Ratio's* are to be reduced to like Antecedents; and if Antecedent, to like Consequents. Now here being two *Ratio's*, *viz.* that of *A* to the Work, as 30 to 1; and that of *B*, as 20 to 1;

First

First I reduce them to like Antecedents: So are they $\frac{600}{20}$ and $\frac{600}{30}$, and in their least Terms $\frac{60}{2}$ and $\frac{60}{3}$: after Reduction I divide this common Antecedent by 30 added to 20, the Sum of the reduced Consequents, or 60 by 5, that is, $3+2$.

Or otherwise, if I work by the *Rule of Three*, my first Question is, If 30 Days of *A* can do 1 Work: what shall 20 Days of *B*? To which the Answer $\frac{2}{3}$ must be added to the Piece of Work to be done; and that $1\frac{2}{3}$ Work shall be the first Number of the second Question, thus: If $1\frac{2}{3}$ Work come of 20 Days when *B* works with *A*, of what shall 1 Work come? *Variety of Operation.*

By Ratio's.

$$\begin{array}{cc} \multicolumn{2}{c}{600} \\ \hline A\ 30 & B\ 20 \\ 1 & 1 \end{array}$$

$$\begin{array}{l} 20 \times 30 = 60|0 \\ 20 + 30 = \ \ 5|0 \end{array} \left(12 \text{ Days.} \right.$$

By the Rule of Three.

	Days.		*Work.*		*Days.*		*Work.*
As	30	.	1	::	20	.	$\frac{2}{3}$

$$\frac{2|0}{3|0} \text{ Work.}$$

	Work.		*Days.*		*Work.*		*Days.*
As	$1\frac{2}{3}$	.	20	::	1	.	12

$$\frac{5}{3}\Big)\frac{20}{1}\Big(\frac{12}{1} \text{ Days.}$$

Example 2. A Conduit hath three Cocks; if the Cistern be full, and Water run out by the greatest Cock, the Cistern will be empty in three Hours; if it run out by the second Cock, the Water will be all run out in four Hours; but if the Water run out by the least Cock, it will be five Hours e're the Cistern be emptied: In what Time will all the Water be run out if it run by all the Cocks together? *Q. Of emptying a Cistern.*

Answ. In 1 Hour and $\frac{13}{47}$ of an Hour. Here the three *Ratio's* given being reduced to like Antecedents, the common Antecedent will be 60, to be divided by 47 the Sum of the reduced Consequents. *Answer.*

$$\begin{array}{ccc} \multicolumn{3}{c}{60} \\ \hline \text{Great Cock } 3, & \text{Second Cock } 4, & \text{Least Cock } 5. \\ 1 & 1 & 1 \\ \hline 20 & 15 & 12 \end{array}$$

$$\begin{array}{l} \text{Antecedents } 3 \times 4 \times 5 = 60 \\ \text{Consequents } 20+15+12 = 47 \end{array} \left(1\tfrac{13}{47} \text{ Hour.} \right.$$

Otherwise, if the Work be wrought by the *Rule of Three*, the first Question is, If three Hours running of the great Cock empty the Cistern: what will four Hours running of that Cock? The Answer to which, $1\frac{1}{3}$ Cistern must be added to the Cistern: And this $2\frac{1}{3}$ shall be the first Number of the second Question, thus; If $2\frac{1}{3}$ Cisterns be run out in four Hours, when will 1 Cistern be run out? Then $1\frac{5}{7}$ Hour, the Answer to this Question, shall be the first Number of the third Question, thus; If $1\frac{5}{7}$ Hour empty 1 Cistern, what will 5 Hours do? And $2\frac{11}{12}$ the Answer to this, shall be added again to the 1 Cistern, and the last Question stands thus; If $3\frac{11}{12}$ Cisterns will be emptied in 5 Hours, when shall 1 Cistern be emptied? The Answer to which is $1\frac{13}{47}$ Hour as before. *Variety of Operation.*

		Hours.		*Cistern.*		*Hours.*		*Cistern.*
1.	As	3	.	1	::	4	.	$1\frac{1}{3}$

$$\frac{4}{3}\Big(1\tfrac{1}{3} \text{ Cistern.}$$

		Cisterns.		*Hours.*		*Cistern.*		*Hour.*
2.	As	$2\frac{1}{3}$	.	4	::	1	.	$1\frac{5}{7}$

$$\frac{7}{3}\Big)\frac{4}{1}\Big(\frac{12}{7} \text{ Hour.}$$

		Hour.		*Cistern.*		*Hours.*		*Cisterns.*
3.	As	$1\frac{5}{7}$	.	1	::	5	.	$2\frac{11}{12}$

$$\frac{12}{7}\Big)\frac{5}{1}\Big(\frac{35}{12} \text{ Cisterns.}$$

		Cisterns.		*Hours.*		*Cistern.*		*Hour.*
4.	As	$3\frac{11}{12}$	.	5	::	1	.	$1\frac{13}{47}$

$$\frac{47}{12}\Big)\frac{5}{1}\Big(\frac{60}{47} \text{ Hour.}$$

Data, 3 Numb. and Sub. used.

Data 3, *Numbers and Substraction needful.*

Q. Of 1 doing a piece of Work.

Example 1. *A* and *B* do a Piece of Work together in 12 Days, which *A* alone can do in 30 Days: in how many Days can *B* do the same if he work alone?

Answer.

Answ. In 20 Days. Here the two *Ratio's* given are as 12 to 1, and 30 to 1; which being reduced to like Antecedents, this common Antecedent is to be divided by the Difference of their reduced Consequents.

Variety of Operation.

Otherwise to work by the *Rule of Three*, the first Question is; If 30 Days of *A* can do 1 Piece of Work, what shall 12 Days of *A* joined with *B* do? To which the Answer $\frac{2}{5}$ of a Piece of Work shall be taken from the whole Piece, and the Residue, which is $\frac{3}{5}$, shall be the first Number of the second Question, thus; If $\frac{3}{5}$ of the Work shall come of 12 Days, of what comes the whole Work?

By *Ratio's*.

$$\overline{360}$$
$$A \text{ and } B \quad \frac{12}{1} \quad A \quad \frac{30}{1}$$

$$\begin{array}{l} 30 \times 12 = 360 \\ 30 - 12 = 18 \end{array} \Big(\; 20 \text{ Days.}$$

By the Rule of Three.

	Days.		*Work.*		*Days.*		*Work.*
As	30	.	1	::	12	.	$\frac{2}{5}$

$$6\Big)\frac{12}{30}\Big(\;\frac{2}{5} \text{ Work.}$$

	Work.		*Days.*		*Work.*		*Days.*
As	$\frac{3}{5}$	.	12	::	1	.	20

$$\frac{\overset{1}{3}}{5}\Big)\frac{\overset{4}{12}}{1}\Big(\frac{20}{1} \text{ Days.}$$

Q. Of filling a Cistern.

Example 2. Water runneth into a Cistern by a Pipe that can fill it in 8 Hours, and runneth out by another Pipe which can empty the Cistern in 22 Hours: in what Time both running will the Cistern be full?

Answer.

Answ. In 12 Hours and $\frac{4}{7}$ of an Hour. Here the *Ratio's* of 8 to 1, and 22 to 1, reduced to like Antecedents, make $\frac{176}{22}$ and $\frac{176}{8}$, and in their least Terms $\frac{88}{11}$ and $\frac{88}{4}$. This common Antecedent 88, divided by 7, the Difference of the Consequents gives in the Quotient $12\frac{4}{7}$ as before.

$$\overline{176}$$
$$\text{Filling Pipe } \frac{8}{1}, \text{ Emptying Pipe } \frac{22}{1}$$
$$\underline{228}$$

$$\begin{array}{l} \text{Antecedents } 8 \times 22 = 176 \\ \text{Consequents } 22 - 8 = 14 \end{array} \Big(\overset{\text{Hours.}}{12\frac{4}{7}}, \text{ or } \frac{88}{7}\Big(12\frac{4}{7}$$

Variety of Operation.

Otherwise, to work by the *Rule of Three*, the first Question is, If 8 Hours fill 1 Cistern, what shall 22 Hours fill? The Answer whereof $2\frac{3}{4}$ shall be substracted from 1; so will there want $\frac{7}{4}$, which shall be the first Number of the second Question to be wrought as the other in the former Example.

	Hours.		*Cistern.*		*Hours.*		*Cisterns.*
As	8	.	1	::	22	.	$2\frac{3}{4}$

$$\frac{\overset{(6}{22}}{8}\Big(\;2\frac{3}{4} \text{ Cisterns.}$$

$$\frac{4}{4} - \frac{11}{4} = \frac{7}{4}$$

	Cistern.		*Hours.*		*Cistern.*		*Hours.*
As	$\frac{7}{4}$	.	22	::	1	.	$12\frac{4}{7}$

$$\frac{7}{4}\Big)\frac{22}{1}\Big(\frac{88}{7}\Big(12\frac{4}{7} \text{ Hours.}$$

Data, above 3 Numb. and Add. used.

Data, *more than 3 Numbers and Addition needful.*

Q. Of 4 Mills grinding.

Example 1. Suppose four Mills, whereof the first grindeth 4 Quarters in 3 Hours; the second 5 Quarters in 4 Hours; the third 6 Quarters in 5 Hours; and the fourth 7 Quarters in 6 Hours: in what Time shall they all grind 30 Quarters?

Answ.

Answ. In 6 Hours and $\frac{2}{33}$ of an Hour. The *Ratio's* in this Proposition having both Terms greater than Units, they are to be reduced to like Consequents, and the Sum of the Antecedents divided by the common Consequent. And because there is a Limitation in the Demand of 30 Quarters, the Quotient of this Division is to be the first Number of a Question to be resolved by the *Rule of Three*, with an Unit in the second Place, and the 30 inquired for in the Third. Answer.

	480	450	432	420
Quarters	4	5	6	7
Hours	3	4	5	6
		360		

$$\begin{array}{ll} \text{Antecedents} & 480+450+432+420=\frac{\overset{34}{1782}\,|2}{360}\Big(4\tfrac{19}{20}\text{ Quarters.} \\ \text{Consequents} & 3\times4\times5\times6= \end{array}$$

Then, If $4\frac{19}{20}$ Quarters be ground in 1 Hour, in what Time shall 30 Quarters be ground? *Facit* as aforesaid, $6\frac{2}{33}$ Hours.

As $\frac{99}{20}$. 1 :: 30 . $6\frac{2}{33}$

$$\frac{\overset{33}{99}}{20}\Big)\frac{\overset{10}{30}}{1}\Big(\frac{200}{33}\Big(6\tfrac{2}{33}$$

The Work by the *Rule of Three*, where the Antecedents and Consequents of the *Ratio's* given are both greater than Units, is different from that where one Term is an Unit in two things. First, In working the several Questions, the Consequents must be made Antecedents, and the Antecedents Consequents. And, 2*ly*, at the last the Consequent is to be divided by the Antecedent, as in the following Operation. *Variety of Operation. The Difference herein from the former.*

1. As $\frac{3}{4}$. 1 :: $\frac{4}{5}$. $1\frac{1}{15}$

$\frac{3}{4}\Big)\frac{4}{5}\Big(\frac{16}{15}$ 1 added. $2\frac{1}{15}$

2. As $2\frac{1}{15}$. $\frac{4}{5}$ :: 1 . $\frac{12}{31}$

$\frac{31}{\underset{3}{15}}\Big)\frac{4}{\underset{1}{5}}\Big(\frac{12}{31}$

3. As $\frac{12}{31}$. 1 :: $\frac{5}{6}$. $2\frac{11}{72}$

$\frac{12}{31}\Big)\frac{5}{6}\Big(\frac{155}{72}$ 1 added. $3\frac{11}{72}$

4. As $3\frac{11}{72}$. $\frac{5}{6}$ :: 1 . $\frac{60}{227}$

$\frac{227}{\underset{12}{72}}\Big)\frac{5}{\underset{1}{6}}\Big(\frac{60}{227}$

5. As $\frac{60}{227}$. 1 :: $\frac{6}{7}$. $3\frac{17}{70}$

$\frac{\overset{10}{60}}{227}\Big)\frac{\overset{1}{6}}{7}\Big(\frac{227}{70}$ 1 added. $4\frac{17}{70}$

6. As $4\frac{17}{70}$. $\frac{6}{7}$ :: 1 . $\frac{20}{99}$

$\frac{\overset{99}{297}}{\underset{10}{70}}\Big)\frac{\overset{2}{6}}{\underset{1}{7}}\Big(\frac{20}{99}$

This $\frac{20}{99}$ the Quotient of the last Work must be set as $\frac{99}{20}$, and then divided, giveth $4\frac{19}{20}$ as before in the Work by *Ratio's* for the first Number of the Question there stated and resolved.

Example 2. Suppose a Conduit, whose Cistern holdeth 1000 Gallons, hath three Cocks; by the first Cock will run out 20 Gallons in 3 Hours; by the second, 30 Gallons in 7 Hours; and by the third, 40 Gallons in 9 Hours: in what Time will there be run out by all the Cocks $973\frac{1}{3}$ Gallons, and the first Cock be opened half an Hour before the other? Q. *Of emptying a Cistern.*

Answ. In $63\frac{1}{2}$ Hours, or 2 Days 15 Hours and an Half. The Work of the *Ratio's* is as in the last Example, by which will be gotten $15\frac{25}{63}$ Gallons to be run out by all the Cocks running together in 1 Hour: but before the Question can be set therewith to know in what Time $973\frac{1}{3}$ Gallons will be run out, there must be known how much runs out in the half Hour the first Cock runs alone, which will be $3\frac{1}{3}$ Gallons: Answer.

lons: this taken from $973\frac{1}{3}$, leaves 970, which shall give 63 Hours, to which the half Hour must be added.

$$\begin{array}{cccc} & 1260 & 810 & 840 \\ \text{Gallons} & 20 & 30 & 40 \\ \text{Hours} & 3 & 7 & 9 \\ \hline & & 189 & \end{array}$$

$$\begin{array}{lr} & \underline{|7} \\ & \cancel{1}\cancel{0}\cancel{2}|\underline{5} \\ \text{Antecedents } 1260+810+840 == & \cancel{2}\cancel{9}\cancel{1}\,0 \\ \text{Consequents } \quad 3 \times 7 \times 9 == & \cancel{1}89 \end{array} \Big(15\tfrac{25}{63} \text{ Gallons.}$$

Hours. *Gallons.* *Hour.* *Gallons.*
If 3 run out 20: what shall $\frac{1}{2}$? *facit* $3\frac{1}{3}$

$$\frac{3}{1}\Big)\frac{20}{2}\Big(\frac{20}{6}\Big(3\tfrac{1}{3} \qquad 973\tfrac{1}{3} - 3\tfrac{1}{3} = 970.$$

Gallons. *Hour.* *Gallons.* *Hours.*
If $15\frac{25}{63}$ run out in 1: in what 970? *facit* 63.

$$\frac{970}{63}\Big)\frac{970}{1}\Big(\frac{63}{1} \qquad \begin{array}{r} \text{Added } \frac{1}{2} \\ \hline 63\frac{1}{2} \text{ Hours.} \end{array}$$

Variety of Operation.

By the *Rule of Three* the Work is as in the last Example till the $15\frac{25}{63}$ Gallons be gotten, and then the other work as above.

As $\frac{1}{20}$: 1 :: $\frac{7}{30}$: $1\frac{5}{9}$

$$\frac{3}{20}\Big)\frac{7}{30}\Big(\frac{14}{9} \qquad \begin{array}{r} 1 \text{ added.} \\ \hline 2\frac{5}{9} \end{array}$$

As $2\frac{5}{9}$: $\frac{7}{30}$ :: 1 : $\frac{21}{230}$

$$\frac{23}{9}\Big)\frac{7}{30}\Big(\frac{21}{230}$$

As $\frac{21}{230}$: 1 :: $\frac{9}{40}$: $2\frac{13}{28}$

$$\frac{21}{230}\Big)\frac{9}{40}\Big(\frac{207}{48} \qquad \begin{array}{r} 1 \text{ added.} \\ \hline 3\frac{13}{28} \end{array}$$

As $3\frac{13}{28}$: $\frac{9}{40}$ :: 1 : $\frac{63}{970}$

$$\frac{97}{28}\Big)\frac{9}{40}\Big(\frac{63}{970} \qquad \frac{970}{63}\Big(15\tfrac{25}{63}$$

Data above 3 Numb. and Sub. used.

Data *more than* 3 *Numbers and Substraction needful.*

Q. Of hunting an Hare.

Example 1. A Gentleman in hunting an Hare, by their tracing in the Snow findeth that the Hare had 60 Lengths of the Hound's Paces before the Hound: And as often as the Hare runneth 8 Paces, the Hound runneth but 6 Paces; but 2 Paces of the Hound are as much as 3 of the Hare's Paces: In how many Paces of the Hound shall he overtake the Hare?

Answer.

Answ. In 540 Paces of the Hound. Here are three Questions included. First, Seeing the 60 Lengths were of the Hound's Paces, how many that made of the Hare's Paces? Secondly, Since 2 Paces of the Hound are equal to 3 of the Hare, that is gain upon the Hare 1 Pace, how many 6 Paces of the Hound shall gain or be equal to? Thirdly, The Gain of 6 Paces of the Hound known, the third Question ariseth, and is resolved by the Quotient of the first Work with the 6 Paces, and the Gain found as hereunder is to be noted.

Hound's *Hare's* *Hound's* *Hare's*
If 2 Paces make 3 Paces: what shall 60 Paces? *facit* 90 Paces.

$$2\Big)\frac{60}{180}\Big(90 \text{ Hare's Paces.}$$

If

Hound's *Hare's* *Hound's* *Hare's*
If 2 Paces make 3 Paces: what shall 6 Paces? *facit* 9 Paces.

$$2\,\Big)\,\frac{6}{18}\,\Big(\,9 \text{ Hare's Paces.}$$

Substract 8 Hare's Paces.
Gain 1

Hare's *Hound's* *Hare's*
If 1 Pace be gotten by 6 Paces: in how many Hound's Paces will 90 Paces be gotten? *facit* 540 Hound's Paces, as before.

As 1 . 6 :: 90 . 540.

Example 2. A Gentleman hunteth an Hare; and as often as the Hound runneth 6 Paces, the Hare runneth 8; and 2 Paces of the Hound make 3 of the Hare, and the Hound overtaketh the Hare in 540 Paces of his own: how many Hound's Paces had the Hare before the Hound? Q. *Of hunting an Hare.*

Answ. 60 Paces of the Hound. Here also are 3 Questions to be resolved, yet with this difference from the former Example, that as there Substraction was made from the Quotient of the second Work, here the Quotient of the first Work shall be substracted from the first Number of the second Work; the rest of the Works are similar to the former, as hereunder is evident. Answer.

Hound's *Hare's* *Hound's* *Hare's*
If 6 Paces make 8 Paces: what shall 2 Paces? *facit* $2\frac{2}{3}$ Paces.

$$6\,\Big)\,\frac{2}{16}\,\Big(\,2\tfrac{2}{3} \text{ Hare's Paces.}$$

Hare's Paces.
$3 - 2\frac{2}{3} = \frac{1}{3}$ loss.

Hare's *Hound's* *Hare's* *Hound's*
If 3 Paces make 2 Paces: what shall $\frac{1}{3}$ Pace? *facit* $\frac{2}{9}$ Pace.

$$\frac{3}{1}\,\Big)\,\frac{2}{3}\,\Big(\,\tfrac{2}{9} \text{ Hound's Paces.}$$

Paces *Hound's* *Paces* *Hound's*
If 2 Hound come of $\frac{2}{9}$ Pace: of what comes 540 Hound? *facit* 60 Paces.

$$\text{For } \frac{2}{9} \times \frac{540}{1} = \frac{120}{1}$$

$$\text{And } \frac{2}{1}\Big)\frac{120}{1}\Big(\frac{60}{1} \text{ Hound's Paces.}$$

Thus, (as was before noted, and might be seen by many other Instances) besides the Rule, due consideration must be had to the Nature of the Question propounded, duly to prepare or dispose the *Data*, and when to add and substract, &c. in such kind of Specifical Propositions as these are; to conclude which, one Example more shall be added.

A Worm in a Well 24 Feet deep, creepeth upwards every Day $5\frac{1}{4}$ Feet, and downwards every Night $4\frac{1}{3}$ Feet: in how many Days shall he creep out of the Well? Q. *Of a Worm creeping out of a Well.*

Answ. In $21\frac{19}{21}$ Days. For in 21 Days the Deduction of the downward Motion taken from the upward, yet leaveth $19\frac{1}{4}$ Feet for the Worm to crawl up: And the other $4\frac{3}{4}$ Feet to make up 24, he creepeth up on the 22*d* Day, without any deduction, he being up $\frac{2}{21}$ of a Day before Night. Answer.

Here $4\frac{1}{3}$ taken from $5\frac{1}{4}$, leaveth $\frac{11}{12}$ the Gain upward in 1 Day: Then because the last Day he gets up to the top of the Well, there is no deduction to be made, but 1 Night's deduction is to be taken from 24; and so $4\frac{1}{3}$ taken from 24, there remaineth $19\frac{2}{3}$.

	Foot.	*Day.*	*Feet.*	*Days.*		*Days.*	*Feet.*	*Days.*	*Feet.*
Then as	$\frac{11}{12}$.	1 ::	$19\frac{2}{3}$.	$21\frac{5}{11}$	And as	$21\frac{5}{11}$.	$19\frac{2}{3}$ ::	21 .	$19\frac{1}{4}$

And

And because $19\frac{3}{4}$ Feet wants $4\frac{1}{4}$ of 24, this must be the Motion of the Worm the last Day in which he got up to the top of the Well.

	Feet.		Day.		Feet.		Day.
Therefore as	$5\frac{1}{4}$	.	1	::	$4\frac{1}{4}$	.	$\frac{17}{21}$

And this $\frac{17}{21}$ is to be added to the 21 Days in which he got up to the $19\frac{3}{4}$ Feet, so both make up $21\frac{17}{21}$ Days as before.

Proof of Specificks Common and Specifical. Of the first sort.

Specificks, besides their Proof in Common with other Works of the *Rule of Three*, (before seen in the foregoing Chapters) have their Specifical Proof.

Specifical Proof of the first sort, is by turning the Fractions with Integers all into Integers, and working therewith.

This for Questions resolved by the *Direct Rule of Three*, is to multiply the Numerator of the first Number of the three given Numbers into the Denominators of the Second and Third, and this Product shall be the first Number of the Probationary Work. The Numerator of the second given Number shall be the second in this Work: And the third Number here shall be the Product of the Numerator of the Third, into the Denominator of the First.

But for Questions resolved by the *Indirect Rule of Three*, multiply the Numerator of the first of the three given Numbers into the Denominator of the Third, and the Product shall be the first Number of this probationary Work. The Numerator of the second given Number, shall be the second in this Work: And the third Number here shall be the Product of the Numerator of the Third into the Denominators of the First and Second.

Examples.

Example in two of the former Instances, where

Ell $1\frac{1}{2}$ at *l.* $1\frac{1}{4}$, is as *Ells* $3\frac{3}{4}$ to *l.* $3\frac{1}{8}$, by the *Direct Rule.*

s. $5\frac{1}{2}$ to *Ounces* $6\frac{6}{11}$, so is *s.* 12 to *Ounces* 3, by the *Indirect Rule.*

The *Data* being reduced into Improper Fractions, stand thus.

1. *Ell* $\frac{3}{2}$. *l.* $\frac{5}{4}$ :: *Ells* $\frac{15}{4}$.

2. *s.* $\frac{11}{2}$. *Ounc.* $\frac{72}{11}$ :: *s.* $\frac{12}{1}$.

Then $3\times4\times4=48$.	First Number.	And $11\times1=11$.	First Number.
5.	Second Number.	72.	Second Number.
$15\times2=30$.	Third Number.	$12\times2\times11=264$.	Third Number.

As $48 . 5 :: 30 . 3\frac{1}{8}$. — As $11 . 72 :: 264 . 3$

8 … 5 First and Third abbrev. 1 … 24

$8\,)\,\frac{5}{25}\,(\,3\frac{1}{8}$ *l.* $\frac{72}{24}\,(\,3$ Ounces.

Of the second sort.

Specifical Proof of the second Sort, is to turn the Geodeticals or Astronomicals into Decimals, or Decimals into them, and so work the Question in both, and compare the Resolutions together; this may be seen in some of the Examples before wrought in their proper Place, so as farther Instance is here purposely omitted.

Of the third sort.

Specifical Proof of the third Sort, is by making two Questions, and after Resolution, adding, substracting, *&c.* the Quotients that are to be added, substracted, *&c.* according to the Nature of the Proposition.

Example.

Example in the first of the former Instances concerning the two Posts, where *A* travelling 18 Miles a Day, and *B* on the contrary 30, they were found distant the third Day 144 Miles.

	A 18		*B* 30	For as 1 . 18 :: 3 . 54
	3		3	And as 1 . 30 :: 3 . 90
That is	54	+	90	144

Specifical

Of the fourth sort.

Specifical Proof of the fourth sort, is by working the Questions by their *Ratio's*, or by the *Rule of Three*; and the Agreement in their Resolutions wrought both ways, declare their Operations right; and because most of the Examples before are so wrought, farther Instances need not here.

CHAP. VI.

The Rule of five Numbers Direct.

Rule of five Numbers. *Why called* Compound. *Why the* Double Rule of Three. *Different from* Specificks.

AFTER *Specificks* follow the next Sort of derived Proportions to be seen in the *Rule of five Numbers*, sometimes called the *Compound Golden Rule*, and sometimes the *Double Rule of Three*: which latter Name is put upon it, because the Proposition may be resolved by a double Operation of the *Simple Golden Rule*, in opposition to which this is called *Compound*. But as the two Questions are Compound in one; so by the *five Numbers* given, from whence it took the first Name, Resolution may be had, and a sixth Proportional found by one Operation. And though they are resolvable by two, yet differ from *Specificks*, as before in the precedent Chapter was, and further in this and the next Chapter may be observed.

Rule of five Numbers, Direct *and* Indirect.

The *Rule of five Numbers* is both *Direct* and *Indirect*; the latter is referred to the next Chapter: to the knowledg of the other are considerable,

Some things { Preparatory in the right Disposition of the *Data*.
Operatory in the Resolution.
Probationary in the Proof.

Preparatory to the Direct.

The Precepts Preparatory, may be these and such-like.

1. All Superfluous to be discharged.

1. Among the *Data* discharge all the Numbers that are superfluous, for sometime the Proposition giveth five Numbers, when two of them may be discharged as more than needful; and the Numbers reduced to 3, the Question may be resolved by the *Simple Rule of Three*.

Example.

As, If 100 *l.* in 12 Months gain 8 *l.* what shall 100 *l.* in 16 Months? Here 100 in both Places may be cancelled, and Resolution had without.

	Months.		*l.*		*Months.*		*l.*
For as	12	.	8	::	16	.	$10\frac{2}{3}$
	:				:		
	3				4		

$$3)\frac{4}{32}(10\tfrac{2}{3}$$

2. How many Conditional and Interrogative.

2. Of the 5 given Numbers, let 3 be Conditional and Antecedents, or Suppositious; and the other 2 Interrogative and Consequents, corresponding to some of the former Antecedents.

Example.

As in the former Instance 100. 12. 8. are Conditional and Antecedent, the other 100. 16. are Interrogative. And because this latter 100 is Consequential and equal to the former Antecedent, 100 in both Places may be cancelled as before.

3. To place the Numbers aright.

3. Let the three Conditionals be first placed towards the left Hand; and then the two Terms on which the Question depends, in the 4*th* and 5*th* Places: So as the first Term be the principal Cause of Loss or Gain, Increase or Decrease, Action or Passion. That whose Sirname or Denomination betokeneth the Space of Time, Distance of Place, *&c.* the second and the third Term the odd Denomination, which is unlike in Nature to all the other Numbers. Then let the fourth Term and first be of one Denomination; likewise the fifth and second.

Example.

As in the former Instance, the first 100 being the principal Cause of the Gain, is first set; the 12 Months being the Space of Time, is the second Number; the 8 *l.* Gain is the odd Denomination, and so set in the third Place: Then the latter 100 *l.* Principal-Money answering in the Interrogative Part to the first Number,

is placed in the 4*th* Place: And 16 Months of like Denomination to the second in the 5*th* Place.

4. Denominations double, which regarded.

4. When any of the Numbers seem to be doubly denominate, the latter Denomination is to be regarded in placing the Numbers, as before noted in the *Direct Rule of Three.*

5. Data when need to be reduced.

5. If any of the Numbers be of several Denominations, as Pounds and Shillings, or Pounds, Shillings and Pence, *&c.* reduce the same and his correspondent Number into the lowest of those given Denominations; as the First and Fourth, Second and Fifth: but if the third Number want Reduction, he having no Correspondent but the Sixth not yet found, is only brought into the lowest Denomination given, without altering any of the other *Data:* And according to the Third, or Reduction thereof, shall the Sixth Number when found be denominate.

Sixth when found, how denominate.

6. Terms if abbreviated, shorten the Work.

6. Sometime the Conditional Terms given may be reduced to lesser Proportional Numbers, and working by them will shorten the Work. As in the former Instance, instead of 100 *l.* 12 Months, 8 *l.* may be taken; 50 *l.* 12 Months, 4 *l.* or 25 *l.* 12 Months, 2 *l.* or 5 *l.* 12 Months, 8 *s.* or 5 *l.* 3 Months, 2 *s.* &c.

7. To find if Resolution be by the Direct Rule, or not.

7. To find whether a Question propounded be resolvable by the *Rule of five Numbers Direct;* consider whether the principal Cause of Loss or Gain, Interest or Decrease, Action or Passion, *&c.* be the odd denominate Number in the Proposition: For if so, then is the Resolution by the *Indirect Rule* as in the next Chapter; but if otherwise, by the *Direct.* As in Questions of Money, the *Direct Rule* enquireth always for the Interest, and gives the Principal Money, the *Indirect*, for Stock or Time of Continuance, and gives the Interest.

8. Operation to be as the Nature of the Data is.

8. According to the Nature of the *Data*, whether Integers, Fractions, *&c.* so let the Operation be: And if any Proposition be specifical, mix the Work in order to the Resolution accordingly.

Resolution by 1 or 2 Works.

The Premisses observed, the Resolution will be had by one or two Operations.

The Rule for one Operation.

How by one.

Multiply the two first Numbers together for the Divisor, and the three last Numbers one into another for the Dividend; and dividing the one by the other, the Quotient shall be the sixth Proportional, and answer the Question.

The Rule for the Double Work.

How by two.

Take the first, third and fourth Numbers, and work as in the *Direct Rule of Three* for another Proportional: Then take this new Proportional for the second Number of the next Work, and the second and fifth Numbers of the five given Numbers for the first and third Numbers of this second Work, and the Quotient of this last working by the *Rule of Three*, shall answer the Question first propounded.

Q. Of Interest-Money.

Example 1. If 400 *l.* be put out for 16 Months, at the Rate of 8 *l. per Centum per Annum*, (accounting 12 Months to the Year) I demand what will the Interest come to?

Answer.

Answ. $42\frac{2}{3}$ *l.* Here the three Conditional Terms are the 100 *l.* that in one Year (or 12 Months) gains 8 *l.* and 100 *l.* the Principal Cause of the Profit or Increase, that shall be the first Number; and the 400 *l.* being also Principal Money the first Consequent, and of like Nature to the 100, shall be set in the fourth place; 12 Months the second Conditional, being the Space of Time in which the Increase groweth, shall be the second Number, and the Consequent 16 Months of like Nature to be 12, shall be in the fifth Place: So as the 8 *l.* Use-Money having none of the five to parallel him, falls into the third Place.

By one Work.

Then to resolve the Question at one Operation, multiply 100 into 12, the Product 1200 is Divisor: And 8 multiplied into 400, and the Product by 16 makes the Dividend 51200, and after Division $42\frac{2}{3}$ found in the Quotient, is denominate like the third Number, and is the Use-Money gained by 400 *l.* in 16 Months, at the Rate aforesaid.

As

	l.	*Months.*	*l.*		*l.*	*Months.*	*l.*
As	100	. 12 .	8	::	400	. 16 .	$42\frac{2}{3}$

```
  100        8
   12      400
 ----     ----
 12|00    3200        3(8
            16        512( 42 2/3 l.
          ----        12
         19200
          3200
         -----
         512|00
```

If the Operation be double, then is 100 *l.* 8 *l.* 400 *l.* the three Numbers of the first Work, 32 the Number gotten thereby the second Number of the second Work, and 12 and 16 the first and third Numbers of the same Work: Thus, *By two Works.*

	l.		*l.*		*l.*		*l.*			*Months.*		*l.*		*Months.*		*l.*
As	100	.	8	::	400	.	32		As	12	.	32	::	16	.	$42\frac{2}{3}$

```
   4                      4
  --                   3)128( 42 2/3
  32
```

If the conditional Terms be reduced to smaller Proportionals, then the Work at one Operation will stand thus. *Example where the Conditionals are reduced.*

	l.	*Mon.*		*s.*		*l.*		*Mon.*		*l.*	*s.*	*d.*
As	5	. 3	.	2	::	400	:	16	:	40 :	13 :	4.

```
   15          32
             ----
              800           5(5
             1200         12800 ( 85|3
             ----          15     -------
            12800          15     42:13:4
                           15
```

Example 2. If 3 Mowers in 6 Days mow $24\frac{1}{2}$ Acres: how many Acres will 8 Mowers mow in 10 Days? *Q. Of Mowers.*

Answ. 108 Acres and $\frac{8}{9}$ of an Acre. *Answer.*

	Mowers.		*Days.*		*Acres.*		*Mowers.*		*Days.*		*Acres.*
As	3	.	6	.	$24\frac{1}{2}$	::	8	:	10	.	$108\frac{8}{9}$

```
     18             245          1(16
                   1960          1960 ( 108 8/9 Acres.
                                  18
```

Specificks.

Example 1. If 5 Guns in 2 Days spend 60 Barrels of Powder: what will 7 Guns spend in 5 Days, which spend $\frac{1}{3}$ part less than the 5 at every shot? *A specifical Question of Powder spent by 7 Guns.*

Answ. 140 Barrels of Powder.

```
Barrel      60( 20
 1/3         3
60 — 20 = 40
```

	Guns.		*Days.*		*Bar.*		*Guns.*		*Days.*		*Bar.*
As	5	.	2	.	40	::	7	.	5	.	140

```
     10             200
                   1400       1400( 140 Barrels.
                                10
```

Example 2. If 12 Penny-worth of Wine satisfy 8 Persons at a Meal, when Wine is at 6 *d.* a Quart: how much Wine at 4 *d.* a Quart will suffice 40 Persons that drink twice as much as the other? *Another of Wine drunk.*

Answ. 80 Pennyworth of Wine. *Answer.*

Wine

Wine 12 *d.*	*Persons. d.quart. d.*	*Persons. d.quart. s. d.*	
Increased 2	As 8 . 6 . 24	:: 40 . 4 . 6 : 8.	
24	48	96	$\frac{3840}{48}$ (80 *d.*
		3840	

Proof of the Rule of five Numbers Direct.

The Operations of the *Direct Rule of five Numbers* wrought at once, and by the double Work, seeing they agree in the Resolution, and identify the Answer, prove each other, as by the Resolution of the Gain of 400 *l.* in 16 Months wrought before both ways is made plain.

CHAP. VII.

The Rule of five Numbers Indirect.

Rule of five Numbers Indirect.

THE Knowledg of this *Indirect Rule of five Numbers*, consists chiefly in the Preparatory Part rightly to dispose the *Data*, for which these Precepts are necessary.

1. All superfluous Numbers to be discharged. 2. How to place the Data.

1. Discharge all the Numbers superfluous among the *Data*, as before in the *Direct Rule of five Numbers*.

2. Of the five given Numbers as before, if three be Conditionals and Antecedents, and the other two Interrogative and Consequents, then dispose them thus; Let the odd Denomination, which here will be the principal Cause of Loss or Gain, Increase or Decrease, Action or Passion, *&c.* be the first Term, and the other two Conditionals the Second and Third, as in the *Direct Rule of five Numbers*; and let the Second and Fourth, and Third and Fifth, be of like Denominations.

3. How to place them if there be 4 Conditionals.

3. But if there be four Conditionals or Terms of Explanation; then place the single or odd denominate Term in the first Place as before; the Term on which the Question depends, set in the second Place; the Number countervailed by the first located Term, let be the Third; the Number denominate like the second located Term, place in the fourth Place; and then the other Term of the *Data* countervailing the Fourth, and denominate like the Third located Term, will supply the fifth Place.

4. Sixth when found, how denominate. Other Precepts before useful here. Resolution by 1 or 2 Works.

4. Let the sixth Proportional, when found, be always accounted of like denomination to the First, or the Reduction thereof if reduced. The rest of the 4*th*, 5*th*, 6*th*, 7*th* and 8*th* Precepts of the *Direct Rule of five Numbers*, consider and make use of as occasion shall require.

The *Data* duly disposed, as aforesaid, the Resolution will be had by one or two Operations.

The Rule for one Operation.

How by 1.

Multiply the third and fourth Numbers together for the Divisor, and the other three Numbers one into another for the Dividend; and dividing the one by the other, the Quotient shall be the desired Number, and the sixth Proportional.

The Rule for the double Work.

How by 2.

Take the Fourth for the First, the Fifth for the Second, and the Second for the Third; and the Quotient of this Work by the *Rule of Three*, make the third Number of the second Work: and for the First, take the Third of the *Data*; and for the Second, the First of the *Data*, and the Result of this Work by the *Rule of Three* shall resolve the Question first propounded.

Q. Of Principal Money.

Example 1. If $42\frac{2}{3}$ *l.* be Interest for 16 Months, after the Rate of 8 *l. per Centum per Annum*: what Principal Money was delivered to raise that Interest?

Answ. 400 *l.* Here being but three Conditionals, the *Data* is to be disposed according to the second Precept. And because 100 is the principal Cause of Gain in the Conditionals, and the single Denomination, it is first placed, the Year or

12

12 Months being the Space of Time must be Second: And 8 the third Conditional shall occupy the Third Place. Then must 16, because agreeing with the Second, be set in the fourth Place; and $42\frac{2}{3}$ in the fifth Place, being of like denomination with the Third, that is both Use-Money: So the Question stands thus.

If 100 *l.* in 12 Months gain 8 *l.* what Principal-Money in 16 Months will gain $42\frac{2}{3}$ *l.*?

Then to resolve the Question at one Operation, 8 is multiplied into 16, and the Product 128 is Divisor. The other three of the *Data* being multiplied, make the Dividend 51200; which divided by 128, give 400 as the Principal-Money to raise that Interest in the Time proposed. *By 1 Work.*

```
      l.   Months.  l.   Months.   l.      l.
As   100 .  12   .  8  ::  16  .  42 2/3 . 400.
      12            16
    ----           ---
    1200            48
      42 2/3         8
    ----           ---
    2400           128
   4800
     400                 51200
     400                 ----- ( 400 l. Principal.
   -----                  128
   51200
```

If the Work be double, then is 16 . $42\frac{2}{3}$. 12 . the three Numbers of the first Work; 32 the Number gotten thereby, shall be the third Number of the second Work, and 8 and 100 the other Numbers of that Work: Thus, *By 2 Works.*

```
Months.   l.    Months.  l.           l.    l.    l.    l.
As 16  .  42 2/3 :: 12 . 32       As  8  :  100  ::  32  :  400
          12                               32
          --                             -------
           8        512                 8)3200(400 l.
          42.8      --- (32 l.
          ---        16
          512
```

Here is worthy to be noted, that in this and several other Instances, though the Resolution at one Operation is by the *Indirect Rule of five Numbers*; yet by the double Work both are sometimes resolved by the *Direct Rule of Three.* *One Work by the Indirect, yet 2 both Direct sometimes.*

If the Conditional Terms be reduced to lesser Proportionals, the Work at one Operation will stand thus. *Example, where the Conditionals are reduced.*

```
     l.   Mon.   l.    Mon.   l.      l.
As   5  .  3  . 1/10 :: 16  :  42 2/3 : 400
     3
    --                 8             1    80
    15           1    16     8       8   640   400
    42 2/3      -- × ---- = ---      - ) --- ( ---  l.
    --          10     1     5       5    1     1
    30           5
   60
     5
     5
   ---
   640
```

Example 2. If 3 Mowers in 6 Days mow $24\frac{1}{5}$ Acres: how many Mowers in 10 Days can mow $108\frac{8}{9}$ Acres? *Q. Of Mowers.*

Answ. 8 Mowers. *Answer.*

	Mowers.	*Days.*	*Acres.*		*Days.*	*Acres.*	*Mowers.*
As	3 .	6 .	$24\frac{1}{2}$	::	10 .	$108\frac{8}{9}$.	8

```
   6          10
  18         240
 108 8/9       5
 144         245
 180
  16                 1960
 1960                ----( 8 Mowers.
                      245
```

Specificks.

A specifical Question of Travel.

Example 1. If in 10 Days of 8 Hours long, a Man may journey 200 Miles: in how many Days twice as long may he travel 500 Miles?

Answer.

Answ. In $12\frac{1}{2}$ Days.

			Days.	*Hours.*	*Miles.*		*Hours.*	*Miles.*	*Days.*
Hours 8		As	10 .	8 .	200	::	16 .	500 .	$12\frac{1}{2}$

```
Hours      8        8          16
Increased  2       80        32,00     |1
          16      500                   8|6
                400,00                  400
                                        ---( 12 1/2 Days.
                                         32
```

Another of Persons drinking.

Example 2. If 12 pennyworth of Wine satisfy 8 Persons at a Meal, when Wine is at 6 Pence a Quart: how many Persons that drink but half so much Wine at a Meal, will 20 pennyworth of Wine suffice when Wine is at 4 *d.* a Quart?

Answ. 40 Persons.

	Persons.	*d.quart.*	*d.*		*d.quart.*	*d.*	*Persons.*
As	8 .	6 .	6	::	4 .	20 .	40

```
Wine 12                  6          4
     --( 6              48         24
      2                 20
 1/2                   960
12—6=6                                   960
                                         ---( 40 Persons.
                                          24
```

Where four Conditionals are proposed.

Examples of 4 Conditionals.

Q. *Of Paris Pence.*

Example 1. If 4 *d.* of *Paris* be worth 5 *d. Tournois*, and 5 *d. Tournois* 6 *d.* of *Savoy*: how many Pence *Paris* are 15 *d. Savoy*?

Answer.

Superfluous Numbers discharged.

Answ. 10 *d. Paris.* Here two of the Terms, that is, both the Pence of *Tournois* are superfluous, and so may be omitted, and the Question with three Numbers stand thus.

	d.Savoy.	*d.Paris.*		*d.Savoy.*	*d.Paris.*
As	6 .	4	::	15 .	10

```
                    4
               6) 60 ( 10 d. Paris.
```

And so will the Resolution be, if the *Data* be disposed according to the third Precept: Thus,

	d.Paris	*d.Sav.*	*d.Tourn.*		*d.Sav.*	*d.Tourn.*	*d.Paris.*
As	4 .	15 .	5	::	6 .	5 .	10

```
  15          6
  60         30
   5                    300
 300                    ---( 10 d. Paris.
                         30
```

Q. *Of Angels and Crowns.*

Example 2. If 2 Angels countervail 20 *s. Sterling*, and 18 *s.* countervail 3 Crowns *French*: how many Angels will countervail 10 Crowns? or how many Crowns will countervail 12 Angels?

In

In these two Questions the *Data* disposed by the third Precept in the former 2 Angels, being the odd Denomination, will stand in the first place, but in the latter 3 Crowns. The Term on which the Question depends, to be set in the second place, must be 10 Crowns in the former, and 12 Angels in the latter. The third Place being filled with the Number countervailing the first located Term, shall be in the former 20 *s.* and in the latter 18 *s.* The Number denominate like him, placed in the Second, to be set in the fourth Place, must be in the former 3 Crowns, and in the latter 2 Angels. Then the remaining Number denominate like the Third, and countervailing the fourth located Terms, will be placed in the fifth Place, which in the former is 18 *s.* and in the latter 20 *s.* Numbers placed.

Single Operation. Operation single.

		Angels.		*Crowns.*		*s.*		*Crowns.*		*s.*	*Angels.*
Q. 1.	If	2	.	10	:	20	::	3	.	18 ?	*facit* 6.
		10				3					
		20				6,0					
		18									
		160						36/6 (6 Angels.			
		20									
		36,0									

Double Operation. Double.

	Cr.		*s.*		*Cr.*		*s.*
As	3	.	18	::	10	.	60
			10				
	3)		180 (60 *s.*				

	s.		*Ang.*		*s.*		*Ang.*
As	20	.	2	::	60	.	6
			60				
	20)		120 (6 Angels.				

Single Operation. Single.

		Crowns.		*Angels.*		*s.*		*Angels.*		*s.*	*Crowns.*
Q. 2.	If	3	.	12	:	18	::	2	.	20 ?	*facit* 20
		12				2					
		36				36					
		20									
		720						720/36 (20 Crowns.			

Double Operation. Double.

	Ang.		*s.*		*Ang.*		*s.*
As	2	.	20	::	12	.	120
			12				
			40				
			20				
	2)		024 (120 *s.*				

	s.		*Cr.*		*s.*		*Cr.*
As	18	.	3	::	120	.	20
			120				
	18)		360 (20				

The Operations of the *Indirect Rule of five Numbers*, are proved as those of the *Direct* for the Resolution by one Operation, and by two is the same, as by the last Examples wrought both ways is sufficiently convincing. *Proof of the Rule of five Numbers Indirect.*

CHAP.

CHAP. VIII.

FELLOWSHIP.

Fellowship how otherwise called.

DERIVATIVES of the third Sort, which deal with some particular Subject, are next to be exhibited; and among them, first, those Operations that converse about *Fellowship*, called also the *Rule of Society* and *Partnership*, because Questions of *Partnership* are resolved thereby.

The Sorts thereof.

Fellowship may be divided into four Sections,
- Without Time, §. 1.
- With Time, §. 2.
- With diversity of Time, §. 3.
- With diversity of Parts, §. 4.

Without Time of two Sorts.

§. 1. *Fellowship* without Time, either propounds the several Disbursments of the particular Partners, and the general Gain or Loss by their Traffique, and requires to know each Man's Part or Share thereof: Or by the general Stock and particular Shares of the Gain or Loss, requires to know their particular Disbursments.

To resolve the First.

To resolve the Propositions of the first Sort, add all the Monies disbursed, or Stock in Partnership together, for the first Number of the *Rule of Three*: Then set the Gain or Loss in the second Place; and each Man's several Stock or Disbursments in the Third: And by so many Operations of the *Rule of Three* as there be Partners, the Desire is obtained.

Q. Of each Partner's Gain.

Example. Three Men, or *A*, *B*, and *C*, consenting to trade together, make a Bank of Money, in which *A* disbursed 100 *l.* *B* 150 *l.* and *C* 250 *l.* And trading therewith, they gained 300 *l.* what is each Man's Part of the Gain?

Answer.

Answ. *A* 60 *l.* *B* 90 *l.* and *C* 150 *l.* Here 100+150+250=500 make the first Number, 300 the Gain the Second; and the several Sums disbursed the Third Numbers, as followeth.

	Stock.
A	100 *l.*
B	150
C	250
	50,0

As 500 *l.* . 300 *l.* :: 100 *l.* . 60 *l.*
300 × 10 = 3000; 3000 ÷ 50 = 60 *l.* *A.*

As 500 *l.* . 300 *l.* :: 150 *l.* . 90 *l.*
300 × 15 = 4500; 4500 ÷ 50 = 90 *l.* *B.*

As 500 *l.* . 300 *l.* :: 250 *l.* . 150 *l.*
300 × 25 = 7500; 7500 ÷ 50 = 150 *l.* *C.*

To resolve the second Sort.

To resolve the Propositions of the latter Sort, place the Gain or Loss for the first Number; the whole Stock for the Second; and the several Parts of the Gain or Loss for the several third Numbers, and proceed by the *Rule of Three*, as before.

Q. Of each Partner's Stock.

Example. *A*, *B*, and *C*, made a Stock to trade with, and all laid in together 560 *l.* wherewith they gain 150 *l.* of which at the end of their Partnership, *A* took 40 *l.* *B* 50 *l.* and *C* 60 *l.* what had each Man in Stock?

Answer.

Answ. *A* $149\frac{1}{3}$ *l.* *B* $186\frac{2}{3}$ *l.* and *C* 224 *l.*

As

$$\begin{array}{lllll} & l. & l. & l. & l. \\ \text{As} & 15\underline{0} & . \; 560 & :: \; 4\underline{0} & . \; 149\tfrac{1}{3} \\ & & \quad 4 & & \\ & & 2240 & & \end{array} \qquad \begin{array}{l} 1 \\ 7(5 \\ \dfrac{2240}{15}\Big(149\tfrac{1}{3}\, l. \; A. \end{array}$$

$$\begin{array}{lllll} \text{As} & 15\underline{0} & . \; 560 & :: \; 5\underline{0} & . \; 186\tfrac{2}{3} \\ & & \quad 5 & & \\ & & 2800 & & \end{array} \qquad \begin{array}{l} 1 \\ 13(1 \\ \dfrac{2800(0}{15}\Big(186\tfrac{2}{3}\, l. \; B. \end{array}$$

$$\begin{array}{lllll} \text{As} & 15\underline{0} & . \; 560 & :: \; 6\underline{0} & . \; 224 \\ & & \quad 6 & & \\ & & 3360 & & \end{array} \qquad \begin{array}{l} \dfrac{3360}{15}\Big(224\, l. \; C. \end{array}$$

§. 2. *Fellowship with Time*, simply propounds the several Disbursments of the particular Partners at once, and the Time they continue in Stock, with the Gain or Loss in general, and requires to know each Man's Part thereof: Or by the whole Gain or Loss, and Particulars of the Stock, and some particular Partner's Time of Trade, requireth to know the Time of the others Continuance: Or by the Gain or Loss of some particular Partner's Stock and Time according to the *Data*, seeketh the particular Gain or Loss, Stock or Time of another Partner: Or by Mixtures of these, proposeth Propositions mixt. *Fellowship with Time, of four Sorts.*

To resolve the Propositions of the first Sort, multiply severally every Man's Money by the Time it continued in Stock; and the Total of all those Products added together, shall be the first Number of the *Rule of Three*; the Gain or Loss the Second; and the several Products shall be the several third Numbers; and by so many Operations of the *Rule of Three* as there be Partners, Resolution will be had. *To resolve the first.*

Example. A, B, and *C* disburse as followeth; *viz. A* 50 *l. B* 60 *l.* and *C* 80 *l.* And *B* drew away his Money from the Stock at 10 Months end: But *A* and *C* continued the Trade till 16 Months; when the Stock was broken up, there was found 400 *l.* Gain: what is each Man's Share thereof? *Q. Of each Partner's Gain.*

Answ. A $119\frac{27}{67}$ *l. B* $89\frac{37}{67}$ *l.* and *C* $191\frac{3}{67}$ *l.* *Answer.*

	Stock.	*Time.*	
A	50 ×	16 =	800
B	60 ×	10 =	600
C	80 ×	16 =	1280
			268,0

$$\begin{array}{lllll} & & l. & & l. \\ \text{As} & 268\underline{0} & . \; 400 & :: \; 80\underline{0} & . \; 119\tfrac{27}{67} \\ & & \quad 80 & & \\ & & 32000 & & \end{array} \qquad \begin{array}{l} (1 \\ 25(0 \\ 522(8 \\ \dfrac{32000}{268}\Big(\overset{l.}{119\tfrac{27}{67}} \; A. \end{array}$$

$$\begin{array}{lllll} \text{As} & 268\underline{0} & . \; 400 & :: \; 60\underline{0} & . \; 89\tfrac{37}{67} \\ & & \quad 60 & & \\ & & 24000 & & \end{array} \qquad \begin{array}{l} (14 \\ 256(8 \\ \dfrac{24000}{268}\Big(\overset{l.}{89\tfrac{37}{67}} \; B. \end{array}$$

$$\begin{array}{lllll} \text{As} & 268\underline{0} & . \; 400 & :: \; 128\underline{0} & . \; 191\tfrac{3}{67} \\ & & \quad 128 & & \\ & & 51200 & & \end{array} \qquad \begin{array}{l} 2(1 \\ 2448(2 \\ \dfrac{51200}{268}\Big(\overset{l.}{191\tfrac{3}{67}} \; C. \end{array}$$

To resolve the Propositions of the second Sort, multiply that Partner's Stock, whose Time is given by the Time for the second Number of the *Rule of Three*: The first Number shall be that Partner's Gain or Loss; And the several third Numbers shall be the other Partner's Gain or Loss respectively. Then proceed as before by the *Rule of Three*, and the Quotients of these Works will be their Money and Time multiplied together; which if divided by their Money respectively, give the Time desired. *To resolve the second Sort.*

Example. A, B, and *C* are in Partnership. *A* layeth in 200 *l.* for 10 Months, *B* layeth in 350 *l.* and *C* 100 *l.* and through ill Adventure sustained the Loss of 160 *l.* Whereupon breaking off their Trade, *A* found himself a Loser 80 *l.* *B* 56 *l.* and *C* 24 *l.* The Question is, how long the Money of *B* and *C* continued in Stock? *Q. Of the Time of Trade.*

Answer. *Answ.* *B* 4 Months, and *C* 6 Months.

Stock. *Time.* *A Loss* *B Loss*

A $200 \times 10 = 2000$. As $80 . 2000 :: 56 . 1400$

$$\frac{\begin{array}{r}56\\ \hline 11200,0\end{array}}{} \qquad \frac{11200}{8}\Big(\frac{1400}{350}\Big(4 \text{ Months } B.$$

A Loss *C Loss*

As $80 . 2000 :: 24 . 600$

$$\frac{\begin{array}{r}24\\ \hline 4800,0\end{array}}{} \qquad \frac{4800}{8}\Big(\frac{600}{100}\Big(6 \text{ Months } C.$$

To resolve the third sort. To resolve the Propositions of the third Sort, commit the Question to the *Rule of five Numbers*, or the *Double Rule of Three*, *Direct* or *Indirect*, as the Case requires.

Q. Of the Stock of 1 Partner. *Example.* *A* and *B* are in Company: *A* putteth in 240 *l.* and gaineth 50 *l.* in 6 Months: what shall *B* put in to gain 30 *l.* in 4 Months?

Answer. *Answ.* 216 *l.*

	l.	*Months.*	*l.*		*Months.*	*l.*	*l.*
As	240	. 6	. 50	::	4	. 30	. 216

$$\begin{array}{r}240\\ 6\\ \hline 1440\\ 30\\ \hline 432,00\end{array} \qquad \begin{array}{r}50\\ 4\\ \hline 2,00\end{array} \qquad \frac{432}{2}\Big(216\ l.$$

To resolve the fourth Sort. Propositions various. The Propositions of the fourth Sort are many and various: For sometime the Stock is enquired after, sometime the Time, and sometime the Gain or Loss; sometime in general, and sometime particularly of one or more of the Copartners: And sometime more than one of them is included in the Question; and accordingly the *Data* with much Variety mixed, as in the Examples following.

1. Example, where are $\begin{cases} \textit{Data} \begin{cases} \text{Stock, generally.} \\ \text{Time and Gain, particularly.} \end{cases} \\ \textit{Quæsita}, \text{ Stocks, particularly.} \end{cases}$

Q. Of each Partner's Stock. *A*, *B*, and *C*, traffique together with a Stock of 638 *l.* wherewith they gain 90 *l.* And *A* having had his Money in Stock 5 Months, *B* 8 Months, and *C* 7 Months; the Gain was parted, to *A* 18 *l.* *B* 12 *l.* and *C* 60 *l.* what Monies had each in Stock?

Answer. *Answ.* *A* 168 *l.* *B* 70 *l.* and *C* 400 *l.*

Here, after the particular Gains are severally divided by the Times of Continuance, the Analogy is,

As the Sum of those Quotients to the whole Stock:

So are those Quotients severally to the several Stocks.

Gains of *A* $\frac{18}{5}\Big(3\frac{3}{5}$ Times   *B* $\frac{12}{8}\Big(1\frac{1}{2}$ *C* $\frac{60}{7}\Big(8\frac{4}{7}$ $3\frac{3}{5} + 1\frac{1}{2} + 8\frac{4}{7} = 13\frac{47}{70}$

$$As\ 13\tfrac{47}{70} . 638 :: \begin{cases} 3\frac{3}{5} . 168 . \text{Stock of } A. \\ 1\frac{1}{2} . \ \ 70 . \text{Stock of } B. \\ 8\frac{4}{7} . 400 . \text{Stock of } C. \end{cases}$$

2. Example, where are $\begin{cases} \textit{Data} \begin{cases} \text{Stock of One.} \\ \text{Time and Gain of Another.} \end{cases} \\ \textit{Quæsita}, \text{ Stocks severally.} \end{cases}$

Q. Of each Partner's Stock. In a joint Trade *B* putteth in 200 *l.* more than *A*; *B* continueth his Stock but 5 Months, *A* $7\frac{1}{2}$ Months; they gain alike: what Money had each of them in Stock?

Answ.

Answ. A 400 *l.* B 600 *l.* Answer.

Here substracting one Time from the other, the Analogy is;

As the Difference of Times, to the Stock propounded:

So are the Times themselves, to the several Stocks of each other.

$$\text{Times of}\begin{cases} A & 7\tfrac{1}{2} - 5 = 2\tfrac{1}{2} \\ B & 5 \end{cases}$$

$$\text{As } 2\tfrac{1}{2} \,.\, 200 :: \begin{cases} 5 \,.\, 400 \,.\, \text{Stock of } A. \\ 7\tfrac{1}{2} \,.\, 600 \,.\, \text{Stock of } B. \end{cases}$$

$$\text{3. Example, where are}\begin{cases} \textit{Data}\begin{cases}\text{Stocks and Gains, particularly.}\\ \text{Time, generally.}\end{cases} \\ \textit{Quæsita}\text{, Times particularly.}\end{cases}$$

Three are in Company, the Stock of A in their Trade is 168 *l.* of B 70 *l.* of C 400 *l.* they gain 90 *l.* whereof A hath 18 *l.* B 12 *l.* and C 60 *l.* they withdrew their Stock severally; but all the several Times their Monies were in Stock, added together, make 20 Months: how long did each Man continue his Money in Stock? Q. *Of the Time of Trade.*

Answ. A 5 Months, B 8 Months, and C 7 Months. Answer.

Here abbreviate every Man's Gains with his Stock to the least Terms, and take these Parts of any Number that will equally be divided by all the Denominators: Or if no such Number come readily to mind, multiply all the Denominators one into the other; and divide the Product by the respective Denominators, and multiply the Quotients by the several Numerators: And these last Products abbreviate with the Sum of them. Then the Analogy is;

As the Sum of these Parts, is to the whole Time given:

So is every Man's Part severally, to his respective Time.

	A.		B.		C.	
Gains	18	3	12	6	60	3
Stock	168	28	70	35	400	20

$$28 \times 35 \times 20 = 19600$$

$$\frac{19600}{28}\Big(\,700 \times 3 = 2100 \qquad \frac{19600}{35}\Big(\,560 \times 6 = 3360 \qquad \frac{19600}{20}\Big(\,980 \times 3 = 2940$$

2100	5	A.
3360	8	B.
2940	7	C.
8400	20	

$$\begin{array}{lccc} & \textit{Parts.} & & \textit{Months.} \\ \text{As} & 20 & . & 20 :: \end{array} \begin{cases} \textit{Parts.} \;\; \textit{Months.} \\ 5 \;.\; 5 \;.\; \text{Time of } A. \\ 8 \;.\; 8 \;.\; \text{Time of } B. \\ 7 \;.\; 7 \;.\; \text{Time of } C. \end{cases}$$

$$\text{4. Example, where are}\begin{cases} \textit{Data}\begin{cases}\text{Stock and Time of one.}\\ \text{Time of another.}\\ \text{Stock of a Third.}\\ \text{Gains of all particularly.}\end{cases} \\ \textit{Quæsita}\begin{cases}\text{Stock of One.}\\ \text{Time of Another.}\end{cases}\end{cases}$$

A in Company with B and C, putteth in 168 *l.* for 5 Months, B putteth in a Sum of Money for 8 Months, and C 400 *l.* for a certain Time; they gain 90 *l.* whereof A must have 18 *l.* B 12 *l.* and C 60 *l.* how much was the Stock of B? and what Time did the Stock of C continue in the Company? Q. *Of the Stock and Time of Trade.*

Answ. The Stock of B was 70 *l.* And the Time the 400 *l.* of C was in Stock was 7 Months. Answer.

Here the Analogies are; As the Gain of one Partner is to his Stock multiplied by his Time: So is the Gain of the other Partners severally to theirs; Which when found, is to be divided by their Time or Stock respectively.

Stock of A . 168
Time 5
840

$$\text{As } 18 \,.\, 840 :: 12 \,.\, \frac{560}{\text{Time } 8}\Big(\,70 \,.\, \text{Stock of } B.$$

$$\text{As } 18 \,.\, 840 :: 60 \,.\, \frac{2800}{400}\Big(\,7 \;\text{Time of } C.$$

5. Example

5. Example, where are *Data* { Stock and Gain of One. / Time and Gain of Another. / Rate on the Hundred of Both. } *Quæsita* { Stock of One. / Time of Another. }

Q. Of the Stock of one, and Time of another.

A joint Stock of *A* and *B* gained them, after the Rate of 20 *l. per Centum per Annum*, 50 *l.* apiece; *A* had 400 *l.* in Stock, and the Stock of *B* continued but 5 Months: what Money had *B* in Stock, and how long did *A* continue in the Company?

Answ. *A* continued $7\frac{1}{2}$ Months, *B* put into Stock 600 *l.*

Here first finding the Gain of the Stock given by the Rate propounded, the Analogies are,

As the Gain according to the Rate, to 1 Year:
So is the Gain received, to the Time required.
And then, As the Time of one Partner is, to the Stock of the other:
So is the Time of the Partner found, to the Stock of the other sought.

As	100	.	20	::	400	.	80	for *A*. Then
	l.		*Months.*		*l.*		*Months.*	
As	80	.	12	::	50	.	$7\frac{1}{2}$.	Time of *A*.
	Time B.		*Stock A.*		*Time A.*		*Stock B.*	
As	5	.	400	::	$7\frac{1}{2}$	.	600 .	Stock of *B*.

6. Example, where are *Data* { Stock, Time, and part of the Gain of One. / Time and part of the Gain of Another. / Time of a third Partner. } *Quæsita*, Stocks of two Partners.

Q. Of the Stock of two Partners.

Three Partners trade together. *A* continuing his Stock in the Trade 5 Months, must have $\frac{1}{5}$ of the Gain. *B* keepeth his Money in Stock 8 Months. *C* layeth in 400 *l.* for 7 Months, and taketh $\frac{2}{3}$ of the Gain: how much Money had *A* and *B* in the Stock?

Answer.

Answ. *A* 168 *l.* and *B* 70 *l.*

Here, after adding the Parts of Gains given together, and taking the Total out of 1, the Analogies are;

As the Part of Gain of one Partner given, is to his Stock multiplied by his Time: So is the Part of Gain of another: And also so is the Remainder of the Parts taken from 1, to his Stock multiplied by his Time. Which divided by the Time, giveth the Stock of that Partner sought respectively.

$$\overset{A}{\tfrac{1}{5}} + \overset{C}{\tfrac{2}{3}} = \tfrac{13}{15} \text{ Total.} \qquad 1 - \tfrac{13}{15} = \tfrac{2}{15} \text{ Remainder.}$$

$$\text{As } \tfrac{2}{3} \;.\; \overset{(400\times7)}{2800} \;::\; \tfrac{1}{5} \;.\; \frac{840}{\text{Time of } A \; 5}\Big(168 \text{ Stock of } A.$$

$$\text{As } \tfrac{2}{3} \;.\; 2800 \;::\; \tfrac{2}{15} \;.\; \frac{560}{\text{Time of } B \; 8}\Big(70 \text{ Stock of } B.$$

Fellowship with diversity of Time of three sorts.

§. 3. *Fellowship*, with diversity of Time, altereth the Stock and Time of some, or all of the particular Partners continuance; and thereby with the general Gain or Loss, requireth their due Proportions thereof: Or by their particular Gains or Losses, and Times of Occupation with the general Stock, seeketh their particular Disbursments: Or by the Gain or Loss of some particular Partners, Stock and Time, enquireth the particular Gain or Loss, Stock or Time of another Partner, according to the *Data*.

To resolve the first.

To resolve the Propositions of the first Sort, multiply the several Disbursments of the particular Partners, by the respective Times those Disbursements continue in Stock, adding or substracting for every Partner according to the Proposition: These Products added together, shall be the first Number of the *Rule of Three*: And the Sum of the Products of each Partner's Stock, multiplied by his respective Times,

Times shall be the third Numbers. The second Number shall be the Gain or Loss as before.

Example. *A*, *B*, and *C*, trade together for 12 Months: *A* putteth in presently 40 *l.* and 4 Months after 30 *l.* more, and 3 Months after 20 *l.* more. *B* at first putteth in 100 *l.* but 3 Months after taketh away 20 *l.* and 5 Months after 20 *l.* more. *C* layeth down first 60 *l.* and five Months after taketh away 10 *l.* but 3 Months after putteth in 20 *l.* They gain 500 *l.* what is each Man's Part thereof? *Q. Of each Partner's Gain.*

Answ. *A* $164\frac{164}{249}$ *l.* *B* $188\frac{188}{249}$ *l.* and *C* $146\frac{146}{249}$ *l.* *Answer.*

$$A\left\{\begin{array}{l} \textit{Stock. Time.} \\ 40 \times 4 = 160 \\ +\ \underline{30} \\ 70 \times 3 = 210 \\ +\ \underline{20} \\ 90 \times 5 = \underline{450} \\ \qquad\qquad 820 \end{array}\right. \quad B\left\{\begin{array}{l} \textit{Stock. Time.} \\ 100 \times 3 = 300 \\ -\ \underline{20} \\ 80 \times 5 = 400 \\ -\ \underline{20} \\ 60 \times 4 = \underline{240} \\ \qquad\qquad 940 \end{array}\right. \quad C\left\{\begin{array}{l} \textit{Stock. Time.} \\ 60 \times 5 = 300 \\ -\ \underline{10} \\ 50 \times 3 = 150 \\ +\ \underline{20} \\ 70 \times 4 = \underline{280} \\ \qquad\qquad 730 \end{array}\right.$$

$$\begin{array}{l} A\ 820 \\ B\ 940 \\ \underline{C\ 730} \\ 249,0 \end{array}$$

As $2490 \cdot 500 :: 820 \cdot 164\frac{164}{249}$ (l. ... l.)

$$\begin{array}{r} 500 \\ \underline{82} \\ 41000 \end{array} \qquad \frac{41000}{249}\left(\ 164\frac{164}{249}\ l.\ A.\right.$$

As $2490 \cdot 500 :: 940 \cdot 188\frac{188}{249}$

$$\begin{array}{r} 500 \\ \underline{94} \\ 47000 \end{array} \qquad \frac{47000}{249}\left(\ 188\frac{188}{249}\ l.\ B.\right.$$

As $2490 \cdot 500 :: 730 \cdot 146\frac{146}{249}$

$$\begin{array}{r} 500 \\ \underline{73} \\ 36500 \end{array} \qquad \frac{36500}{249}\left(\ 146\frac{146}{249}\ l.\ C.\right.$$

To resolve the Propositions of the next Sort; multiply the several Gains or Loss by the respective Times: the Total of these Products place for the first Number; the general Stock for the second Number; and the particular Products for the third Numbers: And proceed as before by the *Rule of Three*. *To resolve the second Sort.*

Example. *A*, *B*, and *C*, together hire certain Pasture-Land for 30 *l.* Rent: And besides their joint Stock of Sheep, equal in Number, *A* feedeth there at first 20 Oxen 80 Days; and taking them away, putteth in 100 other Oxen, which he keepeth there 14 Days. *B* at first putteth in 40 Oxen 50 Days; and removing them, afterward putteth in 16 Oxen for 100 Days. *C* putteth to Pasture only 70 Oxen, and keepeth them there, without alteration, 60 Days: what part of the Rent shall each Man pay? *Q. Of the Pasturage of Cattel, what Rent each shall pay.*

Answ. *A* 8 *l.* 6 *s.* 8 *d.* *B* 10 *l.* and *C* 11 *l.* 13 *s.* 4 *d.* *Answer.*

$$A\left\{\begin{array}{l} \textit{Oxen. Days.} \\ 20 \times 80 = 1600 \\ 100 \times 14 = \underline{1400} \\ \qquad\qquad 3000 \end{array}\right. \quad B\left\{\begin{array}{l} \textit{Oxen. Days.} \\ 40 \times 50 = 2000 \\ 16 \times 100 = \underline{1600} \\ \qquad\qquad 3600 \end{array}\right. \quad \begin{array}{l} \textit{Oxen. Days.} \\ C\ 70 \times 60 = 4200 \end{array}$$

$$\begin{array}{l} A\ 3000 \\ B\ 3600 \\ \underline{C\ 4200} \\ 108,00 \end{array}$$

As $108 \cdot 30 :: 30 \cdot 8\frac{1}{3}$ (l. ... l.)

$$\begin{array}{r} 30 \\ \underline{30} \\ 900 \end{array} \qquad \frac{900}{108}\left(\ 8\frac{1}{3}\ l.\ A.\right.$$

As $108 \cdot 30 :: 36 \cdot 10$

$$\begin{array}{r} 30 \\ \underline{36} \\ 1080 \end{array} \qquad \frac{1080}{108}\left(\ 10\ l.\ B.\right.$$

	l.		l.
As 108 .	30 ::	42 .	11$\frac{2}{3}$
	42		
	1260		

$\frac{1260}{108}$ ($11\frac{2}{3}$ l. C.

To resolve the third Sort.

To resolve the Propositions of the latter sort, where the Time or Stock is diversly given, as before in 2 §. of *Fellowship*, where the Time is simply given: Let the Numbers be orderly disposed, and the Question committed to the *Rule of five Numbers*, *Direct* or *Indirect*, as the Nature of the Question requireth.

Q. Of the Pasturage of Cattel, what one shall pay.

Example. *A* hired Pasturage for 56 Bullocks 150 Days, and was to pay therefore 5 *l.* but meeting with a Market at 90 Days end, selleth off 40, and keeping the Remainder, taketh in 10 Bullocks of *B* at one time for 10 Days, and another time 5 Bullocks for 12 Days; and hireth out Pasturage to *C* for 36 Bullocks for 60 Days: what shall *C* pay for the same?

Answer.

Answ. 1$\frac{2}{7}$ *l.* For if 5 *l.* buy 150 Days Pasturage for 56 Bullocks, then will 1$\frac{2}{7}$ *l.* buy 60 Days Pasturage for 36 Bullocks, by the *Direct Rule of five Numbers.*

	Bull.	*Days.*	*l.*	*Bull.*	*Days.*	*l.*
As	56 .	150 .	5 ::	36 .	60 .	1$\frac{2}{7}$
		84,00		300		
				108,00		

$\frac{108}{84}$ (1$\frac{2}{7}$ l.

If the Question had been, what Cattel to keep.

But if the Paiment of *C* 1$\frac{2}{7}$ *l.* had been given, and the Question had been, How many Bullocks he should have depastured there, then the Question had been resolved by the *Indirect Rule of five Numbers.* If 5 *l.* buy 150 Days Pasturage for 56 Bullocks: what will 1$\frac{2}{7}$ *l.* buy for 60 Days?

Answer.

Answ. Pasturage for 36 Bullocks.

	Bull.	*Days.*	*l.*	*Days.*	*l.*	*Bull.*
As	56 :	150 :	5 ::	60 :	1$\frac{2}{7}$:	36
		56		5		
		900		3,00		
		750				
		8400				
		2400 1$\frac{2}{7}$				
		108,00				

$\frac{108}{3}$ (36 Bullocks.

Questions may be various.

Here may be a Mixture of the Propositions, as in the second Section before.

Fellowship, *with diversity of Parts, of 2 sorts.*

§. 4. *Fellowship* with diversity of Parts, is either when a certain Sum is to be divided among several sorts of Partners, so as Parcel of the Partners have their Dividend in proportion to other Parcel of them: Or else when a certain Sum is to be divided among single Partners, according to the *Ratio* of their single parts: This latter Sort is called *Fractionary Fellowship*, and stored with variety of Examples.

Latter, how called.

To resolve the first.

To resolve the Propositions of the first Sort, multiply the Number of each Parcel of Partners by the Proportional Part each Partner is to receive, and the Total of these Products shall be the first Number of the *Rule of Three*; the Sum to be divided the Second; and the several Products the several third Numbers: And proceed as before by the *Rule of Three.*

Q. Of the Expences of Canons and Vicars.

Example. Suppose in a Cathedral were 30 Canons and 40 Vicars, which have allowed to spend 360 *l. per Annum*: But every Canon is to have 5 *l.* to the Vicar's 3 *l.* How much then are their Annual Expences severally?

Answer.

Answ. The Canons 200 *l.* and the Vicars 160 *l.*

Canons

					l.			l.
Canons	$30 \times 5 = 150$		As	270	360	::	150	200
Vicars	$40 \times 3 = 120$				15			
	270				1800			
					360			
					5400		$\frac{5400}{27}$(	200 l. Canons.

As 270	360	::	120	160
	12			
	720			
	360			
	4320		$\frac{4320}{27}$(	160 l. Vicars.

To resolve the Propositions of the latter Sort, take a Number that will equally divide by all the Consequents or Denominators; or to find one, multiply all the Denominators or Consequents one into another, or so many of them as be not equal Halfs of some other of them, and divide this ultimate Product, or the Number first taken, by the Consequents severally, and multiply these Quotients by the Antecedents or Numerators: Add all these several Products (or the Quotients where one is Antecedent) into one Total, which with the several Addends abbreviate if it may be: Then with the least Terms, or with the Total, and the several Products or Quotients, if 1 be Antecedent, where they will not be abbreviated, proceed as in the other sort of *Fellowship* to the Work of the *Rule of Three*. What other Necessaries occur are inserted, with the several Examples following. *To resolve the second Sort.*

Example 1. A certain Man bequeathed to charitable Uses 100 *l.* to be divided in such Proportion, that *A* should have $\frac{1}{2}$, *B* $\frac{1}{3}$, and *C* $\frac{1}{4}$: how should the 100 *l.* be divided according to the Intent of the Testator? Q. *Of a Legacy.*

Answ. To *A* $46\frac{2}{13}$ *l.* *B* $30\frac{10}{13}$ *l.* and *C* $23\frac{1}{13}$ *l.* Answer.

Here, and in all Questions of like sort, the *Ratio* of the *Data* is to be minded, for otherwise the Question were impossible, seeing $\frac{1}{2}$ and $\frac{1}{3}$ added to $\frac{1}{4}$, after the manner of Fractions, make more than the Whole, the $\frac{1}{2}$ of 100 *l.* being 50 *l.* the $\frac{1}{3}$ more $33\frac{1}{3}$ *l.* with $\frac{1}{4}$ which is 25 *l.* added together, make in all 108 *l.* 6 *s.* 8 *d.* But the Intent of the Question being, that every time *A* had $\frac{1}{2}$ Pound, or Shilling, *B* should have $\frac{1}{3}$ of a Pound or Shilling, and *C* $\frac{1}{4}$; So as to every 6 Pence *A* had, *B* should have 4 Pence, and *C* 3 Pence: Wherefore taking $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of 12, a Number which will be equally parted by 2, 3, 4. or multiplying those Consequents together, they make 24; which divided by 2, 3, and 4 severally, giveth 12, 8, and 6: these added together (their Antecedents being Units) make 26; which abbreviated with them, make 6, 4, 3, and together 13, like to the Parts of 12. This 13 shall be the first Number, and the other the third Numbers of the *Rule of Three*, as followeth. *The Ratio of the Data to be noted, and the Intent of the Question.* *How to proceed.*

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ — 8 — 24

$\frac{24}{2}$(12 $\frac{24}{3}$(8 $\frac{24}{4}$(6

12	6 *A*
8	4 *B*
6	3 *C*
26	13

$\frac{12}{2}$(6 $\frac{12}{3}$(4 $\frac{12}{4}$(3 — 13

			l.			l.		
As	13	.	100	::	6	.	$46\frac{2}{13}$	$\frac{600}{13}$($46\frac{2}{13}$ l. *A.*
			6					
			600					
As	13	.	100	::	4	.	$30\frac{10}{13}$	$\frac{400}{13}$($30\frac{10}{13}$ l. *B.*
			4					
			400					
As	13	.	100	::	3	.	$23\frac{1}{13}$	$\frac{300}{13}$($23\frac{1}{13}$ l. *C.*
			3					
			300					

Example

Q. Of a Ship, what each Partner paid.

Example 2. *A*, *B*, *C*, and *D*, bought a Ship for 890 *l.* and were to have their Parts according to these Proportions, *viz.* *A* $\frac{1}{2}$ and $\frac{1}{10}$, *B* $\frac{1}{4}$ and $\frac{1}{10}$, *C* $\frac{1}{6}$, and *D* $\frac{1}{20}$: what must each Man pay?

Answer.

Answ. *A* 457$\frac{5}{7}$ *l.* *B* 267 *l.* *C* 127$\frac{1}{7}$ *l.* and *D* 38$\frac{1}{7}$ *l.*

What to be done when one hath more Parts than one.

Here, and in such Questions where more Parts or Proportions than one belong to one Partner, I may, after the manner of Fractions, add them into one, and work as before, or otherwise: After Division of the Number taken, or the ultimate Product of the Consequents, add the several Quotients belonging to one Partner together for his third Number of the *Rule of Three*; and in this Example, because 60 is a Number that will be equally divided by all the Consequents, as into Halfs, Fourths, Sixths, Tenths and Twentieth Parts, I may take 60 to divide by the Consequents, or in multiplying the Consequents to find a Number, I may omit 2, because the half of 4, and 10 the half of 20, and take only 4, 6, and 20.

$$\overbrace{\tfrac{1}{2}+\tfrac{1}{10}=\tfrac{3}{5}}^{A} \;\cdot\; \overbrace{\tfrac{1}{4}+\tfrac{1}{10}=\tfrac{7}{20}}^{B} \;\cdot\; \overset{C}{\tfrac{1}{6}} \;\cdot\; \overset{D.}{\tfrac{1}{20}}$$

Thus, or thus, $\frac{60}{5}(12 \times 3 = 36\ A.$ $\quad \frac{60}{20}(3 \times 7 = 21\ B.$ $\quad \frac{60}{6}(10\ C.$ $\quad \frac{60}{20}(3\ D.$

36	*A.*
21	*B.*
10	*C.*
3	*D.*
70	

A $\left\{\begin{array}{l}\frac{60}{2}(30 \\ \frac{60}{10}(6\end{array}\right\}36$ As 70 : 890 *l.* :: 36 : 457$\frac{5}{7}$ *l.*

$$\begin{array}{r} 890 \\ 36 \\ \hline 5340 \\ 2670 \\ \hline 3204{,}0 \end{array} \qquad \frac{3204}{7}(457\tfrac{5}{7}\ l.\ A.$$

B $\left\{\begin{array}{l}\frac{60}{4}(15 \\ \frac{60}{10}(6\end{array}\right\}21$ As 70 : 890 :: 21 : 267.

$$\begin{array}{r} 890 \\ 21 \\ \hline 890 \\ 1780 \\ \hline 1869{,}0 \end{array} \qquad \frac{1869}{7}(267\ l.\ B.$$

C $\frac{60}{6}(10$ As 70 : 890 :: 10 : 127$\frac{1}{7}$.

$$\begin{array}{r} 890 \\ 10 \\ \hline 890{,}0 \end{array} \qquad \frac{890}{7}(127\tfrac{1}{7}\ l.\ C.$$

D $\frac{60}{20}(3$ As 70 : 890 :: 3 : 38$\frac{1}{7}$.

$$\begin{array}{r} 890 \\ 3 \\ \hline 267{,}0 \end{array} \qquad \frac{267}{7}(38\tfrac{1}{7}\ l.\ D.$$

70

Q. Of a Sum to be paid.

Example 3. The Sum of 300 *l.* was to be paid by *A*, *B*, and *C*, in such Proportions, that *A* was to pay 6 *l.* more than $\frac{1}{2}$, *B* $\frac{1}{3}$ and 12 *l.* over, and *C* 8 *l.* less than $\frac{2}{3}$: what must each Man pay?

Answer.

Answ. *A* 102$\frac{2}{3}$ *l.* *B* 76$\frac{4}{9}$ *l.* and *C* 120$\frac{8}{9}$ *l.*

When there is Money overplus, or to be abated.

Here, and in those Questions where there is Overplus-Money, or Money to be abated; From the Sum to be divided, the Overplus-Money is to be substracted, and to that Sum the Abatement must be added, and then proceed as before; and to the respective Portions obtained by the *Rule of Three*, add the Overplus-Money to be added, and substract the other to be substracted.

I 2

$$\underbrace{\frac{1}{2}\ \frac{1}{3}\ \frac{2}{3}}_{6} \qquad \frac{6}{2}(3 \qquad \frac{6}{3}(2 \qquad \frac{6}{3}(2 \quad 2 \quad \overline{4} \qquad \begin{array}{rl} 3 & A. \\ 2 & B. \\ 4 & C. \\ \hline 9 & \end{array}$$

A 6 *l.*
B 12 } +
18
C 8 −
10
300 − 10 = 290

As 9 . 290 *l.* :: 3 . 96 $\frac{2}{3}$ *l.*
290 × 3 = 870

$6(6 \quad \frac{870}{9}(96\frac{2}{3}$ *l.*
6 +
102 $\frac{2}{3}$ A.

As 9 . 290 :: 2 . 64 $\frac{4}{9}$
290 × 2 = 580

$4(4 \quad \frac{580}{9}(64\frac{4}{9}$ *l.*
12 +
76 $\frac{4}{9}$ B.

As 9 . 290 :: 4 . 128 $\frac{8}{9}$
290 × 4 = 1160

$28(8 \quad \frac{1160}{9}(128\frac{8}{9}$ *l.*
8 −
120 $\frac{8}{9}$ C.

Example 4. Three Owners of a Ship set her forth; the Charge whereof *A* payeth $\frac{1}{4}$ Part, *B* 300 *l.* and *C* $\frac{9}{16}$ Parts of the Whole: how much Money paid *A* and *C* for their Parts? and what Part of the Ship had *B*? and how much was the whole Charge? *Q. Of the Charge to set out a Ship.*

Answ. *A* paid 400 *l.* *C* 900 *l.* *B* had $\frac{3}{16}$ Parts of the Ship, and the whole Charge was 1600 *l.* Answer.

Here, and in others alike, where several Questions are interwoven, several things are to be done in order to their Resolution: For first the Parts of *A* and *C* being given, added together, and the Total substracted from an Unit, or the Whole, discovers the Part of *B*; that known, seeing the Charge thereof is given, the Charge of the other Parts will be had by the *Rule of Three:* which when found, added to the Charge of *B*, answers the whole Charge. *How to proceed when Questions are interwoven.*

	A.		*C.*		*Sum.*			*Remain.*	
Parts of	$\frac{1}{4}$	+	$\frac{9}{16}$	=	$\frac{13}{16}$	Unit, or $\frac{16}{16}$ − $\frac{13}{16}$	=	$\frac{3}{16}$	Part of *B*.

As $\frac{3}{16}$. 300 *l.* :: $\frac{1}{4}$. 400 *l.*

$$\frac{3}{16}\Big)\frac{300}{4}\Big(\frac{400}{1}\ l.$$ Charge of *A* for $\frac{3}{4}$.

Particular Charges.

A for $\frac{1}{4}$. 400 *l.*
B for $\frac{3}{16}$. 300 *l.*
C for $\frac{9}{16}$. 900 *l.*
Total 1600 *l.* Charge.

As $\frac{3}{16}$. 300 :: $\frac{9}{16}$. 900.

$$\frac{3}{16}\Big)\frac{2700}{16}\Big(\frac{900}{1}\ l.$$ Charge of *C* for $\frac{9}{16}$.

Example 5. Three Merchants have gained 100 *l.* which they divide in such manner, that the $\frac{1}{2}$ Part of *A* was as much as $\frac{5}{8}$ of what *B* had; and $_{8}$ Part of *B* was equal to $\frac{1}{2}$ Part of *C*: how much had each Man for his Part? *Q. Of Gain parted.*

Answ. *A* 50 *l.* *B* 40 *l.* *C* 10 *l.* Answer.

Here, and in such others as before, any Number may be taken that will equally be divided by the Denominators or Consequents; and if none come to hand, by their Multiplication as before such a Number may be had: then accounting that for *A*, take the Parts thereof for *B* and *C*; and the other Partners, if more, according to the Proposition: add all these Numbers together for the first Number *The first Number how gotten.*

 of

of the *Rule of Three*, the Number firſt taken, and the Parts thereof ſhall be the ſeveral third Numbers, and the Sum to be divided the Second, as before.

Denominators $2 \times 8 = 16$.

$A\ 16$. then $\frac{1}{2}$ is $\overset{l.}{8} = \frac{5}{8}\ B$. And as $\overset{B}{\frac{5}{8}}$. $\overset{l.}{8}$ :: $\overset{B}{1}$. $\overset{l.}{12\frac{4}{5}}$. $\frac{5}{8})\frac{8}{1}(\frac{64}{5}\ B$.

$B\ 12\frac{4}{5}$ then $\frac{1}{8}$ is $1\frac{3}{5} = \frac{1}{2}\ C$. And as $\overset{C}{\frac{1}{2}}$. $\overset{l.}{1\frac{3}{5}}$ :: $\overset{C}{1}$. $\overset{l.}{3\frac{1}{5}}$. $\frac{1}{2})\frac{8}{5}(\frac{16}{5}\ C$.

$C\ 3\frac{1}{5}$

32

As 32 . $\overset{l.}{100}$ :: 16 . $\overset{l.}{50}$

$$\begin{array}{r} 100 \\ 16 \\ \hline 1600 \end{array} \qquad \frac{1600}{32}\left(\overset{l.}{50}\ A.\right.$$

As 32 . 100 :: $12\frac{4}{5}$. 40

$$\begin{array}{r} 100 \\ 12\frac{4}{5} \\ \hline 1200 \\ 80 \\ \hline 1280 \end{array} \qquad \frac{1280}{32}\left(\overset{l.}{40}\ B.\right.$$

As 32 . 100 :: $3\frac{1}{5}$. 10

$$\begin{array}{r} 100 \\ 3\frac{1}{5} \\ \hline 300 \\ 20 \\ \hline 320 \end{array} \qquad \frac{320}{32}\left(\overset{l.}{10}\ C.\right.$$

Q. *Of a Legacy.* *Example 6.* A certain Man on his Death-bed, by his laſt Will and Teſtament, bequeathed 360 *l.* to be thus diſpoſed, *viz.* If his Wife, being then with Child, brought forth a Son, then to his Son the $\frac{1}{2}$, and his Wife $\frac{1}{3}$: But if ſhe brought forth a Daughter, then his Wife to have $\frac{1}{2}$, and his Daughter $\frac{1}{3}$. And it happened that the Woman brought forth both a Son and a Daughter. Now the Queſtion is, how the 360 *l.* ſhould be divided, that the Will of the Teſtator ſhould be fulfilled?

Anſwer. *Anſw.* To the Son $170\frac{10}{19}$ *l.* to the Mother $113\frac{13}{19}$ *l.* and to the Daughter $75\frac{15}{19}$ *l.*

How to get the Proportional Numbers. For here, and in ſuch-like Queſtions, Numbers are to be ſought out that bear ſuch Proportion one to another, as the ſeveral Parts propounded, the Intention of the Teſtator being, that the Mother ſhould not have $\frac{1}{3}$ to the Son's Part, and $\frac{1}{2}$ to the Daughter's Part both, ſeeing theſe Parts added together would be $\frac{5}{6}$, and ſo more than the Son's Part: But as the Son was to have $\frac{1}{2}$, ſo the Mother $\frac{1}{3}$, that is proportionally as 3 to 2; whereby the Son is intended to have as much as the Mother, and half as much more; and the like muſt the Mother have to the Daughter. Therefore to find the Proportional Numbers of this Sort, multiply the firſt and laſt Terms or Parts together, as here 2 by 3, which is 6, and this ſhall be the middle Number for the Mother's Portion. Then multiply this middle Number 6 by the leſſer Term 2, and divide the Product 12 by the greater Term 3, and the Quotient ſhall be the leſſer Term, and being here 4, ſhall be for the Daughter's Portion. Again, multiply that middle Number by the greater Term, and divide the Product by the Leſſer, for the greateſt Portion that is here for the Son; ſo 6 multiplied into 3 produceth 18, and this divided by 2 gives 9 in the Quotient. And theſe Numbers found, are the ſeveral third Numbers of the *Rule of Three*, the Total of them the Firſt, and the ſecond Number the Sum to be divided as before.

2x3= 6 Mother.
6x2=$\frac{1}{2}$(4 Daughter.
6x3=$\frac{18}{2}$(9 Son.
19

As 19 . 360 :: 9 . 170$\frac{10}{19}$ l.
3240 (170$\frac{10}{19}$ Son.

As 19 . 360 :: 6 . 113$\frac{13}{19}$ l.
2160 (113$\frac{13}{19}$ Mother.

As 19 . 360 :: 4 . 75$\frac{15}{19}$ l.
1440 (75$\frac{15}{19}$ Daughter

Proof 360

The proper Proof of *Fellowship* (other than such Propositions as are determined by some single Operation of the *Rule of five Numbers*, which have their Proof in common therewith) is to add into one Total the several Quotients that answer the Proposition; which if the Work be right, will return the second Number. As in the last Example 170$\frac{10}{19}$ *l.* 113$\frac{13}{19}$ *l.* and 75$\frac{15}{19}$ *l.* added together, make 360 *l.* the Sum to be divided. *Proof of Fellowship.*

CHAP. IX.

ALLIGATION.

LEAVING *Fellowship*, wherein was a Mixture of Partners and their Stock; the next Subject Derivatives deal with, is the Mixture of Merchandises, as Corn, Wine, Wool, Metals, Medicines, &c. under the Title of *Alligation*. *Alligation mixeth divers Merchandises.*

Alligation is of two kinds, Medial and Alternate: Medial properly seeketh a Mean in the Price, Quantity or Quality between the Extreams. And Alternate altereth the placing of the Differences falling out between the mean Price and the Extreams: And both proposeth the Numbers and Quantities Homogeneal, or reduceth them into such, and properly intendeth simple Mixtures, or those done but once; those often repeated being either Figurals, or continued Proportions; of which more in the last Chapter of this Part, and the fifth Chapter of the next, in Section 6. *Two sorts of Alligation.* *Numbers to be Homogeneal.*

Medial Alligation contains six Propositions. *Medial the Propositions.*

Prop. 1. By the Quantities to be mixed, and the particular Prices, to find the Price or Value of some part of the Mixture. *1. To find the Price of part.*

Multiply the Quantities to be mixed severally by their own Prices, and divide the Sum of these Products by the Sum of the Quantities mixed. *Rule.*

Example 1. A Merchant would mix 100 Bushels of Rye at 4 *s.* the Bushel, with 40 Bushels of Barley at 3 *s.* 6 *d.* the Bushel, and 60 Bushels of Wheat at 6 *s.* the Bushel: and know what one Bushel of that Miscellane would be worth. *Q. of Miscellane.*

Answ. 4 *s.* 6 *d.* For the Quantities 100, 40, and 60, added, make 200, and these Quantities multiplied severally by the particular Prices, make 400, 140, and 360; which Products added, make 900; this divided by 200, gives 4$\frac{1}{2}$ *s.* as before. *Answer.*

Quantities.	*Prices.*		
	Bushels.	*s.*	
Rye	100	x 4	= 400
Barley	40	x 3$\frac{1}{2}$	= 140
Wheat	60	x 6	= 360
	2,00		9,00

Analogy.

As 200 . 900 :: 1 . 4$\frac{1}{2}$

$\frac{9}{2}$(4$\frac{1}{2}$ *s.* Bushel.

Example

Q. Of the Worth of a Cask of Wine.

Example 2. Five Casks that hold 60 Gallons apiece, are to be filled with Wine, *viz.* 75 Gallons of Sack at 4 *s. per* Gallon, and 225 Gallons of White-Wine at 2 *s. per* Gallon: what shall one Cask of this mixed Wine be worth?

Answer. *Answ.* 7 *l.* 10 *s.* For after the Value of 1 Gallon of the Mixture is found, as before, to be 2 *s.* 6 *d.* the Value of 60 Gallons, the Content of 1 Cask, is had by Multiplication.

	Quantit. *Gallons.*		*Prices.* *s.*		
Sack	75	×	4	=	300
White	225	×	2	=	450
	30,0				75,0

(1
7(5 (2½ s. Gallon.
30

Cask.
60 Gallons.
2½
120
30
15|0
l. 7:10 *s.*

2. To find the Quantity.

Prop. 2. By the particular Prices of the Quantities, and Sum paid or received for a Mixture bought or sold; to find what Quantity of each kind was bought or sold.

Rule. Divide the Sum paid or received for the Mixture bought or sold, by the Sum of the particular Prices.

Q. Of Pieces of Money sent for to the Mint.

Example 1. A certain Noble-man sent his Servant with the King's Majesties Warrant to the Mint-master for 4000 *l.* and he must bring it in Pieces of 12 *d.* 6 *d.* 4 *d.* 2 *d.* 1 *d.* and he must bring of each sort of Pieces a like Number: how many of each sort must he bring?

Answer. *Answ.* 38400 Pieces of each sort: For 4000 *l.* brought into Pence, because the other Pieces are in Pence, the Result 960000 *d.* divided by 25, that is 12 + 6 + 4 + 2 + 1, giveth the Answer aforesaid.

Prices.	*Sum paid.*
12 *d.*	4000 *l.*
6	240
4	160000
2	8000
1	960000 *d.*
25	

22
960000 (38400 Pieces.
25
25
25

Q. Of Spice sold, how much of a Sort.

Example 2. A Grocer sold four sorts of Spice, of each a like Quantity, but at several Rates; *viz.* Large Mace at 8 *s.* 4 *d. per* ℔. Cinamon at 6 *s.* 8 *d. per* ℔. Nutmegs at 5 *s. per* ℔. and Ginger at 2 *s. per* ℔. For what he sold, he received 23 *l.* 2 *s.* The Question is, how many Pounds of each sort he sold to make up the said Sum of 23 *l.* 2 *s.*

Answer. *Answ.* 21 ℔. of each sort: For the Prices given added make 22 *s.* and 23 *l.* 2 *s.* reduced into Shillings that they may be Homogeneal, make 462 *s.* which divided by 22 *s.* give 21 ℔ as before.

	Prices.		*Sum received.*
Large Mace	8 *s.*	: 4 *d.*	23 *l.* 2 *s.*
Cinamon	6	: 8	20
Nutmegs	5	: 0	460
Ginger	2	: 0	2
	22	: 0	462

2 ℔.
462 (21 of each Sort.
22
22

3. To increase or lessen a Mixture.

Prop. 3. By the Quantities of a Mixture, to augment or diminish the Mixture proportionally.

Rule. Sum up the Quantities, and then by the *Rule of Three*, as that Sum is to the Augmentation or Diminution: So is the Quantity of each Parcel of the Mixture, to the Quantity of the Mixture desired.

Q. Of increasing an Ointment.

Example 1. In the Ointment called *Unguentum album Camphoratum*, there is put to Oil of Roses ℥ 12, white Wax ℥ 3, Ceruse ℥ 6, and Camphire beaten with Oil of Roses ʒ 2; which reduced all into Drams, make 96 . 24 . 48 . 2. and in the Total 170. And if I would make the Quantity to contain 210 Drams:

Drams: how much of each Ingredient must be taken?

Answ. Oil $118\frac{10}{17}$ ʒ, Wax $29\frac{11}{17}$ ʒ, Ceruse $59\frac{5}{17}$ ʒ, Camphire $2\frac{8}{17}$ ʒ. Answer.

As 170 . 210 :: 96 . $118\frac{10}{17}$ Oil.

```
   96
 1260        2(1
1890         3 4(0
20160        2 0 1 6( 118 10/17
             1 7
```

As 170 . 210 :: 24 . $29\frac{11}{17}$ Wax.

```
   24
  840        (1
 420         1 6(1
 5040        5 0 4( 29 11/17
             1 7
```

As 170 . 210 :: 48 . $59\frac{5}{17}$ Ceruse.

```
   48
 1680        1 5(5
 840         1 0 0 8( 59 5/17
10080        1 7
```

As 170 . 210 :: 2 . $2\frac{8}{17}$ Camphire.

```
   2
  420        (8
             4 2( 2 8/17
             1 7
```

Example 2. A Pectoral Pouder of 10 ℔. is made up with Sugar ℔ 6, Licorish ℔ 2, Anni-seeds ℔ $1\frac{1}{2}$, and Fennel-seeds ℔ $\frac{1}{2}$: But I would make no more than 4 ℔ of the Pouder; how much of the several Ingredients must be taken? Q. *Of lessening a Pouder.*

Answ. Sugar $2\frac{2}{5}$ ℔, Licorish $\frac{4}{5}$ ℔. Anni-seeds $\frac{3}{5}$ ℔, and Fennel-seeds $\frac{1}{5}$ ℔. Answer.

As 10 . 4 :: 6 . $2\frac{2}{5}$ Sugar.

```
     6
10) 24 ( 2 2/5
```

As 10 . 4 :: 2 . $\frac{4}{5}$ Licorish.

```
     2
10)  8 ( 4/5
```

As 10 . 4 :: $1\frac{1}{2}$. $\frac{3}{5}$ Anniseeds.

```
    1 1/2
     4
     2
10)  6 ( 3/5
```

As 10 . 4 :: $\frac{1}{2}$. $\frac{1}{5}$ Fennel-seeds.

```
    1/2
10)  2 ( 1/5
```

Prop. 4. By the Qualities or Nature of the several Ingredients in a Mixture, to find the Temperament or Emergent Quality of the Mixture. 4. *To find the Temperament of a Mixture.*

Dispose the Quantities of the Mixture severally in Rows, setting orderly them their several Qualities; multiply each Quantity by its own Quality. And if the Qualities of the Ingredients be contrary, substract the contrary Qualities so multiplied one from the other, as Hot from Cold, Moist from Dry, *&c.* and set down the Difference of the Products. Then as the Sum of all the Quantities is, to the Difference of the Products, or the Products where no contrary Qualities are: So is an Unit to the Quality emergent, and always of the same Kind with the greater Product where the Qualities are contrary. Rule.

Example 1. A Goldsmith mixeth 20 Portions of Silver of 6 ℥ fine, with 5 of 8 ℥ fine, 5 of 10 ℥ fine, and 10 Portions of Copper; what Fineness shall the melted Mass be of? Q. *Of the fineness of Metal.*

Answ. $5\frac{1}{4}$ ℥. Answer.

	Quantities. *Portions.*		*Fineness.* ℥.		
Silver	20	×	6	=	120
Silver	5	×	8	=	40
Silver	5	×	10	=	50
Copper	10	×	0	=	00
	40				210

As 40 . 210 :: 1 . $5\frac{1}{4}$

```
21    ℥.
 4 ( 5 1/4 Fine.
```

Example 2. If the Species called *Dianthus*, be made according to the *London Dispensatory*, and the Qualities of the Simples taken according to *Sennertus* in his *Institutes*, *Lib.* 5. *Par.* 1. *Chap.* 3. *Parkinson* in his *Herbal*, and other approved Authors: And it be enquired to know the Quality, Emergent or Temperament of the Composition; then disposing orderly the Quantities and Qualities as below, Q. *Of the Quality of Dianthus.*

below, and multiplying respectively the one by the other, and substracting the contrary Qualities one from the other, the Remains left to be divided by the Total of the Quantities, declare the Medicine of a fine Temperament, *viz.* Hot in the first Degree and somewhat above, and Dry not full a Degree.

Answer.

	Ingredients.	*Quantities.*	*Qualities.* *hot.*	*cold.*	*moist.*	*dry.*	*Products.*			
Dianthus	Rosemary Flowers	ʒ 8, or ℈ 24	× 2	0	0	2	48	0	0	48
	Red Roses	ʒ 6, or . 18	× 0	1	0	1	0	18	0	18
	Violets	ʒ 6, or . 18	× 0	1	2	0	0	18	36	0
	Licorish	ʒ 6, or . 18	× 1	0	1	0	18	0	18	0
	Cloves	4	× 3	0	0	3	12	0	0	12
	Indian Spiknard	4	× 1	0	0	2	4	0	0	8
	Nutmegs	4	× 2	0	0	2	8	0	0	8
	Galanga	4	× 3	0	0	3	12	0	0	12
	Cinamon	4	× 2	0	0	2	8	0	0	8
	Ginger	4	× 3	0	0	3	12	0	0	12
	Zedoary	4	× 2	0	0	2	8	0	0	8
	Mace	4	× 2	0	0	2	8	0	0	8
	Wood of Aloes	4	× 2	0	0	2	8	0	0	8
	Cardamoms the Less	4	× 3	0	0	3	12	0	0	12
	Anniseeds	4	× 2	0	0	1	8	0	0	4
	Dillseeds	4	× 2	0	0	3	8	0	0	12
		126					174	36	54	178

Hot. *Cold.*
$174 - 36 = \frac{138}{126}$ (12) ($1\frac{2}{21}$ Hot. As $126 \,.\, 138 :: 1 \,.\, 1\frac{2}{21}$.

Temperament.

Dry. *Moist.*
$178 - 54 = \frac{124}{126}$ ($\frac{62}{63}$ Dry. As $126 \,.\, 124 :: 1 \,.\, \frac{62}{63}$.

When among the Simples there is some Compound.

When in any Composition among the Simples, some compound Ingredient is mixed, then the Temperament of that Compound being first gotten, the other is to be found in like manner. As in Mastick Pills, because among the Simples *Hiera picra* is used, which is a Compound, the Temperament thereof is first found to be Hot in the second Degree, and Dry in the second Degree, and nigh $\frac{3}{4}$ of a Degree more: And then the Temperament of the Pills is found to be almost two Degrees Hot, and above two Degrees and an half Dry.

	Ingredients.	*Quantities.* ʒ	*Qualities.* *hot*	*dry.*	*Products.*	
Hiera picra simple	Cinamon	6	× 2	2	12	12
	Xylobalsamum	6	× 2	2	12	12
	Roots of *Asarabacca*	6	× 3	3	18	18
	Spiknard	6	× 1	2	6	12
	Mastich	6	× 2	2	12	12
	Saffron	6	× 2	1	12	6
	Aloes $12\frac{1}{2}$ ʒ, or	100	× 2	3	200	300
		136			272	372

$\frac{272}{136}$ (2 Hot.

(100) $\frac{372}{136}$ ($2\frac{25}{34}$ Dry.

	Ingredients.	*Quantities.* ʒ	*Qualities.* *hot.*	*dry.*	*Products.*	
Mastich Pills.	Mastich	2	× 2	2	4	4
	Aloes	4	× 2	3	8	12
	Agarick	$1\frac{1}{2}$	× 1	2	$1\frac{1}{2}$	3
	Hiera Simple	$1\frac{1}{2}$	× 2	$2\frac{3}{4}$	3	$4\frac{1}{8}$
		9			$16\frac{1}{2}$	$23\frac{1}{8}$

$\frac{9}{1}$) $\frac{33}{2}$ ($\frac{11}{6}$ ($1\frac{5}{6}$ Hot.

$\frac{9}{1}$) $\frac{185}{8}$ ($\frac{185}{72}$ ($2\frac{41}{72}$ Dry.

Prop.

Prop. 5. By the Quantities of a Mixture, to find the particular Quantity of any Ingredient in any Part of the Mixture.

5. To find the Quantity, &c. in a simple Mixture.

If the Mixture be Simple, or but once, then by the *Rule of Three*,

Rule.

As the Total of the Ingredients in the Composition is, to the Quantity of the Dose, (or part of the Mixture proposed): So is the Quantity of the Ingredient proposed in the whole Composition, to the Quantity of the Ingredient in the Dose.

Q. Of Miscellane.

Example. If 700 Bushels of Wheat be mixed with 100 Bushels of Rye: how much Rye is there in one Bushel of that Miscellane?

Answer.

Answ. $\frac{1}{8}$ of a Bushel, that is a Gallon.

	Bushels.
Wheat	. 700
Rye	. 100
Miscellane	. 800

As 800 . 100 :: 1 . $\frac{1}{8}$

$\frac{1|00}{8|00}\left(\frac{1}{8}\right.$ Rye.

In a compound Mixture, how best to proceed.

But if the Mixture be compound, that is, often repeated, then the best way is to proceed by figural Proportions, as afterward in the 16*th* Chapter. Otherwise thus, by the *Rule of Three*, proceed to find the Quantity desired after the first Mixture as before. Then proceed accordingly to repeat the like Work upon every Mixture till your Desire be obtained.

Common Way.

Q. Of Wine and Water mixed.

Example. A Merchant hath a Piece of Wine of 128 Gallons, out of which he draweth 16 Gallons, and filleth it up again with Water. Again, he draweth out 16 Gallons, and filleth it up again with Water; and the third time doth the like: how much Wine and Water was at last in the Cask?

Answer.

Answ. $85\frac{1}{4}$ Gallons of Wine, and $42\frac{1}{4}$ Gallons of Water: For by the first Draught there was left but 112 Gallons of Wine in the Cask; which filled up with Water, and 16 Gallons of that Mixture drawn, there was 14 of Wine and 2 of Water drawn out: So upon the second Draught there were but 98 Gallons of Wine in the Cask: Then the Cask filled, there must be 30 Gallons of Water to make up 128. And upon the third Draught there were $12\frac{1}{4}$ Gallons of Wine more drawn out, and $3\frac{1}{4}$ of Water; which $12\frac{1}{4}$ taken from 98, leaves $85\frac{1}{4}$ Gallons of Wine as before, the Residue was Water to fill up the Cask.

	Wine.	
	128	
Wine drawn out	16	Water put in.
Wine remaining	112	at the first Draught.
	14	
Wine remaining	98	at the second Draught.
	$12\frac{1}{4}$	
Wine remaining	$85\frac{1}{4}$	at the third Draught.

Wine. *Water.*

$85\frac{1}{4} + 42\frac{1}{4} = 128$

As 128 . 16 :: 16 . 2

16
96
16
128)256(2 Water run out.
14 Wine run out.
16

As 128 . 30 :: 16 . $3\frac{1}{4}$

16
180
30
128)480($3\frac{1}{4}$ Water run out.
$12\frac{1}{4}$ Wine run out.
16

Quantity of each in part, how discovered.

Now if the Question had desired to know the Quantity of Wine or Water in any smaller Quantity of the Mixture than the Whole; as suppose in 12 Gallons, then the Analogy is thus: For

Wine;

Wine; As 128 . $85\frac{3}{4}$ :: 12 . $8\frac{5}{128}$

$$\begin{array}{r} 12 \\ \hline 170 \\ 859 \\ \hline 1029 \end{array} \qquad \frac{1029}{128}\Big(\,8\tfrac{5}{128}\text{ Gallons in 12.}$$

Water; As 128 . $42\frac{1}{4}$ :: 12 . $3\frac{123}{128}$

$$\begin{array}{r} 12 \\ \hline 84 \\ 423 \\ \hline 507 \end{array} \qquad \frac{507}{128}\Big(\,3\tfrac{123}{128}\text{ Gallons in 12.}$$

6. To find the Quantities mixed, tho unequally. *Prop. 6.* By the Total of a Mixture, with the Total Value, and the Values of the ſeveral Ingredients mixed, to find the ſeveral Quantities mixed, though unequally.

Rule. Multiply the Total of the Mixture by the leaſt Value, ſubſtract the Product from the Total Value, and the Remainer is the firſt Dividend: Then take the ſaid leaſt Value from the greateſt valued Ingredient, and the Remainer is the firſt Diviſor. The Quotient of this Diviſion ſhews the Quantity of the higheſt prized Ingredient, the other is the Complement to the Whole. And when more Ingredients than two are in the Compoſition, the Diviſors are the ſeveral Remains of the leaſt Value taken from the other. The Dividends are the Remains left upon the Diviſions till 0 remain there; which will be one ſhort of the Number of Ingredients, and this defective Ingredient is to be ſupplied as a Complement. And in *Diviſion* no more muſt be taken in every Quotient, than that there may be left enough for the other Diviſors, and the laſt to leave 0 remaining.

Example in a Mixture of two Ingredients.

Q. Of Sack mixed. A Merchant mixeth to the Quantity of 128 Gallons of Sack, which he ſelleth for 37 *l.* 3 *s.* in which Mixture was Malaga at 6 *s.* the Gallon, and Sherry at 4 *s.* the Gallon: how many Gallons of each ſort were in the Mixture?

Anſwer. *Anſw.* $115\frac{1}{2}$ Malaga, $12\frac{1}{2}$ Sherry.

Total Mixture. *Total Value.*

$$\begin{array}{r} 128 \\ 4 \\ \hline 512 \end{array}\ \text{Gallons}\left\{\begin{array}{l} 6\,s.\ \text{Malaga.} \\ 4\,s.\ \text{Sherry.} \\ \hline \text{Divisor } 2 \end{array}\right\} \begin{array}{r} 37\,l.\ \ 3\,s. \\ 20 \\ \hline 743 \\ 512 \\ \hline \text{Dividend } 231 \end{array} \qquad \frac{231}{2}\Big(\,115\tfrac{1}{2}\text{ Malaga.}$$

Complement $12\frac{1}{2}$ Sherry.

128

Example, in a Company of three Sexes.

Q. Of Expences of ſeveral Sexes. Twenty four Perſons, Men, Women, and Children, ſpent 37 *s.* 8 *d.* whereof every Man was to pay 2 *s.* every Woman 18 *d.* and every Child 8 *d.* how many of each Sex were there?

Anſwer. *Anſw.* 15 Men, 2 Women, 7 Children.

Company.

Company. 24 / 8 / 192	{ 24 *d.* a Man. / 18 *d.* a Woman. / 8 *d.* a Child. }	*Total Expence.* 37 *s.* 8 *d.* / 12		24—8=16 First Divisor. / 18—8=10 Second Divisor.

74
378
452
192
First Dividend 260

$\frac{\cancel{26}(0}{\cancel{16}}$ (2, $\cancel{10}$) (15 Men.

Second Dividend $\frac{\cancel{20}}{\cancel{10}}$ (2 Women.

Complement 7 Children.

24

Example in a mixt Sale at four Rates.

A Baker sold 12 Loaves of Bread of four Sorts, for 12 Pence, *viz.* Twopenny-Loaves, Penny-Loaves, Halfpenny-Loaves, and Farthing-Loaves: how many of each Sort were there? *Q. Of Loaves of Bread.*

Answ. 4 Twopenny-Loaves, 2 Penny-Loaves, 2 Halfpenny-Loaves, and 4 Farthing-Loaves. Answer.

Loaves.	*q.*	*Total Sum.*		
12 / 1 / 12	{ 8 Twopenny. / 4 Penny. / 2 Halfpenny. / 1 Farthing. }	12 *d.* / 4 / 48 *q.* / 12	8 / 4 / 2 / 1	7 First Divisor. / 3 Second Divisor. / 1 Third Divisor.

First Dividend 36

$\frac{\cancel{36}}{\cancel{7}}$ (8 (4 Twopenny.

Second Dividend $\frac{\cancel{8}}{\cancel{3}}$ (2 (2 Penny.

Third Dividend $\frac{\cancel{2}}{\cancel{1}}$ (2 Halfpenny.

Complement 4 Farthing.

12

In the Questions falling under this Proposition, two things are to be noted. *What to be noted here.*

First, When there is no definite Number of the Species allotted to be had, (as was in the last Example, where the Demand was limited to four Sorts of Loaves) there is no certainty as to the particular Numbers desired: But the Question may oftentimes be resolved by other Numbers than those found out as above. For in the second Example above, if 10 Men, 10 Women, and 4 Children, spend at the Rates aforesaid, the Sum of 37 *s.* 8 *d.* may be paid by them as exactly, as if there be 15 Men, 2 Women, and 7 Children. *1. If the Number allotted be certain or not.*

Secondly, Sometimes a Question is so propounded, that before a final Resolution thereof, some Operations of the *Rule of Three* must precede. *2. If other Work must precede Resolution.*

As a Merchant sold 24 lb of Pepper, Ginger and Sugar for 48 *s.* *viz.* 4 lb of Pepper for 9 *s.* and so much was 6 lb of Ginger valued at; and 12 lb of Sugar was rated as 9 lb of Ginger: how much of each Sort was sold?

Answ. 18 lb of Pepper, 2 lb of Ginger, and 4 lb of Sugar.

	℔		s.		℔		s.			℔		s.		℔		s.	
As	4	.	9	::	1	.	$2\frac{1}{4}$	Pepper.	As	6	.	9	::	9	.	$13\frac{1}{2}$	Ginger.
As	6	.	9	::	1	.	$1\frac{1}{2}$	Ginger.	As	12	.	$13\frac{1}{2}$	::	1	.	$1\frac{1}{8}$	Sugar.

ob. s.

24 {54 a Pound of Pepper. } 48
27 {36 a Pound of Ginger. } 24
168 {27 a Pound of Sugar. } 192
48 96
648 1152 *ob.*
648
First Dividend 504

54—27=27 First Divisor.
36—27= 9 Second Divisor.

1
23|8
504 (℔ 18 Pepper.
27

Second Dividend 18 (2 Ginger.
9
Complement 4 Sugar.
24

Alligation Alternate.

Alternate Alligation, to declare the due Proportion of every Ingredient entring the Mixture, doth alter or change the Places of such Excesses or Differences as fall out between the mean Price and the Extreams, ascribing that to the greater Extream which proceeds from the Lesser, and the contrary. And for better direction, Lines (called *Lines of Combination)* are commonly drawn to link or tie together a Number greater than the common Price to one Lesser; from which the Name of *Alligation* was first borrowed, and afterwards became common also to Questions of Mixture resolved by *Medial Alligation*, though there be no such tying or alligating the Numbers together, as in this called *Alternate*; which from the interchanging of the Differences, was added to that of *Alligation*, to distinguish the Species from that of *Medial*.

Lines of Combination, *what.*

Whence the Name of Alligation.

Theorems.

Necessary Theorems *to the Resolution.*

1. Let every greater Extream be linked with one lesser.
2. When either of the Extreams be Single, and the other Extreams be Plural, the single Extream must be linked to all the rest.
3. If both greater and lesser Extreams are not single, then they may be linked so diversly, that sundry Differences may be taken, and diversities of Answers to the Question, yet all true. But if one of the Extreams be single, there can be but one Answer.
4. The Numbers being linked, take the Difference of each Number from the mean or common Price, and place this Difference against the Number he is linked to alternately.
5. Every Number linked with more than one, must have all the Differences of the Numbers he is linked to set against him.
6. Those Differences resolve the Question, when the Price of every of the Ingredients is given, without their Quantities, and the Demand be to mix them so as to sell a certain Quantity at a mean Rate.
7. But when a Quantity of one, with the Prices of all the Ingredients is given, and the Demand is to know the Quantities of the other Ingredients, then the *Rule of Three* is to be used.
8. And when the Price of every Ingredient is given, without any of their Quantities, and the Demand be to make up a certain Quantity to be sold at a mean Rate, Then all the Differences added together shall be the first Number in the *Rule of Three*; the whole Quantity to be mixed shall be the second Number; and each Difference apart the several third Numbers: And so many Sorts mixed, so many Operations of the *Rule of Three*.
9. A Question may be so propounded, as both sorts of *Alligation* are needful to the Resolution.

Examples,

Examples, where the mean Rate is required according to the 6th Theorem. *Mean Rate required.*

A Merchant hath Wheat at 28 *d.* the Bushel, Rye at 20 *d.* *Barley* at 14 *d.* and Oats at 10 *d.* and would mix the same so as a Bushel of the Miscellane may be sold for 16 *d.* how much of each sort must be taken? Q. *Of Miscellane.*

Answ. Because two of the Extreams are greater than 16, the common Price, and two are lesser, the Numbers may be linked two ways, and the Mixture accordingly different: For either to 6 Bushels of Wheat may be taken 2 of Rye, 4 of Barley, and 12 of Oats: or to 2 of Wheat may be taken 6 of Rye, 12 of Barley, and 4 of Oats. Answer.

	d.		
d. 16	28	6	Wheat.
	20	2	Rye.
	14	4	Barley.
	10	12	Oats.
		24	Differences.

	d.		
d 16	28	2	Wheat.
	20	6	Rye.
	14	12	Barley.
	10	4	Oats.
		24	Differences.

Sugar-Cakes are made with Sugar of 14 *d.* the Pound, Flower at 2 *d.* the Pound, and Eggs at 1 *d.* the Pound: what Quantities of each may be taken to make the Paste worth 6 *d.* the Pound? Q. *Of Sugar-Cakes.*

Answ. Eggs and Flower of each 8 ℔, of Sugar 9 ℔; for the Differences of 1 and 2 from 6, are 4 and 5, which added make 9, belonging to the greater Extream, being single, and so linked to both the lesser Extreams. Answer.

		d.			
Common Price	d. 6	1	8	8	Eggs.
		2	8	8	Flower.
		14	4+5	9	Sugar.
				25	Differences.

Examples, where the Quantities of some Ingredients are required, as in the 7th Theorem. *Quantities of some Ingredients required.*

Ten Bushels of Wheat at 28 *d.* the Bushel, is to be mixed with Rye at 20 *d.* Barley at 14 *d.* and Oats at 10 *d.* how many Bushels of those other Sorts may be taken to afford a Bushel of the Miscellane at 16 *d*? Q. *Of Miscellane.*

Answ. The Extreams being Plural, and Numbers the same in the first Example above, according to the Differences situate by the different linking the Numbers, so shall the Answer be by help of the *Rule of Three*, in the manner following, according to both the above-mentioned Varieties. Answer.

	d.			*Analogy.*	
d. 16	28	6	Wheat.		
	20	2	Rye.	As 6 . 2 :: 10 . $3\frac{1}{3}$	Rye.
	14	4	Barley.	As 6 . 4 :: 10 . $6\frac{2}{3}$	Barley.
	10	12	Oats.	As 6 . 12 :: 10 . 20	Oats.

	d.				
d. 16	28	2	Wheat.		
	20	6	Rye.	As 2 . 6 :: 10 . 30	Rye.
	14	12	Barley.	As 2 . 12 :: 10 . 60	Barley.
	10	4	Oats.	As 2 . 4 :: 10 . 20	Oats.

One hundred Quarts of Canary at 12 *d.* the Quart, are to be mixed with Malaga at 9 *d.* and White-wine at 6 *d.* how many Quarts of the two latter must be taken to sell a Quart of the Mixture at 10 *d*? Q. *Of Sack mixed.*

Answ. Of each 40 Quarts: For one of the Extreams being single, there can be no Variety in linking the Numbers. Answer.

If

d.	d.			
10	12	1+4	5	Canary.
	9	2	2	Malaga.
	6	2	2	White.

Analogy.

As 5 . 2 :: 100 . 40 Malaga.

$$5\overline{)\;\begin{array}{c}100\\ \hline 200\end{array}}\;(40$$

As 5 . 2 :: 100 . 40 White.

Quantities at a mean Rate required. *Examples, where the Quantities are required at a mean Rate, according to the* 8*th* Theorem.

Q. Of Wool mixed. A Clothier is to mingle 156 stone of Wool of several Colours, *viz.* Crimson of 18 *s.* the Stone, Blew of 14 *s.* Green of 11 *s.* and White of 9 *s.* how much of each may be taken to make a Stone of the Mixture worth 12 *s?*

Answer. *Answ.* According to the Differences of the Nuumbers diversly linked, more or less may be taken of each sort: Thus,

s.	s.	
12	9	6
	11	2
	14	1
	18	3
		12

Analogy.

As 12 . 156 :: 6 . 78 White.
As 12 . 156 :: 2 . 26 Green.
As 12 . 156 :: 1 . 13 Blew.
As 12 . 156 :: 3 . 39 Crimson.

s.	s.	
12	9	2
	11	6
	14	3
	18	1
		12

As 12 . 156 :: 2 . 26 White.
As 12 . 156 :: 6 . 78 Green.
As 12 . 156 :: 3 . 39 Blew.
As 12 . 156 :: 1 . 13 Crimson.

Q. Of Silver mixed. A Goldsmith would mix 90 ℔ of Silver, that the Mixture might hold out 9 ℥ Fine; and taketh of sundry Sorts of Silver, as some of 4 ℥ Fine, some of 5, of 6, of 8, with others of 11 and 12 ℥ Fine: how much of each sort may be taken?

Answer. *Answ.* If the 11 ℥ be alligated to 4 and 5, and the 12 ℥ to 6 and 8, then must be taken of each of 4 and 5, the quantity of $7\frac{19}{23}$ ℔, and of each of 6 and 8 the Quantity of $11\frac{17}{23}$ ℔, and of 11 ℥ Fine $35\frac{5}{23}$ ℔, and of 12 ℥ Fine $15\frac{15}{23}$ ℔.

s.	℥		
9	4	2	2
	5	2	2
	6	3	3
	8	3	3
	11	4+5	9
	12	1+3	4
			23

Analogy.

As 23 . 90 :: 2 . $7\frac{19}{23}$ of 4 ℥
The like $7\frac{19}{23}$ of 5 ℥
As 23 . 90 :: 3 . $11\frac{17}{23}$ of 6 ℥
The like $11\frac{17}{23}$ of 8 ℥
As 23 . 90 :: 9 . $35\frac{5}{23}$ of 11 ℥
As 23 . 90 :: 4 . $15\frac{15}{23}$ of 12 ℥
} Fine.

Data diversly alligated. *Other Varieties of alligating the* Data *in the last Example.*

9	4	3	3
	5	3	3
	6	3	3
	8	2	2
	11	1	1
	12	3+4+5	12
		Differences	24

9	4	2+3	5
	5	2	2
	6	2	2
	8	2	2
	11	1+3+4+5	13
	12	5	5
		Differences	29

9

Mean	Rates	Links	Differences
9	4	2+3	5
	5	3	3
	6	2	2
	8	2	2
	11	1+3+5	9
	12	4+5	9
		Differences	30

Mean	Rates	Links	Differences
9	4	3	3
	5	3	3
	6	2+3	5
	8	2+3	5
	11	1+3	4
	12	1+3+4+5	33
		Differences	23

Mean	Rates	Links	Differences
9	4	2	2
	5	2+3	5
	6	2+3	5
	8	3	3
	11	3+4+5	12
	12	1+3+4	8
		Differences	35

Mean	Rates	Links	Differences
9	4	3	3
	5	2+3	5
	6	2+3	5
	8	2+3	5
	11	1+3+4	8
	12	1+3+4+5	13
		Differences	39

Besides these, the Lines of Combination may be otherwise drawn, according as more or less of any one Ingredient is intended to be used: For by *Alternate Alligation* the Quality of a Mixture may be augmented or diminished, or made finer or coarser at pleasure.

Examples, where both Sorts of Alligation *are needful, according to the ninth* Theorem. *Alligation of both Sorts used.*

A Mint-master hath 10 Penny Weights of Gold of 18 Caracts Fine, 20 of 19 Caracts Fine, 100 of 21 Caracts Fine, and 120 of 22 Caracts Fine; and he would mix them so, that every Penny-weight of the Mixture might be 20 Caracts Fine: whether doth he need to mix any Allay therewith; and if any, how much? *Q. Of Gold allayed.*

Answ. First by *Alligation Medial*, the Mixture of the *Data* will be found $21\frac{1}{5}$ Caracts Fine, so as there must be some Allay. Then by *Alligation Alternate* is found, that for every 20 Penny-weights of the Gold must be taken $1\frac{1}{5}$ Penny-weight of Allay: So as for the whole 250 Penny-weights of Gold, must be 15 Penny-weights of Copper, or other Allay. Answer.

Medial.

$$\begin{array}{rcl} 10 \times 18 & = & 180 \\ 20 \times 19 & = & 380 \\ 100 \times 21 & = & 2100 \\ 120 \times 22 & = & 2640 \\ \hline 25{,}0 & & 530{,}0 \end{array}$$

(5
530($21\frac{1}{5}$ Fine.
25
25

Alternate.

$$20 \begin{cases} 21\frac{1}{5} & 20 \text{ Gold.} \\ 0 & 1\frac{1}{5} \text{ Allay.} \end{cases}$$

As 20 . $1\frac{1}{5}$ :: 250 . 15

$$\begin{array}{r} 250 \\ 1\frac{1}{5} \\ \hline 250 \\ 50 \\ \hline 20)\,300\,(15 \text{ Allay.} \end{array}$$

A Physician hath a Medicine compounded of Simples, Hot, Cold, and Temperate; that is to say, 8 ℥ Hot in the third Degree, 1 ℥ Hot in the Second, 1 ℥ Temperate; 2 ℥ Cold in the Second, and 2 ℥ Cold in the Fourth; and would compound the Medicine to make it Hot in $1\frac{1}{2}$ Degree: what Quantities of the Ingredients must be taken? *Q. Of composing a Medicine.*

Answ. By *Medial Alligation*, the Temperament of the Medicine is found to be Hot iu the first Degree; and then as the Numbers be linked together by *Alternate Alligation*, the particular Quantities may be taken to make the Medicine of the desired Temperament. Answer.

Medial.

	Quant.		*Qual.*			
Hot	8	×	3	=	24	26
	1	×	2	=	2	
Temp.	1	×	0	=	0	
Cold	2	×	2	=	4	12
	2	×	4	=	8	
	14					38

$$\frac{14}{14}\Big(\ 1 \text{ Hot.}$$

Alternate.

	Hot	3	$3\frac{1}{2} + 5\frac{1}{2}$	9
	Hot	2	$1\frac{1}{2}$	$1\frac{1}{2}$
$1\frac{1}{2}$	Temp.	0	$\frac{1}{2}$	$\frac{1}{2}$
	Cold	2	$1\frac{1}{2}$	$1\frac{1}{2}$
	Cold	4	$1\frac{1}{2}$	$1\frac{1}{2}$
			Differences	14

The Differences happening to equalize the Quantities, the Differences serve for the Quantities to be taken without farther work.

Proof of Alligation. *Of the* 1, 2, 4 *and* 6 *Propositions Medial.*

The usual Proof of *Alligation* is according to the Species thereof: Those of the First, Second, Fourth and Sixth Propositions of *Medial Alligation*, have their Proof by Multiplication of the Quantities mixed by the several Rates or Qualities thereof before Mixture; and the whole Quantity or Quantities so mixed by the new Rate or Quality; which when the Work is right, will be both equal.

As in the first Example of the first Proposition.

			s.				
Rye	100	×	4	=	400		Bushels 200 whole Quantity.
Barley	40	×	$3\frac{1}{2}$	=	140		$4\frac{1}{2}$ *s.* new Rate.
Wheat	60	×	6	=	360		800
							100
	200				900 *s.*	=	900 *s.* or 45 *l.*

And in the last Example of the second Proposition.

Mace.	*Cinamon.*	*Nutmegs.*	*Ginger.*		
21 ℔.	21 ℔.	21 ℔.	21 ℔.	21 ℔.	Quantity of each.
$8\frac{1}{3}$ *s.*	$6\frac{2}{3}$ *s.*	5 *s.*	2 *s.*	22 *s.*	Total Price.
168	126	105	42	42	
7	14			42	
175	140			462	

175 + 140 + 105 + 42 = 462 or 23 *l.* 2 *s.*

Of the third Medial, and of Alternate.

Those of the third Proposition in *Medial Alligation*, and those also of *Alligation Alternate*, generally are proved as Operations in *Fellowship* before, by adding all the Quotients together, to return the second Number or Total of the Quantities mixed. Nevertheless those of *Alligation Alternate* may be proved, and it seems the better way, as others of the first Proposition in *Alligation Medial*, by multiplying each Quotient by the Rate or Quality thereof before Mixture, to agree with the whole Quantity mixed, multiplied by the new Rate or Quality.

As in the first Example of the third Proposition.

	Oil.		*Wax.*		*Ceruse.*		*Camphire.*		*Total Quantity.*
Quotients	$118\frac{10}{17}$	+	$29\frac{11}{17}$	+	$59\frac{5}{17}$	+	$2\frac{8}{17}$	=	210 Second Number.

And in the first Example of the Work by the 8th Theorem.

	White.		*Green.*		*Blew.*		*Crimson.*		*Total Quantity.*
Quotients	78	+	26	+	13	+	39	=	156 Second Number.

And by the Rates thus.

White	78	×	9 *s.*	=	702		156 Total Quantity.
Green	26	×	11	=	286		12 *s.* New Rate.
Blew	13	×	14	=	182		312
Crimson	39	×	18	=	702		156
	156				1872	=	1872 or 93 *l.* 12 *s.*

Those

Those of the fifth Proposition, and others depending on Proportions, admit of Proof with them by reversing the Question. *Of the fifth Medial.*

As in the first Example of the fifth Proposition.

	Bushel of Miscellane.		*Rye.*		*Bushels of Miscellane.*		*Rye.*
If	1	contain	$\frac{1}{8}$:	How much Rye is contain'd in	800 ?	*Answ.*	100.

$$\text{As } 1 \;.\; \tfrac{1}{8} \;::\; 800 \;.\; 100 \qquad \frac{800}{8}\Big(100$$

Those resolved by both Sorts of *Alligation*, *Medial* and *Alternate*, have their Proofs respectively where any Difference is. *Of Both.*

CHAP. X.

Barter and Exchange.

BOTH *Barter* and *Exchange* agreeing essentially, are placed together in this Chapter: For *Barter* is but an Exchange of Wares or Merchandises one for another: And Exchange a *Barter* of one sort of Money or Merchandise for another; or the same Merchandise by the Accompt, Weight or Measure of another Countrey. *Barter and Exchange, how they agree.*

Barter, (vulgarly called *Truck* and *Scosing*) and the Concerns thereof relating to the Exchange of one Commodity for another, so as the Merchant may save his own, have Part in Money, or get some Overplus by the Bargain, may be comprised under the 10 following *Cases*. *Barter how called.* *10 Cases thereof.*

Case 1. If the Price of both Commodities be given, to know how much of one Commodity may be given for any Quantity of the other. *1. One Quantity for another.*

By the *Rule of Three*, get the Total Value of the Quantity to be exchanged, and afterward by another Operation of the same Rule, get the Quantity desired. Rule.

Example. *A* and *B* barter; *A* hath 24 Broadcloths, at 10 *l.* the Piece; *B* hath Wheat at 5 *s.* the Bushel: how much Wheat will pay for the Cloth? *Q. Of Wheat for Cloth.*

Answ. First the Value of the Broadcloths is found to be 240 *l.* then at 5 *s.* the Bushel, 240 *l.* will buy 960 Bushels: And so much Wheat ought *A* to have, or else will lose by the Bargain. Answer.

	Cloth.	*l.*	*Cloths.*	*l.*		*s.*	*Bushel.*	*l.*	*Bushels.*
As	1 .	10 ::	24 .	240	As	5 .	1 ::	240 .	960
		24						20	
		240					5)	4800 (	960

Case 2. If the Price in ready-Money, and *Barter*-Price of one be given, to know the *Barter*-Price of the other, and how much at that Price of the one Commodity may be given for any Quantity of the other. *2. Barter-price of one for the other.*

See what is gained on the Shilling, Pound, Hundred, &c. by the one Party: then by the *Rule of Three* the Gain or Overplus of the other is found, as also the quesited Quantity. Rule.

Example. *A* and *B* barter, *A* hath Raisins at 30 *s.* *per* C. ready Money; but in *Barter* will sell for 40 *s.* *per* C. *B* hath Sugar at 12 *d.* *per* lb. ready Money, but would gain proportionally to the other: how therefore must *B* rate his Sugar in *Barter*? and how many lb. of Sugar may he give for 4 C. of Raisins? *Q. Of Sugar for Raisins.*

Answ. The Gain found by the Question to be 10 *s.* in 40 *s.* the first Question stands thus: If 30 *s.* gain 10 *s.* what shall 1 *s.* which is the ready-Money Price of the Sugar? and by the Work 4 *d.* is gained: So must the *Barter*-price of the Sugar be 16 *d.* whereby the first Question is answered. Then if 16 *d.* buy 1 lb. of Sugar, 8 *l.* (the *Barter*-Price of 4 C. of Raisins) shall buy 120 lb. of Sugar; which answereth the second Question. Answer.

As 30 s. . 10 s. :: 1 s. . $0\frac{1}{3}$ s. or As 30 s. . 40 s. :: 1 s. . $1\frac{1}{3}$ s.

	s.		s. d.
A Ready-Money Price	30	*B* Ready-Money Price	1 . 0
Barter Price	40	Barter Price	1 . 4
Difference	10	Difference	0 . 4

As 1 C. . 40 s. :: 4 C. . 160 s. As $1\frac{1}{3}$ s. . 1 ℔. :: 160 s. . 120 ℔. Sugar.

3. *Ready-Money Price of one for another.* *Case* 3. If the Price in Ready-Money, and *Barter* of one Party with the *Barter*-price of the other be given, to know the Ready-Money Price of the other, and how much at that Price of the one will countervail a Quantity of the other Commodity.

Rule. Find by the *Rule of Three* the Ready-Money Price desired; and by another Work of the same Rule the Quantity sought.

Q. *Of Salt for Wine.* *Example.* *A* hath Salt at 4 *s.* the Bushel Ready-Money; but in *Barter* will have 4 *s.* 6 *d.* and will exchange with *B* for Wine at 18 *l.* the Tun: how is the Wine rated in Ready-Money? and how much Salt shall *B* have for 3 Tuns of Wine?

Answer. *Answ.* By resolving the first Question, 16 *l.* is found to be the Ready-Money Price of the Wine: And by resolving the second Question, for 3 Tuns of Wine *B* shall receive 240 Bushels of Salt.

As $4\frac{1}{2}$ s. . 4 s. :: 18 l. . 16 l.   Tuns 3 × 18 = 54 l.   As $4\frac{1}{2}$ s. . 1 Bushel. :: 54 l. . 240 Bushels.

18 × 20 = 360; × 4 = 1440

$\frac{9}{2}\Big)\frac{1440}{1}\Big(\frac{2880}{9}\Big(\frac{32{,}0}{16{:}0\ s.}$

54 × 20 = 1080

$\frac{9}{2}\Big)\frac{1080}{1}\Big(\frac{2160}{9}\Big($ 240 Bushels.

4. *Gains on the 100, which the most.* *Case* 4. If the Rates, both in ready Money and *Barter*, of both Parties be given; to know the Gains of each upon the Hundred, and which is the greatest Gainer.

Rule. After the Gains of each Party on the 100 is found by the *Rule of Three*; substract the one from the other, and the Difference shall be the Gain of one Party above the other.

Q. *Of Figs for Ginger.* *Example.* *A* and *B* will *barter*: *A* hath Figs at 24 *s.* *per* C. ready Money; but in *Barter* will have 30 *s.* *B* hath Ginger at 4 *l.* 5 *s.* *per* C. but in *Barter* will have 4 *l.* 15 *s.* *per* C. how much did each gain on the 100 by the *Barter*? and which must have Money of the other to ballance the *Barter*, and how much?

Answer. *Answ.* *A* will be found to gain 25 *l.* on the 100, and *B* but 11 *l.* 15 *s.* $\frac{5}{17}$; it follows therefore *A* is the greatest Gainer. And to ballance the *Barter*, *B* must have of *A* one half of the Difference, 13 *l.* 4 *s.* 8 *d.* $\frac{8}{17}$.

As 24 s. . 6 s.gain. :: 100 l. . 25 l.gain.

100 × 20 = 2000; × 6 = 12000

24)12000(50,0 / 25:0 l. *A* gains *per* 100

	l.	s.	d.	
A .	25	0	0	gains.
B .	11	15	$3\frac{9}{17}$	gains.
	13	4	$8\frac{8}{17}$	Difference.
	6	12	$4\frac{4}{17}$	half.

As $4\frac{1}{4}$ l. . $\frac{1}{2}$ l.gain. :: 100 l. . $11\frac{13}{17}$ l.gain.

$\frac{17}{4}\Big)\frac{100}{2}\Big(\frac{200}{17}\Big($ $11\frac{13}{17}$ l. *B* gains *per* 100

Case

Case 5. If the Rates, both in ready Money and *Barter*, of one Party be given; and he will have a part of his *Barter*-Price in ready Money; to know how the other Party may rate his Goods to be equal in the *Barter*. 5. Barter-Price and ready Money of one for the other.

Subtract the demanded Part from the *Barter*-Price, and the other Price, and with these two Remains, and the other Party's ready-Money Price, commit the Work to the *Rule of Three*. Rule.

Example. *A* hath Kerseys at 15 *l.* ready Money, but in *Barter* will have 18 *l.* and besides will have $\frac{1}{3}$ of his *Barter*-price in ready Money: *B* hath Linen at 3 *s.* *per* Ell ready Money: how shall *B* rate his Linen to be equal with *A*? Q. Of Kersey for Linen.

Answ. Taking 6, which is $\frac{1}{3}$ of 18, from 15 and 18, the Remains 9 and 12 are the first and second Numbers of the *Rule of Three*, and 3 *s.* the Third; by the Work whereof it appears *B* must rate his Linen at 4 *s.* *per* Ell. Answer.

$\frac{1}{3}$ of 18 = 6   15 − 6 = 9
                18 − 6 = 12

	l.	*l.*	*s.*	*s.*
As	9 .	12 ::	3 .	4
	20	20		
	180 *s.*	240 *s.*		
		3		
		720		

$\frac{72}{18}$(4 *s.* for 1 Ell of Linen.

Case 6. If both the Price in ready Money and *Barter* of one Party be given, and a Part of his *Barter*-Price in ready Money desired; to know how the other may ballance the *Barter*, and gain a Sum on the 100. 6. Both Prices, and part Money, to gain a Sum on the 100.

Take as in the last *Case* the Part desired from both the Prices given, and find, by the *Rule of Three*, the Advance of the other Party's ready-Money Price, according to the Sum to be gained on the 100; this Number found, with the other Remains, commit to another Work of the *Rule of Three*. Rule.

Example. *A* hath Stuffs which he rateth at 25 *s.* the Piece ready Money; but in *Barter* will have 30 *s.* and will have $\frac{1}{4}$ of his *Barter*-Price in ready Money. *B* hath Stockins at 40 *s.* the Dozen ready Money, and would ballance the *Barter*, and gain 10 *l.* *per Cent.* how shall *B* rate the Stockins in *Barter*? Q. Of Stuffs for Stockins.

Answ. Substracting 7 *s.* 6 *d.* which is $\frac{1}{4}$ Part of 30 *s.* from 25 and 30, the Remains are 17 *s.* 6 *d.* and 22 *s.* 6 *d.* And finding the Gain of 40 *s.* at the rate of 10 *l.* *per Cent.* to be 4 *s.* it appears by the *Rule of Three*, *B* must rate his Stockins at $2\frac{29}{35}$ *l.* the Dozen, or reduced lower at 2 *l.* 16 *s.* $6\frac{6}{7}$ *d.* Answer.

       s.   *s.* *d.*
$\frac{1}{4}$ of 30 = 7 : 6

s.   *s.* *d.*   *s.* *d.*
25 − 7 : 6 = 17 : 6
30 − 7 : 6 = 22 : 6

	l.	*l.*	*l.*	*l.*
As	100 .	110 ::	2 .	$2\frac{1}{5}$

$$\begin{array}{r} 2 \\ 100)\overline{220}(2\frac{1}{5} \end{array}$$

	l.	*l.*	*l.*	*l.*	
As	$\frac{7}{8}$.	$1\frac{1}{8}$ ::	$2\frac{1}{5}$.	$2\frac{29}{35}$	Stockins rated *per* Dozen.

$\frac{7}{8}$)$\frac{99}{40}$($\frac{99}{35}$($2\frac{29}{35}$

Case 7. If besides the different Rates given of one Party, he would gain a Sum on the 100, and have a Part ready Money; to know how the other Party shall ballance the *Barter*. 7. Rates and ready Money of one, Gain on the 100 for the other.

Enquire, by the *Rule of Three*, the Gains upon the 100, after the *Barter*-Price; from which take the Part demanded in ready Money, as also from the *Barter*-Price; and with these two Remains, and the other's ready-Money Price, commit the Work to the *Rule of Three*. Rule.

Example. *A* and *B* will *barter*: *A* hath Pease at 2 *s.* *per* Bushel ready Money; but in *Barter* will have 2 *s.* 6 *d.* and will gain 10 *l.* *per Cent.* and have $\frac{1}{3}$ of his over-price in ready Money. *B* hath Flax at 7 *d.* *per* ℔. how shall he rate his Flax to ballance the *Barter*? Q. Of Pease for Flax.

Answ. At 10 *l.* *per C.* 2 *s.* 6 *d.* will be 2 *s.* 9 *d.* of which $\frac{1}{3}$ is 11 *d.* which taken from both, the other Question will stand thus: If 19 *d.* ready Money make Answer.

22 *d.* in *Barter*; what will 7 *d.* in ready Money? The Answer to which is, $8\frac{2}{19}$ *d.* and so must *B* rate his Flax in *Barter.*

	l.	*l.*	*s.*	*d.*	*s.*	*d.*
As	100	. 110 ::	2	: 6 .	2	: 9

$\frac{1}{3}$ of 2 *s.* : 9 *d.* or 33 *d.* = 11 *d.*

$$\begin{array}{r} 240 \\ \hline 240,00 \end{array} \qquad \begin{array}{r} 12 \\ \hline 30 \\ 110 \\ \hline 33,00 \end{array} \qquad \frac{33}{240}\Big(\tfrac{11}{80}\ l. \text{ or } 2\ s. : 9\ d.$$

s.	*d.*				*d.*
2	: 6	—	11	=	19
2	: 9	—	11	=	22

	d.	*d.*	*d.*	*d.*	
As	19 .	22 ::	7 .	$8\frac{2}{19}$	The Rate in *Barter* of 1 ℔. of Flax.

$$\begin{array}{r} 7 \\ \hline 19)154\big(8\tfrac{2}{19} \end{array}$$

8. *Rates of one, Mony and Goods of the other.* Rule.

Case 8. If the different Rates of one Party be given, and the other will have some Money and some Goods, to know the Quantity of the latter.

To the Total Value of the Price in ready Money and *Barter* severally, add the Sum to be paid ready down; by these Totals, with the ready-Money Price of the other, will be found the *Barter*-Price, with which the Quantity of Goods by another Work of the *Rule of Three* will be had accordingly.

Q. Of Ashes for Allum.

Example. A hath 20 Tuns of Ashes, at 54 *l.* the Tun ready Money; but in *Barter* rateth them at $55\frac{1}{2}$ *l.* *B* hath Allum at $12\frac{1}{4}$ *l.* the C. ready Money, and he will have 360 *l.* ready Money of *A*: how must *B* rate the Hundred of Allum in *Barter?* and how much Allum must he deliver for the said 20 Tuns of Ashes, and 360 *l.* in Money?

Answer.

Answ. The ready-Money Price of the Ashes found to be 1080 *l.* and the *Barter*-price 1110 *l.* to both which 360 *l.* added, makes 1440 *l.* and 1470 *l.* And if 1440 *l.* give 1470 *l.* then shall $12\frac{1}{4}$ *l.* give $12\frac{97}{192}$ *l.* the *Barter*-Price of the Allum. Then if $12\frac{97}{192}$ *l.* be for 1 C. of Allum, 1470 *l.* shall be for $117\frac{27}{49}$ C. which is the Quantity desired, and to be delivered for the 20 Tuns of Ashes, with 360 *l.* in Money.

	Tun.	*l.*	*Tun.*	*l.*			
As	1 .	54 ::	20 .	1080	Ready Money	Ashes.	1080 + 360 = 1440
As	1 .	$55\frac{1}{2}$ ::	20 .	1110	*Barter*-Price		1110 + 360 = 1470

	l.	*l.*	*l.*	*l.*	*Allum.*
As	1440 .	1470 ::	$12\frac{1}{4}$.	$12\frac{97}{192}$	*Barter*-Price.

	l.	*C.*	*l.*	*C.*	
As	$12\frac{97}{192}$.	1 ::	1470 .	$117\frac{27}{49}$	Allum.

9. *Time for delivery: how to rate the other.* Rule.

Case 9. If the Proposition include some Time for delivery of the Goods, and propounds the Rates of the one Party, to know how to rate the Goods of the other Party:

State the Question for Resolution by the *Rule of five Numbers*, and what is gotten thereby add to the given Price.

Q. Of Wine for Sugar.

Example. A hath Wine at 24 *l. per* Tun in Money, but in *Barter* 27 *l.* to be delivered at 3 Months. *B* hath Sugar at 5 *l. per* Hundred in Money, to be delivered at 6 Months: how shall *B* rate his Sugar in *Barter?*

Answer.

Answ. At $6\frac{1}{4}$ *l. per* Hundred, for so it will be when $1\frac{1}{4}$ gotten by the Work is dded to the Price given.

	l.	*Months.*	*l.*	*l.*	*Months.*	*l.*
As	24 .	3 .	3 ::	5 .	6 .	$1\frac{1}{4}$

$$\underbrace{24 \cdot 3}_{72} \qquad \underbrace{3 \cdot 5 \cdot 6}_{\begin{array}{c}18 \\ \hline 90\end{array}} \qquad \begin{array}{r}(18 \\ \frac{90}{72}\end{array}\Big(\ 1\tfrac{1}{4}\ l. + 5\ l. = 6\tfrac{1}{4}\ l.$$

l.		
24	Ready-Money Price	of the Wine.
27	*Barter*-Price	
3	Difference.	

l.		
5	Ready-Money Price	of the Sugar.
$6\frac{1}{4}$	*Barter*-Price	
$1\frac{1}{4}$	Difference.	

Case

Case 10. If the Proposition propound a Time for delivery with the *Barter*-Price of both, and the ready-Money Price of one Party, with part of his over-Price; to know the ready-Money Price of the other. — *10. Time and Price of one, for the Price of the other.*

Proceed according to the State of the Questions by the several Cases needful to the Resolution, as in other-like mixture of Questions, *mutatis mutandis*. — Rule.

Example. *A* hath Hemp at 16 *l.* the C. in Money, in *Barter* at 20 *l.* the C. to be delivered at 4 Months, and will have $\frac{1}{4}$ of his *Barter*-Price in ready Money. *B* hath Saffron in *Barter* 10 *l.* $\frac{9}{11}$ *per* ℔. at 6 Months to be delivered: what is the Saffron worth in ready Money? — Q. *Of Hemp for Saffron.*

Answ. 7 *l. per* ℔. For by the 5th *Case* the $\frac{1}{4}$ of the *Barter*-Price, which is 5 *l.* is taken from 16 and 20: Then the Difference of the two Remains with the Times gets a Number to be added to the ready-Money Price, after the $\frac{1}{4}$ Part is taken from it; which with the *Barter*-Price of the Saffron, the ready-Money Price of the Saffron is gotten by the third *Case*. — Answer.

	l.		*l.*		*l.*
Ready Money	16	—	5	=	11
Barter-Price	20	—	5	=	15
			Difference		4

$\frac{1}{4}$ of 20 = 5

	Mon.		*l.*		*Mon.*		*l.*
As	4	.	4	::	6	.	6

11 + 6 = 17

As 17 . 11 :: 10 $\frac{9}{11}$. 7

$$\begin{array}{r} 10\frac{9}{11} \\ \hline 110 \\ 9 \\ \hline 17)\,119\,(\,7 \end{array}$$

7 *l.* Ready-Money Price of the Saffron

Exchange of one sort of Money or Merchandise for another, or the same by different Accompts, and the Necessaries thereof, proper for this Place, though placed by some under *Reduction*, as in *Reduction of Geodeticals* was before noted, may be comprised under the two following *Cases*. — Exchange. *Where placed by some. Two Cases thereof.*

Case 1. If the Proposition be single, as to the Exchange of one sort of Money or Merchandise for another; or the same Merchandise by different Weights, Measures, &c. — *1. Proposal single.*

When the Proportion of one to the other is not readily known, or cannot be reduced to Digits or small Numbers, whereby the Parts may be easily taken, commit the Work to the *Rule of Three*. — Rule, *without Reduction.*

Example. A Merchant transporteth from *London* to *Bourdeaux* 13 C. Weight of Copperas, and the same is exchanged at *Bourdeaux* for 13 C. of Prunes their Weight: The Question is, how much the Prunes weighed *London*-weight. — Q. *Of Copperas for Prunes.*

Answ. Because the Proportion between the Weights of *London* and *Bourdeaux* being as 104 to 94$\frac{3}{4}$, is not easy to be brought to small Parts, therefore the Question is committed to the *Rule of Three*, by which is gotten 1426$\frac{346}{379}$, which divided by 112 the Pounds in 1 C. *Averdupois*, there is found but 12 C. 82$\frac{346}{379}$ ℔ at *London*. — Answer.

	Bourdeaux.		*London.*		*Bourdeaux.*
As	94$\frac{3}{4}$ ℔.	.	104 ℔.	::	1300 ℔.

$$\begin{array}{r} 1300 \\ \hline 31200 \\ 104 \\ \hline 135200 \\ \hline \end{array} \qquad \frac{379}{4}\Big)\,\frac{135200}{1}\Big(\,\frac{540800}{379}\Big(\,1426\tfrac{346}{379}\ \text{℔. } \textit{London.}$$

When the Proportions are easy to be reduced to small Numbers, the Resolution may be had by *Practice*, or the *Rule of Three*. — Rule *with Reduction.*

Examples in Measure.

A Merchant buyeth at *Antwerp* 1440 Ells of Silks, and selleth the same at *London*: how many Ells shall he make out by *London*-Measure? — Examples in Measure. Q. *Of Silks* Antwerp *Measure sold at* London.

Answ. Because by the first Chapter of *Geodeticals* 100 Ells *Antwerp* make but 60 Ells *London*, the Proportion is soon espied to be as 5 to 3. If I multiply 1440 by 3, and divide the Product by 5; or else add $\frac{3}{5}$ of the Number together; or otherwise substract $\frac{2}{5}$ therefrom; either Way make 864, the Number of Ells at *London*. — Answer.

Antwerp.		*London.*		*Antwerp.*		*London.*
As 5 Ells	.	3 Ells	::	1440 Ells.	.	864 Ells.

$$5)\overline{4320}(864 \quad (1440 \times 3)$$

Otherwise by Practice. $\frac{1}{5}$ of 1440 = 288, $\times 3$ = Ells 864 *London.*

As 100 . 60 :: 1440 . 864

$$100)\overline{864|00}(864 \quad (1440 \times 60)$$

$\frac{2}{5}$ = 576; 864 Ells *London.*

London *Ells turned into* Antwerp.

On the contrary, to turn Ells *English* into Ells *Antwerp*, add $\frac{2}{3}$ of the Number thereto.

As if it were desired to know how many Ells *Antwerp* 864 Ells *London* would make; $\frac{2}{3}$ thereof added thereto, make 1440 Ells *Antwerp*, as before.

864 Ells *London.*
$\frac{2}{3}$ { 288, 288 }
1440 Ells *Antwerp.*

Q. *Of Cloth, whether any lost.*

A Merchant buyeth at *London* 738 Yards of Cloth, and at *Antwerp* maketh out but 980 Ells? whether did he lose any by the Way?

Answer. *Answ.* The Proportions being as 75 to 100, or 3 to 4 by the *Rule of Three*, or *Practice*, by adding $\frac{1}{3}$ of 738 thereto, there should be 984 Ells made out at *Antwerp*; so as 4 Ells *Antwerp* were lost.

London.		*Antwerp.*		*London.*		*Antwerp.*
As 3 Yards	.	4 Ells	::	738 Yards	.	984 Ells.

$$3)\overline{2952}(984 \quad (738 \times 4)$$

As 75 . 100 :: 738 . 984

$$75)\overline{73800}(984 \quad (738 \times 100)$$

Otherwise by Practice.
738 Yards *London.*
$\frac{1}{3}$ 246
984 Ells *Antwerp.*

Antwerp *Ells turned into* London *Yards.*

On the contrary, to turn Ells *Antwerp* into Yards *London*, the Proportion being as 4 to 3, take $\frac{3}{4}$ of the *Antwerp*-Measure, either by substracting $\frac{1}{4}$ thereof, or taking half the given Number, and adding thereto half that half.

As if it were desired to know how many Yards *London* 984 Ells *Antwerp* would make: either 246, which is $\frac{1}{4}$ thereof, taken away, leaves the Remain 738; or $\frac{3}{4}$ of the given Number is taken, which is all alike.

984 Ells *Antwerp.*
$\frac{1}{4}$ 246
738 Yards *London.*

984 Ells *Antwerp.*
$\frac{1}{2}$ 492 }
$\frac{1}{4}$ 246 }
738 Yards *London.*

Examples in Money.

Examples in Money.

Q. *Of* Sterling *and* French *Money.*

A Merchant delivereth to the Exchangers at *London* 300 l. *Sterling*, after 54 *d.* the French Crown, (that is 3 *Liures*) and taketh a Bill to receive at *Paris* 60 *Sols Tournois* for every Crown: how much *Tournois* or French Money will pay the said Bill?

Answer. *Answ.* By the *Rule of Three* 4000 *Liures*: and so by *Practice*, if 300 *l.* be brought into Shillings, and $\frac{1}{3}$ taken away, because 54 *d.* to 3 *Liures* in less Terms, is as 18 *d.* to 1; that is, $1\frac{1}{2}$ *s.* for 1 *Liure.*

Sterl. Liure. Sterl. Liures.
As $1\frac{1}{2}$ *s.* . 1 :: 300 *l.* . 4000
20
6000

1 2000
$\frac{3}{2}$) $\frac{6000}{1}$ ($\frac{4000}{1}$

d. *Liures.* *l.* *Liures.*
As 54 . 3 :: 300 . 4000
240
72000
3
54) 216000 (4000

Otherwise by Practice.
l. *s.* *s.*
300 × 20 = 6000 *Sterling.*
$\frac{1}{3}$ 2000
4000 *Liures.*

If in *Sols* 80000
But in Crowns $1333\frac{1}{3}$

On the contrary, to turn French Money into English: So much as the *Liure* is valued above an English Shilling, convert into Parts one or more of a Shilling, and add thereto, and the Total is Shillings *Sterling*. *French Money turned into English.*

As if 4000 *Liures* at 18 *d.* a piece were to be turned into *Sterling* Money, because 18 *d.* is $1\frac{1}{2}$ *s.* that is $\frac{1}{2}$ above a Shilling, $\frac{1}{2}$ of 4000 is added thereto: So is the Total 6000 *s.* or 300 *l.* as before.

But if the *Liure* be rated at 20 *d.* then 8 *d.* above a Shilling being $\frac{2}{3}$ of a Shilling, $\frac{2}{3}$ of 4000 is to be added thereto; and the Total will be $6666\frac{2}{3}$ *s.* or 333 *l.* 6 *s.* 8 *d.*

4000 *Liures.*
$\frac{1}{2}$ 2000 at 18 *d.* per *Liure.*
600,0 *s.*
l. 300:0 *Sterling.*

4000 *Liures.*
$\frac{2}{3}$ $\left\{\begin{matrix}1333\frac{1}{3}\\1333\frac{1}{3}\end{matrix}\right\}$ at 20 *d.* per *Liure.*
$666,6\frac{2}{3}$ *s.*
l. 333:6:8 *d. Sterling.*

A Merchant in *Spain* taketh up 300 Pieces of Eight, at 4 *s.* 4 *d.* a Piece, and payeth for the same in *London* 64 *l.* whether hath he paid his due? *Q. Of Spanish Money paid in London.*

Answ. 300 Pieces of Eight, at 4 *s.* 4 *d.* amount to in *Sterling* Money 65 *l.* by the *Rule of Three*, or *Practice*, multiplying by the Shillings, and adding $\frac{1}{3}$ for the 4 *d.* So it appeareth he paid 20 *s.* too little. *Answer.*

Piece. Sterl. Pieces. Sterl.
As 1 . $4\frac{1}{3}$ *s.* :: 300 . 65 *l.*
$4\frac{1}{3}$
1200
100
130,0 *s.*
l. 65:0 *s. Sterl.*

Piece. *d.* Pieces. *l.*
As 1 . 52 :: 300 . 65
300
15600
22
15600 (65 *l. Sterl.*
240

On the contrary, to turn English Money into Spanish, commit the Question to the *Rule of Three*, because the Proportions are as 60 to 13, uneasy Parts to take of some Numbers that may be given. *English Money turned into Spanish.*

As if 65 *l. Sterling* were to be turned into Pieces of Eight, at 4 *s.* 4 *d.* the Piece, the Work would be thus.

s. *Piece.* *l.* *Pieces of 8.*
As $4\frac{1}{3}$. 1 :: 65 . 300
20
$\frac{13}{3}$ 1300

1 100
$\frac{13}{3}$) $\frac{1300}{1}$ ($\frac{300}{1}$ Pieces of Eight.

Case 2. If the Proposition be double, as to express or include two Questions therein: *2. Proposal double.*

Rule. Commit the Work of either to the *Rule of Three*, or *Practice*, as before, or both to the *Rule of five Numbers*, as the State of the Question will admit.

Q. *Of Braces and Ells.* *Example.* Seeing by *Geodeticals* 100 Ells of *Antwerp* are equal to 108 Silk Braces at *Venice*, and 100 Ells of *Antwerp* make 60 Ells at *London*: how many Braces *Venice* are in 3648 Ells *London*?

Answer. *Answ.* $6566\frac{2}{5}$ Braces, as by the following Works appear.

By the Rule of Three.

London.	*Antwerp.*	*London.*	*Antwerp.*
As 3 Ells .	5 Ells ::	3648 Ells .	6080

$3648 \times 5 = \frac{18240}{3}$ (6080

Antwerp.	*Venice.*	*Antwerp.*	*Venice.*
As 100 Ells.	108 Braces ::	6080 Ells.	$6566\frac{2}{5}$ Braces.

$6080 \times 108 = \frac{656640}{100}$ ($6566\frac{2}{5}$

By the Rule of five Numbers.

Venice.	*London.*	*Antw.*	*Lond.*	*Antw.*	*Venice.*
As 108 .	3648 .	100 ::	60 .	100 .	$6566\frac{2}{5}$

108
29184
3648
393984,00

60,00

$\frac{39398(4}{6}$ ($6566\frac{2}{5}$

Or omitting the superfluous Numbers.

Lond.	*Venice.*	*Lond.*	*Venice.*
As 60 .	108 ::	3648 .	$6566\frac{2}{5}$

$\frac{393984}{6}$ ($6566\frac{2}{5}$

Of Millan Braces. But if the Demand had been of *Millan* Braces for Linen, and not for Silk; then because 60 Ells *London*, or 100 Ells *Antwerp* are equal to 120 Braces used at *Millan* for Linen Cloth, there needed nothing but to double the 3648 Ells given, and 7296 had been the Braces desired.

And on the contrary, to take $\frac{1}{2}$ the *Millan* Braces for Linen, you have forthwith the *London* Ells.

Nevertheless in the Silk Braces at *Millan*, because 141 of them answer to 60 Ells of *London*, which will not be reduced to small Numbers, and so in others of like sort, it is best to work by the *Rule of Three*, or *Five Numbers*, as aforesaid.

Proof of Barter and Exchange. All Questions of *Barter and Exchange*, receiving their Resolution by the *Rule of Three*, *Practice*, *Specificks*, or *Rule of five Numbers*, will be accordingly proved: or the Question may be reversed, which will add to the Demonstration.

CHAP. XI.

Loss and Gain.

Loss and Gain, the Subject thereof. Four Cases thereof. THE Title of this Chapter shews the Subject thereof, to converse with Questions resolving what *Loss* or *Gain* may accrue by Traffick; and this in one of these four Cases following.

1. Bare enquiry of either. *Case* 1. When there is a bare enquiry of *Loss* or *Gain*.

Either the Rates of buying and selling are given, to find the *Gain* or *Loss* in general, or upon the Hundred.

Or on the contrary, to find the Rates of a Quantity, the *Gain* or *Loss* of the Whole, or upon the Hundred is given.

How resolved. In all which, Resolution is to be had by the *Rule of Three*, the *Data* being duly disposed or prepared, according to the State of the Question propounded.

Q. *Of* Loss. *Example* 1. I have 10 Yards of Cloth that cost me 8 *l.* 5 *s.* which I sell again for 15 *s.* the Yard: what do I lose thereby?

Answer. *Answ.* 15 *s.* For by the *Rule of Three*, every Yard bought in, is found to cost 16 *s.* 6 *d.* which is 1 *s.* 6 *d.* a Yard more than the selling Price, and this in 10 Yards amounts to 15 *s.*

As

Yards.	*l.*	*s.*	*Yard.*	*s.*
As 10	8 : 5	::	1	$16\frac{1}{2}$

20

$10)165(16\frac{1}{2}\,s.$

$16\frac{1}{2}\,s.$ Buying Price.
15 Selling Price.
$1\frac{1}{2}\,s.$ Loss in 1 Yard.

10 Yards.
$1\frac{1}{2}\,s.$
10
5
15 *s.* Total Loss.

Example 2. If 1 Yard cost 5 *s.* 4 *d.* and be sold again for 6 *s.* 8 *d.* how much is gained on the 100? *Q. Of Gain on the 100.*

Answ. 25 *l.* For taking the buying Price from the selling Price, the Gain of 1 Yard is had, the rest appeareth by the Work of the *Rule of Three.* *Answer.*

s. *d.*
6 : 8
5 : 4
1 : 4 Gain.

	s.	*Gain.*		*l.*	*Gain.*
As	$5\frac{1}{3}$	$1\frac{1}{3}\,s.$	::	100	25 *l.*

$\frac{16}{3}$

100
20
2000
$1\frac{1}{3}$
2000
$666\frac{2}{3}$
$2666\frac{2}{3}$

$\frac{16}{3})\frac{8000}{3}(\frac{500}{l.\,25:0\,s.}$ Gain on the 100 *l.*

Example 3. If I pay 34 *l.* for 560 Yards of Cloth, and would gain 17 *l.* 6 *s.* 8 *d.* thereby: how must I sell 1 Yard? *Q. Of the Price of a Yard.*

Answ. For 22 *d.* Here adding the Gain to the buying Price, the Total 51 *l.* 6 *s.* 8 *d.* is the second Number of the *Rule of Three*, the other Numbers are the Yards given. *Answer.*

l.
34
17 : 6 : 8
51 : 6 : 8

	Yards.		*l.*		*Yard.*	
As	560	.	$51\frac{1}{3}$	::	1	$\frac{11}{120}\,l.$ or 22 *d.*

$\frac{560}{1})\frac{154}{3}(\frac{11}{120}\,l.$ Price of 1 Yard.

Example 4. If I buy Cloth at 7 *s.* 6 *d.* the Ell, and it proving worse than expected, I am resolved to lose 5 in the 100: how must I rate the Cloth an Ell? *Q. Of the Price of an Ell.*

Answ. 7 *s.* $1\frac{1}{2}$ *d.* For if 100 *l.* lose 5 *l.* then 7 *s.* 6 *d.* shall lose $4\frac{1}{2}$ *d.* which taken from 7 *s.* 6 *d.* leaves 7 *s.* $1\frac{1}{2}$ *d.* *Answer.*

	l.		*Loss.*		*l.*		*Loss.*
As	100	.	5 *l.*	::	$\frac{3}{8}$	.	$\frac{3}{160}\,l.$ in 1 Yard.

$\frac{100}{1})\frac{15}{8}(\frac{3}{160} \times 240 = \frac{720}{160}(4\frac{1}{2}\,d.$

s. *d.*
7 : 6 Buying Price.
$4\frac{1}{2}$ Loss.
7 : $1\frac{1}{2}$ Selling Price.

Case 2. When the Enquiry is of *Loss* or *Gain* with Time. *2. Loss or Gain with Time, how resolved.*

The *Data* duly disposed, Resolution is to be had by the *Rule of five Numbers.*

Example 1. If 1 Ell of Holland cost 2 *s.* 6 *d.* and it be sold for 2 *s.* 8 *d.* to be paid at the end of four Months: what after that rate is gained upon the 100 in 12 Months? *Q. Of Gain in 12 Months.*

Answ. 20 *l.* *Answer.*

s. *d.*
2 : 8 Selling Price.
2 : 6 Buying Price.
0 : 2 Gain.

	l.	*Mon.*	*d.*		*l.*	*Mon.*	*l.*
As	$\frac{1}{8}$	4	2	::	100	12	20

$\frac{1}{2}$

24

$\frac{1}{2})\frac{2400}{1}(\frac{4800}{240}(20\,l.$

Example 2. If a parcel of Goods that cost 15 *l.* be sold again for 14 *l.* 16 *s.* $10\frac{1}{2}$ *d.* to be paid at the end of 3 Months: whether at that rate is there lost 6 in the 100 for 12 Months? *Q. Of Loss in 3 Months.*

Answ.

Anſwer. *Anſw.* No, but 4 *l.* 3 *s.* 4 *d.*

l.	*s.*	*d.*	
15	00	00	Buying Price.
14	16	$10\frac{1}{2}$	Selling Price.
00	3	$1\frac{1}{2}$	Loſs.

l. *Mon.* *s.* *l.* *Mon.* *l.*

As 15 . 3 . $3\frac{1}{8}$ :: 100 . 12 . $4\frac{1}{6}$

45

$37\frac{1}{2}$

45)3750($8,3\frac{1}{3}$ *s.*

l. 4 : 3 : 4 *d.*

3. Loſs *or* Gain, *with Allowance or Rebatement, how reſolved.*

Caſe 3. When the Enquiry is of *Loſs* or *Gain* with Allowance, or Rebatement in the Weight; both paſſing commonly by the Names of *Tare*, *Tret* and *Cloff*.

Addition for the Allowance, and *Subſtraction* for the Rebatement being made, the reſidue of the Work is by the *Rule of Three*, or *Practice*.

Q. *Of the Hundred with Tret.* *Example* 1. At 14 *l.* the 100 Suttle, what will 890 ℔. Suttle be worth, giving 6 ℔. weight upon every 100 for Tret?

Anſwer. *Anſw.* $117\frac{29}{53}$ *l.* For ſo is the Reſolution, adding the Allowance, and working by the *Rule of Three*.

℔.	
100	Suttle.
6	Tret allowed.
106	Sum.

℔. *l.* ℔. *l.*

As 106 . 14 :: 890 . $117\frac{29}{53}$

890

1260

112

12460

(5

280(8

12460($117\frac{29}{53}$ *l.*

106

Q. *Of the Hundred with Tret.* *Example* 2. At 2 *s.* 6 *d.* the Pound, what ſhall 780 ℔. be worth, allowing 4 ℔. Tret on the groſs Hundred?

Anſwer. *Anſw.* $94\frac{4}{29}$ *l.* For by the firſt Work of the *Rule of Three*, or *Practice*, the Value of 1 C. at the rate of 2 *s.* 6 *d.* the ℔. is found to be 14 *l.* And then by another Work of the ſame Rule is gotten $94\frac{4}{29}$ *l.*

℔.	
112	Great Hundred.
4	Tret allowed.
116	Sum.

At 2 *s.* 6 *d.* *per* ℔. what coſts 112 ℔.(14 *l.*

$\frac{1}{8}$ 8

℔. *l.* ℔. *l.*

As 116 . 14 :: 780 . $94\frac{4}{29}$

14

3120

780

116)10920($94\frac{4}{29}$ *l.*

Q. *Of the Hundred rebating for Tare and Cloff.* *Example* 3. If 100 ℔ be worth 38 *s.* what will 860 ℔ be worth, rebating 5 ℔ upon every 100 for Tare and Cloff?

Anſwer. *Anſw.* 15 *l.* $10\frac{23}{50}$ *s.* For according to that Abatement there is but 817 ℔ to be accompted for; which at the rate of 38 *s.* for 100, gives the ſaid Sum.

℔.	
100	Suttle.
5	Tare rebated.
95	Remain.

℔. ℔. ℔. ℔.

As 100 . 95 :: 860 . 817

95

4300

7740

100)81700(817 ℔.

℔. *s.* ℔. *l.* *s.*

As 100 . 38 :: 817 . 15 : $10\frac{23}{50}$

38

6536

2451

100)31046($31,0\frac{23}{50}$

$15:10\frac{23}{50}$

Example

Example 4. If a Merchant ſell two Baskets of Raiſins weighing 183 ℔, and 159 ℔, for 26 *s.* the Hundred, allowing 3 ℔ Tare on each Basket: what do the Raiſins amount to? *Q. Of Baskets of Raiſins with Tare.*

Anſw. 3 *l.* 18 *s.* For ſo it appears by the *Rule of Three*, the Tare being firſt deducted. *Anſwer.*

℔.	
183	
159	
342	Weight in Groſs.
6	Tare rebated.
336	Weight *Netto.*

As 112 ℔ . 26 *s.* :: 336 ℔ . 3 *l.* : 18 *s.*

336 × 26 = 2016 + 672 = 8736

112) 8736 (78 *s.* = *l.* 3 : 18 *s.*

Caſe 4. When the Enquiry is of *Loſs* or *Gain*, with different Allowance or Abatement. *4. Loſs or Gain with different Allowance or Rebatement, how reſolved.*

The *Gain* or *Loſs* of one Side being gotten, by ſome or other of the Ways foregoing, compare it with the other.

Example 1. Whether doth he loſe more that giveth 5 ℔ upon the 100, or he that rebateth 5 ℔ upon the 100? *Q. Of the moſt Loſs, by giving or abating 5 on 100.*

Anſw. Becauſe he that giveth 5 ℔ upon the 100, giveth 105 ℔ for 100 ℔; and he that rebateth 5 ℔ upon the 100, giveth 100 ℔ for 95 ℔: The Queſtion therefore may be ſet thus; If 105 ℔ be given for 100 ℔, what ſhall 100 ℔ be delivered for? The Anſwer to which being $95\frac{5}{21}$ ℔, it appeareth that he who rebateth 5 ℔ on the Hundred is the Loſer, by ſo much as $95\frac{5}{21}$ ℔ is greater than 95 ℔, that is $\frac{5}{21}$ for the other makes $95\frac{5}{21}$ ℔ of the Hundred. *Anſwer.*

100 ℔	
5	Allowance.
105	Sum.
100 ℔	
5	Rebatement.
95	Remain.

As 105 ℔ . 100 ℔ :: 100 ℔ . $95\frac{5}{21}$ ℔

100 × 100 = 10000

105) 10000 ($95\frac{5}{21}$

95 Hundred with Rebate.

$\frac{5}{21}$ Difference.

Example 2. *A* oweth *B* 600 *l.* to be paid in 3 Months; and *B* oweth *A* 500 *l.* to be paid in 4 Months: But if *A* will clear the Score preſently, *B* offereth to take 98 *l.* Which of them, conſidering the Intereſt, loſeth, and how much? *Q. Of the Sum loſt in paying two Debts.*

Anſw. *B* loſeth 3 *l.* For accounting the Intereſt of 600 *l.* for 3 Months, it is 9 *l.* And the Intereſt of 500 *l.* for 4 Months is 10 *l.* accounting the Intereſt at 6 *per Cent.* which abated from the reſpective Sums, 591 *l.* is left for *A* to pay, and 490 *l.* for *B* to pay; the Difference is 101 *l.* due from *A* to *B*: For which if *B* take 98 *l.* he loſeth 2 *l.* of the Principal, and 1 *l.* of the Intereſt. *Anſwer.*

As 100 *l.* . 12 *Mon.* . 6 *l.* :: 600 *l.* . 3 *Mon.* . 9 *l.* *A* Intereſt.

As 100 *l.* . 12 *Mon.* . 6 *l.* :: 500 *l.* . 4 *Mon.* . 10 *l.* *B* Intereſt.

	l.		*l.*		*l.*		
A oweth	600	—	9	=	591		
B oweth	500	—	10	=	490	*Paid.*	*Loſs.*
Due to *B*					101	— 98	= 3

According to the Reſolution of the Propoſitions in *Loſs* or *Gain*, either by the *Rule of Three*, *Practice*, *Specificks*, or *Rule of five Numbers*, ſo ſhall be the Proof: And if any Doubt ariſe, work the Queſtion reverſed in any of the Caſes. *Proof of Loſs and Gain.*

CHAP. XII. *Equation of Paiment.*

Equation of Paiment in 3 Cases.

SOmetime, as well on the Sale of Goods as Loan of Monies, divers Days or Times of *Paiment* are appointed; and upon a new Agreement, the Debtor willing to be discharged, desireth to know when the whole Sum to be paid may be paid all at once, the Interest duly accounted: How therefore the several Days of paiment may be equated and brought all into one, is the Work of this Chapter, and the Contents thereof may be reduced to three *Cases*.

1. Time singly propounded. Rule.

Case 1. When the Time of Paiment is singly propounded.

Multiply the Sums to be paid by the several Times of Paiment, and the Total of the Products divide by the whole Sum to be paid.

Q. Of 90 l. to be paid at once, tho due at several Times.

Example 1. A Debtor owing 90 *l.* agreed with his Creditor to pay it at three several Days, *viz.* 45 *l.* at 4 Months end, 30 *l.* at 8 Months end, and the remaining 15 *l.* at 10 Months end; but receiving Money unexpectedly, was willing to pay all at one Paiment: what Time must then be given him?

Answer. *Answ.* $6\frac{1}{3}$ Months.

	l.		*l.*		*l.*		
Sums to be paid	45	+	30	+	15		
Times of Paiment	4		8		10		
Products	180	+	240	+	150	=	57(0 (3
					Whole Debt		9 0 ($6\frac{1}{3}$ Months.

Resolved by Fractions.

Or if the several Sums to be paid be propounded in Fractions, or else be reduced into Fractionary Parts of the whole Debt, then multiply after the manner of Fractions, and add the Products into one Total.

As because 45 *l.* is $\frac{1}{2}$ of 90, and 30 *l.* is $\frac{1}{3}$, and 15 *l.* is $\frac{1}{6}$; therefore $\frac{1}{2}$ multiplied by 4, and $\frac{1}{3}$ by 8, and $\frac{1}{6}$ by 10, shall be together $6\frac{1}{3}$ as before.

$$\frac{45}{90} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2}\ (2$$
$$\frac{30}{90} = \frac{1}{3} \times \frac{8}{1} = \frac{8}{3}\ (2\tfrac{2}{3}$$
$$\frac{15}{90} = \frac{1}{6} \times \frac{10}{1} = \frac{10}{6}\ (1\tfrac{2}{3}$$
$$6\tfrac{1}{3} \text{ Months.}$$

Q. Of 500 l. some due present, and some at other Times, when to be paid at once.

Example 2. *A* is indebted to *B* 500 *l.* to be paid as followeth, *viz.* 100 *l.* present, 260 *l.* at 6 Months, and 140 *l.* at 9 Months: when are these Paiments on Time to be paid at once? or when shall the whole Debt of 500 *l.* be paid together?

Answer. *Answ.* At $7\frac{1}{20}$ Months, if 100 *l.* be paid present; but otherwise at the end of $5\frac{16}{25}$ Months the whole 500 *l.* shall be paid.

By Integers.

	l.		*l.*		*l.*		
Sums to be paid	100	+	260	+	140		
Times of Paiment.	Present.		6		9		
			1560	+	1260	=	282|0 ($7\frac{1}{20}$ Months.
			Remaining Debt				40|0

(3
28(2|0
Whole Debt 5 0|0 ($5\frac{16}{25}$ Months.

By Fractions.

Present 100 *l.*

$$\frac{260}{400} = \frac{13}{20} \times \frac{6}{1} = \frac{78}{20}\ (3\tfrac{18}{20} \qquad \frac{260}{500} = \frac{13}{25} \times \frac{6}{1} = \frac{78}{25}\ (3\tfrac{3}{25}$$
$$\frac{140}{400} = \frac{7}{20} \times \frac{9}{1} = \frac{63}{20}\ (3\tfrac{3}{20} \qquad \frac{140}{500} = \frac{7}{25} \times \frac{9}{1} = \frac{63}{25}\ (2\tfrac{13}{25}$$
$$\text{Months } 7\tfrac{1}{20} \qquad \text{Months } 5\tfrac{16}{25}$$

Example

Example 3. A Debt of 600 *l.* was to be paid thus; 200 *l.* present, $\frac{1}{3}$ at 8 Months, and $\frac{1}{5}$ at 10 Months, and the Residue at the Year's End: when may this Money be all paid together? Q. Of 600 *l.* due, some present, some afterward, when to be paid at once.

Answ. At $6\frac{4}{15}$ Months. Answer.

By Integers.

$$\begin{array}{lll} \text{Whole Debt} & 600\ l. & \\ \frac{1}{3} & 200 & 200 \times 8 = 1600 \\ \frac{1}{5} & 120 & 120 \times 10 = 1200 \\ \text{Rest } \frac{2}{15} & 80 & 80 \times 12 = 960 \\ & & 3760 \left(6\frac{4}{15} \text{ Months.} \right. \\ & & 200 + 200 + 120 + 80 = 600 \end{array}$$

By Fractions.

Present 200 *l.*

$$\frac{200}{600} = \frac{1}{3} \times \frac{8}{1} = \frac{8}{3} \left(2\frac{2}{3}\right.$$
$$\frac{120}{600} = \frac{1}{5} \times \frac{10}{1} = \frac{10}{5} (2$$
$$\frac{80}{600} = \frac{2}{15} \times \frac{12}{1} = \frac{24}{15} \left(1\frac{3}{5}\right.$$
$$6\frac{4}{15} \text{ Months.}$$

Case 2. When the Time of Paiment is propounded with *Loss* or *Gain*. 2. Time with Loss or Gain.

Equate the Times of Paiment as before, and then by the *Rule of five Numbers* enquire for the *Gains* or *Loss*. Rule.

Example 1. A Merchant buyeth Silks at 10 *s.* the Yard, and selleth the same at 12 *s.* the Yard, giving 2 Days of Paiment; *viz.* 4 Months for the one Half, and 8 Months for the other: what doth he gain on the 100 in 12 Months? Q. Of Silks sold on Time, what Gain.

Answ. 40 *l.* For the Times of Paiment equated make 6 Months: And then if 10 *s.* in 6 Months gain 2 *s.* an hundred Pounds in 12 Months shall gain 40 *l.* Answer.

$$\frac{1}{2} \times \frac{4}{1} = \frac{4}{2} (2$$
$$\frac{1}{2} \times \frac{8}{1} = \frac{8}{2} (4$$
$$6 \text{ Months.}$$

$$\begin{array}{l} \quad l.\ \ Mon.\ \ s.\ \ \ l.\ \ Mon.\ \ l. \\ \text{As } \frac{1}{2} \,.\, 6 \,.\, 2 :: 100 \,.\, 12 \,.\, 40 \end{array}$$

$$\underbrace{\tfrac{1}{2}\,.\,6}_{3} \qquad \underbrace{2 :: 100\,.\,12}_{24}$$

$$3 \left) 2400 \right(\begin{array}{l} 80|0\ s. \\ 40{:}0\ s. \end{array}$$

Example 2. A Merchant buyeth 20 Cloths, at 6 *l.* the Cloth ready Money; and afterwards selleth 5 of them for 7 *l.* the Cloth to be paid at 4 Months; and the other 15 he selleth for 8 *l.* the Cloth, and giveth 6 Months Time for the Paiment: how much is gained thereby on the 100 in 12 Months? Q. Of Cloth, some for ready Money, and some on Time, what Gain.

Answ. $63\frac{7}{86}$ *l.* For the Money paid for the Cloths bought being 120 *l.* and to be paid for them sold being 155 *l.* there is 35 *l.* difference; and the Times equated being $5\frac{17}{31}$ Months; by the *Rule of five Numbers* is obtained $63\frac{7}{86}$ *l.* Answer.

$$\begin{array}{llll} \text{Cloths bought} & 20 & \text{Cloths sold} & 5 + 15 \\ \text{Price of one} & 6\ l. & \text{Price of one} & 7\ l. \quad 8\ l. \\ & 120 & & 35 + 120 = 155 \\ \text{Sums to be paid} & & 35 + 120 & \\ \text{Times of Paiment} & & 4 \quad\ \ 6 & \\ & & 140 + 720 = \frac{860}{155} & \left(5\frac{17}{31} \text{ Months.}\right. \end{array}$$

(85

$$\begin{array}{l} \quad\ l. \qquad Mon. \qquad l. \qquad l. \qquad Mon. \qquad l. \\ \text{As } 120 \,.\, 5\frac{17}{31} \,.\, 35 :: 100 \,.\, 12 \,.\, 63\frac{7}{86} \end{array}$$

$$420$$

$$\frac{20640}{31} \left) \frac{42000}{1} \right(\frac{130200}{2064} \left(63\frac{7}{86}\ l. \right.$$

Example

Q. Of Cloves sold on Time, Gain on the 100. *Example* 3. A Merchant buyeth Cloves at 8 *s.* the Pound ready Money: how shall he sell the hundred Weight thereof to gain, after the Rate of 10 *l.* on the 100 for a Year, and be paid $\frac{1}{2}$ at 3 Months, and the rest at 6 Months end?

Answer. *Answ.* For 46 *l.* 9 *s.* $7\frac{1}{5}$ *d.* where the Times being equated for one Paiment, make $4\frac{1}{2}$ Months; And the Rate of 1 C. at 8 *s.* the lb, being found to be 44 *l.* 16 *s.* the Ready-money Price, Then by the *Rule of five Numbers* is found 1 *l.* 13 *s.* $7\frac{1}{5}$ *d.* to be added to the Ready-money Price.

Paiments $\frac{1}{2} \times \frac{3}{1} = \frac{3}{2}\ (1\frac{1}{2}$
$\frac{1}{2} \times \frac{6}{1} = \frac{6}{2}\ (3$
$4\frac{1}{2}$ Months

lb.	s.		lb.	l.	s.
As 1	. 8	::	112	. 44	: 16

4
l. 44|8 Primes.

	l.	*Mon.*	*l.gain.*		*l.*	*Mon.*	*l.*	*s.*	*d.*
As	100	. 12	. 10	::	$44\frac{4}{5}$	. $4\frac{1}{2}$	. 1	: 13	: $7\frac{1}{5}$

1200

$$1200)\overline{2016}\left(1\tfrac{17}{25}\ l. \times 20 = \frac{340}{25}\left(13\tfrac{3}{5}\ s. \times 12 = \frac{36}{5}\left(7\tfrac{1}{5}\ d.\right.\right.\right.$$

	l.	*s.*	*d.*
Ready-money Price	44	: 16	: 00
Gain to be added	1	: 13	: $07\frac{1}{5}$
	46	: 09	: $07\frac{1}{5}$ Selling Price of the Hundred.

3. Time sooner for Part, and later for other Part. *Case* 3. When the Time of Paiment is anticipated for Part of the Debt, and procrastinated for the Residue.

Rule. Substract the Paiment or Paiments from the whole Debt, and if the Times of Paiment be more than one, equate them; and this equated Time take from the Time of Paiment, and commit the Work to the *Rule of Three:* But if one Time be propounded, substract the Time of Paiment from the Time when to be paid, and the Sum paid from the whole Debt, and work with the Remainers. Or instead of the Difference of the Time equated or not, if the Times of Paiment be substracted from the Time when the Whole was to be paid, and the Remains be equated, this may be taken to work with.

Q. Of 480 l. if 280 paid before the Day, when the rest. *Example* 1. *A* is indebted to *B* 480 *l.* to pay it in $5\frac{1}{2}$ Months; and at 4 Months end he will pay 280 *l.* upon Condition that *B* will stay (accounting by the Interest) for the other 200 *l.* as long after the Time as this 280 *l.* is paid before the Time: when shall the 200 *l.* be paid?

Answer. *Answ.* $2\frac{1}{10}$ Months after the Expiration of $5\frac{1}{2}$ Months.

	l.		*Months.*
Whole Debt	480	to be paid in	$5\frac{1}{2}$
Paiment	280	made in	4
Residue	200	Difference	$1\frac{1}{2}$

	l.	*Months.*		*l.*	*Months.*
As	200	. $1\frac{1}{2}$	::	280	. $2\frac{1}{10}$

$1\frac{1}{2}$
280
140
$20)\overline{42,0}(2\frac{1}{10}$ Months.

Q. Of 240 l. some paid before due, when the rest. *Example* 2. A Merchant oweth 240 *l.* to be paid in 6 Months; but $1\frac{1}{2}$ Month past he payeth 60 *l.* and 3 Months after that he payeth 80 *l.* more: in what Time shall he pay the Rest, considering his Time was 6 Months?

Answer. *Answ.* $3\frac{9}{10}$ Months after the end of 6 Months.

	l.		*Months.*	*Months.*
Whole Debt	240	to be paid in	6	6
Paiments	60	in	$1\frac{1}{2} \times 60 = 90$	
	80		$4\frac{1}{2} \times 80 = 360$	(3
Residue	100	Paiments	140)450(0	$3\frac{3}{14}$ Time equated.
				$2\frac{11}{14}$ Difference.

As

$$\begin{array}{lcccccc} & l. & \textit{Months.} & & l. & \textit{Months.} \\ \text{As} & 100 & . & 2\tfrac{1}{14} & :: & 140 & . & 3\tfrac{9}{10} \end{array}$$

$$\begin{array}{r} 2\tfrac{1}{14} \\ \hline 280 \\ 110 \\ 10)39,0(3\tfrac{9}{10}\ \text{Months.} \end{array}$$

Example 3. A Merchant is indebted 300 *l.* to be paid the 24*th* of *May*; whereof he payeth the 29*th* of *April* 80 *l.* and the 9*th* of *May* after 120 *l.* upon what Day shall he pay the remaining 100 *l?* *Q. Of 300 l. paid, part before due; when the rest.*

Answ. 38 Days after the 24*th* of *May*, which will be the first Day of *July*. *Answer.*

Whole Debt	300 *l.* to be paid in 25 Days.		25 Days.
Paiments	{ 80 Present 120 in	$10 \times 120 = 1200$	
		Paiments 200	(6 Time equated.
Residue	100		19 Difference.

$$\begin{array}{lcccccc} & l. & & \textit{Days.} & & l. & & \textit{Days.} \\ \text{As} & 100 & . & 19 & :: & 200 & . & 38 \end{array}$$

$$\begin{array}{r} 19 \\ 100)38,00(38\ \text{Days.} \end{array}$$

Or because 80 *l.* was paid *April* 29, which was 25 Days before *May* 24, when the Whole was to be paid; and 120 *l.* being paid *May* the 9*th*, which was 15 Days before *May* 24; if 25 and 15 be equated with the Paiments, the Time equated will be 19 Days, and the Resolution as above. *How otherwise wrought.*

Whole Debt 300 *l.* to be paid in 25 Days, that is from *April* 29, to *May* 24.

$$\text{Paiments}\ \left\{\begin{array}{ll} 80\ \text{before} & 25\times 80=2000 \\ 120\ \text{before} & 15\times120=1800 \end{array}\right\}\ \begin{array}{r} 3800 \\ \text{Paiments}\ 200 \end{array}\left(\begin{array}{l} \textit{Days} \\ 19\ \text{before the Time.} \end{array}\right.$$

As 100 . 19 :: 200 . 38.

The Operations of the first *Case* are to be proved thus: Rate the Interest of the several Sums to be paid, according to the Times of Paiment, and the Total thereof shall be equal to the Interest of the Total Sum accounted to the Time equated. *Proof of Equation of Paiment.*

As in the first *Example* of the first *Case*, the Interest reckoned at 6 *l. per Cent. per Annum*, in both is found 57 *s.* *Of the first Case.*

$$\begin{array}{l} \quad l.\quad \textit{Mon.}\quad l. \\ \text{As } 100 . 12 . 6 :: \end{array} \begin{array}{ccccc} l. & & \textit{Mon.} & & s. \\ 45 & . & 4 & . & 18 \\ 30 & . & 8 & . & 24 \\ 15 & . & 10 & . & 15 \end{array}\Bigg\}\ \text{Interest of the several Paiments in their Times.}$$

$$\text{As } 100 . 12 . 6 :: 90 . 6\tfrac{1}{3} . \underline{57}\ \ \text{Interest of the Total Paiment at the Time equated.}$$

In the Operations of the second *Case*, so far as concerns the Equations of the Paiments, is to be proved as the first. And what further refers to the *Rule of Three*, or *Rule of five Numbers*, admits of their Proof. *Of the second Case.*

The Operations of the third *Case* are to be proved much like those of the First: For account the Interest of the whole Sum till the Time of Paiment thereof to be made, and this shall be equal with the Interest of the several Sums paid, reckoned till the Times of their Paiments. *Of the third Case.*

As in the first *Example* of the third *Case*, rating the Interest at 6 *l.* in the Hundred, the Sum found for both is 13 *l.* 4 *s.*

			l.	*Mon.*	*l.*	*s.*	
l.	*Mon.*	*l.*	280 .	4 .	5 :	12	Interest of 280 *l.* for 4 Months, and 200 *l.* for $7\frac{3}{5}$ Months (that is $5\frac{1}{2}$ and $2\frac{1}{10}$) when paid.
As 100 .	12 .	6 ::					
			200 .	$7\frac{3}{5}$.	7 :	12	
As 100 .	12 .	6 ::	480 .	$5\frac{1}{2}$.	13 :	4	Interest of 480 *l.* the whole Debt to $5\frac{1}{2}$ Months.

CHAP. XIII. *Factorship.*

Factorship.

How placed by some.

What to be noted therein.

QUestions relating to Merchants and Factors, close up the third Sort of Derivatives, and are treated of in this Chapter under the Title of *Factorship*, but with some placed under *Fellowship*.

In *Factorship* are to be minded two things.

First, The Estimation of the Factor.

Secondly, The Partition of the Gains between the Merchant and the Factor, according to their Agreement or Bargain.

Estimation of the Factor, what.

How to be accounted, with or without Stock.

Estimation of the Factor, is an Allowance to the Factor for his Pains or Imploiment, to countervail some part of the Merchant's Stock. And when the Factor layeth in no Stock, his Estimation is in such *Ratio*, or Proportion to the Merchant's Stock, as the Gains of the said Factor are to the Gains of the Merchant. But when the Factor layeth in Stock with the Merchant, then, after the whole Estimation of the Person of the Factor with his Stock is valued according to the Merchant's Stock, the Factor's Stock is to be deducted, and the Remain is the Estimation of his Person.

Examples of both.

Q. *Of the Factor's Estimation without Stock.*

Answer.

1. A Merchant delivereth unto his Factor 900 *l.* to govern in the Trade of Merchandise, upon Condition that he should have $\frac{1}{3}$ of the Gain: what is his Estimation esteemed at?

Answ. 450 *l.* For that Sum beareth the same *Ratio* to 900 *l.* which $\frac{1}{3}$ doth to $\frac{2}{3}$, the Residue of the Gains belonging to the Merchant.

$$\text{As } \tfrac{2}{3} \;.\; \overset{l.}{900} \;::\; \tfrac{1}{3} \;.\; \overset{l.}{450}.$$

$$\tfrac{2}{3}\Big)\overset{\frac{1}{3}}{\frac{300}{1}}\Big(\frac{900}{2}\Big(\overset{l.}{450}\ \text{Estimation.}$$

Q. *Of the Factor's Estimation with Stock.*

Answer.

2. A Merchant delivered unto his Factor 600 *l.* and the Factor layeth in Stock therewith 250 *l.* besides his Personal Estimation; therefore it is agreed he shall have $\frac{2}{5}$ of the Gain: what is the Estimation of his Person?

Answ. 150 *l.* For the whole Estimation of his Stock and Person is 400 *l.* from which 250 *l.* the Factor's Stock deducted, leaves 150 *l.* for the Estimation of his Person.

$$\text{As } \tfrac{3}{5} \;.\; \overset{l.}{600} \;::\; \tfrac{2}{5} \;.\; \overset{l.}{400}$$

$$\tfrac{3}{5}\Big)\overset{\frac{2}{5}}{\frac{240}{1}}\Big(\frac{1200}{3}\Big(\overset{l.}{400}\ \text{Whole Estimation.}$$

250 Factor's Stock.

150 Factor's Personal Estimation.

Partition of the Gains how accounted at, or out of the sending of the Merchant.

Partition of the Gains enquired, propounds the Estimation or Agreement between the Merchant and his Factor: And sometimes accounts the Estimation according to the Merchant's Stock, called *Estimation* upon, or at the sending of the Merchant. And sometimes the Gain is to be parted proportionally, according to the Merchant's Stock and the Factor's Estimation added together, called *Estimation* out of the sending of the Merchant.

Examples

Examples of both.

1. A Merchant putteth into his Factor's Hands to improve in the way of Merchandise 800 *l.* upon condition that the said Factor shall have $\frac{1}{4}$: And after certain Time they found in Profit 135 *l.* 6 *s.* 8 *d.* how much shall the Factor have thereof? *Q. Of Gain parted to the Merchant and Factor.*

Answ. Because $\frac{1}{4}$ is understood to be $\frac{1}{4}$ of the Gains; the Gains shall be divided, $\frac{3}{4}$ to the Merchant, and $\frac{1}{4}$ to the Factor: That *Answer.*

		l.	*s.*	*d.*
$\frac{3}{4}$	To the Merchant	101	10	00
$\frac{1}{4}$	To the Factor	33	16	08
	Total Gains	135	06	08

But then his Estimation is but $\frac{1}{3}$, because $\frac{1}{4}$ of the whole Gains is but $\frac{1}{3}$ of $\frac{3}{4}$ the Merchant's Part.

And if the Factor's Estimation were $\frac{1}{4}$, then must he have but $\frac{1}{5}$ of the Gains, and the Merchant $\frac{4}{5}$; and so must the Gains be parted thus,

	l.	*s.*	*d.*
To the Merchant	108	05	04
To the Factor	27	01	04
	135	06	08

The Reason whereof is, because if his Estimation had been but $\frac{1}{4}$, then the $\frac{1}{4}$ of 800 *l.* being 200 *l.* had made the 800 *l.* to be 1000 *l.* of which $\frac{1}{5}$ is equal to $\frac{1}{4}$ of 800 *l.* the Merchant's Stock.

2. A Merchant delivereth to his Factor 800 *l.* upon Condition that his Factor shall have the Profit of 160 *l.* as though he laid in so much ready Money: what Portion of the Gains shall the said Factor take up for himself at the Reckoning? *Q. Of Gains how parted to the Merchant and Factor.*

Answ. If it be intended that the said 160 *l.* shall be reckoned as Stock to the other 800 *l.* then the whole Stock maketh 960, of which 160 is $\frac{1}{6}$, and such part of the Gains shall the Factor have. *Answer.*

But if the Intent of their Covenants between them were, that the Factor should have the Gains of 160 *l.* of the 800 *l.* then shall the Factor take $\frac{1}{5}$ of the Gains; for 160 to 800, is as 5 to 1.

Examples mixt. *Mixt Examples.*

1. A Merchant puts into his Factor's Hands 390 *l.* to trade with, and the Factor's Estimation is accounted 60 *l.* how much Money must the Factor put in Stock, that he may have $\frac{1}{4}$ of the Gain? *Q. Of Stock laid in by the Factor.*

Answ. 70 *l.* For $\frac{1}{4}$ of the Gain is understood of the whole Gain, Estimation and all accounted thereto: So as $\frac{3}{4}$ Gain for the Merchant coming of 390 *l.* the $\frac{1}{4}$ Gain for the Factor must arise of 130 *l.* that is 60 *l.* Estimation, and 70 *l.* Stock. *Answer.*

		l.			*l.*	
As	$\frac{3}{4}$.	390	::	$\frac{1}{4}$.	130	
					60	Factor's Estimation.
					70	Factor's Stock.

2. A Merchant hath 400 *l.* to trade with in Merchandise; and agreeth with a Factor, that if he put in 90 *l.* to the Stock, then he shall have $\frac{1}{3}$ for his Pains. Afterward another Merchant desireth to be a Copartner, and putteth in Stock 350 *l.* and promiseth to observe the same Agreement made with the Factor: when the Trade is over they find 250 *l.* gained; how must the Gains be divided? and what is the Factor's Pains esteemed at? *Q. Of Gains how to be parted, and of the Factor's Estimation.*

Answ. The Factor's Estimation being valued according to the first Merchant's Stock, is 110 *l.* his Stock 90 *l.* which together with the Stock of both Merchants, make 950 *l.* by which the Gains are to be divided as in *Fellowship.* *Answer.*

As

					l.				*l.*	
				As	$\frac{2}{3}$	. 400	::	$\frac{1}{3}$	. 200	
									90	Factor's Stock.
	Stock.								110	Factor's Estimation.

A .	400 *l.*					
B .	350				*l.*	
Factor	200	*l. gain.*		400	. $105\frac{5}{19}$ *A*	Gain divided.
As	950	. 250	::	350	. $92\frac{2}{19}$ *B*	
				200	. $52\frac{12}{19}$ Factor	

But if the Factor's Estimation had been valued according to both the Merchants Stocks at $\frac{1}{3}$, then his 90 *l.* Stock and Estimation had been together 375 *l.* and his Stock substracted, his Estimation would have been 285 *l.* and the Gain divided by this Account, the Factor will have almost as much as the First, and more than the second Merchant.

					l.				*l.*	
				As	$\frac{2}{3}$	. 750	::	$\frac{1}{3}$	. 375	
									90	Factor's Stock.
	Stock.								285	Factor's Estimation.

A .	400 *l.*					
B .	350				*l.*	
Factor	375	*l. gain*		400	. $88\frac{8}{9}$ *A*	Gain divided.
As	1125	. 250	::	350	. $77\frac{7}{9}$ *B*	
				375	. $83\frac{1}{3}$ Factor	

Proof of Factorship.

The Proof of nothing here need be spoken to, except the finding of the Estimation; forasmuch as all the other Operations of this Chapter depend on *Proportions*, as the *Rule of Three*, *Fellowship*, &c. the Proof whereof hath been set forth at large where they have been handled.

In particular of finding the Estimation.

To prove the finding of the Estimation, add the Estimation to the whole Stock, as well of the Factor, if any be, as of the Merchant's; and take the Part thereof which the Factor was to have, and it will agree with the Estimation found: But where the Factor had Stock, this Part taken shall include his Stock with his Estimation, otherwise his bare Estimation only.

As in the first and last *Examples* of this Chapter.

	First.			*Last.*	
Stock	900 *l.*	Merchant.	Stock	400 *l.*	Merchant.
Estimation	450	Factor.	Stock	90	Factor.
$\frac{1}{3}$ Part)	1350	(450 *l.*	Estimation	110	Factor.
			$\frac{1}{3}$ Part)	600	(200 *l.* Stock and Estimation.

CHAP. XIV. *Falshood or Position.*

Falshood or Position.

THE Comparative Elements of the third Sort of Derivatives done with, yet remains for this Chapter the handling of the fourth Sort, (which have some peculiar Operation requisite to their Resolution) called *Falshood* or *Position.*

Why so called.

Falshood is not so called, because it teacheth any Deceit, but for that by false Positions, or erroneous Numbers taken to work with, the Truth is found out. And though in some other of the foregoing Elements, the Numbers wrought with were not the true, yet were they in just Proportion to the True; but here they are taken at adventure; and if examining thereby the Question, according to the Tenor thereof, your Position fall true, the Question is answered: but because sometime false, it gave the Rule the Name of the *Rule of false Position.*

Of two sorts.

This *Rule of false Position*, or *Falshood*, is twofold, *viz.* *Single* and *Double.*

Single Position *How known.*

The first is called *Position Single*, because sometimes at one working the Question propounded may be resolved, which happens when there is in the Proposition some partition of Numbers in Parts proportional.

The

The Process herein is thus, Imagine a Number at pleasure, and proceed to work therewith according to the Purport of the Question, as if it were the true Number; and what Proportion there is between the false Conclusion, and the false Position, such Proportion hath the given Number to the Number sought: Therefore the Number found by Argumentation shall be the first Number of the *Rule of Three*, and the Hypothetical or supposed Number shall be set in the second Place, and in the third Place shall be the given Number. *What to be done.*

Example 1. *A*, *B*, and *C*, consent together to buy a Ship for 220 *l.* so that *B* must pay twice so much as *A*, and *C* four times so much as *B*: How much must each Man pay? *Q. Of a Ship bought, what each paid.*

Answ. I suppose *A* paid 8 *l.* then must *B* pay 16 *l.* and *C* four times so much, which is 64 *l.* But all these Numbers added together make no more than 88 *l.* and there should be 220 *l.* yet by this Number supposed I proceed to work, saying, *Answer.*

If 88 *l.* come of 8 *l.* of what comes 220 *l*?

Where in the Work is gained 20 *l.* for *A*; then must *B* pay 40 *l.* and *C* 160 *l.* all which added together, produce 220 *l.* the Number propounded.

			l.	*Position false.*	*l.*	*Conclusion true.*	
		As	88 .	8 *l.* ::	220 .	20 *l.*	*A*
Position	8			220		40 .	*B*
Double	16		88)	1760 (20		160 .	*C*
Quadruple	64					220 Proof.	
	88						

Example 2. A Mercer buyeth 30 Yards of Taffety, and 40 Yards of Sattin for 330 *s.* but every Yard of Sattin cost twice so much as a Yard of Taffety: what did a Yard of Taffety cost? *Q. Of Taffety and Sattin, the Price.*

Answ. Suppose a Yard of Taffety cost 4 *s.* then must a Yard of Sattin cost 8 *s.* at which Rates 30 Yards of Taffety will cost 120 *s.* and 40 Yards of Sattin 320 *s.* but 120 and 320 make 440, which should be but 330. I therefore say, As 440 . 4 :: 330 . 3 . So is 3 *s.* gotten for a Yard of Taffety, and then the Yard of Sattin shall cost 6 *s.* *Answer.*

s. *Position.* *s.* *Conclusion.*

If 440 come of 4 *s.* of what comes 330? *facit* 3 *s.*

4

Position 4 Doubled 8 440) 1320 (3 *s.* doubled 6 *s.*

Taffety 30 Sattin 40 Taffety 30 Sattin 40

120 + 320 = 440 Proof 90 + 240 = 330 *s.*

If the Question hath a Fraction or more therein, it is best, for more facility in proceeding, to take such a Number for the *Position* as may be equally parted by the Parts exprest in the Question. *If the Question have Fractions.*

Example 1. A Captain ordered upon Service with a Party, being demanded how many Souldiers he had in his Party? answered, that $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{4}{5}$ of them added together, made 245: how many Men had he? *Q. Of a Captain and his Party, how many.*

Answ. I suppose 30, (a Number that will be equally parted by 2, 3, and 5) and adding the $\frac{1}{2}$ which is 15, with the $\frac{1}{3}$ which is 10, and $\frac{4}{5}$ which is 24, find but 49, and it should be 245: Therefore tho I have erred in supposing, yet I commit the Work to the *Rule of Three*, and obtain 150 the Number sought. *Answer.*

Men. *Position.* *Men.* *Conclusion.*

If 49 come of 30: of what comes 245? *facit* 150.

			245		
Position	30	49)	7350 (150 Souldiers.	$\frac{1}{2}$	75
Half	15			$\frac{1}{3}$	50
Third	10			$\frac{4}{5}$	120
4 Fifths	24				245 Proof.
	49				

Q. Of Sheep left when some gone. *Example* 2. A Man having 1000 Sheep sold at one Time, $\frac{1}{2}$ so many as he now hath, and at another Time $\frac{1}{3}$ so many; and at another Time lost $\frac{1}{4}$ so many: The Question is, how many Sheep he hath yet remaining?

Answer. *Answ.* Suppose he had 12 Sheep left, the Half which is 6, added to $\frac{1}{3}$ which is 4, and $\frac{1}{4}$ which is 3, make with the 12 remaining but 25 instead of 1000; then by the Analogy is found 480 Sheep to be left.

As 25 . 12 :: 1000 . 480 . Sheep left.

Position	12	1000			
$\frac{1}{2}$	6	25)12000(480	$\frac{1}{2}$	240	. Sold.
$\frac{1}{3}$	4		$\frac{1}{3}$	160	. Sold.
$\frac{1}{4}$	3		$\frac{1}{4}$	120	. Lost.
	25			1000	. Proof.

If a Number propounded may be left or taken out of the Whole. When in the Question some Number is propounded to be left or taken out of the Whole, then the Parts proposed may be added together as Fractions, and taken out of the whole Integer: Or the whole Numbers by Argumentation substracted out of the *Position*, and the rest serve for the Analogy as before; only in the former Way the Number so proposed to be left or taken out of the Whole, shall be the second Number.

Q. Of a Man's Estate. *Example* 1. A Man having spent 50 *l.* had yet $\frac{3}{4}$ and $\frac{1}{5}$ of his Estate remaining: what was his Estate at first?

Answer. *Answ.* 1000 *l.* For if I add $\frac{3}{4}$ and $\frac{1}{5}$, they make $\frac{19}{20}$, which taken from $\frac{20}{20}$, the whole Substance, there remaineth $\frac{1}{20}$; which if it be 50 *l.* shall make the Whole 1000 *l.*

Or if I suppose 20 *l.* then $\frac{3}{4}$ is 15, and $\frac{1}{5}$ is 4; which added are 19, and substracted from 20, the *Position* leaves 1; And if 1 come of 20, then 1000 shall come of 50.

$\frac{3}{4} + \frac{1}{5} = \frac{19}{20}$ If $\frac{1}{20}$ be 50 *l.* what shall 1? *facit* 1000 *l.*

$\frac{20}{20} - \frac{19}{20} = \frac{1}{20}$ $\quad \frac{1}{20}\Big)\frac{50}{1}\Big(\frac{1000}{1}$

	1000
$\frac{3}{4}$	750
$\frac{1}{5}$	200
Spent	50
	1000 Proof.

Otherwise.

Position	20
$\frac{3}{4}$	15
$\frac{1}{5}$	4
Total	19
Remain	1

As 1 . 20 *l.* :: 50 . 1000 *l.*

50
20
1000 *l.*

Q. Of Stock, what it was. *Example* 2. One having spent $\frac{2}{3}$ and $\frac{1}{5}$ of his Stock, had only 36 *l.* remaining; what was his Stock?

Answer. *Answ.* 270 *l.* Here the Fractions added make $\frac{13}{15}$, which taken from the whole $\frac{15}{15}$, leaves $\frac{2}{15}$; and this being equal to 36, makes the whole 270.

Or by supposing 15, then $\frac{2}{3}$ is 10, and $\frac{1}{5}$ is 3, which make 13; that taken from 15 leaves 2: therefore as 2 to 15, so is 36 to 270.

$\frac{2}{3} + \frac{1}{5} = \frac{13}{15}$ If $\frac{2}{15}$ be 36 *l.*: what shall 1? *facit* 270 *l.*

$\frac{15}{15} - \frac{13}{15} = \frac{2}{15}$ $\quad \frac{2}{15}\Big)\frac{36}{1}\Big(\frac{270}{1}$ *l.*

	270
$\frac{2}{3}$	180
$\frac{1}{5}$	54
Left	36
	270 Proof.

Otherwise.

Otherwise.

Position	15
$\frac{2}{3}$	10
$\frac{1}{5}$	3
Total	13
Remain	2

As 2 . 15 :: 36 . 270 *l.*

15 × 36: 90, 45, 540

2)540(270 *l.*

If a Number may be set by.

Sometime a Number in the Question propounded abideth unalterable by the Fractions given, and so may be substracted from the Sum given, and set by for a Time till Operation be made with the rest, and then restored again.

Q. Of Cattel, what Number.

Example. A Grasier said of his Stock, If I had as many more Head of Cattel as I have with the half, third and fourth Parts thereof, and 1 overplus, I should have just 630: how many Cattel had he?

Answer.

Answ. I set by the 1, and so remaineth 629; when I have found therefore a Number, which being twice taken with the $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ Parts thereof make 629, I add the 1 thereto: As supposing 12, the Parts whereof accordingly taken, and added to the double thereof, make 37, by which is found 204 the Number desired.

Position	12
Double	24
$\frac{1}{2}$	6
$\frac{1}{3}$	4
$\frac{1}{4}$	3
	37

Position.

As 37 . 12 :: 629 . 204 Cattel.

37)7548(204

204	
204	
102	$\frac{1}{2}$
68	$\frac{1}{3}$
51	$\frac{1}{4}$
1	added.
630	Proof.

Two Numbers, yet single Position.

Sometime two Numbers are demanded, and in the Work two Numbers are taken, yet the Resolution by *Single Position*, because one of the Numbers so taken proves true, and not suppository only.

Q. Of two Numbers.

Example. What two Numbers are they, whose $\frac{2}{3}$ of the one is $\frac{3}{4}$ of the other?

Answer.

Answ. I take 12, a Number that hath such Parts in it; $\frac{2}{3}$ of 12 is 8: Then seeking what Number 8 is $\frac{3}{4}$ of, I suppose 20; but $\frac{3}{4}$ of 20 is 15, and I should have but 8: Nevertheless the *Analogy* will hold; If 15 come of the *Position* 20, then will 8 come of $10\frac{2}{3}$ the other Number desired: So shall 12 and $10\frac{2}{3}$ be the Numbers demanded.

$\frac{2}{3}$ of $12 = 8$

Position 20

$\frac{3}{4}$ 15

As 15 . 20 :: 8 . $10\frac{2}{3}$

8

15)160($10\frac{2}{3}$

Proof.

$\frac{2}{3}$ of $12 = 8 = \frac{3}{4}$ of $10\frac{2}{3}$

$\frac{1}{3}$	4	$\frac{1}{2}$	$5\frac{1}{3}$
$\frac{1}{3}$	4	$\frac{1}{4}$	$2\frac{2}{3}$
	8		8

Double Position, how known.

If there be no Partition in the Numbers to make a Proportion, then must be used the *Rule of Double Position.*

What to be done.

In *Double Position* make a Supposition twice, proceeding therein according to the State of the Question; and if either Number of them supposed happen to resolve the Question, the Work is done: But if not, observe the Errors, and whether they be greater or lesser than the Resolution requireth, and mark the Errors accordingly with the Signs + or —.

Errors to be marked.

Multiplied crosswise into the Positions.

Then multiply contrarywise the one *Position* by the other Error: And if the Errors be both too great or both too little, substract the one Product from the other, and the one Error from the other, and divide the Difference of the Products by the Difference of the Errors.

Errors like or unlike, and then what to do.

But if the Errors be unlike, as the one + and the other —, add the Products, and divide the Sum thereof by the Sum of the Errors added together: For the Proportion of the Errors, is the same with the Proportion of the Excesses or Defects of the Numbers supposed, to the Numbers sought.

Q. Of the Number of Tenements.

Example 1. *A* and *B* diſcourſing of their Tenements; *A* ſays to *B*, If I had 2 of your Tenements I ſhould have double the Number you have: To whom *B* replies, If I had 2 of your Tenements I ſhould be equal with you: How many Tenements had each of them?

Anſwer.

For Anſwer, ſuppoſe *A* had 16, to which 2 added makes 18, which is double the Number of 9; but having taken 2 from thence, it appeareth by this Suppoſition *B* had 11; wherefore 2 taken from 16 and added to 11, make *B* 13, and leave *A* 14. But they ſhould be equal, therefore the *Poſition* is erroneous, and the Error too much by 1.

Again, ſuppoſe *A* had 20, to which 2 being put, it's 22, and double the Number of 11; but from thence 2 being taken, *B* by this Suppoſition muſt have 13. Now take 2 from 20 and put to 13, it's 15 for *B*, and leaves 18 for *A*, which is not equal; therefore I have erred again by 3 too much.

Errors both +.

Then multiplying 16 the firſt *Poſition* by 3 the ſecond Error, and likewiſe 20 the ſecond *Poſition* by 1 the firſt Error, the Product 20 is taken from the Product 48, becauſe the Errors are both +, and the Remainer 28 is the Dividend; and abating 1 the leſſer Error from 3 the Greater, there is 2 for the Diviſor; the Quotient of which Diviſion will be 14, the Number ſought for *A*, and then by Conſequence muſt *B* have 10: for 2 taken from 10, and added to 14, makes 16, which is double to 8; and 2 taken from 14, and put to 10, makes both 12 alike.

Firſt.   *Second.*
Poſition 16 X 20 Poſition.
Error 1+   3+ Error.
20   48
Products 48 — 20 = 28 (14 *A.*
Errors 3 — 1 = 2 (10 *B.*

Proof.
14+2=16
10−2= 8 Half.
14−2=12
10+2=12 Equal.

Reſolution by other Poſitions, where the Errors are both —.

If the Suppoſitions had been 12 and 10, ſuppoſing 12 for *A*, then muſt *B* have 9; and taking 2 from 9 to put to 12, makes *A* 14, which is double to 7 for *B*; but taking 2 from 12 to put to *B*, makes *B* 11, and leaves but 10 for *A*, which is an Error too little by 1. And ſuppoſing 10 for *A*, then muſt *B* have 8; and taking 2 from 8 to put to 10, makes 12 for *A*, double to 6 for *B*; but then taking 2 from 10 to put to 8 makes *B* 10, and leaves *A* but 8, which is an Error too little by 2, for they ſhould be equal: And becauſe the Errors are both alike, the Work is as before.

Firſt.   *Second.*
Poſition 12 X 10 Poſition.
Error 1—   2— Error.
10   24

Products 24—10=14 (14 *A.*
Errors 2— 1= 1 (10 *B.*

Reſolution by other Poſitions, where the Errors are + and —.

But if 20 and 10 had been the *Poſitions*, then the Errors as before being found unlike, that is 3 + and 2 —, the Total of the Products muſt be the Dividend, and the Total of the Errors the Diviſor.

Firſt.   *Second.*
Poſition 20 X 10 Poſition.
Error 3+   2— Error.
30   40

Products 30+40=70 (14 *A.*
Errors 3+ 2= 5 (10 *B.*

Q. Of 3 Debts, what they were.

Example 2. *A*, *B*, and *C*, were indebted to *D*, who hath forgotten their Particular Debts, but remembreth the Debt of *A* and *B* added together made 50 *l.* and *C* and *B* together owed 80 *l.* and the Debt of *A* and *C* together was 70 *l.* The Queſtion is, what each Man's particular Debt was?

Anſwer.

Anſw. Suppoſe *A* did owe 15 *l.* then muſt *B* owe 35 *l.* (for both their Debts made 50 *l.*) And if *B* owe 35 *l.* then did *C* owe 45 *l.* (becauſe both their Debts made 80 *l.*) But then the firſt Man's 15, and the third Man's 45, make together but 60 *l.* and they ſhould be 70 *l.* So is the Error 10 —.

Then

Then suppose *A* did owe 26 *l.* it will follow that *B* owed 24 *l.* and *C* 56 *l.* But now 56 and 26 arise to 82 *l.* and it should be but 70 *l.* so here is an Error of 12 +.

Now multiplying crosswise as before, the Products 260 and 180 added, the Errors being unlike, make 440 to be divided by 22 the Sum of the Errors. So is 20 *l.* found the Debt of *A*, and consequently *B* 30 *l.* and *C* 50 *l.*

First Position. *A* & *B*. *Consequence.* *B* & *C*. *Consequence.* *A* & *C*.
$A\,15 + B\,35 = 50$ $B\,35 + C\,45 = 80$ $A\,15 + C\,45 = 60$
True Debts 70 Error 10 —

Second Position. *A* & *B*. *Consequence.* *B* & *C*. *Consequence.* *A* & *C*.
$A\,26 + B\,24 = 50$ $B\,24 + C\,56 = 80$ $A\,26 + C\,56 = 82$
True Debts 70 Error 12 +

First.		*Second.*
Position 15	X	26 Position.
Error 10 —		12 + Error.
260		180

Products $260 + 180 = \frac{440}{22}$ ($\overset{l.}{20}$ *A*.
Errors $10 + 12 = 22$

Proof.

$A\,20 + B\,30 = 50$ $B\,30 + C\,50 = 80$ $A\,20 + C\,50 = 70$

Example 3. A Parcel of Linen Cloth, *viz.* Lockram and Canvas, was sold to the Number of 30 Ells for 51 *s.* The Canvas was rated at 18 *d.* the Ell, and the Lockram at 2 *s.* how many Ells of each sort were there? *Q. Of Lockram and Canvas, how much of each.*

Answ. By supposing 10 Ells of Canvas, the Error will be 4 +: And by supposing 6 Ells, the Error will be 6 +; the Products therefore of 6, the *Position* into 4 and of 6, the Error into 10, taken one from the other, leave 36 the Dividend, and the Difference of the Errors 2 is the Divisor: So is there found to be 18 Ells of Canvas, and by Consequence 12 Ells of Lockram. Answer.

First Position.		*s.*	*d.*		*s.*
Canvas	10 Ells ×	1	6	=	15
Lockram	20 Ells ×	2	0	=	40
	30				55
		Paiment			51
		Error			4 +

Second Position.		*s.*	*d.*		*s.*
Canvas	6 Ells ×	1	6	=	9
Lockram	24 Ells ×	2	0	=	48
	30				57
		Paiment			51
		Error			6 +

First.		*Second.*
Position 10	X	6 Position.
Error 4 +		6 + Error.
24		60

Products $60 - 24 = \frac{36}{2}$ (18
Errors $6 - 4 = 2$

Proof.

		s.	*d.*		*s.*
Canvas	18 Ells ×	1	6	=	27
Lockram	12 Ells ×	2	0	=	24
	30				51

Example 4. A Carpenter was to build a Piece of Work in 30 Days, and by Agreement was to receive for every Day he wrought 3 *s.* but for every Day he did not work within that Time, he was to be amerced 2 *s.* When the Work was done he received but 25 *s.* for his Work: The Question is, how many Days he worked, and how many Days he plaid? *Q. Of a Carpenter's Work and Play.*

Answ. By the Suppositions of 20 and 12, the Errors will be 15 + and 25 —; the Products added make 680, and the Errors added 40: So by Division is found he worked 17 Days and plaid the rest. Answer.

First Position.

		s.		s.
Wrought	20 Days	× 3	=	60
Played	10 Days	× 2	=	20
	30			40
		Received		25
		Error		15 +

Second Position.

		s.		s.
Wrought	12 Days	× 3	=	36
Played	18 Days	× 2	=	36
	30			00
		Received		25
		Error		25 −

First. Position 20 ╳ 12 Position. *Second.*

Error 15 + ╳ 25 − Error.

180   500

Products $180 + 500 = 680$ (17

Errors $15 + 25 = 40$

Proof.

			s.
Working Days	17 at 3*s.* a Day	=	51
Playing Days	13 at 2*s.* a Day	=	26
	30		25 Received.

Notes. 1. *Suppose Number easy to be parted.* *Note* 1. As in *Single*, so in *Double Position*, though the Number supposed be never so false, Resolution may be had thereby: Yet for more ease in the Work, suppose a Number likely or apt to be parted equally into so many Parts as are necessary to the Resolution of the Question.

2. *Second Position Homogeneal to the first.* 2. Let the second *Position* always be *Homogeneal* to the first, that is, belong both to one Man, one Thing, *&c.* For though the Operator is at liberty to suppose for which he will of the Positives sought in the Question; yet if he suppose first for one, and then for another, the Resolution will be confused.

3. *Both Errors if equal & unlike in Signs.* 3. If both the Errors be equal in Numbers, and yet their Signs unlike, half of both the *Positions* is the Sum desired.

Q. *Of Apples given 3 Maids, how many.* *Example.* A Man having a certain Number of Apples, met three Maids who desired some of his Apples; whereupon he giveth *A* $\frac{1}{4}$ of his Apples, and she giveth him three Apples again. To *B* he giveth $\frac{1}{3}$ of his remaining Apples, and she giveth him two Apples again. To *C* he giveth $\frac{1}{7}$ of his remaining Apples, and she giveth him one again; so had he 13 Apples left: how many Apples had he at first?

Answer. *Answ.* 20: For $\frac{1}{4}$ thereof is 5, and 3 returned again leaves 18; of which $\frac{1}{3}$ is 6, and 2 returned leaves him 14; of which $\frac{1}{7}$ is 2, and 1 returned leaves 13.

And if the Suppositions made for Resolution be 16 and 24, the Errors will be $1\frac{5}{7}$ — and $1\frac{5}{7}$ +: So 20 the half of 40 (which is the Total of 16 and 24) is the Number sought.

4. *Double Position most useful.* 4. All the Propositions resolved by *Single Position*, will be resolved by *Double*; and several by other of the Comparative Elements foregoing.

Example by Specificks *of the fourth Sort, as well as Position.*

Q. *Of filling a Cistern, the Time.* At *Bronze* is a Statue in form of a Lion standing upon a Fountain, with this *Epigram*; If I let the Water pass out of my right Eye, I can fill the Cistern (holding 732 Gallons) in 2 Days: If I let it pass out of my left Eye, I can fill it in three Days: If it pass out of my Feet, the Cistern will be filled in 4 Days: But if it pass out of my Mouth, I can fill the Cistern in 6 Hours: In what time then shall I fill it, if I pour forth Water by all the Passages at once?

Answer. *Answ.* In 4 Hours and 44 Minutes *ferè*: For in that time cometh out of his Mouth 576 Gallons, out of his right Eye 72, out of his Left 48, and out of his Feet 36.

By Position.

1. Position 3 Hours, Error $266\frac{7}{8}$ —

Then as		*Hours.*		*Gallons.*		*Hours.*		*Gallons.*
	Right Eye	48	.	732	::	3	.	$45\frac{3}{4}$
	Left Eye	72						$30\frac{1}{2}$
	Feet	96						$22\frac{7}{8}$
	Mouth	6						366
								$465\frac{1}{8}$

$732 - 465\frac{1}{8} = 266\frac{7}{8}$

2. Position

		Hours.		*Gallons.*		*Hours.*		*Gallons.*
2. Position 4 Hours. Error $111\frac{5}{6}$ —	Then as	Right Eye	48	. 732	::	4	.	61
		Left Eye	72					$40\frac{2}{3}$
		Feet	96					$30\frac{1}{2}$
		Mouth	6					488
								$620\frac{1}{6}$

$732 - 620\frac{1}{6} = 111\frac{5}{6}$

$266\frac{7}{8} - 111\frac{5}{6} = 155\frac{1}{24}$

$$\begin{array}{ll} 4 & 3 \\ \hline 1064 & 333 \\ 3\frac{1}{2} & 2\frac{1}{2} \\ \hline 1067\frac{1}{2} - & 335\frac{1}{2} = 732 \end{array} \qquad \begin{array}{l} 732 \times 24 = \\ 155 \times 24 + 1 = \end{array} \begin{array}{l} |2684 \\ \cancel{17568} \\ \hline \cancel{3721} \end{array} \Big(4\tfrac{44}{61} \text{ Hours.}$$

By Ratio's *after the manner of* Specificks: *Thus,* By Specificks.

$$\begin{array}{ccccccccc} & & & 1990656 & & & & & \\ \hline \textit{Right Eye.} & & \textit{Left Eye.} & & \textit{Feet.} & & \textit{Mouth.} & & \\ 48 & \times & 72 & \times & 96 & \times & 6 & = & 1990656 \\ \vdots & & \vdots & & \vdots & & \vdots & & \\ 1 & & 1 & & 1 & & 1 & & \\ \hline 41472 & + & 27648 & + & 20736 & + & 331776 & = & 421632 \end{array} \Big(4\tfrac{44}{61} \text{ Hours.}$$

Example by Position *and* Equation of Paiment.

A is indebted to *B* 800 *l.* to be paid at 4 Months; 1 Month being past, he payeth 200 *l.* and $1\frac{1}{2}$ Month after the first Month he payeth 300 *l.* more; when shall the other 300 *l.* be paid? *Q. Of a Debt paid, part before due, when the rest.*

Answ. 5 Months after the last Paiment, that is, $3\frac{1}{2}$ Months after the end of the 4 Months. Answer.

By Position.

	l.			
Debt	800	multiplied by the Time of Paiment	$800 \times 4 =$	3200
Paid	200	multiplied by the Time of Paiment	$200 \times 1 =$	200
Rest	600	multiplied by the Time of Paiment	$600 \times 2\frac{1}{2} =$	1500
Paid	300	more, so remaineth yet 300, which suppose *A* kept 2 Months	$300 \times 2 =$	600
				2300
		Suppose *A* kept the 300 *l.* 4 Months;	Error	900—
		Then instead of the 600 above, must be 1200 added to 1500 and 200, which		2900
			Error	300—

$$\begin{array}{ll} 900 - & 300 = 600 \\ 4 & 2 \\ \hline 3600 - & 600 = 3000 \end{array} \qquad \frac{3000}{600}\Big(5 \text{ Months after } 2\tfrac{1}{2} \text{ Months.}$$

By Equation of Paiment *thus.* By Equation of Paiment.

Debt 800 *l.* in 4 Months.

$$\text{Paid}\left\{\begin{array}{l} 200 \times 1 = 200 \\ 300 \times 2\frac{1}{2} = 750 \end{array}\right\} \frac{950}{\text{Paid } 500}\Big(\begin{array}{l} 4 \text{ Months.} \\ 1\frac{9}{10} \text{ Time equated.} \\ \hline 2\frac{1}{10} \text{ Difference.} \end{array}$$

Rest 300

$$\text{As } 300 \,.\, 2\tfrac{1}{10} :: 500 \,.\, 3\tfrac{1}{2}$$

$$\begin{array}{r} 2\frac{1}{10} \\ \hline 1000 \\ 50 \\ 300)\overline{1050}\Big(3\tfrac{1}{2} \text{ Months after 4 Months.} \end{array}$$

Endless were the Questions that might be propounded for Resolution by *Position*; and several Authors are found stored with much variety of such Propositions, *Other Questions chosen out of other Books.*

ons, besides those already answered, some of the choice Ones here selected may serve instead of the rest.

1. Of the Travel of two Posts. *Example* 1. There are two Towns distant one from the other 200 Miles, from whence two Posts that depart upon one Day from the one Town unto the other, and one goeth two Miles a Day more than the other, they meet in five Days; how many Miles doth each Post travel in a Day?

Answer. *Answ.* The one went 19, and the other 21 Miles in a Day; which in 5 Days made the One to have gone 95, and the Other 105, and together 200 Miles.

2. Of 3 Silver Cups, with a Cover. *Example* 2. A Goldsmith hath three Silver Cups with a Cover of 18 Ounces, and the second Cup weigheth half as much as the First and third Cups. If the Cover be put to the first Cup, it weighs as much as all the three Cups; and if joined to the Second, it will weigh as much as the second Cup twice, and the third Cup once: But if put to the third Cup, it weigheth twice as much as the first and second Cups: what then was the Weight of each Cup?

Answer. *Answ.* The first Cup weighed 6 ℥, the Second 8 ℥, and the Third 10 ℥, that is together 24 ℥: And so much is the Cover and first Cup $18 + 6$. The Cover and second Cup is 26, that is $18 + 8$, which is equal to the third Cup, and double the Second $10 + 8 + 8$. And the Cover and third Cup is 28 the double of 14, the Weight of the first and second Cups.

3. Of Crowns caught up in a Difference. *Example* 3. Two Partners had in Accompt between them 400 French Crowns, whereof one should have 230, and the other 170: But in parting them they fell so at Variance, that he had most that could catch most. Yet afterward being reconciled, they agreed that he that had most should lay down $\frac{1}{2}$ of them he had, and he that had least should lay down $\frac{1}{3}$, and the Sum of both should be equally divided between them, and so should each Man have his due: The Question is, how many of the Crowns each Man caught up?

Answer. *Answ.* One caught up 280, and the other 120: for $\frac{1}{2}$ of 280 is 140, and there remaineth 140; and $\frac{1}{3}$ of 120 is 40, and there remaineth 80; this 40 and that 140 added are 180, the Half whereof is 90 for each; and this added to the Remains, makes for one 230, that is $90 + 140$, and for the other 170, that is $90 + 80$.

4. Of Money and Cloth, how much. *Example* 4. A Merchant buying Cloth, findeth if he take 12 Cloths, he shall want 42 *l.* to pay for them; but if he take 9 Cloths, then he hath 84 *l.* too much: How much Money had he, how much did a Cloth cost, and how many Cloths bought he?

Answer. *Answ.* He had 462 *l.* which will pay for 11 Cloths at 42 *l.* a Cloth.

5. Of the Money Partners had. *Example* 5. *A*, *B*, and *C*, buy a Ship for 200 *l.* If *A* have $\frac{1}{2}$ of what *B* pays, then he can pay for the Ship alone: And if *B* have $\frac{1}{4}$ of *C*, or *C* have $\frac{1}{3}$ of *A*, then they respectively can pay for the Ship: what Monies had each of them?

Answer. *Answ.* *A* had 120 *l.* which lacking 80 *l.* of 200 *l.* *B* must be double to 80, that is 160 *l.* this wanting 40 *l.* of 200 *l.* *C* must have 4 times 40, that is 160 *l.* as *B* had. And then 160 *l.* and 40 *l.* which is $\frac{1}{3}$ of *A*, will make up 200 *l.* also.

6. Of Hiero's *Crown.* *Example* 6. *Hiero* King of *Syracuse* in *Sicily*, had caused to be made a Crown of Gold to be offered for his good Success in Wars; in making whereof his Goldsmith fraudulently took out a Portion of the Gold, and put in Silver for it, yet so that there was nothing thereof to be seen or abated of the full Weight. The King suspecting the Fraud, propounds the Doubt to *Archimedes*; *viz.* How he might discover the Fraud without breaking the Crown. *Archimedes* not knowing presently how to answer the King's Desire; a while after as he chanced to enter into a Bath, he observed, as his Body entred into the Bathing-Vessel, the Water ran over; and thereupon apprehending a Reason of Solution to the King's Question, as the Story reports, was so rejoiced, that forgetting he was naked, ran Home, crying, εὕρηκα, εὕρηκα, *I have found, I have found*: And caused two massy Pieces, one of Gold and the other of Silver, to be prepared, of the same Weight with the Crown; and considering that Gold being heavier, and more compact by Nature than Silver, and so occupying less room, if it were put into a Vessel brim full of Water, would cause less Water to run over than a Mass of the same Weight of Silver would do: Whereupon trying both, he noted the Quantities of Water at each Time so run over, and learned thereby what Proportion in Quantity is between Gold and Silver of equal Weight. And then putting the Crown into the Vessel brim full of Water, (as before) marked how much Water run over then; and comparing it with the Water that ran over when the Gold was put

put in, noted how much it did exceed that; and likewise comparing it to the Water that ran over when the Silver was put in, marked how much it was less than that; and by these Proportions found out the just Quantity of Gold that was taken out of the Crown, and how much Silver was put in instead thereof.

Vitruvius who writeth the History, doth not declare the Particulars: But suppose the Crown to weigh 8 ℔, and so the other Masses, and imagine when the Gold was put in, there ran over 2 ℔ of Water; and when the Silver was put in, there ran over $3\frac{1}{2}$ ℔; and when the Crown was put in, there ran over but $2\frac{1}{4}$ ℔. Now supposing there was 2 ℔ of Silver in the Crown, then must there be 6 ℔ of Gold: And accordingly by the *Rule of Three*,

	℔ *Gold*	℔ *Water.*		℔ *Gold*	℔ *Water.*
As	8	2	::	6	$1\frac{1}{2}$

$$2 \times 6 = 12;\quad 8)\,12\,(1\tfrac{1}{2}$$

	℔ *Silver.*	℔ *Water.*		℔ *Silver.*	℔ *Water.*
As	8	$3\frac{1}{2}$	::	2	$\frac{7}{8}$

$$3\tfrac{1}{2} \times 2 = 6\tfrac{1}{1} = 7;\quad 8)\,7\,(\tfrac{7}{8}$$

$\frac{7}{8}$
$2\frac{3}{8}$ Sum.
$2\frac{1}{4}$ when the Crown was put in.
$\frac{1}{8}$ Error +

Supposing again the Silver 1 ℔, then must the Gold be 7 ℔; and the Proportions by the *Rule of Three*, thus:

	℔ *Gold.*	℔ *Water.*		℔ *Gold.*	℔ *Water.*
As	8	2	::	7	$1\frac{3}{4}$

$$2 \times 7 = 14;\quad 8)\,14\,(1\tfrac{3}{4}$$

	℔ *Silver.*	℔ *Water.*		℔ *Silver.*	℔ *Water.*
As	8	$3\frac{1}{2}$	::	1	$\frac{7}{16}$

$$\tfrac{8}{1})\,\tfrac{7}{2}\,(\tfrac{7}{16}$$

$\frac{7}{16}$
$2\frac{3}{16}$ Sum.
$2\frac{1}{4}$ when the Crown was put in.
$\frac{1}{16}$ Error —

Products.

$$\frac{2}{1} \times \frac{1}{16} = \frac{1}{8}$$

$$\frac{1}{1} \times \frac{1}{8} = \frac{1}{8}$$

$$\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

Products $\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$
Errors $\frac{1}{8} + \frac{1}{16} = \frac{3}{16}$ $\quad \frac{3}{16})\frac{1}{4}(\frac{4}{3}(1\frac{1}{3}$ ℔ Silver.

So had the Crown, if 8 ℔ in all $\left\{\begin{array}{l} 1\frac{1}{3} \text{ ℔ Silver} \\ 6\frac{2}{3} \text{ ℔ Gold} \end{array}\right\}$ By the Suppositions abovesaid.

Yet *Archimedes* needed not to have the Masses of equal Weight with the Crown; for by other Quantities of the same Metal, though of unequal Weight, the Solution may be had, as *Record* hath well observed.

Proof of Position of both sorts.

The Proof of all the Works wrought by *Position*, is when the Number desired is found, to add, substract, multiply, or divide thereby according to the tenor of the Question: and if in Conclusion there be Agreement in all things with the *Data* in the Proposition, the Work is right; as by the Proof of several of the Works before in this Chapter is fully approved.

CHAP. XV. *Proportions doubled.*

Figural Proportions.

Hitherto this second Part of the fourth Book hath spoken of plain *Disjunct Proportions*; now therefore a little is to be seen of *Figural*, which are so named, because the Proportion between the Numbers given and required, lies pro-

perly between them as they are, or are to be Figurate, or to the Resolution make use of Figural Numbers: So as being Figurate to the Square Quantity, they are called *Doubled Proportions*; if cubed, *Tripled Proportions*, &c.

Doubled Proportions how taken.

Doubled Proportions; are not here to be strictly taken only for the Proportions about Squares, but take in among them other Plain or Superficial Figures, as Circles, Triangles, *&c.* and consider as well the *Proportions* between the *Parts* of such Figures one to another, as the *Proportions* between the *Data* and *Quæsita* in Propositions concerning them.

Circle, the Principal Parts, and Analogies thereof.

The principal Parts of a Circle, wherein an Analogy is requisite, are three; The *Diameter*, the *Circumference* (called also *Peripherie* and *Perimeter*) and the *Area*: And because the Product of half the Circumference into half the Diameter is the *Area*, accompting the Diameter to the Circumference, according to the aforementioned *Archimedes*, or *Oughtred* from *Ludolph van Ceulen*, mentioned before also in *Figural Numbers*, Chap. 2.

The Analogies *are:*

As 7, to 22. or 1, to 3,1416	}	So is the Diameter to the Circumference.
As 14, to 11. or 1, to 0,7854	}	So is the Square of the Diameter to the *Area*.
As 88, to 7. or 1, to 0,0795775	}	So is the Square of the Circumference to the *Area*.

Propositions.

Q. Of the Circumferance and Area.

A Circle whose Diameter is 21: What shall be the Circumference, and what the *Area* by *Archimedes*?

Answer.

Answ. 66 Circumference, and $346\frac{1}{2}$ *Area*.

As 7 . 22 :: 21 . 66 Circumference.
As 14 . 11 :: 441 (Q. 21) . $346\frac{1}{2}$ *Area*.

Q. Of the Diameter & Area.

A Circle whose Circumference is 66: what shall be the Diameter, and what the *Area* by *Archimedes*?

Answer.

Answ. 21 Diameter, and $346\frac{1}{2}$ *Area*.

As 22 . 7 :: 66 . 21 . Diameter.
As 88 . 7 :: 4356 (Q. 66) . $346\frac{1}{2}$ *Area*.

Q. Of the Diameter and Circumference.

A Circle whose *Area* is $346\frac{1}{2}$: what shall be the Diameter, and what the Circumference by *Archimedes*?

Answer.

Answ. 21 Diameter, and 66 Circumference.

As 11 . 14 :: $346\frac{1}{2}$. 441 (Q. 21) . Diameter.
As 7 . 88 :: $346\frac{1}{2}$. 4356 (Q. 66) . Circumference.

Triangles, *their Sides and Angles*, &c.

Touching *Triangles*, their principal Parts are their Sides, Angles and *Areas*; but (that *Geometry* be not too far intruded upon, to which properly these Speculations belong) it will be enough to say something here of the Sides and Angles of Right-angled Triangles only.

Of a Right Angle.

The Doctrine of Triangles acquaints us, that in all plain Right-angled Triangles, the Square of the Side that subtendeth the Right-angle, is equal to the two Squares of both the other Sides. Wherefore in such Propositions, where the *Data* and Number sought represent the three Sides of such a Triangle, the Resolution of either in question is accordingly to be found.

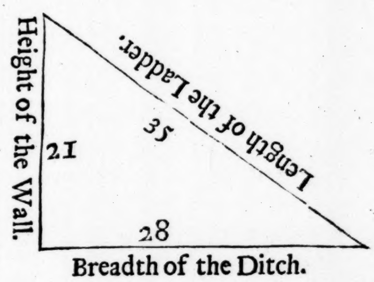
Q. Of the length of Scaling Ladders.

Example 1. The Walls of a City 21 Feet high, which hath a Ditch at the bottom 28 Feet broad, are to be scaled; and the scaling Ladders are commanded to be made a Foot longer than will reach from the outermost Brink of the Ditch, to the Top of the Wall: how long must the Ladders be?

Answer.

Answ. 36 Feet long: For the Squares of 21 and 28, which are 441, and 784 added, make 1225, whose Square Root is 35 the length, that will reach from the Brink of the Ditch to the Top of the Wall; to which 1 Foot added, because the Ladders were to be so much longer, makes the whole length of the Ladder 36 Feet.

Height



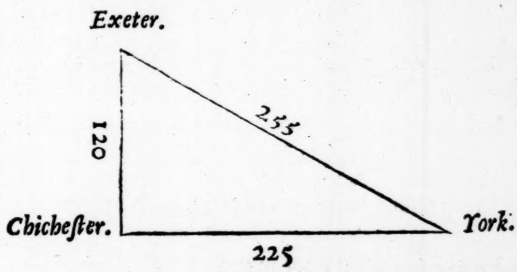
$$\left.\begin{array}{l} 21 \times 21 = 441 \\ 28 \times 28 = 784 \end{array}\right\} = 1225 \,(\, 35$$

325 — 1 added.

Length of the Ladders 36

Example 2. There are two Towns, as ſuppoſe *Chicheſter* and *York*, which lie South and North, diſtant one from the other 225 Miles; a third Town, as *Exeter* lying plain Weſt from *Chicheſter*, is diſtant from *York* 255 Miles: how far is *Chicheſter* diſtant from *Exeter* in a right Line? Q. *Of the Diſtance of two Cities.*

Anſw. 120 Miles: For here the number ſought being not repreſented by the *Hypotenuſal* Line, but by one of the Sides, containing the Right Angle; the Square of the other Side containing the ſame Angle, is to be taken from the Square of the ſubtending Side, that is, 50625, the Square of 225 from 65025 the Square of 255; ſo is the Remain 14400, the Square of 120. Anſwer.



$$\begin{array}{l} 255 \times 255 = 65025 \\ 225 \times 225 = 50625 \\ \hline \qquad\qquad\quad\; 14400 \,(\, 120 \text{ Miles.} \\ \qquad\qquad\quad\; 144 \end{array}$$

The *Area* of ſuch Triangles being always half the Product of the 2 Sides, containing the Right Angle: If it were inquired how many Acres of Ground there were in the Triangle laſt mentioned, conſidering that one Engliſh Mile contains 5280 Feet or 320 Perches, and one Acre 160 Square Perches, the Sides 120 and 225 multiplied by 320, and the Products one into another, and the Half divided by 160, make ſuch a Triangle to contain 8640000 Acres of Land. Q. *Of the Area of ſuch a Triangle.* Anſwer.

Sides of the Triangle	120		225 Miles.	
Perches in an Engliſh Mile	320		320	
	2400		4500	
	360		675	Product.
Perches in the Sides	38400	×	72000	= 2764800000
			Half	1382400000

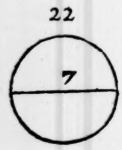
$$\frac{1382400000}{160} \,(\, 8640000 \text{ Acres.}$$

Sometime it is needful to turn a Circle into a Rectangle-Triangle, which is commonly performed thus; Make the Perpendicular of the Triangle equal to the Semidiameter of the Circle, and the Ground or Baſe-Line of the Triangle equal to the *Peripherie* of the Circle. *To turn the Triangle into a Circle.*

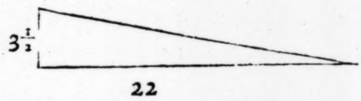
Example. If a Circle have the Diameter 7, and *Peripherie* 22: what ſhall the Sides of a Rectangle-Triangle be, that is equal in *Area* to the Circle, according to *Archimedes*? *Example.*

Anſw. The Perpendicular $3\frac{1}{2}$ and the Baſe 22. Anſwer.

22



11
$3\frac{1}{2}$
33
$5\frac{1}{2}$
Area of $38\frac{1}{2}$ the Circle.



22
$3\frac{1}{2}$
66
11
77
Area of $38\frac{1}{2}$ the Triangle.

Quadrangles, *what considered in them.*

Quadrangles, whether exact Squares, or other Rectangle Figures, (so far as concerns *Arithmetick*) have considered, their Sides, their *Area's*, Values, and Alteration of their Forms: But because all these, more or less, have been touched before, as among *Figural Numbers* in the second Part of the second Book, or in the *Indirect Rule of Three*, or *Specificks*, in this second Part of the fourth Book, they must be more sparingly remembred now.

Of their Sides and Area's.

Touching the Sides and *Area's* of *Quadrangles*.

Square.

Of the Square.

If the *Data* be the { Side / *Area* } and the *Quesita* the { *Area*. / Side.

For the Area. To resolve the First, square the Side, and the Product is the *Area*.

For the Side. To resolve the Second, extract the Square Root of the *Area*, as before in *Figural Numbers*.

If another Denomination be sought.

But if the Proposition require the Number sought in another Denomination to that given; Then if the Denomination be greater than that given, divide the Number found by so many of the Lesser, as are contained in one of the Greater. And if the Denomination required be lesser than that given, multiply the Number given or found accordingly.

Q. *Of the Acres in a Field.*

Example 1. A Field is 36 Rods Square every way: how many Acres doth that Field contain?

Answer.

Answ. $8\frac{1}{10}$ Acres: Here, because Acre is an higher Denomination than Rod, after 36 the Side is multiplied into it self, the Product 1296 being the *Area* in Rods, is divided by 160 the Rods in an Acre, and the Quotient is $8\frac{1}{10}$ as above.

Side $36 \times 36 = 1296$ ($8\frac{1}{10}$ Acres.
Perches in an Acre 160

Ex. *If the Yards or Feet therein be sought.*

But if the Proposition had demanded, how many Yards or Feet, &c. there had been, then these Denominations being lesser than Rods, either 36 must be multiplied by $5\frac{1}{2}$ for Yards, and by $16\frac{1}{2}$ for Feet, and the Products squared; or 1296 must be multiplied by $30\frac{1}{4}$ for Yards, and by $272\frac{1}{4}$ for Feet, because so many make a Square Rod.

Answer.

36
$5\frac{1}{2} \times 5\frac{1}{2} = 30\frac{1}{4}$; 1296
180 ; 38880
18 ; 324
$198 \times 198 = 39204$ Yards.

36
$16\frac{1}{2} \times 16\frac{1}{2} = 272\frac{1}{4}$; 1296
216 ; 2592
36 ; 90724
18 ; 25922
3
$594 \times 594 = 352836$ Feet.

Q. *Of the Rods in one Side.*

Example 2. A Square Field containeth 39204 Yards: how many Rods doth one Side contain?

Answer.

Answ. 36 Rods: For Rod being an higher Denomination than Yard, 198 the Square Root of 39204 extracted, is divided by $5\frac{1}{2}$, or 5,5 the Yards in one Rod, and the Quotient is 36 Rods as before.

```
  . . .        3 3
 39204 ( 19 8,0 ( 36 Rods.
 1        5,5
Gnomon  261      5,5
Gnomon   3104
```

But if the Proposition had been to know how many Rods or Yards in the Side of a Field that is $8\frac{1}{10}$ Acres: Then, because Rods and Yards are lesser Denominations, $8\frac{1}{10}$ is multiplied by 160 for Rods, and 4840 for Yards, because so many are in 1 Acre, and the Root of each shall be the Side in those Denominations. *Ex. If the Rods or Yards of the Side be sought, and Acres given. Answer.*

```
  160                    4840
    8 1/10                  8 1/10
 ------                 ------
  1280                  38720
    16                    484
 ------ ( √            ------ ( √
  1296  ( 36 Rods.      39204 ( 198 Yards.
```

Of Oblongs.

Oblongs.

Such Rectangle Figures as have their opposite Sides parallel.

If the *Data* be $\left\{\begin{array}{l}\text{Both Sides}\\ \textit{Area}\text{ and Side}\end{array}\right\}$ and the *Quesita* $\left\{\begin{array}{l}\textit{Area.}\\ \text{Other Side.}\end{array}\right.$

In the first Case multiply the Sides together. *For the Area.*

But if the Proposition require the Number sought in another Denomination to that given: then, as before in Squares, divide or multiply accordingly by so many of the Lesser as are contained in one Greater. *If another Denomination be sought.*

Example 1. A Board is 1,17 Feet broad, and 16,32 Feet long: how many square Feet doth it contain? *Q. Of the Feet in a Board.*

Answ. Almost 19,1 Feet, as the Product of 16,32 by 1,17 makes appear. *Answer.*

```
      16,32 Length.
       1,17 Breadth.
     -------
     11424
     1632
    1632
    -------
Area 19,0944 Square Feet.
```

Example 2. A tiled Roof hath the Breadth $16\frac{1}{4}$ Feet, and the Length 47 Feet: how many Squares of Tiling doth the whole Roof contain? *Q. Of the Squares of Tiling in a Roof.*

Answ. 15,275 Squares, or $15\frac{11}{40}$: Here the Length 47 being doubled (for both Sides) is 94; which multiplied by $16\frac{1}{4}$, or 16,25, the Product 1527,5 is divided by 100 (or 10 x 10) the Feet in a Square. *Answer.*

```
 16,25 Breadth.
    94 Double length.
 -------
  6500
 14625                  1527,500
 -------                -------- ( 15,275 Squares.
 1527,50 Feet.             100
```

Example 3. A Pavement broad 17,35 Feet, and long 30,5: how many Square Yards doth it contain? *Q. Of the Yards in a Pavement.*

Answ. A small matter above 58,797 Yards: For the Product of the Length into the Breadth 529,175 divided by 9, (or 3 x 3) the square Feet in 1 Yard give 58,797, and 2 is left remaining on the Division. *Answer.*

17,35 Breadth.
30,5 Length.
8675
52050
529,175 Feet.

7786(2
$\frac{529,175}{9}$ (58,797 Yards.

If the Demand were for Inches.

But if in any of these *Examples*, the Demand had been to know the Inches, then forasmuch as Inches are of lesser Denomination than the Feet given in the Proposition, the Feet found should have been multiplied by the Inches in 1 square Foot, that is (12×12 or) 144, Or the Sides given multiplied by 12, and then multiplied one into the other, would have produced the same Effect.

For the Side unknown.

In the second Case, when the *Area* and Side is given, to find the other Side, divide the *Area* by the given Side. But oftentimes the *Area* is given implicitly, and to be understood in the Denomination demanded.

Q. *Of Length to make a Foot of Board.*

Example 1. A Board is 6 Inches broad: how much Length thereof shall make a square Foot.

Answer.

Answ. 24 Inches, or two Feet: For either 144 the Inches in 1 square Foot divided by 6, or 1 Foot by $\frac{1}{2}$ in Fractions, effecteth the Desire; and the Analogy in all such Questions is reciprocal, and resolved by the *Indirect Rule of Three*, as in Plain Proportions before.

	Breadth.		*Length.*		*Breadth.*		*Length.*
As	12	.	12	::	6	.	24

6) 144 (24 Inches.

Or as 1 Breadth . 1 Length :: $\frac{1}{2}$ Breadth . 2 Length.

$\frac{1}{2}$) $\frac{1}{1}$ ($\frac{2}{1}$ Feet.

Q. *Of Length to make a Square of Tiling.*

Example 2. A Roof $16\frac{1}{4}$ Feet broad: how much Length thereof shall make a Square?

Answer.

Answ. $6\frac{2}{13}$ Feet?

	Breadth.		*Length.*		*Breadth.*		*Length.*
As	10	.	10	::	$16\frac{1}{4}$	.	$6\frac{2}{13}$

$\frac{65}{4}$) $\frac{100}{1}$ ($\frac{400}{65}$ ($6\frac{2}{13}$ Feet.

Q. *Of Length to make a Yard of Pavement.*

Example 3. A Pavement $16\frac{1}{2}$ Feet broad; how much Length thereof shall make a Square Yard?

Answer.

Answ. $\frac{6}{11}$ of a Yard.

	Breadth.		*Length.*		*Breadth.*		*Length.*
As	3	.	3	::	$16\frac{1}{2}$	.	$\frac{6}{11}$

9

11 3
$\frac{33}{2}$) $\frac{9}{1}$ ($\frac{6}{11}$ Yard.

Duplicate Ratio's.

Among like Plains in a Duplicate *Ratio*, that is, as the Squares of their Homologal Sides: If 3 Numbers be given, in which as the Square of the First is to the Square of the Second, so the Third to a Number sought. Then as the first Square to the third Number; so is the second Square to the Number sought.

Q. *Of the Number of paving Tiles.*

Example 1. Two like Rectangled *Area's* or Plains, the Greater in Length 40 Feet, the Lesser 24, each paved with paving Tiles, the Greater hath 1200 Tiles: how many hath the Lesser?

Answer.

Answ. 432: Because 1600, the Square of 40 to 576 the Square of 24, is as 1200 to 432, that is as 25 to 9.

As

$$\text{As } \underset{(Q.\,40)}{1600} \;.\; 1200 \;::\; \underset{(Q.\,24.)}{576} \;.\; 432 \text{ Tiles.}$$

$$\begin{array}{r} 576 \\ 12 \\ \hline 1152 \\ 576 \\ \hline 16)\,6912\,(432 \end{array}$$

Example 2. How many Acres of Wood-land, measured with a Perch of 18 Feet, are there in 73 Acres of Land measured with a Perch of $16\frac{1}{2}$ Feet? Q. *Of Acres Woodland Measure.*

Answ. 61,34, and somewhat over: For 18 and $16\frac{1}{2}$ reduced to their least Terms, are as 12 to 11: Wherefore, Answer.

$$\text{As } \underset{(Q.\,12)}{144} \;.\; 73 \;::\; \underset{(Q.\,11)}{121} \;.\; 61{,}34 \text{ Acres.}$$

$$\begin{array}{r} 121 \\ 73 \\ \hline 363 \\ 847 \\ \hline 8833 \end{array} \qquad 8833{,}00\,(61{,}34$$

Concerning the Values of Quadrangles in exchange or otherwise, the Analogy lies between the two *Area's*, Direct if the Price be the second Number, but Reciprocal if the Quantity. *For the Values*

Example 1. A Merchant hath 40 Yards of Cloth of $1\frac{1}{4}$ Yard broad, and would exchange for another Piece of $\frac{3}{4}$ Broad: how many Yards of this ought he to have for the other? Q. *Of Cloth of one Breadth for another.*

Answ. $66\frac{2}{3}$ Yards: For the 40 multiplied by $1\frac{1}{4}$, being the Content of 1 Piece, shall be divided by $\frac{3}{4}$, as in the *Indirect Rule of Three.* Answer.

$$\text{As } \underset{(40\times1\frac{1}{4})}{50} \;.\; 1 \;::\; \tfrac{3}{4} \;.\; 66\tfrac{2}{3} \text{ Yards.}$$

$$\frac{3}{4}\Big)\frac{50}{1}\Big(\frac{200}{3}\Big(66\tfrac{2}{3}$$

Example 2. How many Rods of Land, 4 Rods broad, shall I have for a Piece of Land that is 3 Rods broad and 12 Rods long? Q. *Of a Piece of Land for another.*

Answ. 9 Rods in length. Answer.

$$\text{As } \underset{(3\times12)}{36} \;.\; 1 \;::\; 4 \;.\; 9 \text{ Rods.}$$

Example 3. A Gentleman bespeaks a Piece of Wainscot of 36 Feet 3 Inches long, and 8 Feet 4 Inches broad; and agreeth to pay for it by the Yard square, at 10*s.* every Square: what doth this Piece of Work come to? Q. *Of the Price of a Piece of Wainscot.*

Answ. 16 *l.* : 15 *s.* : $7\frac{7}{9}$ *d.*: For the Length multiplied into the Breadth, is $302\frac{1}{12}$ square Feet: And 9 square Feet making 1 square Yard, the Analogy is; As 9 to 10*s.* so is $302\frac{1}{12}$, to 16 *l.* : 15 *s.* : $7\frac{7}{9}$ *d.*: Or reducing the square Feet into square Yards, it is, As 1 . 10 *s.* :: $33\frac{61}{108}$. 16 *l.* : 15 *s.* : $7\frac{7}{9}$ *d.* Answer.

Length,

Length. *Breadth.*

$36\frac{1}{4} \times 8\frac{1}{3}$ Or, $\frac{145}{4} \times \frac{25}{3}$ $\Big\} = \frac{3625}{12}$

(6
38(1
Or $\frac{3625}{12\times9} = \frac{3625}{108}$ (*Square Yards.* $33\frac{61}{108}$.

s. l. s. d.
As $9 . 10 :: \frac{3625}{12} . 16 : 15 : 7\frac{7}{8}$

(7
$\frac{9}{1}\Big)\frac{36250}{12}\Big(\frac{3625(0}{108}\Big($ 33| 5 s.
l. 16:15$\frac{3}{5}\frac{5}{4}$

s. l. s.
As $1 . 10 :: 33\frac{61}{108} . 16 : 15\frac{3}{5}\frac{5}{4}$

Q. Of a Piece of Heben Wood, the Price.

Example 4. If a Foot Square of *Heben* Wood be worth 7 *s.* 3 *d.* what will that Piece be worth that is $9\frac{1}{2}$ Feet long, and $4\frac{1}{3}$ Feet broad, but of equal Thickness with the other?

Answer.

Answ. 14 *l.* 18 *s.* $5\frac{1}{2}$ *d.* For $9\frac{1}{2}$ the Length, multiplied into $4\frac{1}{3}$ the Breadth, produce $41\frac{1}{6}$ square Feet; the Rest is resolved by the *Rule of Three Direct.*

Length. *Breadth.*

$9\frac{1}{2} \times 4\frac{1}{3}$ Or, $\frac{19}{2} \times \frac{13}{3}$ $\Big\} = \frac{247}{6}$

Sq.Foot. *s.* *Sq.Feet.* *l.* *s.* *d.*
As 1 . $7\frac{1}{4}$ :: $\frac{247}{6}$. 14 : 18 : $5\frac{1}{2}$

2(1
230(1
$\frac{7163}{24}\Big($ s. $29{,}8\frac{1}{3}\frac{1}{4}\times12 = \frac{13(2}{24}\Big($ $5\frac{1}{2}$ *d.*
(1
l. 14:18:$5\frac{1}{2}$

Concerning the Forms of Quadrangles.

About the Alteration of the Forms of *Quadrangles*, besides what may be understood in some of the Premisses, more may be seen in some Military Propositions, wherein some of the other Propositions are allso mixed.

Military Propositions.

Military Propositions principally concern the Forming of Battels.

Battels how considered.

Battels are considered in respect to the Number of Men, or Form of the Ground.

Square Battel of Men.

Square Battel of Men, is that which hath an equal Number of Men in Rank and File, or (as some phrase it) *Front* and *Flank.*

Square Battel of Ground.

Square Battel of Ground, is that which hath the Rank as long as the File, though the Men in Rank be more than in File.

Of the first four Sorts.

In respect of Men, there is a fourfold Variety, *viz.* a Square Battel, a Double Battel, a Battel of the Grand Front, which is called a *Quadruple Battel*, and a Battel of any Proportion of the Number in Rank to the Number in File.

1. Square Battel of Men.

1. For a Square Battel of Men, extract the Square Root out of the whole Number of Men, the same shall be the Number of Souldiers to be set in Rank, and likewise in File.

Q. Of two Brigades, how many the Front, &c.

Example 1. A Captain-General hath under his Command two Brigades, one of *Poles* consisting of 7225, and the other of *Switzers* consisting of 6084 Men; and upon occasion of Service would form each of them into Square *Battalia*; and again, all of them into one Square Battel: how many Men shall be in the Front, and accordingly in the Flank of every of them?

Answer.

Answ. Of the *Poles* 85, of the *Switzers* 78. And in Square Battel of the Total 115; and there will be 84 Men spare.

8
7225 (85
64
80
25

21
6084 (78
49
112
64

7225
6084
13309

22|84
13309 (115
1
21
1125

Q. Of an Army marching, what in Rank and File.

Example 2. An Army marching in a narrow Passage, were 20 in Rank, and 1620 in File; and when they come to pitch the Field, they would embody into a square Form: how many must there be in Rank and File?

Answ.

Answer. *Answ.* 180: For multiplying 1620 by 20, the whole Number of the Men is found to be 32400; which is a Square Number, and hath 180 for the Root.

```
 1620          26
   20          32400 ( 180
 -----         1
 32400         16
                64
```

2. Double Battel. 2. For a double Battel of Men, extract the Square Root of half the Number of Men, and the same doubled shall be the Number of Souldiers to be set in a Rank.

Example. *Example.* 1458 shall give 54, that is double of 27, the Root of 729, the half of 1458.

3. Quadruple Battel. 3. For a Quadruple Battel, extract the Square Root of a Quarter of the Number of Men, and the same quadrupled, shall be the Number of Souldiers to be set in a Rank.

Example. *Example.* 1024 shall give 64, that is Quadruple of 16, the Root of 256, the Quarter of 1024.

4. Battel of any other Form. 4. For a Battel required of any other Form, that is, if a *Ratio* be given according to which the Number of Men in Rank shall be to the Number of Men in File; Multiply the 2 Terms of the *Ratio* given: Then as the Product is to the Square of the Term which is for the Rank; or as the Term which is for the File, is to the Term which is for the Rank; so is the whole Number of Souldiers, to the Square of the Number of Men to be placed in a Rank. *Analogies.*

For in *Species* FR . Rq :: F . R.

Example. *Example.* 1944 Souldiers are to be marshalled, so that the Number of the Rank be to the Number of the File, as 8 to 3; that is, for 8 in Rank, 3 in File: what shall then be the Number of Men in the Rank?

Answer. *Answ.* 72: For the Terms of 8 multiplied into 3 is 24, and the Square of 8 is 64: Therefore as 3 to 8, or as 24 to 64; so is 1944 to 5184, which is the Square of 72.

```
     File.  Rank.  Number.   Q. Rank.
As    3  .   8  ::  1944   .  5184
                       8       2  √
                3 ) 15552 ( 5184 ( 72 Rank.
                           49
                           284
```

Proof thereof. And for Proof thereof, If there be 72 in Rank, which is 9 times 8, then there must be 27 in File, which is 9 times 3: And 72 multiplied into 27, produce 1944 the Number given.

Battels of the second Sort. In respect of the Form of Ground, the Battel is either a Square form of Ground, or longer one way than the other. For the Distance or Order of Souldiers marshalled in Array, may be distinguished either into *Open-Order*, or *Order*.

Open-Order, what. *Open-Order*, (as *Barriffe* in his *Military Discipline* tells us) is when the very Centers of their Places are distant 6 Feet asunder, both in Rank and File.

Order, what. *Order*, is when the Centers of their Places are distant 3 Feet, but some will have it 3 Feet in Rank and 6 Feet in File, which last *Order* and whatsoever *Order* else there is, in which the Distance of the Ranks one from another is greater than the Distance of the Files, causeth that a Square of Men maketh not a Square of Ground, but the Ground is longer on the File than on the Rank. So,

Distance, in Rank . R. in File . F. Number, in Rank . R. in File . F.

Square Battel of Ground. 1. If then it be a Square Battel of Ground, the Centers of the Distances being 3 Feet in Rank and 6 in File, because 3 is the Half of 6, the *Ratio* of the Distances is as 1 to 2. and seeing the Number in Rank, to the Number in File is reciprocal to the Distances, the *Ratio* of the Number of Men in Rank, to the Number of Men in File, shall be as 2 to 1. and the Rule may be as in the 4*th* sort of Battels abovementioned: As the Term of the File to the Term of the Ranks; *Analogies.*

So is the whole Number of Souldiers, to the Square of the true Number of the File.

RF . Fq :: R . F :: F . R.

Example. *Example.* 1352 Souldiers are to be set in a Square of Ground, that their Distances may be $3\frac{1}{2}$ Feet in a Rank, and 7 Feet in a File: what shall the File be?

Answer. *Answ.* The *Ratio* of the Rank to the File shall be reciprocally, as 7 to $3\frac{1}{2}$, that is, as 2 to 1. Therefore,

F . R :: N . Rq.
1 . 2 :: 1352 . 2704

2
1)2704(52 File.

Proof thereof. So the File being 52, the Rank must be half so much, which is 26, and $52 \times 26 = 1352$, a Proof of the Truth thereof.

Battel when the Ground is a long Square. 2. If a Battel, wherein the Distance in Rank is unequal to that in File, that is, be longer one way than the other, according to any *Ratio* given, there is to be considered a double *Ratio*, one reciprocal in respect of the Distances, the other according to the Form of the Ground: wherefore to find the *Ratio* of Men in Rank to the Men in File, multiply the 2 Terms of the Rank for the Rank, and the 2 Terms of the File for the File; and then the Rule shall be as in the 4*th* sort of Battels above.

Analogies.

Example. *Example.* 10290 Souldiers are to be set in *Battalia*, so that they may stand only 3 Feet asunder in Rank, and 7 Feet in File, and the Length of the Ground for the Rank to the Length of the Ground for the File, shall have the *Ratio* of 5 to 2: what number of Men shall be in the Front?

Answer. *Answ.* 245. For the *Ratio* of the Rank to the File, is as 7 to 3, in respect of the Distances, but in respect of the Ground, the *Ratio* of the Rank to the File, is as 5 to 2. So the like Terms multiplied, make 35 and 6. Then as 6 . 35 :: 10290. 60025. whose Square Root is 245 the Number of Men to be set in Rank, briefly expressed in Species thus;

Ff . Rr :: N . Rq   or, Rr. Ff :: N . Fq.

Proof thereof. And for Proof of the Truth thereof, seeing for every time 35 in the Rank (that is 7×5) there must be 6 in the File (that is 3×2) there being then 7 times 35 in 245, there must be 7 times 6, that is 42, in the File, and $42 \times 245 = 10290$ the whole Number of Souldiers.

How to enlarge or diminish a Plot of Ground. Hence arise Propositions, touching enlarging of a Plot of Ground for a Camp, Building, &c. or the contrary.

Example 1. *Example* 1. If 1000 Souldiers may be lodged in a Square of 300 Feet; how many must be the side of a Square which will serve to lodg 5000?

Answer. *Answ.* Almost 671, for the Analogy is thus.

N.S Q:G N.S Q:G
As 1000 . 300 × 300 :: 5000 . 450000

90000   (1
5000   96(1
1000)450000,000(450 0(00(670 +
36
889

Example 2. *Example* 2. An Architect projecting a Building, at first layeth out a Plot of Ground of 58 Feet square every way; but afterwards enlarging his Intentions, findeth it necessary to take in double the Ground: what must then the Side of that Square be that is double to the former?

Answer. *Answ.* 82 Feet and a small matter over: For seeing the Ground is to be doubled, the Square of 58 is to be doubled, which is 6728, and the greatest square Root therein 82, as before.

$$\text{Root } 58 \times 58 = \overset{\textit{Square.}}{3364} \times 2 = \overset{\textit{Doubled.}}{\cancel{6}\cancel{7}28} \left(\begin{matrix} 4 \\ 82 \end{matrix} \right. \sqrt{\ } \text{ Side.}$$

$$\begin{matrix} 64 \\ 324 \end{matrix}$$

But if the Propoſition had been to double the Side of the Ground, then the Square muſt have been the Square of 116 the Double of 58, which is four times as much as the former.

$$\text{Root } 58 \times 2 = 116 \times 116 = \overset{\textit{Square.}}{13456} \textit{ Area. } \frac{13456}{3364} \Big(4$$

Proof of doubled Proportions.

In all theſe Operations, ſo far as concerns the Extraction of Roots, or *Rule of Three*, have the Proof of their Work already ſpoken to in other Chapters; the reſt proper to this Chapter, hath the Proof in the Anſwers to the Propoſitions particularly illuſtrated.

CHAP. XVI. *Proportions tripled, &c.*

Tripled Proportions.

AFTER the Proportions about Plain Figures, come thoſe uſed about Bodies, or Solids.

Proportions of Bodies large.

Proportions about Bodies taking in the *Ratio* of the ſeveral Parts, are of themſelves enough to fill a large Treatiſe, eſpecially if the word Body be largely taken.

Of Man's Body. *Meaſures taken from thence.* *Proportions of* Noah*'s Ark.*

Albertus Dureus hath wrote a whole Book of the Meaſures of Man's Body, and *Pythagoras* calls Man *the Meaſure of all Things*, becauſe he is the moſt Perfect among all ſublunary Bodies; and according to the Maxim of Philoſophers, that which is moſt Perfect, and the firſt in Rank, meaſureth all the reſt. And we know that the Meaſures of an Inch, Foot, Cubit, Pace, *&c.* have taken their Names and Quantities from Humane Bodies: Yea, ſuch is the admirable Symmetry and Concordancy in the Parts of Man's Body, that ſome well-proportioned Works have been faſhioned and compoſed after his Proportion. As *Noah*'s Ark was in Length 300 Cubits, in Breadth 50, and in Height 30; ſo that the Length doth contain the Breadth 6 times, and ten times the Depth: and ſuch Proportion will be found in the Body of a Man being meaſured.

Of Man's Body.

Obſervations of Man's Body.

1. The Length of a Man well made (which is commonly called his Height) is equal to the Diſtance, from the one end of his longeſt Finger to the other, when the Arms are extended as far as they may.
2. The Breadth of a Man (or the Space which is from one ſide to another) makes the ſixth part of all the Body, taken in Length or Height.
3. The Thickneſs of the Body (taken from the Belly to the Back) is the tenth Part of the whole Body, ſome ſay the ninth.
4. The Length of the Face, is equal to the Length of the Hand, taken from the Wriſt to the extremity of the longeſt Finger.
5. The Height of the Brow, the length of the Noſe, the ſpace between the Noſe and the Chin, the Length of the Ears, and the greatneſs of the Thumb, are perfectly equal the one to the other.
6. If a Man have his Feet and Hands extended or ſtretched forth in Form of St. *Andrew*'s Croſs, placing one Foot of a Pair of Compaſſes upon his Navel, there may be deſcribed a Circle which will paſs by the ends of his Fingers and Toes; and drawing Lines by the Terms of them, there may be inſcribed a Square within a Circle.

What Bodies are here intended.

But theſe and ſuch like are ſet aſide, the intent of this Chapter being to call over thoſe Proportions, converſant about inanimate Bodies or Geometrical Figures that have Solidity; and among them, ſuch as are moſt uſeful in meaſuring of Timber, Stone and other Solids, Queſtions of Gunnery, Pyrotechnie, Gauging

ing of Vessels, and other Mathematical Propositions; most of which depend upon the Knowledg of the Cube and Globe, or Sphere.

Of Globes and Cubes. Globes have their capacities one to another, as the Cubes of their Diameters; for by *Euclid* 18 *Prop.* 12 Book, they bear triple that Proportion their Diameters do. And Cubes by *Euclid*, 19 *Prop.* 8 Book, bear triple Proportion in comparison of their Sides. *Triplicate Ratio's, what.* Hence like *Solids*, are said to be in a Triplicate *Ratio*, when they are as the Cubes of their Homologal Sides; and so not unfitly may this Chapter, concerned in the Proportions of such Bodies, bear the Title of *Tripled Proportions*.

This agreement between Cubes and Globes, occasions the Resolutions to several Propositions concerning them, to agree also in the Method of Resolution.

Propositions relating to these and other Bodies, commonly concern either their Solidity, Gravity, Value, or Form, or a Mixture of one with the other; some more particularly to one Body, and others to another.

Globe or Sphere.

Particularly of the Globe *or* Sphere.

Principal Parts. Because the Product of the *Axis* into the *Periphery* is the *Superficies*, and that again by $\frac{1}{6}$ of the *Axis* is the *Solidity*: Therefore,

If the *Data* be the { *Axis*, *Superficies*, *Axis*, *Solidity*, } and the *Quesita* be the { *Superficies*, *Axis*, *Solidity*, *Axis*,

According to *Archimedes* and *Oughtred* before mentioned,

Analogies therein.

The Analogies *are:*

As 7, to 22, or 1, to 3,1416	So is the Square of the *Axis*, to the *Superficies*.
Contrary, as 22, to 7, or 1, to 0,31831	So is the *Superficies*, to the Square of the *Axis*.
As 21, to 11, or 1, to 0,5236	So is the Cube of the *Axis*, to the *Solidity*.
Contrary, as 11, to 21, or 1, to 1,90986	So is the *Solidity*, to the Cube of the *Axis*.

Propositions.

Q. Of the Superficies and Solidity. A Sphere whose *Axis* is 14; what is the Superficies thereof? and what the *Solidity* by *Archimedes*?

Answer. *Answ.* 616 Superficies, and $1437\frac{1}{3}$ *Solidity.*

(Q. 14)
As 7 . 22 :: 196 . 616 *Superficies.*
(C. 14)
As 21 . 11 :: 2744 . $1437\frac{1}{3}$ *Solidity.*

Q. Of the Axis. A *Sphere*, whose *Superficies* is 616: what is the *Axis* by *Archimedes*?

Answer. *Answ.* 14 *Axis.*

As 22 . 7 :: 616 . 196 (Q. 14) *Axis.*

Q. Of the Axis. A *Sphere*, whose *Solidity* is $1437\frac{1}{3}$: what is the *Axis* by *Archimedes*?

Answer. *Answ.* 14 *Axis.*

As 11 . 21 :: $1437\frac{1}{3}$. 2744 (C. 14) *Axis.*

Cube.

Particularly of the Cube.

Principal Parts. If the *Data* be the { Side or Root, Solidity } and the *Quesita* be the { Solidity. Side or Root:

For

For the first, multiply the side cubically, and you have the Solidity. *For the Solidity.*

For the second, extract the Cube Root of the Solidity, as before in Figural Numbers. *For the Root.*

But if the Proposition require the Number sought, in another Denomination to that given: then, as in the Chapter last before about Squares and long Squares, so here in Cubes and long Cubes, *&c.* divide for a Greater, and multiply for a lesser Denomination. *If another Denomination be sought.*

Example 1. A piece of Timber is 18 Inches Square: how many solid Feet are therein? *Q. Of the Feet in a Piece of Timber.*

Answ. $3\frac{3}{8}$ Feet: for seeing Feet are an higher Denomination than Inches, the Cube of 18, is divided by the solid Inches in one Foot, that is, 1728 (or $12 \times 12 \times 12$). And if 18 be turned into $1\frac{1}{2}$ Feet, the Triple *Ratio* will effect as much. *Answer.*

$$\text{Side } 18 \times 18 \times 18 = \frac{5832}{1728}\,(648\ (3\tfrac{3}{8}\text{ Feet} \qquad \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{27}{8}\,(3\ (3\tfrac{3}{8}$$

Solid Inches in 1 Foot

Example 2. A Cubical Body is $1\frac{1}{2}$ Foot Square every way: how many solid Inches are therein? *Q. Of the Inches in a Cube.*

Answ. 5832 Inches: For Inches being lesser than Feet, the $3\frac{3}{8}$ Feet, Solid Content, are multiplied by 1728, or $1\frac{1}{2}$ Foot is reduced into Inches, and multiplied cubically. *Answer.*

$$\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{27}{8}\,(3\tfrac{3}{8}\text{ Feet.}$$

$$\begin{array}{r} 1728 \\ 3\frac{3}{8} \\ \hline 5184 \\ 648 \\ \hline 18 \times 18 \times 18 = 5832 \text{ Inches.} \end{array}$$

Example 3. A Rock exactly cubical, containeth 1728 solid Feet: how many Inches does one Side contain? *Q. Of Inches in the Side of a Rock.*

Answ. 144 Inches: The Cube Root of 1728 Feet extracted, is 12 Feet; but Inches being of lesser Denomination, 12 is multiplied by 12, the Inches in a Foot. *Answer.*

$$\begin{array}{l} 1728\,(12 \times 12 = 144 \text{ Inches.} \\ 16 \\ \;\;128 \end{array}$$

Example 4. A Mount of Cubick Form containeth 2985984 solid Inches: how many Feet doth one Side contain? *Q. Of Feet in the Side of a Mount.*

Answ. 12 Feet: The Cube Root of 2985984 is 144, to be divided by 12, because Feet desired is the higher Denomination. *Answer.*

$$\begin{array}{l} 1241 \\ 2985984\,\left(\frac{144}{12}\right(12 \text{ Feet.} \\ 1 \\ 1744 \\ \;\;241984 \end{array}$$

Those Bodies that are cubical, yet not exact Cubes, order as Cubes. *Cubical Bodies.*

Example 1. A Tree hewed Square, 2 Feet broad, $1\frac{1}{2}$ Foot thick, and 3 Yards long: how many solid Feet are therein? *Q. Of Feet in a Tree.*

Answ. 27 Feet: For $2 \times 1\frac{1}{2}$ produces 3 Feet for the *Area* of the Plain; which multiplied into the Length 9 Feet, every Yard being 3, makes the Product 27 as before for the Solidity. And in like manner the Solidity of other Bodies, not regular Cubes, is to be found. *Answer.*

$$\begin{array}{lr} \text{Feet } 2 \times 1\frac{1}{2} = & 3 \\ \text{Yards } 3 \times 3 = & 9 \\ \hline & 27 \text{ Feet solid.} \end{array}$$

Q. Of Feet in a Plank.

Example 2. A Plank 20 Inches broad, $2\frac{1}{2}$ Inches thick, and 3 Yards long: how many ſolid Feet are therein?

Anſwer.

Anſw. $3\frac{1}{8}$ Feet: For 50 the *Area* of the Plain, made by the Breadth and Thickneſs, multiplied into the Length, which is 108 Inches, gives the Solidity in Inches 5400; this divided by 1728 the ſolid Inches in a Foot, gives in the Quotient $3\frac{1}{8}$ as before.

$$\begin{array}{lll} \text{Inches} & 20 \times 2\frac{1}{2} = & 50 \\ \text{Yards} & 3 \times 36 = & 108 \\ & \text{Solid} & \overline{5400}\ \text{Inches.} \end{array} \qquad \begin{array}{c} (216 \\ \cancel{5400} \\ \hline \cancel{1728} \end{array}\Big(3\tfrac{1}{8}\ \text{Feet.}$$

What Part of ſuch Bodies, &c.

In theſe and ſuch other Bodies, if it be deſired to know what Portion of Length there muſt be to any Breadth and Thickneſs given to make a Foot cubical, or ſuch other ſolid Content: then by the *Area* made of the Breadth and Thickneſs, divide the Cubick Foot, or other Solid Content; for the Analogy is reciprocal, as in the *Indirect Rule of Three*.

Q. Of Length for a Foot.

Example 1. A Piece of Timber is 8 Inches broad, and 9 Inches thick: how much in Length ſhall make a Foot thereof?

Anſwer.

Anſw. 24 Inches, or 2 Feet: For the *Area* of 8×9, is 72; which dividing 1728, the ſolid Inches in 1 Foot, the Quotient is 24: Or by the *Indirect Rule of Three*, As 144 to 12; ſo 8×9 to 24.

$$8 \times 9 = \begin{array}{c} 28 \\ \cancel{1728} \\ \hline \cancel{72} \end{array}\Big(24\ \text{Inches. Or as}\ \underset{(12\times12)}{144} \ .\ 12 :: \underset{(8\times9)}{72}\ .\ 24\ \text{Inches.}$$

$$72)\underline{1728}(24$$

Q. Of the Length for a Yard of a Wall.

Example 2. A Bricklayer hath undertaken to build a Brick-Wall of 3 Feet thick and 4 Feet high, to be paid for his Work by the Yard cubick; and would know how much Length ſhall make a cubical Yard of that Wall?

Anſwer.

Anſw. $2\frac{1}{4}$ Feet: For 27 the Yard cubick divided by 12, the Product of the Height and Thickneſs, gives $2\frac{1}{4}$ in the Quotient.

$$\begin{array}{r} 3 \times 3 \times 3 = \\ 3 \times 4 = \end{array} \begin{array}{c} (3 \\ \cancel{27} \\ \hline \cancel{12} \end{array}\Big(2\tfrac{1}{4}\ \text{Feet: Or as}\ \underset{(3\times3)}{9}\ .\ 3 :: \underset{(3\times4)}{12}\ .\ 2\tfrac{1}{4}\ \text{Feet.}$$

$$12)\underline{27}(2\tfrac{1}{4}$$

Jointly, of the Gravity, Value *and* Form.

For the Weight, Value & Form. Analogy between the Weight of Bodies of the ſame Kind.

The *Gravity* or Weight of Bodies of the ſame Kind, is according to the Cubes of their Diameters or Axis in Spherical Bodies, and of their Sides in cubical Bodies in direct Proportion.

Q. Of the Weight of a Bullet.

Example 1. Suppoſe a Bullet 8 Inches Diameter weigh 30 ℔: what ſhall a Bullet of the ſame Kind weigh that is but 4 Inches Diameter?

Anſwer.

Anſw. $3\frac{3}{4}$ ℔: Here the Cube of 8 to 30, is as the Cube of 4 to $3\frac{3}{4}$: Or in leſs Terms, the Cube of 2 to 30, is as the Cube of 1 to $3\frac{3}{4}$: For 8 to 4, is as 2 to 1.

$$\frac{8}{4} = \frac{2}{1} \quad \text{As}\ \underset{}{\overset{(C.8)}{512}}\ .\ \overset{℔}{30} :: \overset{(C.4)}{64}\ .\ \overset{℔}{3\tfrac{3}{4}} \qquad \text{Or, As}\ \overset{(C\,2)}{8}\ .\ \overset{℔}{30} :: \overset{(C.1)}{1}\ .\ \overset{℔}{3\tfrac{3}{4}}$$

$$512)\begin{array}{c} 64 \\ \hline 1920 \end{array}(3\tfrac{3}{4} \qquad \begin{array}{c} (6 \\ \cancel{30} \\ \hline 8 \end{array}\Big(3\tfrac{3}{4}$$

Q. Of the Weight of a Bullet.

Example 2. If a Gun of $5\frac{1}{2}$ Inches Diameter in the Mouth, ſhoot a Bullet of $20\frac{51}{64}$ ℔: what ſhall that Bullet weigh that ſerveth for a Gun of 8 Inches Diameter?

Anſwer.

Anſw. 64 ℔: Here the Cubes may be reduced into Halfs; that is, 8 into 16, and $5\frac{1}{2}$ into 11: Or elſe the *Analogy* ſet as in Fractions;

As

(C.11) lb (C.16) lb
As 1331 . $20\frac{51}{64}$ :: 4096 . 64
$20\frac{51}{64}$
81920
3264
1331)85184(64

(C.$5\frac{1}{2}$) lb (C.8) lb
Or, As $\frac{1331}{8}$. $\frac{1331}{64}$ :: 512 . 64

And by cancelling the like Numerators 1331, after the manner of Fractions, it shall be,

As $\frac{1}{8}$. $\frac{1}{64}$ :: 512 . 64

Example 3. Let a Cube of Metal of 4 Inches square, weigh 8 lb: what shall a Cube of the same Metal weigh that is 5 Inches square? *Q. Of the Weight of a Cube.*

Answ. $15\frac{5}{8}$ lb: For the Cube of 4 to 8, is as the Cube of 5 to $15\frac{5}{8}$. *Answer.*

(C.4) lb (C.5) lb
As 64 . 8 :: 125 . $15\frac{5}{8}$
8
64)1000($15\frac{5}{8}$.

Example 4. When a Cube of Metal, that is $3\frac{1}{2}$ Inches every way, weigheth 64 Ounces *Troy* Weight: what shall a Cube of the same Metal weigh that is $5\frac{1}{4}$ Inches? *Q. Of the Weight of a Cube.*

Answ. 216 Ounces *Troy*: For the Cube of 14 the Quarters in $3\frac{1}{2}$, is to 64, as the Cube of 21 the Quarters in $5\frac{1}{4}$ is to 216. *Answer.*

(C.14) ℥ (C.21) ℥
As 2744 . 64 :: 9261 . 216
64
37044
55566
592704

x64
4396
592704(216
2744
2744
2744

Contrariwise; If the Gravity of one Body be given, to find the Side of another Cubical, or the Axis of a Spherical Body: Then as the Roots of the given Numbers one to the other; so shall the Side or Axis given be, to that required: Or as the given Numbers one to another; so shall the Cube of the Side or Axis given be, to that required. *Analogy to find the Side or Axis.*

Example 1. If a Cube of 64 Ounces *Troy*, have the Side $3\frac{1}{2}$ Inches: what shall the Side of that Cube be that weigheth 216 Ounces *Troy*? *Q. Of the Side of a Cube.*

Answ. $5\frac{1}{4}$ Inches: The Cube Roots of 64 and 216, which are 4 and 6, set in less Terms are 2 and 3: Therefore as 2 to $3\frac{1}{2}$; so is 3 to $5\frac{1}{4}$. *Answer.*

Cubes	64	216
Roots	4	6
Least Terms	2	3

Inches. *Inches.*
As 2 . $3\frac{1}{2}$ :: 3 . $5\frac{1}{4}$
$\frac{2}{1}$)$\frac{21}{2}$($\frac{21}{4}$

Example 2. In case a Bullet of 4 Inches in the Axis weigh $3\frac{3}{4}$ lb *Averdupois*: what shall be the Axis of a Bullet of the same Kind that weigheth 30 lb? *Q. Of the Axis of a Bullet.*

Answ. 8 Inches: Where the first and third Numbers not being cubical, 4 is cubed; and after *Multiplication* and *Division*, the Root of the Quotient is extracted. *Answer.*

(C.4)
As $3\frac{3}{4}$. 64 :: 30 . 8 Inches.
30
$\frac{15}{4}$)$\frac{1920}{1}$($\frac{7680}{15}$(512($\sqrt{\ }$8

Among like Solids in a Triplicate *Ratio*, that is as the Cubes of their Homologal Sides: If 3 Numbers be given, in which as the Cube of the First is to the Cube of the Second; so the Third to a Number sought: Then as the first Cube to the third Number; so is the second Cube to the Number sought. *Triplicate Ratio's.*

Example

Q. Of Powder to charge a Gun.

Example 1. If $\frac{43}{100}$ ℔ of Gunpowder suffice to charge a Gun, whose Concave Diameter is $1\frac{1}{2}$ Inch: how many Pounds of the same Powder will suffice to charge a Gun whose Concave Diameter is 7 Inches?

Answer.

Answ. $43,7\frac{1}{135}$ ℔: For as the Cube of $1\frac{1}{2}$, or 1,5 to the Cube of 7; so is 0,43 to 43,7 and $\frac{1}{135}$ of a Prime.

(C. 1,5) (C. 7)
As 3,375 . 0,43 :: 343 . $43,7\frac{1}{135}$ Gunpowder.

343
1 29
172
129
147,49

(2
~~12~~ ~~365~~(5 ℔
~~147,490~~ 0($43,7\frac{1}{135}$
~~3,375~~
~~3,375~~
~~3,375~~

Q. Of Powder to charge a Gun.

Example 2. Suppose 43,7 ℔ of Gunpowder are sufficient to charge a Gun, whose Diameter in the Concave is 7 Inches: And there is another Sort of Gunpowder more strong, that is to the former as 5 to 2: how much of this will suffice to charge a Gun of 4 Inches Diameter?

Answer.

Answ. 3,26 ℔ and somewhat more: As appears, first, by the Proportions of the Powder, seeking how much of that strong Powder is enough to charge a Gun of 7 Inches Diameter; and then, by a second Work, with the Proportions of the Diameters.

℔ ℔
As 5 . 43,7 :: 2 . 17,48

2
87,4

~~32~~
~~87,40~~(℔ 17,48
5

(C. 7) ℔ (C. 4.) ℔
As 343 . 17,48 :: 64 . $3,26\frac{54}{343}$

64
69 92
1048 8
1118,72

~~21~~(5
8~~9~~ ~~1~~(4 ℔
~~1118,72~~($3,26\frac{54}{343}$
343

Proportions of Magnitudes and Gravities of Bodies, a curious Enquiry.

The Proportions as well of the Magnitudes, as of the Gravities of Bodies one to another, is a nice Inquisition; and among Authors some difference is found, according as the Experiments of the one have been more or less curious than the other.

What by Van Etten.

Henry van Etten, in his *Mathematical Recreations*, acquaints us, That a Quantity of Water to an equal Quantity of Metal, is in Proportion as,

Water.	*Tin.*	*Iron.*	*Copper.*	*Silver.*	*Lead.*	*Gold.*
10 .	75 .	81 .	91 .	104 .	$116\frac{1}{2}$.	$187\frac{1}{2}$.

What by Alsted.

Herewith agreeth the Learned *Alsted*, save that *Iron* he makes to be but $80\frac{22}{37}$, and adds, Oil 9, Honey 15, and Quicksilver 150.

So as a Magnitude of Gold, weighing $187\frac{1}{2}$ ℔. shall have an equal magnitude of Silver weigh but 104 ℔. and the like of the rest. But for that there is not only difference in the Weight of Fresh Water and Salt, but between Fresh Waters themselves, some being more Mineral than others, according to the Tincture they receive by the Mines, along which they glide in the Bowels of the Earth; therefore some accompt this but an uncertain Basis, and as unstable as the Water it self.

What by Mr. Oughtred, *according to* Ghetald.

Mr. *Oughtred* (often already mentioned) in his *Circles of Proportion*, pag. 67, 68, *&c.* tells us, That pieces of Metal, if they be of equal Magnitude, have their Weights in direct Proportion; but if of equal Weight, they have their Magnitudes in Proportion reciprocal. And inserts the Proportions of their Weights, according to the Experiments of *Marinus Ghetaldus*, in his Tractate, called *Archimedes Promotus*, thus:

☉ Gold	3990	Wherefore	☉ . ☿ :: 7 . 5 :: 3990 . 2850.
☿ Quickſilver	2850		☉ . ♄ :: 38 . 23 :: 3990 . 2415.
♄ Lead	2415		☉ . ☽ :: 57 . 31 :: 3990 . 2170.
☽ Silver	2170		☉ . ♀ :: 19 . 9 :: 3990 . 1890.
♀ Braſs	1890		☉ . ♂ :: 19 . 8 :: 3990 . 1680.
♂ Iron	1680		☉ . ♃ :: 95 . 37 :: 3990 . 1554.
♃ Tin	1554		

Therefore if 4 Pieces of Metals, whereof the Third is of the ſame kind with the firſt, and the fourth of the ſame kind with the ſecond, are Proportional; their Gravities alſo are Proportional.

And again, if there be 4 Pieces of Metals, whereof the Third is of the ſame kind with the Firſt, and the Fourth of the ſame kind with the Second, and the Firſt and Second of equal Greatneſs, and the Third and Fourth of equal Weight; the Gravities of the Firſt and Second ſhall be reciprocal to the Magnitudes of the Third and Fourth.

Weight of a Cylinder of Tin of an Inch long, &c.

The aforeſaid *Ghetald* (uſing the antient Roman Foot, which by his Accompt ſeems very little leſs than ours) found a Cylinder of Tin of 1 Inch thick and long, to weigh 1824 Grains, whereof $\frac{2}{3}$ is 1216 for the Weight of a Sphere of that Thickneſs; becauſe every Sphere is $\frac{2}{3}$ of a Cylinder that hath the Height and Diameter of the Baſe, the ſame with the *Axis* of the *Sphere*.

Weight of a Sphere.

(C. 10) (C. 12)
Then as 1000 . 1216 :: 1728 . 2101,248. the Weight of a Sphere whoſe Diameter is $\frac{1}{10}$ of a Foot; or ſet according to the manner of other Proportions, the Queſtion will ſtand thus: If a Sphere whoſe Diameter is $\frac{1}{12}$ of a Foot weigh 1216 Grains, what ſhall that Sphere weigh, whoſe Diameter is $\frac{1}{10}$ of a Foot? which in reaſon becauſe biggeſt muſt weigh moſt; and ſo the Cube of 10 being leſs than the Cube of 12, will be Diviſor as before.

Weight of a Cube.

And by the ſame Accompt, a cubed Inch of Tin weigheth 2322,3887324; and a cubed tenth Part of a Foot 4013,0877296.

How thereby to find the Weight of a Sphere of another Axis,

Accordingly to find the Weight of a Sphere of Tin of any other *Axis*, multiply the Cube of the *Axis* given, by 1216, if it be in Inch-meaſure, or by 2101,248, if it be by Decimal Parts of a Foot; and the Product will be the Weight of that Sphere.

And the Weight of a Sphere of another Metal.

But to find the Weight of a Sphere of any other Metal at any Diameter, aſſigned either in Inch-meaſure, or Decimal Parts of a Foot: ſeek the Weight of a Sphere of Tin at that Diameter given; and then as the proportional Number of Tin is to the Weight of the Sphere of Tin; ſo is the Proportional Number of the other Metal, to the Weight of the Sphere propoſed.

Example in Iron for the Weight.

Example. Suppoſe a Sphere of Iron have the Diameter 3 Inches: what ſhall the Weight be?

Anſw. 35494,054 Grains, and ſome ſmall Overplus.

(C. 1) (C. 3)
As 1 . 1216 :: 27 . 32832. Grains in the Sphere of Tin.
♃ ♂
As 1554 . 32832 :: 1680 . 35494,054. Grains in the Sphere of Iron.

To find the Diameter or Axis of a Sphere of other Metal.

And to find the Diameter of a Sphere of any Metal in Inch-meaſure, or Decimal Parts of a Foot by the Weight, ſeek the Cube of the Diameter of a Sphere of Tin of that Weight: and then as the Proportional Number of the other Metal, is to the Cube of the Diameter found; ſo is the Proportional Number of Tin, to the Cube of the Diameter of the Sphere propoſed.

Example in Iron for the Axis.

Example. A Sphere of Iron weigheth 35494,054 Grains: what is the Diameter thereof?

Anſw. 3 Inches.

Cube Cube
As 1216 . 1 :: 35494,054 . 29,189189144, *&c.*
♂ ♃
As 1680 . 29,18918914 :: 1554 . 27 Root 3.

Of the Value of Bodies.

The Value of all Bodies, is accompted according to their Solidity or Gravity; ſo as the Solidity or Gravity gotten, the Value is to be had by the *Rule of Three Direct.*

Q. Of the Worth of an Iron Bullet.

Example 1. At 15 *l.* the Tun, what is an Iron Bullet worth of 3 Inches Diameter?

Anſwer.

Anſw. 0,618967 *s.* or 7 Pence Farthing and ſomewhat above: For by the Work above, the Weight of ſuch a Bullet is found to be 35494,054 Grains; and then if one Tun coſt 15 *l.* or 1 Hundred 15 *s.* which is all as one, becauſe 20 Hundred are 1 Tun, the Weight of the Bullet, ſhall give the Sum aforeſaid.

	Grains in 1 C.	*s.*	*Grains in the Bullet.*	*s.*
As	860160 .	15 ::	35494,054 .	0,618967, *&c.*

Q. Of the Worth of a Piece of Timber.

Example 2. A Piece of Timber is 2 Feet broad, 3 Feet thick, and 6 Feet long: what doth it come to at 20 *s.* a Tun?

Anſwer.

Anſw. 18 *s.* For the Solidity found as before to be 36 ſolid Feet; and 40 Feet being a Tun, it ſhall be worth 18 *s.*

$$2 \times 3 \times 6 = 36 \qquad \text{As } 40 \,.\, \overset{s.}{20} :: 36 \,.\, \overset{s.}{18}.$$

Of the Form of Bodies, 2 Things.

Touching the Form of Bodies, Propoſitions of two ſorts are uſual.

1. To increaſe or diminiſh the Body, and yet keep the ſame Form: Or,
2. To alter the Form, and yet keep the ſame Solidity or Gravity.

1. To increaſe or leſſen, &c.

To increaſe a Body Spherical or Cubical, Double or Triple, *&c.* as is deſired, the Solidity; and to diminiſh the ſame, do the contrary.

Q. Of the Axis of a Globe, and Side of a Cube, doubled.

Example 1. A Globe whoſe *Axis* is 14, and a Cube of the ſame ſide, are both to be doubled: what ſhall the *Axis* of the One, and the Side of the other ſo doubled be?

Anſwer.

Anſw. The deſired *Axis* and Side ſhall be between 17 and 18, becauſe the Solidity of the Globe, whoſe *Axis* is 14, being according to *Archimedes* $1437\frac{1}{3}$, doubled is $2874\frac{2}{3}$; and this Solidity ſhall have for the *Axis* the Cube Root of 5488, which is the Double of 2744 the Cube of 14.

Cube.
Root $14 \times 14 \times 14 = \underline{2744}$
Doubled $\underline{5488}$

(5
~~2~~9(7
~~4~~ ~~3~~ 1(5
~~5~~ ~~4~~ 8 8(17 575
1
21
147
343

Periphery.
Axis $14 \times 44 = 616$ Superficies.
$\frac{1}{6}$ of the Axis $\underline{2\frac{1}{3}}$
1232
$\underline{205\frac{1}{3}}$
$\underline{1437\frac{1}{3}}$ Solidity.
Doubled $\underline{2874\frac{2}{3}}$

As $11 \,.\, 21 :: 2874\frac{2}{3} \,.\, 5488$ (C. 17 575)
$\underline{21}$
2874
5748
$\underline{14}$
11)60368(5488

Q. Of the Axis of a Globe, and Side of a Cube halfed.

Example 2. A Globe and *Cube*, whoſe Axis and Side is 14, are both to be made but half ſo much: what then ſhall the *Axis* of the one, and the Side of the other be?

Anſwer.

Anſw. 11 and a ſmall matter over: For ſo half 2744 the Cube of 14, and half $1437\frac{1}{3}$ the ſolid Sphere of 14, according to *Archimedes*, will both give.

Root

Root 14. Cube $\underline{2744}$
Half $\underline{1372}$

(41
1372(11 41
1331

Axis 14 . Solidity $1437\frac{1}{3}$
Half $\underline{718\frac{2}{3}}$

As 11 . 21 :: $718\frac{2}{3}$. 1372 (C. 11 41
21
718
1436
14
11)15092(1372.

How to increase or lessen the Burden of Ships.

And thus the Book called, *The Treasure of Travellers*, teacheth in the Building of Ships, how to increase or diminish their Burden, by increasing or diminishing accordingly their Length of the Keel, Breadth at the Main-Beam, and Depth in the Hold: and also to fit their Cables, Ropes, Masts, *&c.* though as to the Tackle and Rigging of Ships, Nautical Experience gives the best Directions.

Example.

So if a Ship of 60 Tuns, have the Length of the Keel 32 Feet within the Post; and another were to be built of 120 Tuns, or of 30 Tuns: Then the Cube of 32, which 32768 doubled for the Greater, and halfed for the Lesser, and the Roots accordingly taken, the Keel of the Greater will be somewhat above 40 Feet, and of the Lesser 25 Feet and better.

Keel 32. Cube 32768, Doubled 65536, Halfed 16384.

1
65536(40 1536
64

8|759
16384(25 759
8
7625

What the doubling or halfing of the Side of Cubical Bodies effects.

In all Cubical Bodies, the doubling of one Side doubles the whole Body, and halfing of one Side accordingly lessens the Body by half: So as in Bodies that are not exact Cubes, if the Increase or Decrease be only desired; then their Solidity is to be increased or decreased accordingly: But if their Form be to be kept, then the best way is to increase or decrease one Side.

Example in the Double.

As if a Piece of Timber 2 Feet broad, three Feet thick, and 6 Feet long, be doubled; then the Solidity 36 doubled shall be 72 equal to a Piece of 4 Feet broad, 3 Feet thick, and 6 Feet long; or 2 Feet broad, 6 Feet thick, and 6 Feet long; or 2 Feet broad, 3 Feet thick, and 12 Feet long.

Breadth.	*Thickness.*	*Length.*	*Solidity.*
2 ×	3 ×	6 =	36
		Doubled	72

$$\left.\begin{array}{l} 4\times3\times6 \\ 2\times6\times6 \\ 2\times3\times12 \end{array}\right\} = 72$$

Example in the Half.

Likewise if the same Piece of Timber were to be halfed, Then as the Half of 36 is 18; so shall be the Product of half one of the Sides multiplied into the rest.

Breadth.	*Thickness.*	*Length.*	*Solidity.*
2 ×	3 ×	6 =	36
		Half	18

$$\left.\begin{array}{l} 1\times3\times6 \\ 2\times1\frac{1}{2}\times6 \\ 2\times3\times3 \end{array}\right\} = 18$$

2. How to alter the Form, and keep the Solidity or Gravity.

To alter the Form, and keep the Solidity or Gravity, is most usual in irregular Bodies, to reduce them to Cubes or Spheres; which when their Solidity or Gravity is had thereby, the Axis of the one, or Side of the other, is to be had as in the *Examples* before.

And not only Irregulars, but others also, as Cones, Cylinders, Pyramids, Prisms, *&c.* may be thus reduced without any Difficulty, when their Solidity or Gravity is found; but in getting thereof they differ one from another, as is well known by the Rudiments of *Geometry*, to which their Measure properly belongs: Yet it may not be unprofitable here to see of some of them, some of their *Analogies*.

Of

Cone.

Of a Right Cone.

Becaufe a Sphere is equal to two *Cones* that have their Height, and the Diameter of their Bafe the fame with the Axis of the Sphere: The *Analogies*, by *Archimedes*, common to the *Cone*, are:

Analogies thereof.

As 7 to 22; fo is half the Diameter multiplied in the Side, to the Conical Superficies.

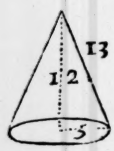
As 14 to 11; fo is the Square of the Diameter multiplied in $\frac{1}{3}$ of the Axis, to the Solidity.

To the Conical Superficies, if the *Area* of the Bafe be added, the Total fhall be the Superficies of the whole *Cone*.

Example.

Wherefore if a right *Cone* have the Diameter at the Bafe 10, and the Height or Axis 12; then fhall the Side be 13, the Conical Superficies $204\frac{2}{7}$, the Total Superficies $282\frac{6}{7}$, and the Solidity $314\frac{2}{7}$.

The Meafure of a Cone.



$5 \times 5 = 25$
$12 \times 12 = 144$ √
$169($13 Side.

As $7 . 22 :: 10 . 31\frac{3}{7}$ *Periphery.*

$5 \times 15\frac{5}{7} = 78\frac{4}{7}$ *Area* of the Bafe.

(5×13)
As $7.22::65.204\frac{2}{7}$ Conical Superficies.
$78\frac{4}{7}$ *Area* of the Bafe added.
$282\frac{6}{7}$ Total Superficies.

(Q.10×4)
As $14 . 11 :: 400 . 314\frac{2}{7}$ Solidity.
11
$14)4400(314\frac{2}{7}$

Variety.

So if the Side and half the Periphery be multiplied, the Product is the Conical Superficies.

And if $\frac{1}{3}$ of the *Area* at the Bafe be multiplied into the Height, or $\frac{1}{3}$ of the Height into the *Area*, the Solidity is had.

$13 \times 15\frac{5}{7} = 204\frac{2}{7}$ Conical Superficies.

$\left.\begin{array}{l} 26\frac{4}{21} \times 12 \\ \text{or} \\ 78\frac{4}{7} \times 4 \end{array}\right\} = 314\frac{2}{7}$ Solidity.

Cylinder.

Of a Right Cylinder.

Becaufe $\frac{2}{3}$ of a *Cylinder* is equal to a Sphere, that hath the Height and Diameter of the Bafe the fame with the Axis of a Sphere: The Common *Analogies* by *Archimedes* in the *Cylinder* are;

Analogies thereof.

As 7, to 22; fo is the Diameter multiplied in the Axis, to the Superficies-Cylindrical: to which add the *Area* of both the Bafes for the Total Superficies.

As 14, to 11; fo is the Square of the Diameter multiplied in the Side to the Solidity.

Or as 88, to 7; fo is the Square of the *Periphery* multiplied in the Side to the Solidity.

Example.

Therefore if a Right *Cylinder* have the Diameter 14, and the Height or Axis as much; then fhall the *Area* of each Bafe be 154, the Cylindrical Superficies 616, the Total Superficies 924, and the Solidity 2156.

The Meafure of a Cylinder.

D
As 7 . 22 :: 14 . 44 *Periphery.*
Half $7 \times 22 = 154$ *Area* of the Bafe.
Doubled 308 Bafes.

D L
As $7 . 22 :: 14 \times 14 . 616$ Cylindrical Superficies.
308 Bafes added.
924 Total Superficies.

$\left.\begin{array}{l} \quad\text{(Q.14 D) L} \\ \text{As } 14 . 11 :: 196 \times 14. \\ \quad\text{(Q.44 P) L} \\ \text{Or as } 88 . 7 :: 1936 \times 14. \end{array}\right\} 2156$ Solidity.

Variety.

So alfo if the *Periphery* be multiplied into the Height or Axis; the Product is the Cylindrical Superficies.

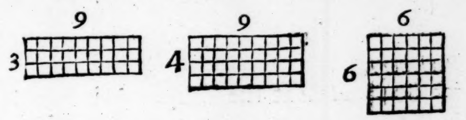
And if the *Area* of one Bafe be multiplied into the Axis, the Solidity is found.

$44 \times 14 = 616$ Cylindrical-Superficies.
$154 \times 14 = 2156$ Solidity.

Hereby

Hereby appeareth, 1. That it is not safe to judg the Value of Bodies by their Surface or Perimeter, but by their Solidity or Gravity, as was said before: For Plain Figures, called *Ipsoperimeters*, and also Bodies of Equal Surface, may be vastly different in their *Area's* and Solid Contents; as those of different Sides and Diameters may have their *Area's* and Solidities equal. *How to value Ipsoperimeters and Bodies of equal Surfaces.*

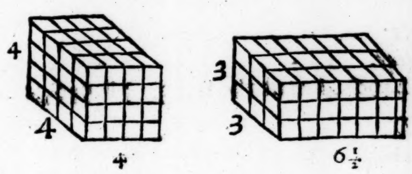
For let there be among plain Rectangle Figures, one whose Sides are 3 and 9, another 4 and 9, and a Square 6 and 6; it is evident that the Sides of the First are Ipsoperimetral with the Sides of the Square, that is, both 24: Yet the *Area* of the one is but 27, a Quarter less than 36 the Square *Area*. And though the *Perimeter* of the Second be 26, when the *Perimeter* of the Square is but 24; yet the *Area* of the second Figure is 36, equal to the *Area* of the Square. *Examples in Plains.*



$$6 + 6 \times 2 = 24$$
$$6 \times 6 = 36$$

Perimeter 24	*Perimeter* 26	*Perimeter* 24
Area 27	*Area* 36	*Area* 36

So in Solids, if one Body whose Length is $6\frac{1}{2}$ Feet, Breadth and Thickness alike 3 Feet, be examined and compared with the Cube of 4 Feet; their Surfaces will be equal, but their Solidities as different as $58\frac{1}{2}$ from 64. *Examples in Solids.*



Cube.

Superficies $16 \times 6 = 96.$
Solidity $4 \times 4 \times 4 = 64.$

Long Cube.

Solidity $3 \times 3 \times 6\frac{1}{2} = 58\frac{1}{2}$

Superficies $\left\{\begin{matrix} 3 \times 3 \times 2 \\ 6\frac{1}{2} \times 3 \times 4 \end{matrix}\right\} = 96.$

Again, let a Cube whose side is 14, be compared with a Sphere of the same Diameter, and a Cone and Cylinder of equal Diameter and Height; both the Superficies and Solidities will be thus different.

		Superficies	Solidities
14	Side or Root of the Cube,	1176 .	2744
	Diameter or Axis of the Sphere,	616 .	$1437\frac{1}{3}$
	Height and Diameter of the Cone,	$498\frac{6}{31}$.	$718\frac{2}{3}$
	Height and Diameter of the Cylinder,	924 .	2156

Wherefore it is evident, if *Sempronius* borrow of *Caius* a Sack of Corn 4 Feet high, and 2 Feet over, and pay him again with 2 Sacks of Corn, each 4 Feet high and 1 Foot over; yet hath *Caius* been paid but one Quarter of his Corn. *Caius paid too little.*

As 7 . 22 :: 2 . $6\frac{2}{7}$
$1 \times 3\frac{1}{7} = 3\frac{1}{7}$
4
Lent $12\frac{4}{7}$

As 7 . 22 :: 1 . $3\frac{1}{7}$ Paid.
$\frac{1}{2} \times 1\frac{4}{7} = \frac{11}{14} \times 4 = \frac{44}{14}(3\frac{1}{7}$
Paid $3\frac{1}{7}$ due $9\frac{3}{7}$.

2ly. It is evident, that if a Body partake of different Forms, the true Measure of that Body must be mixt accordingly; as in the Gaging of Vessels will be further clear. *How to measure Bodies of mixt Forms.*

A Wine or Beer Vessel is in form of a *Spheriode*, partly like a Sphere, and partly *Oval* or *Cylindrical*: Wherefore measure the two Diameters, *viz.* that at the Head, and that at the Bung, and also the Length within their Vessel, either in Inches or in Decimal Parts of a Foot, and by the Diameters find out the Circles. Then, *Spheroide, what.* *To gage Vessels.*

Add together 2 third Parts of the greater Circle, and 1 third Part of the lesser Circle, and multiply the Total by the Length; and this shall produce the Content or Solidity.

To find $\frac{2}{3}$ and $\frac{1}{3}$ of any Circle, the Analogies by the Common Way of *Archimedes*, and also by Mr. *Oughtred*, from *Van Ceulen*, are thus: *Analogies.*

For $\frac{2}{3}$ As 21, to 11 . Or as 1, to 0,5236 } So is the Square of the Diameter, to the $\frac{2}{3}$ of the Circle.

For $\frac{1}{3}$ As 42, to 11 . Or as 1, to 0,2618 } So is the Square of the Diameter, to the $\frac{1}{3}$ of the Circle.

Example.

Example. A Vessel whose Diameter at the Bung is 32 Inches, and at the Head 18 Inches, is in Length 40 Inches: what Content shall this Vessel be of?

Answ. By the common Proportions $24849\frac{11}{21}$ Solid Inches, and by the other 24839,584 Solid Inches; as by the following Operations of both appears.

Common Way.

Solidity by Archimedes.

As 7 . 22 :: 32 . $100\frac{4}{7}$ *Periphery.*
$16 \times 50\frac{2}{7} = 804\frac{4}{7}$ *Area* at the Bung. $\frac{2}{3}$ is $536\frac{8}{21}$
As 7 . 22 :: 18 . $56\frac{4}{7}$ *Periphery.*
$9 \times 28\frac{2}{7} = 254\frac{4}{7}$ *Area* at the Head. $\frac{1}{3}$ is $84\frac{6}{7}$

(Q.32.)
As 21 . 11 :: 1024 . $536\frac{8}{21}$ }
Or, (Q.18) } as above.
As 42 . 11 :: 324 . $84\frac{6}{7}$ }

Total $621\frac{5}{21}$
Length 40
Solidity $24849\frac{11}{21}$

Other Way.

Solidity by Van Ceulen.

As 1 . 3,1416 :: 32 . 100,5312 *Periphery.*
$16 \times 50,2656 = 804,2496$ *Area* at the Bung. $\frac{2}{3}$ is 536,1664
As 1 . 3,1416 :: 18 . 56,5488 *Periphery.*
$9 \times 28,2744 = 254,4696$ *Area* at the Head. $\frac{1}{3}$ is 84,8232

(Q.32)
As 1 . 0,5236 :: 1024 . 536,1664 }
Or, (Q.18) } as above.
As 1 . 0,2618 :: 324 . 84,8232 }

Total 620,9896
Length 40
Solidity 24839,5840

Content how otherwise to be found.

The Content of a Vessel in Cubick Inches is also to be had thus; to 2 Squares of the Diameter at the Bung, add the Square of the Diameter at the Head; and dividing the Total into 3 Parts, say, As 452 to 355; so is $\frac{1}{3}$ of the said Total multiplied by the Length, to the Content in Cubick Inches.

And so $18 \times 18 = 324$ And $32 \times 32 = 1024$
Double of 1024 is 2048
3)2372($790\frac{2}{3} \times 40 = 31626\frac{2}{3}$

And as 452 . 355 :: $31626\frac{2}{3}$. 24839, *&c.* as before.

To turn the Solidity into Concave Measure. Example.

Then having the Cubick Inches, or Cubick tenth Parts of a Foot in one Gallon of Wine or Beer, the Content of the Vessel in Gallons may be found.

281 For there being in 1 Foot 1000 Cubick 10*th* Parts, that is $10 \times 10 \times 10$; And 1728 Cubick Inches, that is $12 \times 12 \times 12$: And as Mr. *Oughtred* and others write, 231 Cubick Inches in a Wine Gallon, and $272\frac{1}{4}$ in a Beer Gallon; and the *Ratio* between them in lesser Terms, as 14 to $16\frac{1}{2}$: If Cubick Inches be given, the *Analogy* is,

(C.12) (C.10)
As 1728 . 272,25 :: 1000 . 157,5521— Cubick 10*th* Parts in a Beer Gallon.
As 1728 . 231 :: 1000 . 133,6805+ Cubick 10*th* Parts in a Wine Gallon.

But if Cubick 10*th* Parts be given, the *Analogy* is contrary.

(C.10) (C.12)
As 1000 . 157,5521— :: 1728 . 272,25. Cubick Inches in a Beer Gallon.
As 1000 . 133,6805+ :: 1728 . 231. Cubick Inches in a Wine Gallon.

Wherefore if the Solidity of the Vessel aforesaid were desired in Cubick 10*th* Parts: Then, by the

Common

Common Way.

As 1728 . 24849,5238 :: 1000 . 14380,5114, *&c.*

Other Way.

As 1728 . 24839,584 :: 1000 . 14374,7592, *&c.*

And if the Content of the same Vessel were desired in Gallons Wine-measure or Beer-measure; then accordingly division is to be made by 231 for Wine, if Cubick Inches, or by 133,6805, if Cubick 10*th* Parts be given: And by $272\frac{1}{4}$ or 272,25 for Beer, if Cubick Inches be given; or by 157,5521, if Cubick 10*th* Parts: So will be found in the said Vessel by the

Common Way { Beer Gallons 91, 27 / Wine Gallons 107, 57 } and somewhat above.

Other Way { Beer Gallons 91, 23 / Wine Gallons 107, 53 } and somewhat above.

By one of the Officers of Excise, I have heard that their Gage, or Rule by which they measure the Brewers Tuns and Vessels, is computed after the Rate of 282 Cubick Inches to a Gallon of Beer; which if so, seems to be for allowance of Lees, *&c.*

3ly. Hence also is apparent, That by the help of Figural Proportions, not only Bodies of mixed Forms may be measured, but also one and the same Body, whose Solidity or Gravity is mixed, may be measured, and in higher Proportions than tripled. *How to measure a mixt Body in Higher Proportions.*

Example 1. Suppose a Merchant have a Piece of Wine of 128 Gallons, and draw out thereof 16 Gallons, and fill it up again with Water: And again draw out 16 Gallons, and fill it again with Water, and do the like the third and fourth Time: How much Wine and Water were then in the Vessel? Q. *Of Wine mixt with Water four times.*

Answ. $75\frac{1}{32}$ Gallons of Wine, and the residue Water. Answer.

Here after the Numbers 128 and 16 are set in their least Terms, that is, 8 and 1, the Lesser is taken from the Greater, so it is 8 and 7; and the like will be if 16 be taken from 128, and the Remain 112 abbreviated with 128, both which are to be Figurate to the 4*th* Quantity, because the Mixture was fourfold, (for always the Figuration must be according to the Mixture) and then the Analogy is,

As the greater Figural Number to the whole Quantity: So is the lesser Figural Number to the Quantity desired.

(QQ. 8)		(QQ. 7)		
As 4096 .	128 ::	2401 .	$75\frac{1}{32}$	Wine.
		Complement.	$52\frac{31}{32}$	Water.
			128 Gallons.	

Example 2. If a Goldsmith have an Ingot of Silver 12 Penny Weights Fine, weighing 8 Marks, and cut off a Mark thereof, and melt with the residue a Mark of Copper; and from this mingled Mass cut off another Mark, and put thereto again a Mark of Copper, and do the like the third Time: The Question is, how much a Mark as it is mix'd will hold? Q. *Of Silver mixt with Copper.*

Answ. $8\frac{5}{128}$ Penny Weights fine. Answer.

For here the Proportion between the Cubes of 8 and 7, because the Mixture is but triple, guide the Analogy.

Whole	8	Marks.	(C. 8)	(C. 7)
Cut off	1	Mark.	As 512 . 12 ::	343 . $8\frac{5}{128}$ Fine.
Difference	7			

Thus such Mixtures are more easily resolved than by *Alligation*, as in that Chapter was before noted: and hereby also the Value of such Mixtures are as easily discovered. *These Mixtures better resolved here than by Alligation; also their Value.*

Example

Q. Of the Worth of mixed Wine and Water.

Example 1. A Merchant had 128 Gallons of Wine in a Cask, worth 5 *s.* the Gallon, and draweth out thereof 16 Gallons, and filleth up the Cask again with Water; and then draweth out 16 Gallons more, and filleth up again the Cask with Water: what will a Gallon of this mixture be worth?

Answer.

Answ. $3\frac{53}{64}$ *s.* for the Analogy is, as the whole Quantity Figurate, according to the Number of Mixtures, is to the highest Price; so is the Difference between the whole Quantity and the first Draught so figurate as the other, to the Price of the Mixture.

Whole. *First Draught.* *Difference.*

128 ——— $16 === 112$ abbreviated $\frac{112}{128} = \frac{7}{8}$ Figurate twice $\frac{49}{64}$.

Therefore as $64 \;.\; \overset{s.}{5} \;::\; 49 \;.\; \overset{s.}{3\frac{53}{128}}$.

Q. Of the Worth of Wine mixt with other Wine.

Example 2. If a Merchant have 128 Gallons of Wine, worth 5 *s.* a Gallon, and draw out thereof 16 Gallons, and fill up the Vessel again with Wine of 4 *s.* the Gallon, and afterwards draw out 16 Gallons, and fill up the Cask again with the Wine of 4 *s.* the Gallon, and so do the third time: what shall a Gallon of this mingled Wine be worth?

Answer.

Answ. $4\frac{343}{512}$ *s.* After 16, the first Draught, is taken from 128 the whole Quantity, and the Remain abbreviated therewith to their least Terms, and each of them figurate according to the Mixture, because the Wine put in was of Value as well as that drawn out, the *Analogy* here may be,

As the greater Figural Number, to 1; so is that greater Figural Number multiplied by the lowest Price given, and added to the lesser Figural Number, to the Price desired.

$128 — 16 = 112$ Abbreviated $\frac{112}{128} = \frac{7}{8}$ Figurate thrice $\frac{343}{512}$.
And $512 \times 4 + 343 = 2391$
Therefore $512 \;.\; 1 \;::\; 2391 \;.\; 4\frac{343}{512}$ Price desired.

Proof of this Work.

And the Truth hereof may appear, if, according to the Work of *Alligation* before, it be examined: for thereby being found upon the third Mixture in the Vessel $85\frac{3}{4}$ of the highest Price, and $42\frac{1}{4}$ of the lowest Price, and each of them multiplied respectively by their several Prices, and the Total of the Products divided by the whole Quantity so mixed, will bring forth in the Quotient the mean Price as before.

$$\text{Gallons}\begin{cases} 42\frac{1}{4} \times \overset{\textit{Prices.}}{4} === 169 \\ 85\frac{3}{4} \times 5 === 428\frac{3}{4} \end{cases}$$

And as $128 \;.\; 1 \;::\; 597\frac{3}{4} \;.\; 4\frac{343}{512}$.

$$\frac{128}{1}\Big)\frac{2391}{4}\Big(\frac{2391}{512}\Big(\overset{s.}{4\frac{343}{512}}$$

Proof of tripled Proportions.

The Operations of this Chapter being composed of *Figurals* and *Plain Proportions*, have their Proofs accordingly spoken to in other Places. And by reversing the Questions and Varieties of Work, the Truth of the Conclusions will sufficiently appear without further Assay or Example.

Partis secundæ Libri quarti

FINIS.

The Third PART of the Fourth BOOK.

CHAP. I.

Of PROPORTIONS Continued.

SImple *Disjunct Proportions* have at large, with their *Comparative Elements*, been unravelled in the foregoing Part; therefore this will concern it ſelf with *Continual Proportions*.

Continual Proportions their Comparative Elements.

Continual Proportions, as before in the firſt Chapter of this Fourth Book, were obſerved to be both *Arithmetical* and *Geometrical*, that having the Difference or Exceſs, and this the *Ratio* between every two Numbers or Terms equal. As,

Arithmetical and Geometrical, how different.

Examples of both.

Arithmetical	1 .	2 .	3 .	4 .	5 .	6.	&c.	*Difference*	1.
	1 .	3 .	5 .	7 .	9 .	11.	&c.	*Difference*	2.
	2 .	4 .	6 .	8 .	10 .	12.	&c.	*Difference*	2.
Geometrical	1 .	2 .	4 .	8 .	16 .	32.	&c.	*Ratio*	2.
	1 .	3 .	9 .	27 .	81 .	343.	&c.	*Ratio*	3.
	2 .	8 .	32 .	128 .	512 .	2048.	&c.	*Ratio*	4.

Both theſe are called *Progreſſion*, becauſe the Numbers go forward from the firſt Term, and have their Progreſs in an orderly Series, increaſing by an equal and continued *Difference* in the one, and *Ratio* in the other.

Both called Progreſſion, and why.

They both agree alſo in their placing or ſetting down, either the firſt Term to be ſet to the left Hand, and the reſt in order to the Right, as above: Or elſe the firſt Term at top, and the other underneath in order, as in *Addition of Integers*.

Wherein they agree.

How placed.

Yet further, the firſt Term in both is called the *Antecedent*, and all the reſt in reference thereto *Conſequents:* And moreover the firſt and laſt Terms in both are the *Extreams*, and all the other *Means*.

Antecedent and Conſequent.

Extreams and Means.

As they herein agree among themſelves, ſo in ſome things they agree with *Disjunct Proportions*; that is to ſay, *Arithmetical Continued*, with *Arithmetical Disjunct*; and *Geometrical Continued*, with *Geometrical Disjunct*.

Agreement with Disjunct Proportions.

1. Numbers in *Arithmetical Proportion Continued*, or *Disjunct*, have the *Aggregate* of the *Extreams*, equal to the *Aggregate* of the *Means*.

1. Total of the Extreams and Means equal.

Example.

Arith. Proportion Continued.

3 . 6 . 9 . 12 . 15

Aggre-18-gate.

Arith. Proportion Disjunct.

4 . 7 :: 5 . 8

Aggre-12-gate.

For in the *Disjunct*, as 4+8, ſo 7+5 are 12. And in the *Continued*, as 3+15 are 18, ſo alſo 6+12; and ſo likewiſe 9, the odd Mean added to himſelf, makes alſo 18.

2. Numbers in *Geometrical Proportion Continued*, or *Disjunct*, have the *Product* of the *Extreams* equal to the *Product* of the *Means*.

2. Product of the Extreams and Means equal.

Example.

Geom. Proportion Continued.

2 . 4 . 8 . 16 . 32

Pro-64-duct.

Geom. Proportion Disjunct.

5 . 15 :: 6 . 18

Pro-90-duct.

$$\left.\begin{array}{l}\text{For } 2 \times 32 \\ \text{and } 4 \times 16\end{array}\right\} = 64 \qquad \left.\begin{array}{l}\text{Also } 5 \times 18 \\ \text{and } 15 \times 6\end{array}\right\} = 90$$

And so 8 the odd Mean multiplied by himself, produceth 64.

3. Four Proportionals added to or taken from 4 others, the Consequence.

3. If 4 Numbers in *Arithmetical Proportion Continued* or *Disjunct*, be added to, or substracted from 4 other alike Proportional, the Totals and Remains will respectively be like Proportionals; and in the Totals the Excess will also be added, but in the Remains diminished.

Example.

Arith. Proportion continued.

	1	4	7	10	Excess	3
	4	8	12	16	Excess	4
Totals	5	12	19	26		7
Remains	3	4	5	6		1

Arith. Proportion disjunct.

	3 . 5	:	7 . 9		Excess	2
	2 . 4	:	5 . 7		Excess	2
Totals	5 . 9	:	12 . 16			4
Remains	1 . 1	:	2 . 2			0

4. Four Proportionals multiplied or divided by 4 others, the Consequence.

4. If 4 Numbers in *Geometrical Proportion continued*, or *disjunct*, be multiplied or divided by 4 other Numbers respectively proportional, the Products and Quotients shall be accordingly proportional; and the *Ratio* in the Products will likewise be multiplied, but in the Quotients divided.

Example.

Geom. Proportion continued.

	1	2	4	8	Ratio	2
	3	6	12	24	Ratio	2
Products	3	12	48	192		4
Remains	3	3	3	3		1

Geom. Proportion disjunct.

	7 . 28	::	9 . 36	Ratio	4
	8 . 32	::	9 . 36	Ratio	4
Products	56 . 896	::	81 . 1296		16
Remains	$1\frac{1}{7}$. $1\frac{1}{7}$	::	1 . 1		1

Computation of Continual Proportions, and their Issues.

Wherein further both Sorts of *Continued Proportions* agree, or disagree with themselves, or *Disjunct Proportions*, may be observed in uncovering their several *Comparative Elements*. Those *Arithmetical* in the next Chapter, and those *Geometrical* in the Fifth following, between which is placed the Computation of what issues from the First, and afterwards the Proceeds of the Second in the Order following, *viz.*

Continued Proportions
- *Progression Arithmetical*, Chap. 2.
  - *Transposition*, Chap. 3.
  - *Technologie*, Chap. 4.
- *Progression Geometrical*, Chap. 5.
  - *Transmutation*, Chap. 6.
  - *Anatocism*, Chap. 7.

CHAP. II. *Progression Arithmetical.*

Arithmetical Progression. *New Proportionals how gotten. Common Way of proceeding.*

Because in every *Arithmetical Progression*, the Antecedent substracted from the Consequent, shews the Excess or Difference; therefore to beget new Proportionals, the Excess is added to the Antecedent successively for the new Consequents. So as the Common Way, and indeed the most of what old Authors have left concerning this Sort of *Progression*, is, to set down orderly all the Increase in their due Places, and then add them together as in common Addition of *Integers*.

Q. Of Sattin sold encreasing the Price of every Yard.

For Example-sake: Suppose a Merchant selleth 15 Yards of Sattin, to be paid for the first Yard 4 *s.* for the second Yard 6 *s.* and so for every Yard an orderly Increase of 2 *s.* And it were demanded, what the 15 Yards of Sattin did amount to?

Answer.

Answ. By setting down orderly all the Terms, and adding them together, into one Total; the Sum is found 270 *s.*; or 13 *l.* : 10 *s.*, after the Common Way: Thus,

Terms

Terms or Places.

1.2.3. 4. 5. 6. 7. 8. 9.10.11.12.13.14.15

First Term, 4.6.8.10.12.14.16.18.20.22.24.26.28.30.32 Last Term.

Excess or Difference 2. Total or Sum 270*s.* or 13 *l.* 10 *s.*

Or thus;

Yards of Sattin	1	4 *s.*	First or least Term.
	2	6	
	3	8	
	4	10	
	5	12	
	6	14	
	7	16	Excess or Difference 2.
	8	18	
	9	20	
	10	22	
	11	24	
	12	26	
	13	28	
	14	30	
Terms of Places	15	32	Last or greatest Term.
		270	Total or Sum.

How the Terms are made. In this Example, the first Term, and as it were the Root, is 4, the second is made of the first and one Difference, the third of the first and 2 Differences, and generally every Term is made of the first and the Sum of the Differences, the Number of which is less by 1 than the Number of Terms, and so the thirteenth Term shall be 4 and twelve Differences, that is, $4 + 24$, seeing in this Example the Difference is 2.

The Number of Differences is noted by $T - 1$.
The Sum of the Differences by $TX - X = \omega - \alpha$.

What to be noted. But for a more orderly Proceeding in the Computation of this first sort of Progression, two things in general are to be noted.

1. That there are 5 Principals in every *Arithmetical Progression.*
2. That any 3 of the 5 being given, the other 2 may be found.

What the five Principals are. The 5 Principals are.

1. The first Term and his Species. 1. The first or least Term of any *Progression*, which in the Example above is 4, this is noted in Species with the Greek α, being the first small Letter of their Alphabet called *Alpha.*

2. The last Term and his Species. 2. The last or greatest Term noted in Species with the Greek ω, being the last small Letter of their Alphabet called *Omega*, and in the Example above is 32.

3. The Number of Terms and Species thereof. 3. The Number of Terms or Places in the whole *Progression*, this in the Instance above is 15, and in Species noted commonly with the Capital Roman T.

4. The Excess or Difference, and his Species. 4. The common Difference or Excess (*sometime called Increase*), whose Note in Species is X, another Capital Letter of the Roman Alphabet, in the Example above is 2.

5. The Total or Sum, and his Species. 5. The Total or Sum of all the Terms in the *Progression*, which above in the Example is 270; for this the Common Note in Species is Z, the last Capital Roman Letter in their Alphabet.

How by any 3 of these the other may be found. The Invention of any 2 of the 5 by 3 given, constitutes 20 Propositions, set down in Species by Mr. *Oughtred*, *p.* 78, 79, and 80, *of his Clavis*, that is 4 Propositions for every of the 5 Principals: So as every Question may be varied 20 several ways, though the thing sought be one of those 5.

To find the first Term. 1*st*. To find the first Term (or α) of any *Arithmetical Progression.*

1. Data. ω. T. X. 1. If the last Term, the Number of Terms, and the Excess be given, that is, ω. T. X. Then $\omega + X - TX = \alpha$.

Rule. That is, from the Sum of the second Principal added to the Fourth, substract the Product of the third Principal, multiplied by the Fourth.

Example. As in the former Instance 32 . 15 . & 2 . to find 4.

$$\begin{array}{lll} 32\ (\omega) & 15\ (T) & 34\ \text{Total.} \\ \underline{2\ (X)} & \underline{2\ (X)} & \underline{30}\ \text{Product.} \\ \underline{34}\ (\omega+X) - & \underline{30}\ (TX) = & \underline{4}\ (\alpha) \end{array}$$

2. Data. ω. T. Z. 2. If the last Term, the Number of Terms, and the Sum be given, that is, ω . T . Z.

$$\text{Then } \frac{2Z}{T} - \omega = \alpha.$$

Rule. That is, divide the double of the fifth Principal by the Third, and from the Quotient take the Second.

Example. As in the former Instance 32 . 15 . and 270 . to find 4.

$$\begin{array}{llll} 270\ (Z) & & & \\ \underline{2} & & 9 & \\ & & 540\big(\ 36 & \\ \underline{540}\ (2Z) & (T) & 15 & \underline{32}\ (\omega) \\ & & & 4\ (\alpha) \end{array}$$

3. Data. ω. X. Z. 3. If the last Term, the Excess and the Sum be given, that is, ω . X . Z.

$$\text{Then } \tfrac{1}{2}X \pm \surd : \omega q + \omega X + \tfrac{1}{4}Xq - 2ZX = \alpha.$$

$$\text{As } \alpha \text{ shall happen to be} \left\{ \begin{array}{l} \text{greater} \\ \text{lesser} \end{array} \right\} \text{than } \tfrac{1}{2}X$$

Rule. That is, add together the Square of the second Principal, with the Product of the second multiplied into the Fourth, and the fourth Part of the Square of the Fourth; and from the Total, take the Product of the fourth and fifth doubled: to the Square Root of the Remain, add half the Fourth, except the first Term (or Number sought) be less than half the Fourth, and then this Root is to be taken from half the Fourth.

Example. As in the former Instance 32 . 2. and 270 . to find 4.

$$\begin{array}{llll} 32\ (\omega) & & 2\ (X) & 270\ (Z) \\ \underline{32} & & \underline{2} & \underline{2}\ (X) \\ \underline{64} & 32\ (\omega) & 4\ (Xq) & 540\ (ZX) \\ \underline{96} & \underline{2}\ (X) & & \underline{2} \end{array}$$

$$1024\ (\omega q) + 64\ (\omega X) + 1\ (\tfrac{1}{4}Xq) = 1089 - 1080\ (2ZX) = 9 \left(\begin{array}{l} \surd \\ 3 \\ \underline{1\ (\tfrac{1}{2}X)} \\ \underline{4\ (\alpha)} \end{array} \right.$$

The half of X is here added, because 4 the First Term is greater than 2 the Excess.

4. Data. T. X. Z. 4. If the Number of Terms, the Excess, and the Sum be given, that is, T . X . Z.

$$\text{Then } \frac{2Z}{2T} - \frac{TX}{2} + \frac{X}{2} = \alpha.$$

Rule. That is, divide the double of the fifth Principal, by the double of the Third; to the Quotient add half the Fourth, and from the Total, substract half the Product of the Third multiplied into the Fourth.

Example. As in the former Instance 15 . 2 . and 270 . to find 4.

$$\begin{array}{lllll} 270\ (Z) & 15\ (T) & & & 15\ (T) \\ \underline{2} & \underline{2} & \frac{540}{30}\Big(\ 18 & & \underline{2}\ (X) \\ \underline{540}\ (2Z) & \underline{30}\ (2T) & \quad\ \underline{1\ (\tfrac{1}{2}X)} & & \underline{30}\ (TX) \\ & & \quad\ 19 - & & 15 = 4\ (\alpha) \end{array}$$

To find the last Term. 2*dly*. To find the last Term (or ω) of any *Arithmetical Progression*.

1. Data. α. T. X. 1. If the first Term, the Number of Terms, and the Excess be given, that is, α . T . X.   Then $\alpha + TX - X = \omega$.   or $T - 1 \times X + \alpha = \omega$.

Rule. That is, to the Product of the third Principal multiplied by the Fourth, add the first and substract the Fourth; or multiply the Number of Terms lacking 1, by the fourth Principal, and to the Product add the First.

As

As in the former Instance 4 . 15 . and 2 . to find 32. Example.

15 (T)	15—1=14 (T—1)
2 (X)	2 (X)
30 (TX)	28 (T—1×X)
4 (α)	4 (α)
34 (TX+α)	32 (ω)
2 (X)	
32 (ω)	

2. If the first Term, the Number of Terms, and the Sum be given, that is, α . T . Z. 2. Data. α.T.Z.

$$\text{Then } \frac{2Z - T\alpha}{T} = \omega.$$

That is, double the fifth Principal, and take out thereof the Product of the third multiplied by the First; then divide the Remainder by the Third. Rule.

As in the former Instance 4 . 15 . and 270 . to find 32. Example.

270 (Z) 15 (T)
2 4 (α)
540 (2Z) — 60 (Tα) = 480 / (T) 15 (32 (ω)

3. If the first Term, the Excess, and the Sum be given, that is, α . X . Z. 3. Data. α. X. Z.

$$\text{Then } \surd : \alpha q - \alpha X + \tfrac{1}{4} Xq + 2ZX : - \tfrac{1}{2} X = \omega.$$

That is, Square the first Principal, double the Product of the fifth multiplied by the Fourth, and add them together with the fourth Part of the Square of the Fourth: From the Total, take the Product of the first multiplied by the Fourth, and from the Square Root of the Remain take half the Fourth. Rule.

As in the former Instance 4 . 2 . & 270 . to find 32. Example.

			Z X
		XX	270×2
4 (α)	4 (α)	2×2	540 (ZX)
4	2 (X)	4 (Xq)	2
16 (αq)	— 8 (αX) +	1 ($\frac{1}{4}$X) +	1080 (2ZX) = 1089 ($\surd$ 33

1 ($\frac{1}{2}$X)
32 (ω)

4. If the Number of Terms, the Excess, and the Sum be given, that is, T . X . Z. 4. Data. T.X.Z.

$$\text{Then } \frac{2Z}{2T} + \frac{TX}{2} - \frac{X}{2} = \omega.$$

That is, divide the fifth Principal doubled, by the double of the Third; to the Quotient add half the Product of the Third multiplied by the Fourth, and from the Total take half the Fourth. Rule.

As in the former Instance 15 . 2 . and 270 . to find 32. Example.

270 (Z)	15 (T)	15 (T)	
2	2	2 (X)	540 / 30 (18
540 (2Z)	30 (2T)	30 (TX)	15
		15 (2)	33

1 ($\frac{1}{2}$X)
32 (ω)

3*dly*. To find the Number of Terms (or T) of any *Arithmetical Progression*. *To find the Number of Terms.*

1. If the first Term, the last Term, and the Excess be given; that is, α . ω . X. 1. Data. α.ω.X.

$$\text{Then } \frac{\omega - \alpha}{X} + 1 = T.$$

6 V That

Rule. That is, substract the first Principal from the Second, divide the Remain by the Fourth, and to the Quotient add an Unit.

Example. As in the former Instance 4 . 32 . & 2 . to find 15.

$$\overset{\omega}{32} - \overset{\alpha}{4} = \frac{28}{\underset{(X)}{2}} \Big(14 + 1 = 15\,(T)$$

2.Data. α. ω.Z. 2. If the first Term, the last Term, and the Sum be given; that is, α . ω . Z.

$$\text{Then } \frac{2\ Z}{\omega + \alpha} = T.$$

Rule. That is, divide the Double of the fifth Principal, by the First added to the Second.

Example. As in the former Instance 4 . 32 . & 270 . to find 15.

$$\overset{\alpha}{4} + \overset{\omega}{32} = 36 \Big) \overset{2\,Z}{540} \Big(15\ (T)$$

3.Data. α. X.Z. 3. If the first Term, the Excess, and the Sum be given; that is, α . X . Z.

$$\text{Then } \surd : \frac{\alpha q - \alpha X + \frac{1}{4}Xq + 2ZX :}{Xq} - \alpha + \tfrac{1}{2}X = T.$$

Rule. That is, square the first Principal, double the Product of the fifth multiplied by the Fourth, and add them together with the fourth Part of the Square of the Fourth; From the Total take the Product of the First multiplied by the Fourth, divide this Remainer by the Square of the Fourth; and from the Square Root of the Quotient take the First, and add half the Fourth.

Example. As in the former Instance 4 . 2 . & 270 . to find 15.

$$\begin{array}{cccc} & & XX & Z\quad X \\ 4\ (\alpha) & 4\ (\alpha) & 2\times 2 & 270\times 2 \\ 4 & 2\ (X) & 4\ (Xq) & 540\ (ZX) \\ & & & 2 \end{array}$$

$$16\,(\alpha q) - 8\,(\alpha X) + 1\,(\tfrac{1}{4}Xq) + 1080\,(2ZX) = \frac{1089}{\underset{(Xq)}{4}} \Big(\overset{\surd}{\frac{33}{2}}$$

$$\frac{33 - \overset{\alpha}{4} + \overset{\frac{1}{2}X}{1}}{2} = 15\,(T)$$

4.Data. ω.X.Z. 4. If the last Term, the Excess, and the Sum be given; that is, ω . X . Z.

$$\text{Then } \frac{\omega + \frac{1}{2}X -}{X} + \surd : \frac{\omega q + \omega X + \frac{1}{4}Xq - 2ZX}{Xq} = T.$$

As α shall happen to be $\left\{\begin{array}{l}\text{greater}\\ \text{lesser}\end{array}\right\}$ than $\frac{1}{2}X$.

Rule. That is, substract the Product of the fourth and fifth Principals doubled, from the Product of the Second and Fourth, added to the Square of the Second and Fourth Part of the Square of the Fourth; divide the Remainer by the Square of the Fourth, and take the Square Root of the Quotient from the Quotient of the Second added to half the Fourth, and divided by the Fourth; except the first Term be less than half the Fourth, for then this Root is to be added.

Example. As in the former Instance, 32 . 2 . & 270 . to find 15.

$$\begin{array}{lllll} 32\ (\omega) & 32\ (\omega) & & XX & Z\quad X \\ 1\ (\frac{1}{2}X) & 32 & & 2\times 2 & 270\times 2 \\ & 64 & 32\ (\omega) & & 540 \\ 33\ (\omega + \frac{1}{2}X) & 96 & 2\ (X) & 4\ (Xq) & 2 \\ 2\ (X) & & & & \end{array}$$

$$\frac{1024\,(\omega q) + 64\,(\omega X) + 1\,(\tfrac{1}{4}Xq) - 1080\,(2ZX)}{4\,(Xq)} = \tfrac{9}{4} \Big(\overset{\surd}{\tfrac{3}{2}}$$

$$\tfrac{33}{2} - \tfrac{3}{2} = 15\,(T)$$

The

The Root $\frac{3}{4}$ is substracted here, because 4, the First Term, is greater than 2 the Excess.

4thly. To find the Excess (or X) of any Arithmetical Progression. *To find the Excess.*

1. If the first Term, the last Term, and the Number of Terms be given; that is, $\alpha . \omega . T$. 1. Data. $\alpha.\omega.$T.

$$\text{Then } \frac{\omega - \alpha}{T - 1} = X.$$

That is, Take the first Principal from the Second, and divide the Remainer by the Third made less by an Unit. Rule.

As in the former Instance, 4 . 32 . & 15 . to find 2. Example.

$$\begin{array}{rrl}
15\ (T) & 32\ (\omega) & \\
1 & 4\ (\alpha) & \\
\hline
14\ ) & 28 & (\ 2\ (X)
\end{array}$$

2. If the first Term, the last Term, and the Sum be given; that is, $\alpha . \omega . Z$. 2. Data. $\alpha.\omega.$Z.

$$\text{Then } \frac{\omega q - \alpha q}{2Z - \omega - \alpha} = X.$$

That is, Substract the Square of the first Principal from the Square of the Second, and divide the Remainer by double the Fifth, lacking the First and Second. Rule.

As in the former Instance, 4 . 32 . & 270 . to find 2. Example.

$$\begin{array}{r}
32\ (\omega) \\
32 \\
\hline
64 \\
96 \\
\hline
1024\ (\omega q) \\
4 \times 4 = 16\ (\alpha q) \\
\hline
1008
\end{array}$$

$$\begin{array}{r}270\\ 2\\ \hline 540\end{array}(2Z) - \begin{array}{r}32+4\\ \hline 36\end{array}(\omega+\alpha) = \begin{array}{r}1008\\ \hline 504\end{array}\Big(\ 2\ (X)$$

3. If the first Term, the Number of Terms, and the Sum be given; that is, $\alpha . T . Z$. 3. Data. α.T.Z.

$$\text{Then } \frac{2Z - 2T\alpha}{Tq - T} = X.$$

That is, Take the Product of double the first Principal multiplied by the Third from the Fifth doubled, and divide the Remain by the Remain of the Third substracted from his Square. Rule.

As in the former Instance, 4 . 15 . & 270 . to find 2. Example.

$$\begin{array}{lll}
270\ (Z) & \underline{15(T) \times 4(\alpha)} & \\
\underline{\quad 2} & 60 & \\
540 & \underline{\quad 2} & \\
540 & 120\ (2T\alpha) = & 420 \\
T\ \ T & & \overline{}\ \Big(\ 2\ (X) \\
15 \times 15 = 225 (Tq) - 15 (T) = & & 210
\end{array}$$

4. If the last Term, the Number of Terms, and the Sum be given, that is, $\omega . T . Z$. 4. Data. ω.T.Z.

$$\text{Then } \frac{2\,T\omega - 2Z}{Tq - T} = X.$$

That is, double the Product of the third Principal multiplied by the Second, and substract therefrom the double of the Fifth: divide the Remain by the Remain of the Third substracted from his Square. Rule.

As in the former Instance 32 . 15 . and 270 . to find 2. Example.

$$\begin{array}{l} 32\ (\omega) \\ \underline{15\ (T)} \\ 160 \\ \underline{32} \\ 480 \qquad\qquad 270\ (Z) \\ \underline{\ \ 2\ \ } \qquad\qquad\quad \underline{\ 2\ } \\ 960\ (2\,T\omega) - 540\ (2Z) ===== \frac{420}{\ }\Big(\ 2\ (X) \\ \frac{\ }{T\ T} \\ 15 \times 15 = 225\ (Tq) - 15\ (T) = 210 \end{array}$$

To find the Sum. 5*thly*. To find the Sum (or Z) of any *Arithmetical Progression.*

1. Data. $\alpha.\omega.T.$ 1. If the first Term, the last Term, and the Number of Terms be given, that is, $\alpha.\omega.T.$ Then $T\omega + T\alpha = 2Z$ or $\frac{T\omega + T\alpha}{2} = Z$

Rule. That is, the Product of the third Principal multiplied by the second, and added to the Product of the Third multiplied by the First, shall be equal to twice the Sum: or if the two Products be halfed, or if half the first and second be multiplied by the Third, or the whole first and second by half the Third, the Sum will be had.

Example. As in the former Instance, 4. 32. and 15. to find 270.

$$\begin{array}{l} 15\ (T) \\ \underline{32\ (\omega)} \\ 30 \qquad\quad 15\ (T) \\ \underline{45} \qquad\quad \underline{4\ (\alpha)} \\ 480\ (T\omega) + 60\ (T\alpha) = \frac{540}{2}\Big(\ 270\ (Z) \end{array}$$

$$\begin{array}{l} \ \ \omega \quad\ \alpha \qquad\ \tfrac{1}{2}T \\ 32 + 4 = 36 \times 7\tfrac{1}{2} = 270\ (Z) \\ \qquad\quad\ \ 18 \\ \qquad\quad\ \ \underline{15} \\ \qquad\quad\ \ 90 \\ \qquad\quad\ \ \underline{18} \\ \qquad\quad\ \ 270\ (Z) \end{array}$$

2. Data. $\alpha.\omega.X.$ 2. If the first Term, the last Term, and the Excess be given, that is, $\alpha.\omega.X.$

Then $\frac{\omega q - \alpha q}{X} + \omega + \alpha = 2Z.$

Rule. That is, substract the Square of the first Principal from the Square of the Second, divide the Remain by the Fourth; to the Quotient add the First and Second, and take half the Total.

Example. As in the former Instance, 4. 32. and 2. to find 270.

$$\begin{array}{l} 32\ (\omega) \\ \underline{32} \\ 64 \qquad\quad 4\ (\alpha) \\ \underline{96} \qquad\quad \underline{4} \\ 1024\ (\alpha q) - 16\ (\alpha q) == \frac{1008}{2}\Big(\ 504 + \overset{\omega}{32} + \overset{\alpha}{4} = \frac{540}{2}\Big(\ 270\ (Z) \\ \qquad\qquad\qquad\qquad\quad (X) \end{array}$$

3. Data. $\alpha.T.X.$ 3. If the first Term, the Number of Terms, and the Excess be given; that is, $\alpha.T.X.$ Then $TX - X + 2\alpha$ in $T == 2Z.$

Rule. That is, multiply the third and fourth Principals; from the Product take the Fourth, to the Remain add double the First, multiply the Total by the Third, and half the Product.

Example. As in the former Instance 4. 15. and 2. to find 270.

$$\begin{array}{l} 15\ (T) \\ \underline{2\ (X)} \\ 30\ (TX) - \overset{X}{2} = 28 + \overset{2\alpha}{8} = 36 \times \overset{T}{15} = \frac{540}{2}\Big(\ 270\ (Z) \end{array}$$

4. Data. $\omega.T.X.$ 4. If the last Term, the Number of Terms, and the Excess be given, that is, $\omega.T.X.$ Then $2\omega + X - TX$ in $T == 2Z.$

That

That is, from the doubled Sum of the second Principal added to the Fourth, take the Product of the third multiplied by the Fourth; multiply the Remain by the third and half the Product. *Rule.*

As in the former Instance 32 . 15 . and 2 . to find 270. *Example.*

$$
\begin{array}{l} 32\,(\omega)\\ \;\;2\\ \hline 64 \\ \;\;2\,(X) \\ \hline 66\,(2\omega+X) \end{array} - \begin{array}{l}15\,(T)\\ \;\;2\,(X)\\ \hline 30\,(TX)\end{array} = 36 \times \overset{T}{15} = \frac{540}{2}\Big(\;270\,(Z)
$$

Touching these Principals and Propositions, two things more come under farther observance. *What further to be noted.*

First, That by every three of the 5 Principals given, both the other 2 wanting are to be found. *1. Both the two unknown found by three given.*

As *Data.*	*Quesita.*			*Propositions.*		
α . ω . T	Z & X	1	of the	5 . & 1	of the	4
α . ω . X	T & Z	1		3 . & 2		5
α . ω . Z	T & X	2		3 . & 2		4
α . T . X	ω & Z	1		2 . & 3		5
α . T . Z	ω & X	2		2 . & 3		4
α . X . Z	ω & T	3		2 . & 3		3
ω . T . X	α & Z	1		1 . & 4		5
ω . T . Z	α & X	2		1 . & 4		4
ω . X . Z	α & T	3		1 . & 4		3
T . X . Z	α & ω	4		1 . & 4		2

Secondly, That all Questions duly propounded in *Arithmetical Progression*, give 3 of the 5 Principals, and require sometime one, sometime both the other, and sometime one or other of the middle Terms; sometime the Increase is inverted, and sometime one Question includes another, so as 5 Cases will compleat all needful to this sort of *Progression*. *2. The Data and Quesita in every Question. Five Cases.*

Case 1. When but one of the 5 is sought, the Proposition under which the Resolution falls, is to be used, as by the Examples above is largely to be seen; alteration only to be made for Fractions and Decimals, where occasion requires, according to their Nature and Use. *1. If but one sought, then as before. Alteration for Fractions and Decimals, &c.*

Case 2. When 2 of the Principals are required in the Question, after one is found, the other is to be sought; and because the Propositions get 2 of the Principals, and some of the Propositions are more easy than others, sometimes it happens that the Principal found in the Work of the First, will more easily procure the other sought, and so may be used as if one of the *Data*. *2. If 2 be sought, after one found the easiest to be chosen.*

Example 1. A Grocer selleth 80 ℔. of Spice conditionally, to be paid for the first Pound 2 *d.* for the second 5 *d.* and so increasing by 3: The Question is, what was paid for the last Pound, and for the whole 80 ℔? *Q. Of Spice sold, what paid for the last Pound, and for the Whole.*

Answ. For the last Pound 239 Pence, and for the whole 9640, or 40 *l.* 3 *s.* 4 *d.* *Answer.*

In this Question are,

$$
\text{The}\left\{\begin{array}{l} \textit{Data}\quad \alpha\;.\;T\;.\;X \\ \qquad\quad\;\; 2\;.\;80\;.\;3 \\ \textit{Quesita}\left\{\begin{array}{l}\omega\\ Z\end{array}\right\}\begin{array}{l}(2)\ \text{Principal}.\,1\\ (5)\ \text{Principal}.\,3\end{array}\right\}\!\!\!\!\!\!\!\!\begin{array}{c}\textit{Resolution.}\\ \end{array}\text{of the}\left\{\begin{array}{c}2\\ 5\end{array}\right\}\text{Propositions.}
$$

$$
\text{Wherefore}\quad\begin{array}{l} 80\,(T)\\ \;\;3\,(X)\\ \hline 240\\ \;\;2\,(\alpha)\\ \hline 242\,(TX+\alpha)\\ \;\;3\,(X)\\ \hline 239\,(\omega)\end{array}\qquad\qquad \text{And}\quad\begin{array}{l} 80\,(T)\\ \;\;3\,(X)\\ \hline 240\\ \;\;3\,(X)\\ \hline 237\,(TX-X)\\ \;\;4\,(2\alpha)\\ \hline 241\end{array} \times 80 = \frac{19280}{2}\Big(\;9640\,(Z)
$$

Q. Of how many Men a Sum was received, and what each paid.

Example 2. If a Man receive 84 *l.* of certain Men, by an orderly Increase, remembring only the Paiment of the first Man to be 9 *l.* and the last 33 *l.* and would know of how many Men he received the said 84 *l.* and what each Man paid one more than another: what shall the Answer be?

Answer.

Answ. Of 4 Men, and each Man paid 8 *l.* more than the other.

Here are $\left\{\begin{array}{l} \textit{Data}\ .\ \alpha\ .\ \omega\ .\ Z \\ \qquad 9\ .\ 33\ .\ 84 \\ \textit{Quesita} \left\{\begin{array}{l} T \\ X \end{array}\right\} \left\{\begin{array}{l} (3)\ \text{Principal}\ .\ 2 \\ (4)\ \text{Principal}\ .\ 2 \end{array}\right\} \text{of the} \left\{\begin{array}{l} 5 \\ 4 \end{array}\right\} \text{Propositions.} \end{array}\right.$ *Resolution.*

Wherefore

$$\overset{\alpha}{9}+\overset{\omega}{33}=42)\overset{2Z}{168}(4\ (T)$$

And

$$33\ (\omega) \times 33 = 99,\ 99,\ 1089\ (\alpha q)$$
$$9\times 9 = 81\ (\alpha q)$$
$$\frac{1008}{126}\Big(8\ (X)$$

$$84\ (Z) \times 2 = \underline{168}\ (2Z) - \overset{33+9}{42}\ (\omega+\alpha) = 126$$

Q. Of Gain in the first and last Months.

Example 3. Suppose a Man gain every Month in the Year 30 *s.* more than he did the first Month, and at 12 Months End found the Whole to amount to 280 *l.* what were his Gains the first and last Months of the 12?

Answer.

Answ. The first Month 15 *l.* : 1 *s.* : 8 *d.* : And the last Month 31 *l.* : 11 *s.* : 8 *d.*

Here are $\left\{\begin{array}{l} \textit{Data}\ .\ T\ .\ X\ .\ Z. \\ \qquad 12\ .\ 30\ .\ 5600\ \text{the Shillings in 280}\ l. \\ \textit{Quesita} \left\{\begin{array}{l} \alpha \\ \omega \end{array}\right\} \left\{\begin{array}{l} (1)\ \text{Principal}\ .\ 4 \\ (2)\ \text{Principal}\ .\ 4 \end{array}\right\} \text{of the} \left\{\begin{array}{l} 1 \\ 2 \end{array}\right\} \text{Propositions.} \end{array}\right.$ *Resolution.*

Wherefore

$$5600\ (Z) \times 2 = 11200\ (2Z) \qquad 12\ (T) \times 2 = 24\ (2T) \qquad \frac{11200}{24}\Big(466\tfrac{2}{3} + 15\ (\tfrac{1}{2}X) = 481\tfrac{2}{3}$$
$$12\ (T) \times 30\ (X) = 360\ (TX)\ s. \qquad 481\tfrac{2}{3} - 180 = 301\tfrac{2}{3}\ (\alpha)$$

And

$$5600\ (Z) \times 2 = 11200\ (2Z) \qquad 12\ (T) \times 2 = 24\ (2T) \qquad 12\ (T) \times 30\ (X) = 360\ (TX),\ \frac{360}{2} = 180$$
$$\frac{11200}{24}\Big(466\tfrac{2}{3} + 180 = 646\tfrac{2}{3} - \overset{\frac{1}{2}X}{15} = 631\tfrac{2}{3}\ (\omega)\ s.$$

Q. Of the Number and Ages of Children.

Example 4. A Man had divers Children; the Youngest 6 Years old, and the Eldest 40, and every one elder than the other by 2 Years: how many Children had he, and what was the Sum of all their Ages?

Answer.

Answ. He had 18 Children, and their Ages together were 414 Years.

Here are $\left\{\begin{array}{l} \textit{Data}\ .\ \alpha\ .\ \omega\ .\ X \\ \qquad 6\ .\ 40\ .\ 2 \\ \textit{Quesita} \left\{\begin{array}{l} T \\ Z \end{array}\right\} \left\{\begin{array}{l} (3)\ \text{Principal}\ .\ 1 \\ (5)\ \text{Principal}\ .\ 2 \end{array}\right\} \text{of the} \left\{\begin{array}{l} 3 \\ 5 \end{array}\right\} \text{Propositions.} \end{array}\right.$ *Resolution.*

Wherefore

$$\overset{\omega}{40} - \overset{\alpha}{6} = \frac{34}{(X)\ 2}\Big(17 + 1 = 18\ (T)$$

And

$$40\ (\omega) \times 40 \qquad 6\ (\alpha) \times 6$$
$$1600\ (\omega q) - 36\ (\alpha q) = \frac{1564}{(X)\ 2}\Big(782 + \overset{\omega}{40} + \overset{\alpha}{6} = \frac{828}{2}\Big(414\ (Z)$$

In all these Examples the Demands being double, after the first is found, he may be taken with two other of the *Data* for finding the second Demand: As in the last Example after T, the Number of Terms was found to be 18; Resolution of the

the second Demand Z might have been found by $\alpha . T . X .$ or $\alpha . \omega . T .$ or $\omega . T . X .$ as well as by $\alpha . \omega . X .$ the first *Data.* And if by $\alpha . \omega . T .$ that is the first Proposition of the 5*th* Principal, the Work had been shorter than that above; and the like is to be understood of others.

Data.

$\alpha . \omega . T$

$6 . 40 . 18$ $\qquad \overset{\omega}{40} + \overset{\alpha}{6} = \underline{46}$ $\quad T \quad$ *Quesita.*

$(\frac{1}{2})\ 23 \times 18 = 414\ (Z)$

Case 3. When together with a Question in *Progression* is involved, explicitly or implicitly, another Question, whose Resolution belongs to some other Element of Numbers, Operation is to be made accordingly. *3. If one Question be included in another.*

Example. If 100 Eggs be placed, every one a Yard distant from other in length, and the first a Yard distant from a Basket: whether one might gather up the Eggs one after another, still returning to the Basket to put them in, before one can run 4 Miles. *Q. Of gathering up 100 Eggs.*

Here are given in this Question the first Principal α, which is 2 Yards, (*viz.* a Yard from the Basket to the first Egg, and as much back again to the Basket) the third Principal T. that is, the 100 Eggs, and the fourth Principal X or 2 Yards, (*viz.* a Yard forth and a Yard back): And Z the fifth Principal is desired, that is, the Number of Yards he that doth gather up the Eggs runneth in all forward and backward. And then because another Question is included, that is, whether this Number of Yards will reach in Length 4 Miles or not? the Number found is to be compared with the Yards in 4 Miles, and allowing 1760 Yards to an English Mile, Z or 10100 Yards found to be run in gathering up the Eggs, is seen to amount to 5 Miles and almost 3 quarters of a Mile more. *Answer.*

Data

$\alpha . T . X .$

$2 . 100 . 2$

$$\begin{array}{r} 100\ (T) \\ \underline{2\ (X)} \\ 200 \\ \underline{2\ (X)} \\ 198 \\ \underline{4\ (2\alpha)} \\ 202 \times 100 = \frac{20200}{2}\left(\begin{array}{l}\textit{Yards.}\\ 10100 (Z)\end{array}\right. \end{array}$$

Quesita *Yards.* *Mile.* *Yards.* *Miles.*

As $1760 . 1 :: 10100 . 5\frac{65}{88}.$

Miles.

$4 \times 1760 = 7040.$

Case 4. When the Increase is inverted, that is, turned into Decrease, and the Question propounded with every Term succeeding the First less than the First; then accompt the last Term the First, and the First the Last, till the Work of the Proposition be ended. *4. If the Increase be inverted.*

Example. A Scout-Master-General being commanded to discover the Quarters of an Enemy, returned this Accompt; If moving (saith he) from the Place where we now are the first Day 30 Furlongs, the second Day 28, the third Day 26, and so every Day lessening 2 Furlongs, in the 15*th* Day we shall come to the Enemy; but they would meet in 9 days: how far shall they march in a Day one day with another, to overtake the Enemy in their Quarters in 9 days? *Q. Of marching to overtake an Enemy.*

Here the first Work being to find out the Distance of the two Armies, or Z the 5*th* Principal, to the finding whereof is given α, the first day's march, 30 Furlongs; T the 15 days, and X the Difference of their March, which is 2 decreasing; so is the increase of the *Progression* inverted, wherefore α the 30 shall be ω the 15*th* Term, and that Term 2 (implicit in the Question) shall be instead of α. Then either by $\alpha . \omega . T .$ or $\alpha . T . X .$ or $\omega . T . X .$ may Z be found, which will be 240 Furlongs; and then the other Question will be resolved by dividing 240 by 9, and their March thereby found to be $26\frac{2}{3}$ Furlongs in a Day. *Answer.*

Data.

$\omega . T . X .$

$30 . 15 . 2$

Quesita.

$$\begin{array}{r} 30\ (\omega) \\ \underline{2} \\ 60 \\ \underline{2\ (X)} \\ \underline{62}\ (2\omega + X) \end{array} - \begin{array}{r} 15\ (T) \\ \underline{2\ (X)} \\ \underline{30}\ (TX) \end{array} = \overset{T}{32 \times 15} = \frac{480}{2}\left(\frac{\overset{Z}{240}}{9}\left(26\frac{2}{3}\ \text{Furlongs.}\right.\right.$$

Case 5.

5. If a middle Term be sought.

Case 5. When any middle Term (that is, intermediate between the first and last Terms) is demanded, the same is to be found according to the *Data*. For seeing the Number of such middle Term may be represented by T, the whole Number of Terms in a *Progression*, the Sum of such Term may be represented by ω the last Term of the *Progression:* so as with little alteration all the Propositions before, for finding the second Principal ω, may serve to find the middle Term desired.

Examples.

As if α and X be given, then by the first Proposition of the second Principal, take an Unite from the Term desired, and multiply the Remain by the Excess, and to the Product add the first Term.

And so in the foregoing Instance, if 4 and 2 be given to find the 12*th* Term of that *Progression*, that is, 26: An Unite taken from 12, and the Remain 11 multiplied by 2, is 22, to which 4 added, the Total is 26 desired.

But if Z be one of the *Data*, let it be understood to be the Sum of the *Progression* to that middle Term desired, and not the Total Sum of the whole *Progression*; and then Operation may be made therewith, as in the other 3 Propositions of the second Principal.

And if ω, or the last Term of the whole *Progression* be given, for one of the *Data*; then invert the Terms in the *Progression*, and multiplying 1 less than the Term desired by the Excess, substract the Product from ω.

So the 12*th* Term of the former Instance shall be the 4*th* Term, from which 1 taken, and the rest 3 multiplied by 2, the Excess shall be 6, which taken from 32, the Remain will be 26 as before.

Terms	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	
	4.	6.	8.	10.	12.	14.	16.	18.	20.	22.	24.	26.	28.	30.	32.	
	15.	14.	13.	12.	11.	10.	9.	8.	7.	6.	5.	4.	3.	2.	1.	inverted.

Proof of Arithmetical Progression.

The Proof of the Operations in this sort of *Progression*, is by placing every Increase in its due Seat or Term, and by common Addition of Integers to collect the Numbers into one Total, as in the first Example of this Chapter.

CHAP. III. *Transposition.*

Transposition what, and the Sorts.

THE first-born of *Arithmetical Progression* is *Transposition*, which is an orderly disposing of some Parts of a Number, so as there may be an equal Difference between the Parts so placed, or a disposing of a Number so that the Parts desired may be taken, and the other left.

The first Sort what it doth resemble. How wrought.

The former sort resembles *Division*, in which though the Dividend be taken a thousand Times by the Divisor, yet the Divisor continues intire. This *Transposition* is to be done with due observation of the Parts: for if the Parts placed be some Aliquot Part of a Number, and the Places be equal to the Divisor, that Aliquot Part is the desired Number; but if 2 or more different Parts be taken, then half the Number of the Divisor of one sort, and half the Number of the Divisor of the other sort of Parts, must be taken.

Examples.

Examples of both follow.

Q. Of Souldiers, how dispos'd, that a like Number may face the Side of a Fort, when some are entertained, and others discharged.

A certain Passage of Square Form had 4 Gates opposite one to the other, that is, in the middle of each side one; and there were appointed 9 Men to defend each Front thereof, some at the Gates, and the other at each Corner or Angle: so each Angle served to assist two Faces of the Square if need required. Now this Square Passage being thus mann'd with 9 at each Side, it happened that 4 Souldiers coming by, desired of the Governor to be entertained into Service, who told them he could not admit of more than 9 upon each Side of the Square: to whom one of the Souldiers being skilled in the Art of Numbers replied, If he would take them into Pay, they would place themselves amongst the rest, and yet keep still the Order of 9 for each Face of the Square; to which the Governor agreed, and they were admitted. But afterwards liking not their Service, they indeavoured to remove themselves and also draw away each Man his Comrade, yet would leave 9 to defend each Side of the Passage: and how may this be?

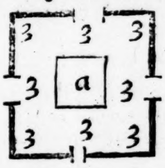
Answer.

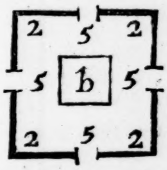
Answ. The Square having 4 Gates and 4 Angles, that is, 8 in all, 3 of each Side including the Angles, this 8 multiplying 3, produceth 24 for the whole Number,

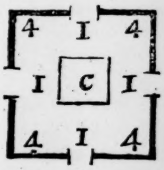
Number, whose Aliquot Part desired is 3; and so many were placed at each Gate and Angle before the 4 Souldiers were admitted, as at *a*.

Then each of the 4 placing themselves at the Gates, removed from every Angle to every Gate one Man, as at *b*. where the whole Number being 28, that is, 4 times 7, and 4 being half the Number of 8 the Divisor, must be now taken, because 8 will not evenly divide 28; and so it must be, that 7 shall be the Number belonging both to a Gate and an Angle, that is, 5 to one and 2 to the other, and 4 times 5 and 4 times 2 make up the 28.

But then 8 being taken away, and 20 Souldiers only left, which 20 will not be evenly divided by 8, but by 4 the half thereof; and therefore 5 the Quotient shall be the Number belonging both to an Angle and a Gate, that is, 4 to one, and 1 to the other, as at *c*. So as upon their departure, 2 were removed from the Gates, and placed at the Angles.



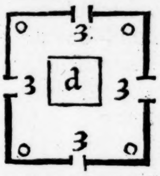


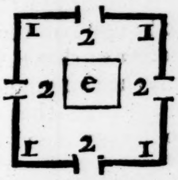


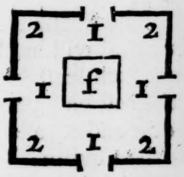
And the like is to be observed on the contrary, when the Parts of one and the same Number are to be so transposed, that their Fronts shall be more or less.

Example, where with a like Number the Side may have more or less.

As the Number 12 may be so parted and placed about a Square, that the Fronts every Way shall have 3, 4, or 5, as at *d. e. f.*







The other sort of *Transposition* to dispose a Number so that the Parts desired may be chosen, is used in Lottery, where sometime the 4, 5, 7, 20, &c. Person or Thing is respected to be secured, and this is performed thus: Take as many Units or Ciphers as there be Persons or Things on which the Lot is to pass, and dispose them on a Paper in a Row; then set a Prick on the respected Place as the Lot is to fall upon, whether the 3, 4, 5, &c. and so count along on the remaining Places, still pricking the alloted Place till the Complement of the Lottery be marked out; and then you shall see how to place the Persons or Things, that the Lot may fall where desired.

The other Sort of Transposition used in Lottery.

Example 1. If 24 Persons have done a Villany, and 6 of them more guilty than the rest, and being apprehended and found Guilty at Trial, the Judg determined by a Lot, that every 7*th* Man should die, till the Number of 6 were executed: how then should the Men be disposed, that so counting by 7, the Lot of Escape should always fall upon one of the 6 that were most notoriously guilty?

Q. Of Malefactors condemned, which to allot to suffer.

Answ. By setting down 24 Ciphers, and telling along by 7 and 7, the 6 Men designed to die are seen to be set in the 4, 7, 12, 14, 20 and 21 Places.

Answer.

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 . 10.11.12.13.14.15.16.17.18.19.20.21.22.23.24.
o.o.o.o.o.o.o.o.o.o.o.o.o.o.o.o.o.o.o.o.o.o.o.o.
1 2 3
4 5 6

Example 2. A Ship at Sea wanting Provision, having 15 Christians and as many Turks, and to preserve all from famishing, resolve to die to feed each other, yet so as by Lot, and that counting by 9 and 9 the ninth Person should die: Now the Master being a Christian, and desirous to save the Christians, disposed them so as the ninth Person still fell out to be a Turk: how may this be done?

Q. Of Christians and Turks, how to save the Christians.

Answ. 4 Christians 5 Turks, 2 Christians 1 Turk, and so along according to the Vowels in *Populeam virgam mater Regina ferebat*, counting *a* 1, *e* 2, *i* 3, *o* 4, *u* 5. Or by the former way;

Answer.

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 . 10.11.12.13.14.15.16.17.18.19.20.21.22.23.24.25.26.27.28.29.30.
14 4 . 7 . 12 . 1 10 5 2 . 8 13.15.11 6 . 3 9

Proof of Transposition.

Transposition of both sorts, carries along with the Demonstration above such Evidence of the Truth of the Work, that nothing can be added for Proof more convincing.

CHAP. IV. *Technology.*

Technology, how called. What it doth import.

THE latter Issue of *Arithmetical Progression* called *Technology*, goes commonly under the Names of Sports and Pastimes; but the Learned *Alsted* entitles it *Technology*, which imports as much as an Artificial discourse or discovery of Numbers or other things concealed.

How defined by Alsted, and the Sorts.

Geysius, and *Alsted* from him, defines *Technology* to be a *Progressional Arithmetical Division*, and that either by *Progressional Arithmetical Divisors*; Or *2ly* of *Progressional Dividends Arithmetically*: Or *3ly*, into *Progressional Quotients Arithmetically*.

In the first Sort six things.

§. 1. In Division by *Progressional Arithmetical Divisors*, the Work is to find such Numbers to be divided as will leave the Remains of the Division, either first Equal; or *2ly*, Progressional; or *3ly*, Equal till the last Division, and then nothing; or *4ly*, Progressional till the Last, and then nothing; or *5ly*, Distant from the Divisor above an Unite; or *6ly*, Disorderly.

In all which there are many Numbers of such Properties, so as the Enquiry is sometimes for the least Number of that Property, and sometime for any such Number at random.

1. To find Remains equal.

1st. To find Numbers whose Remains shall be equal continually, multiply all the Divisors one into another, and to the last Product add the common Remain.

The least Number of such Property.

And if among the Divisors there be none of them compound one of another, then this Total will be the least Number of that Property, otherwise not. Therefore to get such least Number, omit the compound Divisors, and double the Product of the rest being multiplied one into another, if the compound Divisors be 4 or 6, or 4 and 6, and add thereto the common Remain.

But if the Divisors be 8, 9, 10, being 8 and 9 are doubly compound, besides the doubling multiply by 6, and so proceed accordingly for other Compounds: And having gotten the least Number, add him, lacking the Remain, to himself successively, and other like Numbers will be produced.

Q. Of a Number whose Remain 1, Divisors 2, 3, 4, 5, 6, 7. Answer.

Example. One trying whether a Number was Prime or Compound, found that by Division with 2 there was 1 remaining, and so likewise dividing by 3, 4, 5, 6 and 7, there was still left 1 for the Remain: what was that Number?

Answ. If the Number quesited be intended the least of that sort, it shall be 421; but if any Number of that Property, add to 421 continually 420, and other like Numbers will be produced.

Here the Divisors 2, 3, 4, 5, 6, 7, multiplied one into another, produce 5040, to which 1 the common Remain added, the Total is 5041: but this is not the least Number of that Property, because among the Divisors 4 is compound of 2, and 6 of 3, wherefore both 4 and 6 being omitted, the Product of the rest is 420, to which 1 the Remain added, makes 421 for the least Number of that Property.

$$\overset{6}{2\times3}\times\overset{24}{4}\times\overset{120}{5}\times\overset{720}{6}\times7 = 5040$$

1 Remain added.

5041

$$\frac{\overset{1}{5041}}{2}\Big(2520 \quad \frac{\overset{22}{5041}}{3}\Big(1680 \quad \frac{\overset{12}{5041}}{4}\Big(1260 \quad \frac{5041}{5}\Big(1008 \quad \frac{\overset{2}{5041}}{6}\Big(840 \quad \frac{\overset{1}{5041}}{7}\Big(720$$

$$\overset{6}{2\times3}\times\overset{30}{5}\times7 = 210$$

2

420 Doubled.

1 Remain added.

421 The least Number of that Property.

$$\frac{421}{2}\Big(210 \quad \frac{421}{3}\Big(140 \quad \frac{421}{4}\Big(105 \quad \frac{421}{5}\Big(84 \quad \frac{421}{6}\Big(70 \quad \frac{421}{7}\Big(60$$

It

If the Divisors proposed be 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 . 10 . 11 . then the least Number which divided by them will still leave 1 remaining, is found to be 47721. For the Uncompound or Prime Numbers, 2 . 3 . 5 . 7 . 11, multiplied together, make 2310, which doubled for 4 and 6 makes 4620; and then for 8, 9 and 10, the other 3 Compounds multiplied by 6, that is 2 × 3, produceth 27720; to which 1 the common Remain added, is 27721: And by Addition of 27720 thereto, other-like Numbers will be produced. *Example in more Divisors.*

```
     6   30  210
2 × 3 × 5 × 7 × 11  ==  2310   1   2
                          2 . 4 . 6
                        ------------------
                        4620   1   2   3
                           6 . 8 . 9 . 10
                        ------------------
                        27720
                            1 Remain added.
                        -----
                        27721 Least Number.
```

2*ly*. To find Numbers whose Remains shall be Arithmetically Progressional: Multiply as before the Divisors, if they be all Prime, one into another; and from the last Product substract the Difference between the Divisor and Remainer given, and this Remain shall be the least Number of that Property; to which if the Product be added, other-like Numbers will be produced. *2. To find Remains Progressional.*

And if some of the Divisors be compound one of another, then as before omit them; and from the Product of the rest doubled, or otherwise multiplied, according to the Number of Compounds as before, take the Difference: For otherwise if all the Divisors be multiplied, the Number will be higher than the least. *The least Number of such Property.*

Example. Suppose one desire to know what Number that is, which being severally divided by 2, 3, 4, 5, 6, 7, the respective Remains will be 1,2,3,4,5,6. *Q. Of a Number whose Remains are 1, 2, 3, &c. Divisors 2, 3, 4, &c.*

Answ. The least Number of that Property will be found to be 419; to which 420 added successively, other Numbers will be produced. *Answer.*

Here 4 and 6 among the Divisors, being compound of 2 and 3 as before, they are omitted; and from 420 the double Product of the rest, 1 is substracted, so is 419 obtained.

```
  6  30
2 × 3 × 5 × 7 = 210
                  2
               ----
                420 Doubled.
                  1 Difference substracted.
               ----
                419 The least Number of that Property.

 (1         12(2        (3         (4         5(5        6(6
419(209    419(139    419(104    419(83     419(69     419(59
 2          3          4          5          6          7

    6   24  120  720
2 × 3 × 4 × 5 × 6 × 7 = 5040
                           1 Difference substracted.
                        ----
                        5039

2 2(1       222(2       12(3        (4          25(5        16(6
5039(2519   5039(1679   5039(1259   5039(1007   5039(839    5039(719
 2           3           4           5           6           7
```

If the Divisors proposed were 2, 3, 4, 5, 6, 7, 8, 9, 10, 11; then multiplying 2, 3, 5, 7, 11, and doubling the Product, and multiplying the Double by 6 as before; and from this Product 27720 taking 1, the least Number of that Property is found to be 27719: And if 27720 be added, other-like Numbers are produced. *Example in more Divisors.*

2 X

$$\overset{}{2}\times\overset{6}{3}\times\overset{30}{5}\times\overset{210}{7}\times 11 = 2310 \quad 1 \quad 2$$

$$2 \,.\, 4 \,.\, 6$$

$$4620 \quad 1 \quad 2 \quad 3$$

$$6 \,.\, 8 \,.\, 9 \,.\, 10$$

27720
1 Difference substracted.

27719 Least Number.

3. To find Remains equal till the last.

3*ly*. To find Numbers whose Remains shall be equal till the last Division, and then 0 left; multiply all the Divisors except the last one into another, and to the Product add the common Remain.

The least Number of such Property.

But if among the Divisors more be compound one of another than 4, the least of that Nature, this Number will not be the least of that Property; but the Product of the Prime Divisors doubled as before, for 4 and 6, and for the next 3 multiplied by 6, and so according to the Number of Compounds; and then dividing thereby the Number above-gotten, will leave the least Number remaining.

Q. Of Eggs broken.

Example. A Maid carrying Eggs to Market, met with an unruly Fellow who broke them; and afterward by determination of the Justice, was enforced to pay for them; and thereupon the Maid being demanded, how many Eggs she had? answered, That counting them by 2 and 2, there remained 1; and so likewise by 3 and 3, and 4 and 4, and 5 and 5, and 6 and 6; but when she counted them by 7, there rested 0: How many Eggs had she at least?

Answer.

Answ. 301: For the Product of all the Divisors, except 7, is 720; to which 1 added is 721, a Number of like Property, but not the least of that Property: Wherefore because there were 2 compound Numbers, if the double Product of all the Prime Divisors, which is 420, be substracted from 721, the Remain will be 301. And by addition of 420 to 301, other-like Numbers will be produced.

$$2\times\overset{6}{3}\times\overset{24}{4}\times\overset{120}{5}\times 6 = 720 \qquad 2\times\overset{6}{3}\times\overset{30}{5}\times 7 = 210\times 2 = 420$$

1 Remain.

721 — 420 = 301 Least Number desired.

$$\frac{\overset{1}{301}}{2}(150 \quad \frac{301}{3}(100 \quad \frac{\overset{2}{301}}{4}(75 \quad \frac{301}{5}(60 \quad \frac{301}{6}(50 \quad \frac{\overset{2(0}{301}}{7}(43$$

Example in more Divisors.

And if the least Number were desired, which being divided by 2, 3, 4, 5, 6, 7, 8, 9 and 10, would still leave 1 remaining, but would be evenly divided by 11; then 3628801, the Product of all the first 9 Divisors with the common Remain, divided by 27720, the Product of the Prime Divisors doubled and multiplied by 6, the Remain will be 25201, the least Number desired of that Property.

$$2\times\overset{6}{3}\times\overset{24}{4}\times\overset{120}{5}\times\overset{720}{6}\times\overset{5040}{7}\times\overset{40320}{8}\times\overset{362880}{9}\times 10 = 3628800$$

1 Remain.

3628801

$$2\times\overset{6}{3}\times\overset{30}{5}\times\overset{210}{7}\times\overset{2310}{11}\times\overset{4620}{2}\times 6 = 27720$$

(25
856|2
3628 8|01 (130
2772 2 20
2777
27

4. To find Remains Progressional till the last.

4*ly*. To find Numbers whose Remains shall be *Arithmetically Progressional* till the last Division, and then 0 left; because all Numbers that will be evenly divided by a greater Compound Number, will be evenly divided by the Lesser whereof he is so compound, it follows therefore properly that the last Divisor given ought to be a Prime Number, and then the Rule may be thus: For Divisors to 4 multiply the

The least Number of such Property.

Prime 2 into himself and abate 1, so shall 3 be divided by 2 and leave 1, by 3 and leave

leave o; for Divisors to 5, multiply the Prime Numbers 2 and 3 into themselves, and square the Product, from which substract 1; so shall 35 be divided by 2 and leave 1, by 3 and leave 2, by 4 and leave 3, by 5 and leave o. But for Divisors higher than 5, multiply all the Divisors except the two last one into another, and from the Product take the Difference as aforesaid, and this for 7 gets the least Number of that Property; and to get the least Number of such Property for higher Divisors, multiply only the Prime Numbers, and for 4 and 6 double the Product for 8, 9, 10, the next 3 Compounds; multiply the last Product by 6, and so for 13 the like; and this dividing the former Number shall leave the least Number remaining.

Example. There are Numbers which being divided by 2, 3, 4, 5, 6, 7, and 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, will leave their Remains orderly in *Arithmetical Progression* till the last Division, upon which o will remain: what are the least Numbers of those Properties? *Q. Of Numbers divided by 2, 3, 4, &c. whose Remains 2, 3, 4, &c. at last o.*

Answ. For 7 the least Number is 119, which is the Product of the first 4 Divisors lacking 1, and by continued Addition of 420 thereto, other like Numbers are produced: but for 11, the least Number is 2519, and by Addition of 27720 thereto, other like Numbers are produced. *Answer.*

6 24
2 × 3 × 4 × 5 = 120
1
119

(1 ... 119 / 2 (59 ; 2(2 ... 119 / 3 (39 ; 3(3 ... 119 / 4 (29 ; (4 ... 119 / 5 (23 ; 5(5 ... 119 / 6 (19 ; 4(0 ... 119 / 7 (17

6 30 210
2 × 3 × 5 × 7 × 2 = 420 Number to be added.

6 24 120 720 5040 40320
2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 = 362880
1
362879

6 30 210 2310 4620
2 × 3 × 5 × 7 × 11 × 2 × 6 = 27720 Number to be added.

(25
856|1
3628 7|9 (13
27722 0
277

And if the least Number were desired, which would evenly be divided by 13; but by all the other Numbers from 1, would leave the Remains in *Arithmetical Progression*; this accordingly will be found to be 277199: to which if 360360 be continually added, other-like Numbers will be produced.

6 24 120 720 5040 40320 362880 3628800
2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 × 11 = 39916800
1
39916799

6 30 210 2310 30030 60060
2 × 3 × 5 × 7 × 11 × 13 × 2 × 6 = 360360 Divisor.

(277
3880|1
39916 7|99 (110
36036 6 60
3603 3
360

11 1(1 277199 / 2 (138599 ; 122(2 277199 / 3 (92399 ; 3133(3 277199 / 4 (69299 ; 22 4(4 277199 / 5 (55439

3155(5 277199 / 6 (46199 ; 6466(6 277199 / 7 (39599 ; 3537(7 277199 / 8 (34649 ; 88(8 277199 / 9 (30799

(9 277199 / 10 (27719 ; 11(10 5200 277199 / 11 (25199 ; 31 (11 277199 / 12 (23099 ; 1423(0 277199 / 13 (21323

5ly. To find Numbers that leave the Remains Progressional throughout, or Progressional till the last Division, and then o; but distant from the Divisor above 1, as sometimes 2, 3, 4, &c. *5. To find Remains Progressional, &c. but distant above 1 from the Divisor.*

Progressional throughout.

Proceed for the Remains Progressional throughout, as in the second *Case* before, only in getting the least Numbers after the Prime Numbers are multiplied, their Product is to be multiplied by the Prime Numbers omitted from the Unit to the first Divisor multiplied one into another; and for higher Divisors than 7 by 6 besides. And for ease it may be observed, that the least Numbers to the half of the greatest Divisor, is but to abate the Difference between the Divisor and Remainer from the least Number that hath the Difference but an Unit. Other Necessaries may be discerned by the *Examples* following.

Examples to 7.

Examples to 7.

Remains.	12 60 360	*Difference. Numbers desired.*
1.2.3.4.5	$3 \times 4 \times 5 \times 6 \times 7 = 2520 - 2 = 2518$	. Greater.
3.4.5.6.7	$3 \times 5 \times 7 \times 4 = 420 - 2 = 418$	. Least.
Divisors.	15 105	
	20 120	
1.2.3.4	$4 \times 5 \times 6 \times 7 = 840 - 3 = 837$	. Greater.
4.5.6.7	$5 \times 7 \times 12 = 420 - 3 = 417$	. Least.
	35	
1.2.3	30	
5.6.7	$5 \times 6 \times 7 == 210 - 4 = 206$	. Least.
1.2		
6.7	$6 \times 7 == 42 - 5 = 37$	. Least.

Examp. to 11.

Examples to 11.

1.2.3.4.5.6.7.8.9	Greater Number 19958398	
3.4.5.6.7.8.9.10.11	15 105 1155 4620	Least Number.
$2 \times 2 = 4$	$3 \times 5 \times 7 \times 11 \times 4 \times 6 = 27720 - 2 = 27718$	
1.2.3.4.5.6.7.8	6652797	
4.5.6.7.8.9.10.11	35 385 4620	
$2 \times 2 \times 3 = 12$	$5 \times 7 \times 11 \times 12 \times 6 = 27720 - 3 = 27717$	
1.2.3.4.5.6.7	1663196	
5.6.7.8.9.10.11	35 385 4620	
$2 \times 2 \times 3 = 12$	$5 \times 7 \times 11 \times 12 \times 6 = 27720 - 4 = 27716$	
1.2.3.4.5.6	332635	
6.7.8.9.10.11	77 4620	
$2 \times 2 \times 3 \times 5 = 60$	$7 \times 11 \times 60 \times 6 = 27720 - 5 = 27715$	
1.2.3.4.5	55434	
7.8.9.10.11	77 4620	
$2 \times 2 \times 3 \times 5 = 60$	$7 \times 11 \times 60 \times 6 = 27720 - 6 = 27714$	
1.2.3.4	7913	
8.9.10.11	660	
$2 \times 2 \times 3 \times 5 = 60$	$11 \times 60 \times 6 = 3960 - 7 = 3953$	
1.2.3	90	
9.10.11	$9 \times 10 \times 11 == 990 - 8 == 982$	
1.2	$10 \times 11 == 110 - 9 == 101$	
10.11		

Progressional till the last.

For the Remains that are Progressional till the last Division, and then 0, proceed as in the 4*th Case* before, to find the least Number proper to the given Divisors, with 1 the Difference between the Divisor and Remainer; and this least Number multiply by the Difference of the Divisors given, till the half of the greatest Divisor, and the other having but few Divisors, are soon had: or multiply the Prime Numbers, except the two last Divisors, and their Product by the Prime Numbers omitted from the Unit, multiplied with the Difference, and 2 and 6, according to the Compounds as aforesaid, taking in 4 among the Prime Divisors for

for 7; but halfing the Difference, and next omitting it in higher Divisors in this last Multiplication, when in 7 the first Divisor is next above the greater Half of the greater or last Divisor, and in higher Divisors than 7 a Place further, and from the Product take the Difference. What else needful may be observed in the Instances following.

Examples to 7. *Examples to 7.*

Remains. 12 60 Difference 2 × 2 omitted.
1.2.3.4.0 3 × 4 × 5 × 4 == 240
3.4.5.6.7 2 2
Divisors. omitted. 238 Number desired.

Remains. 20 Difference 3 × 2 × 3 omitted.
1.2.3.0 4 × 5 × 18 == 360
4.5.6.7 2+3 3
Divisors. omitted. 357 Number desired.

Remains. Difference $4\frac{1}{2}$ 2 × 2 × 3 omitted.
1.2.0 5 × 12 == 60
5.6.7 2+3 4
Divisors. omitted. 56 Number desired.

1.0
6.7 is 7 the Number given.

Also 119 being found by the 4*th Case* before to be the Number sought, where 1 is the Difference between the Divisor and Remainer: If therefore 119 be multiplied by the Differences 2 and 3, there will be produced 238 and 357, as before.

Examples to 11. *Examp. to 11.*

1.2.3.4.5.6.7.8.0 15 105 420 840
3.4.5.6.7.8.9.10.11 3 × 5 × 7 × 4 × 2 × 6 == 5040
2 Difference.
2 omitted × 2 Difference = 4 5038 Number.

1.2.3.4.5.6.7.0 35 630 1260
4.5.6.7.8.9.10.11 5 × 7 × 18 × 2 × 6 == 7560
3 Difference.
2+3 omitted × 3 Difference = 18 7557 Number.

1.2.3.4.5.6.0 35 840 1680
5.6.7.8.9.10.11 5 × 7 × 24 × 2 × 6 == 10080
4 Difference.
2+3 omitted × 4 Difference = 24 10076 Number.

1.2.3.4.5.0 1050 2100
6.7.8.9.10.11 7 × 150 × 2 × 6 == 12600
5 Difference.
2+3+5 omitted × 5 Difference = 150 12595 Number.

1.2.3.4.0 1260 2520
7.8.9.10.11 7 × 180 × 2 × 6 == 15120
6 Difference.
2+3+5 omitted × 6 Difference = 180 15114 Number.

1.2.3.0 300
8.9.10.11 150 × 2 × 6 == 1800
2+3+5+7 omitted. 7 Difference.
So 2×3×5×5 (omitting 7) = 150 1793 Number.

1.2.0 60
9.10.11 30 × 2 × 6 == 360
2+3+5+7 omitted. 8 Difference.
So 2 × 3 × 5 (omitting 7) = 30 352 Number.

1.0

1.0
10.11 is 11 the Number given.

Also 25 9 by the 4*th Case* before, being found to be the Number sought, where 1 is the Difference; if then 2519 be multiplied by the Differences 2, 3,4,5 and 6, the Numbers will be found as above 5038. 7557. 10076. 12595. and 15114.

Other Diversities may be observed in higher Divisors, but for that seldom more than 11 are propounded in any Question, this may suffice.

6. To find Remains disorderly.

6*ly*. To find the Numbers which divided by the given Divisors leave the Remains disorderly, let it be observed, that the greatest Divisor is the Excess of an *Arithmetical Progression*, and the Remain of that Divisor, if any, the first Term thereof if 0 remain; the greatest Divisor is both the first Term and the Excess, and the Number sought is the last Term of the *Progression:* add therefore the Excess to the first Term continually, till proving every Term by all the Divisors, a Number be found that will leave the Remains proposed.

Q. *Of a Sum divided to* 4 *Children*, &c.

Example 1. A Man distributeth a Sum of Money among his 4 Children equally, and hath 3 *l.* left for himself: but if that Sum had been equally parted among 7, he would have had 5 *l.* left: what was that Sum?

Answer.

Answ. The least Number of that Property is 19, which is found by adding 7 the greatest Divisor to 5 the first Number, and again, to 12 the second Term, because 12 divided by 4 leaves 0.

Q. *Of a Number thought on.*

Example 2. One thinking on a Number, desires me to tell him what it is, whereupon I bid him divide it by 3, 5, 7, and tell me the Remains, which done, he declares them to be 2, 3, 6: what then is the least Number of that Property?

Answer.

Answ. 83: For by adding 7 the Excess to 6, the first Term successively, I find no Numbers till 83 of that Property.

6 2(2 5 (3
13 8 3 (27 12 19 (4
20 3 19 4
27
34 3 (5
41 8(3 19 (2
48 (16 7
55 5
62
69 1(6
76 8 3 (11
83 7

Other Examples.

To get other Numbers of like Property, multiply all the Divisors given one into another, and add the Product to the least Number found as above. So in the first Example 28, that is 4×7, added to 19; and in the second Example 105, that is 3×5×7; added to 83, shall produce other Numbers of like sort.

The Reason of leaving the Way used by others.

The *Appendix* added by *Alsted* out of *Geysius*, directeth to get the least Numbers by multiplying the Remains into certain Multiplicands, and dividing the Total of the Products by the Divisors multiplied one into another, the least Number shall be left remaining: but because sometime the least Number will be more than the Product of all the Divisors, that way is not so generally approved. For 32 divided by 4 and 7, leaves the Remains 0, 4; but 28, that is 4×7, dividing any Number, can never have 32 left a Remain: wherefore though the Way here above mentioned be tedious in many Divisors, yet being general and holding true in all Cases, it is to be chosen before the other, which by reason of the wonderful Variety without multiplicity of Rules and Exceptions, cannot be made good but in some particular Cases.

In division of Progressional Dividends, four things.

§. 2. In Division of *Progressional Dividends*, the work is to divide so that the Quotients may be *Arithmetically Progressional* in their Natural Order, but the Remains Nothing or Equal, Ordinate or Perturbate; in which latter only the Quotients are often interrupted in their *Progression*.

1. To find Nothing remain.

In every *Arithmetical Progression*, the Remains will be 0 when the first Term is equal to the Difference: For then the Difference will exactly divide all the Terms, as 3, 6, 9, 12, 15, &c. this is a Natural *Progression*; and 3 the first Term being equal to the Difference, dividing all the Terms, the Quotients will be in their Natural Order, 1, 2, 3, 4, 5, and 0 remain.

(0

Example.

(0 (0 (0 (0 (0

$\frac{3}{3}$(1 $\frac{6}{3}$(2 $\frac{9}{3}$(3 $\frac{12}{3}$(4 $\frac{15}{3}$(5

2. To find equal Remains.

In an *Arithmetical Progression*, when the Difference will not exactly divide the Terms, this is called sometimes an *Artificial Progression*, and shall have the Quotients *Progressional*, but all the Remains equal; as 3, 5, 7, 9, 11, &c. divided by 2, the Excess shall have the common Remain 1, but the Quotients 1, 2, 3, 4, 5, in their Natural Order.

Example.

(1 (1 (1 (1 (1

$\frac{3}{2}$(1 $\frac{5}{2}$(2 $\frac{7}{2}$(3 $\frac{9}{2}$(4 $\frac{11}{2}$(5

3. To find ordinate Remains.

In Division of an *Arithmetical Progression*, when the Divisor equal to the Number of Terms shall be but an Unit greater than the first Term in a Natural *Progression*, the Remains shall be Ordinate or Retrograde, *Arithmetically Progressional*; but the Quotients orderly, as 5, 10, 15, 20, 25, 30, divided by 6, makes the Quotients 0, 1, 2, 3, 4, 5, but the Remains 5, 4, 3, 2, 1, 0.

Example.

(4 (3 (2 (1 (0

$\frac{5}{6}$(0 $\frac{10}{6}$(1 $\frac{15}{6}$(2 $\frac{20}{6}$(3 $\frac{25}{6}$(4 $\frac{30}{6}$(5

4. To find pertur-bate Remains.

In Division of an *Arithmetical Progression*, when the Divisor equal to the Number of Terms shall be more than an Unit greater than the first Term in a Natural *Progression*, the Remains shall be Pertubate, yet have all the Natural Numbers to the Divisor, as 5, 10, 15, 20, 25, 30, 35, divided by 7; the Remains shall be 5, 3, 1, 6, 4, 2, 0, which are all the Numbers to 7.

Example.

(3 (1 (6 (4 (2 (0

$\frac{5}{7}$(0 $\frac{10}{7}$(1 $\frac{15}{7}$(2 $\frac{20}{7}$(2 $\frac{25}{7}$(3 $\frac{30}{7}$(4 $\frac{35}{7}$(5

In division into Progressional Quotients, two things.

§. 3. The Division into *Progressional Quotients Arithmetically*, is not properly where the Dividends are *Progressional*, as in the last Section, though *Geysius* placeth the 2 last sorts above here: but the Work is to find a Number which being divided, and the Remains continually by certain Divisors given, the Integers in the Quotients shall be in their Natural *Progression* descending to 1. As to divide 110 by 24, and the Remain 14 by 4, and the Remain 2 by 1; the Quotients will be 4, 3, 2.

In this Division is considerable the Invention of $\begin{cases}\text{Divisors.}\\ \text{Dividends.}\end{cases}$

1. To find the Divisors.

For the finding of Divisors, let be noted.

1*st*, The last Divisor will always be 1.

2*ly*, Therefore to 1 add the first Quotient, and the Total is the *Penult Divisor*.

3*ly*, To the Product of the first Quotient by the *Penult Divisor*, add the second Quotient and 1 for the *Antepenult Divisor*.

4*ly*, To the Product of the first Quotient by the *Antepenult Divisor*, add the Product of the second Quotient into the *Penult Divisor*; and the third Quotient and 1 for the *Proantepenult Divisor*.

Examples.

As if the Quotients were desired to be 5, 4, 3, 2; then shall

1 be the *Ultimate Divisor.*

$5 + 1 = 6$ *Penult Divisor.*

$5 \times 6 + 4 + 1 = 35$ *Antepenult Divisor.*

$5 \times 35 + 4 \times 6 + 3 + 1 = 203$ *Proantepenult Divisor.*

2. To find the Dividends.

For the finding of the Dividends.

Add together the several Products of the Divisors, multiplied by their respective Quotients.

Example.

As if a Number be desired, which divided by 203, the Quotient shall be 5; and the Remain of that Division divided by 35, the Quotient shall be 4; and the Remain of that Division divided by 6, the Quotient shall be 3; and the Remain of that Division divided by 1, the Quotient shall be 2: Then shall 1175 be obtained thus.

$$\begin{array}{r} 203 \times 5 = 1015 \\ 35 \times 4 = 140 \\ 6 \times 3 = 18 \\ 1 \times 2 = 2 \\ \hline 1175 \end{array}$$

(160   (2   (2   (0
1175 (5   16(0 (4   20 (3   2 (2
203   35   6   1

Q. Of discovering three Things, hidden. Suppose *A*, *B*, *C* have three Things, as a Ring, a Thimble, and a Bodkin; which they hiding, exchange one with another: how may the Order of their hiding be discovered?

Rule, how called. This Rule is called *Bis cæca*, or *twice Blind*, because it is grounded on blind or unknown Proportions, and therefore requires the more careful Observation of

What to be done. these six Particulars.

1. Progressionals taken. 1. For the Number of Things so hidden or concealed, take Numbers Progressional, as 1, 2, 3, &c. So let the Ring be 1, the Thimble 2, and the Bodkin 3.

2. Divisors found. 2. According to the third Section above, find the peculiar Divisors for those Progressional Quotients, always 1 less than the Quotients; which in the Example above shall be 4, 1; that is, 1 the Last, and 3 + 1 the *Penult Divisor*.

3. Multiplicands found. 3. Multiplicands are to be found in a certain Order, thus; The last Multiplicand is taken at pleasure; from which Number substract the Divisors in their order, and the Remains shall be the other Multiplicands in their order: As in the Example above, taking 6 for the last Multiplicand for *C*, then shall the other Multiplicands be 2 and 5, that is, 6 — 4, and 6 — 1: So shall the three Multiplicands be, for *A* 2, for *B* 5, for *C* 6.

4. Progressionals multiplied. 4. Cause every one of the Persons hiding the Things, to multiply the Progressional Number of the Thing they have so hidden by their proper Multiplicands, and to tell you the Sum of the Products. So if *A* have the Thimble, or 2; and *B* the Ring, or 1; then shall *C* have the Bodkin, or 3: And then multiplying 2 by 2, and 5 by 1, and 3 by 6, the Product of *A* is 4, *B* 5, *C* 18, and the Total 27.

5. Total of the Products substracted. 5. Having the Total, substract the same from the Product of the last Multiplicand by the Sum of the Progressionals: So 6 being the last Multiplicand in the above-mentioned Example, it shall be multiplied in 6 the Progressionals; that is, 1 + 2 + 3 from this Product 36; the Total 27 substracted, leaves 9 remaining.

6. Remain divided, gives discovery. 6. This Remain divided by the first Divisor, the Quotient sheweth the Thing hid by *A*; the Remain of this Division divided by the next Divisor, sheweth the Thing hid by *B*: And so successively for all the rest till the last, where there are many Divisors, which last Thing shall be the Complement. Wherefore in the former Example, 9 the Remain divided by 4 for *A*, the Quotient shall be 2, and shew *A* to have the Thimble; the 1 remaining divided by 1, shall shew *B* to have the Ring, or first Thing; and by Consequence the other Thing remaining, which is the Bodkin, must be in the Custody of *C*.

Example in four Progressionals.

Example in four Things hidden. Persons *A*, *B*, *C*, *D*. Things hid 1, 2, 3, 4.

Divisors 24 . 5 . 1. that is, 1 the last Divisor; 5, which is 4+1, the *Penult Divisor*; and 24 made of 4×5+3+1, the *Antepenult Divisor*.

The last Multiplicand suppose 12; then the other Multiplicands shall be

	12		12		12		12
	24		5		1		0
for *A* —	12	*B*	7	*C*	11	*D*	12

Supposing *A* hath hid	2 .	*B*	3 .	*C*	1 .	*D*	4.
Then the Products are —	12		7		11		12
For *A* —	24 .	*B*	21 .	*C*	11 .	*D*	48

The Total 21 + 11 + 48 — 24 = 56.

The last Multiplicand 12. Progressionals 1, 2, 3, 4 = 10

Product of 12 × 10 = 120 — 56 = 64.

Then

Then dividing 64 by 24, the Quotient 2 shews *A* to have the second Thing hid; the Remain 16 divided by 5, gives 3 in the Quotient for *B*, and denotes he the third Thing hid; the Remain 1 divided by 1, shews *C* hath the first Thing; and by Consequence *D* must have the fourth Thing.

$$\overset{(16}{\frac{\cancel{64}}{\cancel{24}}}\Big(\,2\,A. \qquad \overset{(1}{\frac{\cancel{16}}{\cancel{8}}}\Big(\,3\,B. \qquad \frac{\cancel{1}}{\cancel{1}}\Big(\,1\,C. \qquad D\,4.$$

Hence the Ground of other Methods, to find Things hid, and Numbers thought on.

The Things premised in the foregoing Sections of this Chapter being the Ground, as well of discovering the Dividends by the Divisors and Remains, concealing the Quotients, as by the natural Progression of Numbers from an Unit, and the Products of them by certain Multiplicands in one Sum, (concealing the Order of Multiplication) to find the Order of Multiplication, gave being principally to the several Inventions of other Methods for finding Numbers thought upon, and Things hidden. Some Examples whereof follow.

To find a Number thought on.

1. *To find a Number thought upon, or to tell a Man how many single Pence, or Pieces of Money he hath in his Purse.*

Bid the Person that thinketh, that he quadruple the Number thought on, and to the Product add 6, 8, 10, or any other Number at pleasure, and tell you the Half of the Total: For then if you take away half the Number which you willed him to add, there will remain double the Number at first thought upon.

Example:

The Number thought, suppose	6
The Quadruple thereof	24
To which if 10 be added, it makes	34
The Half of it is	17
From which 5 half the Number added being substracted, there remaineth	12
One half of which returneth the first Number	6

Otherwise.

Or bid him that thinketh, to double his Number; to which let him add 5, and then multiply the Total by 5; and having the Product, cut off the Right-hand Figure, or Cipher, and from the Residue substract 2.

Example:

The Number thought upon	8
The double of it	16
Addition of 5 makes it	21
Multiplied by 5, produceth	105
Right-hand Figure cut off, leaves	10
From which 2 substracted, rests	8

Otherwise.

Otherwise, bid him that thinketh to double the Number, and add 4 thereto, and multiply the Total by 5, and add to the Product 12, and multiply that Total by 10, and declare to you this last Product: Then from the same withdraw 320, and the Remainder in the hundred Place shall be the Number desired.

Example.

The Number thought	6
Doubled is 12, to which 4 added is	16
Multiplied by 5 produceth 80, and 12 added is	92
That multiplied by 10, produceth	920
From which 320 substracted, leaves	600
The Remainder in the hundred Place is	6

Otherwise.

Otherwise, bid him that thinketh, to triple his Number thought: then ask him if it be Even or Odd; if Odd, give him 1 to make it Even: and if 1 be given, reserve 1 in your Mind; and after the Number tripled is made Even, let him cast away half, and then triple that half: Then ask him again, if it be Even or Odd: if Odd, give him 1 again to make it Even; and for this 1 given upon the last tripling, reserve 2 in your Mind, (so that if you give 1 at both triplings, then have you 3 reserved in Mind); afterwards let him cast away half, and tell you how many Nines he can give you out of the other half; and for every 9 accompt 4, to which add the reserved Numbers if the Triples fell odd.

The

Example.

The Number thought ———	15	
The Triple ———	45	
Because it is Odd, 1 is added, and it is ———	46	1 Reserved.
The Half ———	23	
The Triple ———	69	
Because it is Odd, 1 is added, and it is ———	70	2 Reserved.
The Half ———	35	
Out of 35 can be given but 3 Nines, which at 4 for 9, returneth ———		12
		15 Total.

Otherwise. Otherwise, bid him that thinketh that he break the Number in two Parts, and square each part, and add the Products together; and then let him multiply the Parts one by another, and add the Product doubled to the former, and tell you the whole Sum; then take the Square Root thereof for the desired Number.

Example.

The Number thought ——— 7
The Parts suppose 3 and 4

The Square of $\left\{\begin{matrix}3 \text{ is } 9\\ 4 \text{ is } 16\end{matrix}\right\}$ together — 25

The Parts 3 and 4 multiplied make — 12
Doubled is 24, added to 25 is ——— 49
The Square Root thereof is ——— 7

Otherwise. Otherwise, bid the Party that thinketh, that multiply the Number thought by what Number you please; then bid him divide the Product by another Number, and multiply that Quotient by some other Number; and that Product again divide by some other as often as you will: and in like manner take a Number at Pleasure, and secretly multiply and divide by the same Multipliers and Divisors as oft and in the Order he did, then bid him divide the last Quotient by the Number first thought, and in like manner do you; so will the Quotients be both alike; a Thing which seems admirable to those ignorant of the Cause: then bid him to his Quotient add his Number thought on, and demand the Sum (as if you knew nothing of his Quotient), from which substract your Quotient, and you have your desire.

Example.

The Number thought ——— 8
Divided by 4, the Quotient is ——— 2
Which multiplied by 8, is ——— 16
This divided by 4, is ——— 4
Which multiplied by 5, is ——— 20
This divided by the first 8, is ——— $2\frac{1}{2}$

$$\frac{8}{4}\Big(2 \quad 8 \quad \frac{16}{4}\Big(4 \quad 5 \quad (4 \quad \frac{20}{8}\Big(2\tfrac{1}{2}$$

In like manner, if 12 be taken and divided and multiplied by the same Numbers, and the last Product divided by 12, the Quotient will be $2\frac{1}{2}$ as the other.

$$\frac{12}{4}\Big(3 \times 8 = \frac{24}{4}\Big(6 \times 5 = \frac{30}{12}\Big(2\tfrac{1}{2}.$$

To find a Number thought on, without asking any Questions.

2. *To find a Number thought upon without asking any Questions, certain Operations being done.*

Bid him that thinketh add half of his Thought-Number thereto; and if it be an odd Number and cannot be halfed evenly, let him take the bigger half to add to it, and if so, keep 3 in Mind; then bid him take half the whole Sum and add thereto, and if it be an odd Number again as before, let him take the bigger Half, and for this odd one reserve 2 in Mind, so as if both be odd, 5 must be reserved: when this is done, let him substract from the last Total double the Number thought upon, and from the Remainer will him to cast away half if he can; if it be odd, then reject 1, which I reserve, and so perpetually halfing it till he come to 1: for then mark how many halfs there were after the double Number was substracted, and for the first half accompt 2, for the second 4, for the third 8, &c. and add unto these Numbers the Units rejected upon the last halfings (noting the first

1

1 rejected is but 1, the second is to be reckoned 2, the third 4, *&c.*) multiply this Sum by 4, and from the Product deduct the Numbers reserved upon the first halfings, if any.

Example in even Numbers.

Examples in even Numbers.

The Number thought ——	8	The Number thought ———	18	
The half added 4, is ——	12	The half added 9, is ———	27	
The half of 12 is 6, added is	18	The bigger half added makes ——	41	2 Reserved.
Double of 8, is ———	16	Double of 18, is ———	36	
Substracted, leaves ———	2	Substracted, leaves ———	5	
The half ———	1	The first half ———	2	1 Rejected.
		The second half ———	1	
For this one halfing is accompted 2, which multiplied by 4, returneth the first Number	8	For this second halfing is accompted 4, which with 1 rejected is 5; this multiplied by 4 is 20, and the 2 reserved deducted, there rests	18	

Example in odd Numbers.

Examples in Odd Numbers.

The Number thought —	7		The Number thought —	17	
Greater half 4 added, is-	11	3 Reserv'd.	Greater half 9 added, is -	26	3 Reserv'd.
Greater half 6 added, is-	17	2 Reserv'd.	Half 13 added, is ———	39	
Double of 7, is ———	14		Double of 17, is ———	34	
Substracted, leaves ———	3		Substracted, leaves ———	5	
Lesser half ———	1	1 Rejected.	Lesser half ———	2	1 Rejected.
For this half 2 being taken, and the Rejected 1 is 3, multiplied by 4 is 12, from whence the Reserved 5 deducted, leaves	7		Second half ———	1	
			For this half 4 being taken, and the Rejected 1 is 5, multiplied by 4 is 20; from whence the Reserved 3 deducted, leaves	17	

3. *Two Numbers being proposed unto two Persons, to tell which of those Number is taken by each of the Parties.*

To find which of two Numbers taken.

Admit the two Numbers proposed be one Even and the other Odd, as 11 and 12; and when they have privately accepted which they please thereof, triple the one and double the other, and add their Products together: Contrariwise, double that Number you tripled, and triple that Number you doubled, and add their Products together, the one will be Even and the other Odd; then bid A to triple the Number he took, and bid B to double the Number he took, and add both their Products together; and if A can half the Sum evenly, he took the Even Number, otherwise B accepted thereof.

Numbers proposed, 11, 12.

Example.

A taketh ———	11	A taketh ———	12
B ———	12	B ———	11
A tripleth ———	33	A tripleth ———	36
B doubleth ———	24	B doubleth ———	22
Total Odd ———	57	Total Even ———	58

4. *To discover whether Even or Odd Numbers be taken when they are not proposed.*

To find if the Number taken be Even or Odd.

Bid A to triple his Number and B to double his, and add both the Products together, and if they can give you the Even half, A took an Even Number, and B an Odd; if not, understand the contrary.

Example.

A		B			A		B	
4		3			3		4	
3		2	Even.		3		2	Odd.
12	+	6	= 18		9	+	8	= 17.

5. *To know several Numbers thought upon by one or sundry Persons.*

To find several Numbers thought on.

Bid them add the first and second together, and tell you the Total; likewise the Total of the Second and Third, and so further if there be more Numbers, and then tell you the Total of the first and last Numbers added together: For then

 by

by the Rule of *Falshood* or *double Position*, as before in the second Part of this 4th Book, *Chap.* 14. Resolution may be had.

For Odd. Or if the many Numbers thought on be Odd, as 3 Numbers, 5, 7, *&c.* having the Totals given as before, place them in order, and add together all those that stand in the odd Places, *viz.* the First, Third, Fifth, *&c.* keep this Number apart from which you must make Substraction. In like manner add all those Numbers together which are in the even Places, *viz.* the Second, Fourth, *&c.* and substract this Total from the former, the Remain shall be double the first Number; which found, the rest are easily known, because the Sum of the First and Second is given.

For Even. But if the many Numbers thought on be Even, as 4 Numbers, 6, 8, *&c.* then having the Totals of each two Numbers as before, with the Total of the second and last Numbers, add the Numbers in the odd Places, except the First, and take the Sum from the Sum of the Numbers in the even Places, the Remain shall be double the second Number thought upon: And this being known, the rest are easily obtained.

Example in Odd.

Example in five Numbers.

Suppose the Numbers thought ——— 3 . 6 . 4 . 8 . 10
First and Second, are ——— 9
Second and Third ——— 10
Third and Fourth ——— 12
Fourth and Fifth ——— 18
First and Last ——— 13
Numbers in the odd Places, are 9 . 12 . 13. together ——— 34
Numbers in the even Places, are 10 . 18. together ——— 28
Difference ——— 6
The Half is the first Number 3.
This taken from 9, the First and Second leaves 6, *&c.*

Example in Even.

Example in six Numbers.

Suppose the Numbers thought ——— 4 . 6 . 7 . 9 . 10 . 11
First and Second, are ——— 10
Second and Third ——— 13
Third and Fourth ——— 16
Fourth and Fifth ——— 19
Fifth and Sixth ——— 21
Second and last ——— 17
Numbers in the even Places are 13 . 19 . 17 . together ——— 49
Numbers in the odd Places are, 16 . 21 . together ——— 37
Difference ——— 12
The Half is the second Number 6.
This taken from 10, the First and Second, leaves 4, *&c.*

To find Digits thought on, or Points of Dice.

6. *To declare one or more of the Digits thought upon, or the Points cast by two or more Dice.*

Let the first Number be doubled, and thereto add 5; multiply the Sum by 5, then add 10, and the next Number thought upon; multiply this Sum by 10, and thereunto add the next Number, and so proceed. Now if he give you the last Sum, then if he thought but upon one Number, substract only 35 from it, and the Remain in the Place of Tens is the Number thought upon: If he thought on 2, then take 350, and the Remainers in the 100 and 10 Places are the desired Numbers, *&c.* And because the Right-hand Figures, where above 3 are thought upon, will be the same Numbers thought upon, the more to conceal the Secret, you may ask for half the last Sum, or bid him put 14, 15, or some other Number thereto, which afterward may be easily deducted.

Example.

Numbers thought, suppose ———	4 . 6 . 8 . 7 . 9.			
Double of 4 is ———	8			
Addition of 5 makes it ———	13			
Multiplication of 5 produceth ———	65			
Addition of 10 makes it ———	75	— 35	=	40
Next Number thought added ———	81			
Sum multiplied by 10, is ———	810	— 350	=	460
Next Number 8, added, is ———	818			
Sum multiplied by 10, is ———	8180	— 3500	=	4680
Next Number 7 added, is ———	8187			
Sum multiplied by 10, and 9 added, makes it ———	81879	— 35000	=	46879

7. *To tell what Numbers remain, after certain Operations done, without asking any Questions.*

To find what Remains, &c.

Let him that thinketh on a Number, multiply it by what Number you please: to the Product bid him add another Number, (which you must be sure may be equally divided by that Number before multiplied by) then let him divide the Sum by the Number he first multiplied by, and from the Quotient substract the Number thought; and this Remainer shall be equal to your Quotient, if you divide that Number which was added by that which multiplied.

Example.

Number thought, suppose ———	8	
Multiplied by 5, is ———	40	
Adding 20, it is ———	60	So 20, divided by 5, yieldeth 4
Which divided by 5, gives ———	12	
Number thought taken away, leaves remaining ———	4	

8. *A Ring hidden among 9 or 10 Persons; how to discover the Person that hath the Ring, and upon which Hand, Finger, and Joint.*

To find what Hand, Finger and Joint a Ring is on.

When the Ring is disposed among the Company, you being absent, cause the Persons to sit down in a Row; and let one of them who is privy to the Ring's Disposal, double the Number of the Person, and thereto add 5; then let him multiply the Addition by 5, to the Product bid him add the Number of the Finger: And lastly, to the right Hand adjoin the Number of the Joint, and add to the whole Sum, for Secrecy-sake, 7, 8, 9, *&c.* which done, require the Total; whence take the Number last added, and from the Residue substract 250, and you shall have 3 Figures left; the First whereof to the left Hand shall signify the Person, the middle Number the Finger, and the Third the Joint. And if after Substraction of 250 there rest 0 in the Place of Tens, then is the Ring on the Tenth or little Finger of the left Hand; and so must 1 be abated from the Place of Hundreds.

Example.

Suppose *A*, *B*, *C*, *D*, *E*, *F*, or 6 Persons.
And *E* the 5*th* Person had the Ring on his left Hand, on the middle Joint of the 9*th* Finger, accompting from the Thumb of the Right.

Then 5 doubled is 10, and 5 added makes ———	15
This multiplied by 5, produceth ———	75
The Finger added, it is ———	84
The Joint 2 adjoined, makes it ———	842
And if for Secrecy 7 be added, the Total is ———	849—257=592
From this given Total 7, and 250 taken, there remaineth 592, representing ———	*Person.* the 5, *Finger.* the 9, *Joint.* the 2.

Otherwise.

Otherwise, observing an Order of the Persons, and likewise of their Hands as before, calling the Right the First, and the left the Second; after the Number of the Person is doubled, 5 added, and that Sum multiplied by 5 as before, bid the Party add 10, and the Number of the Hand; which Sum bid him also to multiply by 10, and then add the Number of the Finger; and then again multiply by 10, and add the Number of the Joint, and what Number you please afterward for secrecy-sake: Then demanding the Total, deduct from thence the Number last added, and 3500, and the remaining Figures represent the Person, Hand, Finger and Joint desired.

Example.

Suppose the 6*th* Person hath the Ring on the left Hand, third Finger, and second Joint.

Then 6 doubled is 12, and 5 added makes	17
This multiplied by 5 is 85, and 10 added is	95
The Hand added is 97, and multiplied by 10 is	970
The Finger added is 973, and multiplied by 10 is	9730
The Joint added, makes the Total	9732
From this 3500 taken, leaves	6232

Representing the 6*th* Person, second Hand, third Finger, and second Joint.

To find the Points cast on Dice.

9. *Two or more Dice being cast, how by Art to discover the Number of Points that may arise.*

Suppose one had cast three Dice, bid him add the Points that were upmost together, and put one of the Dice apart with the same Side upmost; let him add to the Sum the Points under the other two, then bid him cast those two Dice, and add the Points cast to the former Sum; and put one of the two Dice away, not changing the Side; and marking the Points under the other Dye, add it to the former Sum: Lastly, throwing that one Dye, whatever appears upwards, add it to the former Sum, and let the Dice remain without Alteration. This done, coming to the Table, note what Points appear upward on the three Dice; to which add 3 times 7, (for every Dye cast, always 7 is to be added) and this addition of 7 for every Dye, and the Points lying almost on the Dice, shall be equal to all the Operations made by the other Party privately.

Example.

As if three Dice being cast, there should appear 4, 6, 3, which added together make 13; then laying by one of the Dice, as suppose 4, and adding to 13 the Points under 6 and 3, which will be 1 and 4, (for always the Point, and his Opposite, or that above and underneath, makes 7) the Sum will be 18: Then throwing the 2 Dice, suppose there appear 5 and 2, which makes (being added) 25; and laying by 5, and adding the Point under 2, which will be 5, it makes the former Sum 30: Then throwing the one Dye, suppose there appear 4, then is the Sum 34. So the Points lying upward, are 4, 5, 4; which when I find, I add for each Dye 7, and that 21 with 13, the Points lying upward, make up 34 as before.

To find the Number of Pieces of Money, &c. in the Hand.

10. *If one hold in each Hand as many pieces of Money, Stones, &c. as in the other, how to find the Number.*

Bid him that holds the same, that he put out of one Hand into the other what Number you think convenient (provided it may be done): then bid him take out of the Hand that he put the Number into as many as remain in the other Hand, and put into that Hand; for then be assured that in the Hand which was put the first taking away, there will be found just double the Number taken away at first.

Example.

As admit in each Hand were 10 pence, then suppose 4 were taken out of the right Hand and put into the Left; then was there 14 in the Left, and but 6 remaining in the Right; then if 6 be taken from 14 and put to the other Hand, there will be left but 8 the Double of 4, the Number first substracted.

To find how many Counters, &c. three Persons have taken.

11. *Three Persons having taken Counters, Cards or other Things; to find how many each hath taken.*

Cause the third Party to take a Number which may be divided by 4; and as often as he takes 4, let the Second take 7, and the First 13; then cause them to put all together and declare to you the Total, which divide by 3, and the Quotient is the Double of the third Number, or Sum which the third Person did take.

Otherwise.

Or cause the First to give to the Second and Third as many as each of them hath; then let the Second give to the First and Third as many as each of them hath; Lastly, let the Third give to the First and Second as many as each of them hath, and ask how much one of them hath (for then they will have all alike); so half that Number is the Number the third Person had at first; which once known, all is soon known.

Example the first Way.

Example by the first way: Suppose the Third took 8 Counters, which is twice 4, then must the Second take 14, which is so many times 7, and so consequently the First 26, which is twice 13; all added together is 48, which divided by 3 yieldeth 16, the half of which 8 is the third Person's Number.

Example

	A	B	C
Example by the latter way: Numbers supposed to be taken	26	14	8.
A giving to *B* and *C* like Sums, that is, *B* 14 and *C* 8, they have	4	28	16
B then giving to *A* and *C* like Sums, that is, *A* 4 and *C* 16, they have	8	8	32
C then giving to *A* and *B* like Sums, that is, *A* 8 and *B* 8, they have	16	16	16

Example the latter Way.

12. *Three Cards chosen out of a Pack, to find how many Points they contain if the Pack be full.*

To find the Points in three Cards.

Let him that hath chosen the 3 Cards, accompt the Points in each Card, and bid him take as many Cards as will make up 15 to each Number of the Points on the Cards first chosen, then will him to give you the remaining Cards; for 4 of them being rejected, the rest shew the Number of Points that were on the 3 Cards so chosen.

As if the Cards were 7, 6, 5: now 7 wanting 8 of 15, therefore he took 8 Cards to make up the Points on the first Card 15; so is there 9 Cards taken from 52 (the whole Pack) for the first Card, then 6 wants 9 of 15; therefore taking 9 Cards with the Card of 6 Points, that is, 10 more from the whole Pack, so will but 33 Cards remain of the 52; then 5 doth want 10 of 15, therefore 11 Cards more taken from 33 there will be left but 22; which Number of Cards given, substract 4 from, and the Remain 18 is the Number of the Points 7, 6, 5, on the Cards first chosen. *Example.*

But when 4, 5, 6, or more Cards be chosen, and the Number of Cards out of which they are taken be more than 52, and the Term be 15, 14, 12, or such like; then multiply the Term by the Number of Cards taken at first, and to the Product add the Number of Cards so taken; then this Total substract from the whole Number of Cards, the Remain shall be the Number which must be taken from the Cards which remain to make up the Game: if there remain 0 after Substraction, then the Number of Cards which remain, do declare the Number of Points in the Cards first chosen: if the Substraction cannot be made, then substract the Number of Cards from that Number, and the Remainer added to the Cards that did remain, will be the Number of Points in the Cards first taken. *If more than 3 Cards be taken.*

As if the Cards were 8, 6, 3, 2, and the Term given 14. I see the first or 8 wants 6 of 14, the second or 6 wants 8 of 14, and 3 wants 11 of 14, and 2 wants 12, which taken, the Party delivers you the rest of the Cards; and if the Pack were whole, they are but 11; for 7, 9, 12 and 13 are 41: then do you multiply 14 by 4 (the Number of the Cards chosen at first) which makes 56, to which 4 (the said Number of Cards) added is 60, from which 52 substracted, leaves 8, which with 11 is 19, and so many were the Points of the Cards, *viz.* 8, 6, 3, 2. *Examples.*

Admit the Cards were 7, 10, 5, 8, and the Term given 12, then doth 7 want 5 of 12, ten 2, five 7 and eight 4, which with the 4 Cards first taken make 22, this taken from 52 leaves 30: Now multiplying 12 by 4, is produced 48, to which 4 added, the Total is 52, which substracted from 52 leaves 0: therefore doth 30 (the Cards left) represent the Numbers first taken, *viz.* 7, 10, 5, 8, which together make up 30.

Again, suppose 3 Cards be taken, as 7, 9, 5, their differences to 15 are 8, 6, 10, which Numbers of Cards taken from 52, the Remain is but 25; then multiply 15 by 3, to the Product 45 add 3 (for the first Cards taken) it is 48, which taken from 52, there rests 4; this taken from 25 (the remaining Cards) leaves 21 the Sum of 7, 9, 5.

13. *Three Things and three Persons proposed; to find which of them hath either of the three Things.*

To find which of 3 Persons hath things hidden.

Suppose the Persons be *A. B. C.* and the 3 Things be a Ring, a Thimble, and a Bodkin, and the Persons mutually consent to change the Things among themselves; then take 24 Counters (if more be taken the Matter may seem the more secret, but no more are needful) and lay them before the Parties, and causing them to sit or stand in a Row, give *A* with the Ring 1 Counter, *B* with the Thimble 2 Counters, and *C* with the Bodkin 3 Counters, leaving the rest of the Counters with them: retire apart, and let them change the Things, but not the Counters given them, and bid them, that he that hath the Ring after the Change, take up so many Counters as you gave him at first, and him that then shall have the Thimble to take up 2 for every Counter given him at first, and he that shall have the Bodkin to take up for every one given him at first 4: Then returning, consi-

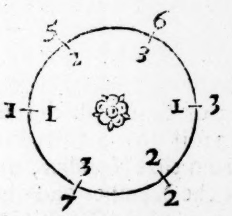
der the remaining Counters left out of the 24; for if the Parties have followed your Directions, there will be left either 1, 2, 3, 5, 6 or 7, and no other Number. Now if 1 remain, then hath there been no Exchange, but as you delivered them so they remain; if 2 remain, then hath *A* the Thimble, *B* the Ring, and *C* the Bodkin: If 3 remain, then hath *A* the Ring, *B* the Bodkin, and *C* the Thimble, and so further as is expressed in the following Table.

By the Table.

Remaining Counters.	*Persons.*	*Things hid.*	*Remaining Counters.*	*Persons.*	*Things hid.*
	A	Ring.		*A*	Thimble.
1	*B*	Thimble.	5	*B*	Bodkin.
	C	Bodkin.		*C*	Ring.
	A	Thimble.		*A*	Bodkin.
2	*B*	Ring.	6	*B*	Ring.
	C	Bodkin.		*C*	Thimble.
	A	Ring.		*A*	Bodkin.
3	*B*	Bodkin.	7	*B*	Thimble.
	C	Thimble.		*C*	Ring.

Otherwise by the Circle.

Otherwise, the same may be wrought without the Table by the help of a Circle, so divided into 6 Parts, wrote with 1 within, and 1 without; 2 within, and 5 without, *&c.* as followeth.



So if the Number of Counters remaining of the 24, be found in the upper Semicircle without; then that which is opposite within shews the First, and the next the Second, *&c.* as counting the Ring, Thimble and Bodkin, 1, 2, and 3: And suppose 6 Counters remain, 6 being found without at top, the opposite Figure within is 3; which imports that the first Man hath the Bodkin, or third Thing; and so going to the left Hand, the second Man hath the Ring, or first Thing, *&c.* But if the Remainer be found at the Bottom, as 2, then the Opposite within is 2, declaring the first Man to have the second Thing, or Thimble; and so going backward, contrary to the Former, the second Man hath the first Thing, or the Ring, *&c.* As if the former Table were thus figured as at *A*, or in Transmutations of Names as at *B*.

Table how otherwise set.

A

	A	1		*A*	2
1	*B*	2	5	*B*	3
	C	3		*C*	1
	A	2		*A*	3
2	*B*	1	6	*B*	1
	C	3		*C*	2
	A	1		*A*	3
3	*B*	3	7	*B*	2
	C	2		*C*	1

B

	1	*A*		1	*B*
1	2	*B*	5	2	*C*
	3	*C*		3	*A*
	1	*B*		1	*C*
2	2	*A*	6	2	*A*
	3	*C*		3	*B*
	1	*A*		1	*C*
3	2	*C*	7	2	*B*
	3	*B*		3	*A*

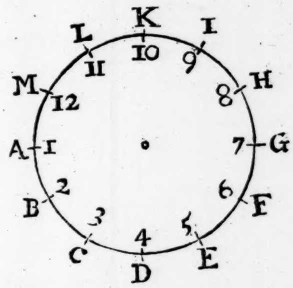
To find which Number on a Circle was thought on.

14. *Many Numbers disposed circular (or otherwise) to find which of them any one thinks upon.*

Suppose that having ranked 10 or 12 Things in a Row, as A, B, C, D, E, F, G, H, I, K, L, M, or Circular, (as the Figure sheweth); and one hath thought upon G, which is the 7*th*, then ask the Party at what Letter he will begin to accompt, (for accompt he must, otherwise it cannot be done); which suppose at K, that is the 10*th* Place; to which 10 add the Number of the Circle 12, which together is 22; so bid him to accompt 22 from K backwards, beginning his Accompt

compt with that Number he thought, so shall he accompt I to be 8, H 9, &c. And so will the Accompt of 22 end on the Letter G exactly, the Thing or Number sought.

Example.



To divide Liquors equally without Fusion.

To close up this Chapter, it is worthy note; 1*st*. That when a Question is propounded, of dividing equally among a Number of Persons, certain Liquors in Cask without Fusion; if the Number of Vessels in which the Liquors are, will not be equally divided by the Number of Persons, without a Remain, the Resolution is impossible: but if 0 remain upon the Division, break the Quotient into as many several Parts as Partners; so as being added together, they will make the Whole, and according to these Parts shall the Cask be divided.

Q. Of 21 Cask with Wine divided.

Example. *A*, *B*, and *C*, bought a Quantity of Wines, each paid alike, and was to have alike: It happened (some being sold) there was to be parted among them at last 21 Hogsheads, of which 7 were full, 7 half full, and 7 empty: how must they share the Vessels with the Wine in them, that each of them may have as many Vessels, and as much Wine one as another?

Answer.

Answ. Because 21 will be equally divided by 3, the Quotient 7 is to be broken into Parts, as 3, 3, 1, or 2, 2, 3; either of which Partitions make 7, and each of the Parts less than the Half: So may *A* have 3 Vessels full, 3 empty, and 1 half full; or 2 full, 2 empty, and 3 half full: then will he have 7 Vessels and $3\frac{1}{2}$ Hogsheads of Wine in Quantity, *B* the like; and then will be left for *C*, after the first Division, 1 full Vessel, 1 empty, and five half full; or after the second Division, 3 full, 3 empty, and 1 half full. And because this, as other-like Questions, will be truly resolved by more Divisions than one, the Proposition, however placed by some, is most proper for *Technology*.

	Full.	*Empty.*	*Half.*		*Full.*	*Empty.*	*Half.*
A	3	3	1	*A*	2	2	3
B	3	3	1	*B*	2	2	3
C	1	1	5	*C*	3	3	1

To equalize the Sums for which divers Things sold at different Rates.

2*ly*. That in Questions about equalizing the Sums for which different Numbers of Commodities may be sold at different Rates, besides a Progression given, an occult Progression in the Proposition is necessary to be discovered for the Resolution; which is often done by adding the different Rates, and thereby dividing the least Term of the given Progression, and by the Integers in the Quotient multiplying the highest Rate for the Increase of the Progression sought from an Unit. Yet it is questionable whether it will hold generally; but the Artist when it fails, may supply the defect by his Ingenuity.

Q. Of Apples sold differently, yet the Sums alike.

Example. Suppose 3 Women carry Apples or Oranges to Market; as *A* 20, *B* 30, and *C* 40: how can it be that they can sell as many for a Penny the one as the other, and when they had sold all, have one as much Money as another and no more, the Rates they sell at increasing as 3 to 1?

Answer.

Answ. 3 and 1 added make 4, which dividing 20 give 5; this multiplied into 3 the highest Rate, produceth 15 for the Increase of the Progression sought: So if *A* sold 1 Apple for 1 Penny, then adding 15 to 1, *B* shall sell 16 for Pence apiece; and adding 15 to 16, *C* shall sell 31 for Pence apiece; and the rest of their Apples being all sold at 3 Pence apiece, shall make up each of their Monies 4 *s.* 10 *d.* or 58 *d.*

If the Sum be not propounded, the Resolution divers.

But it is to be further noted, that if the whole Sum of the Money made be not propounded in the Question, these as other Propositions in *Technology*, are capable of being truly resolved by several Numbers, even as many as the last Parcel at the highest Price is more than an Unit, if the first Parcel at the lowest Price

begin

begin at the Unit. And therefore in the former Example, because 31 wants 9 of 40, the Question may be resolved 9 ways: Nevertheless the Sum of Money will be different in every of them, as here followeth.

Examples.

Price	A . B . C		A . B . C		A . B . C	
1*d*	1 . 16 . 31	58 *d.*	2 . 17 . 32	56 *d.*	3 . 18 . 33	54 *d.*
3	19 . 14 . 9		18 . 13 . 8		17 . 12 . 7	
d.	*A . B . C*		*A . B . C*		*A . B . C*	
1	4 . 19 . 34	52 *d.*	5 . 20 . 35	50 *d.*	6 . 21 . 36	48 *d.*
3	16 . 11 . 6		15 . 10 . 5		14 . 9 . 4	
d.	*A . B . C*		*A . B . C*		*A . B . C*	
1	7 . 22 . 37	46 *d.*	8 . 23 . 38	44 *d.*	9 . 24 . 39	42 *d.*
3	13 . 8 . 3		12 . 7 . 2		11 . 6 . 1	

Proof of Technology.

The Practice of most of the Works in this Chapter carry the Evidence of their Proof in their Operation, the Conclusions proving themselves true, so as other Demonstrations would be vain.

CHAP. V. *Progression Geometrical.*

Geometrical Progression.

BEsides what hath been said of this sort of *Progression*, *Chap.* 1. *Part* 1. and *Chap.* 1. *Part* 3. of this 4*th* Book, it is necessary to add, that in every *Geometrical Progression*, the Consequent divided by the Antecedent, shews the *Ratio* in the Quotient; so as to beget new Proportionals of this sort, is to multiply the Antecedent by the *Ratio*: but this is to be understood of Integers in Fractions, and Decimals the contrary.

New Proportionals, how gotten.

Ratio, *improperly called the* Excess.

The *Ratio* (called in some Authors the *Excess*, but improperly, the Excess being the Difference of 2 Numbers in *Arithmetical Progression*) is alike between all the Terms; so as if the second Number contain the first 2, 3, or 4 times or more, then shall the Third contain the Second so many times also, and the Fourth the Third, *&c.*

To double it,&c. *what.*

To double, triple or multiply how often soever any *Ratio* is, so often to put together the Space or Distance between the Terms.

Common way of Proceeding.

The common Proceedings by most in this *Progression* is by the first Term, the *Ratio*, and the Number of Terms, to place all the Terms orderly as they increase, and collect their Total by common Addition.

Q. *Of Wheat sold, increasing the Price of every Bushel.*

Example. A Farmer selleth a Quarter of Wheat (containing 8 Bushels) to be paid for the first Bushel 2 Farthings, for the second 8 Farthings, and so increasing by a fourfold Proportion or *Ratio*: the Question is, what was to be paid for the Wheat by that Agreement?

Answer.

Answ. 43690 Farthings or 45 *l.* 10 *s.* 2 *d.* $\frac{1}{2}$. For by setting down orderly, as at *A. B.* or *C.* all the Terms, and adding them together, the Sum is so found.

	d			*B* *q*
A. Least Term or Extream	$0\frac{1}{2}$	1	Bushel of Wheat.	2
	2	2		8
	8	3		32
	2:8	4		128
Ratio 4.	10:8	5		512
	2:2:8	6		2048
	8:10:8	7		8192
Greatest Term or Extream	34:2:8	8	Terms or Places.	32768
Total or Sum	$45:10:2\frac{1}{2}$			43690

(2 *q* | *q* 4) 43690 (~~1092~~ (2 *d* / ~~12~~ (91,0 / $45:10:2\frac{1}{2}$

C

Terms or Places.

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 .

Least Term 2.8.32.128.512.2048.8192.32768. Greatest Extream.

Ratio 4. Sum 43690 *q.*

And

And in Species thus, $\alpha \,.\, \beta \,.\, \frac{\beta q}{\alpha} \,.\, \frac{\beta c}{\alpha q} \,.\, \frac{\beta qq}{\alpha c} \,.\, \frac{\beta qc}{\alpha qq} \,.\, \frac{\beta cc}{\alpha qc} \,.\, \frac{\beta qqc}{\alpha cc} \,.\, \because$

In the Computation of this second sort of *Progression* is to be observed as in the Sections following. *What to be noted.*

§ 1. That as in *Arithmetical Progression*, so in *Geometrical*, there are 5 principal Things, *viz.* *1st. The five Principals.*

1. The first Term or least Extream, which in the Example above is 2, and noted in Species with α as the least Term of an *Arithmetical Progression.* *1. The first Term, &c.*

2. The last Term or greatest Extream, being the last of the *Progression*, noted also sometime as in *Arithmetical Progression* with ω: but if all the Terms be set down, then it is according to the Power of the Multiplication of the second Term, to be divided by the next inferior Power of the First. So as in the former Example 32768 the last Term is expressed by $\frac{\beta qqc}{\alpha cc}$. *2. The last Term, &c.*

3. The Number of Terms or Places in the whole *Progression*, which in the Instance above is 8, and in Species noted commonly as in *Arithmetical Progression* by *T*, and sometimes by *N*. *3. The Number of Terms, and Species thereof.*

4. The *Ratio* marked sometime with *R*, sometime with *r*, sometime with *Rat.* and sometime with $\frac{R}{S}$ or R to S; and when α is 1, is always equal to β, and so sometime marked therewith. This in the foregoing Example is *Quadruple*, or as 4 to 1; the Number of *Ratio*'s in every *Progression*, or the Number of Spaces or Distances between the first and last Terms is always less by 1 than the Number of Terms, and so marked with T—1, or N—1. *4. The Ratio, and several Species thereof.*

5. The Sum or Total of all the Terms in the *Progression*, which above in the Example is 43690. and for this the Note in Species is Z, in common with *Arithmetical Progression.* *5. The Sum or Total, &c.*

§. 2. By any 3 of the 5 Principals, the other two may be found. As, *2ly. To find the two unknown.*

1. *To find the first Term* (or α) *of a Geometrical Progression.* *To find the first Term.*

1. If the last Term, the Number of Terms, and the *Ratio* be given; that is, ω . T . R. and in the former Instance 32768 . 8 . and 4. to find 2. *1. Data. ω. T. R.*

Then either 1*st*, divide the second Principal by the Fourth so many times lacking one, as there be Units in the Third: *Rules.*

Or, 2*ly*, divide the second Principal by the Fourth, exalted to the next inferior Power of the Third, taking the Third for an Index.

Examples.

$\frac{\omega}{R}$ to T−1 = α   4) 32768 (8192 (2048 (512 (128 (32 (8 (2
(1 . 2 . 3 . 4 . 5 . 6 . 7 = 8 − 1)

$\frac{\omega}{R \text{ figurat to } T - 1} = \alpha$ $\frac{32768}{4.16.64.256.1024.4096.16384}$ (2
1 . 2 . 3 . 4 . 5 . 6 . 7 = 8 − 1

2. If the last Term, the Number of Terms, and the Sum be given; that is, ω . T . Z . and in the former Instance 32768 . 8 . & 43690 . to find 2. *2. Data. ω. T. Z.*

Then without respect to the third Principal, take the greatest Common Divisor between the Second and Fifth, marked sometimes in Species with M: So 2 shall be the Number sought, because it is the greatest Common Divisor between 32768 and 43690. *Rule.*

Example.

M) $\frac{\omega}{Z} = \alpha$ $\frac{43690}{32768}$ (1, $\frac{32768}{10922}$ (3, 2   2) $\frac{32768}{43690}$ ($\frac{16384}{21845}$

3. If the last Term, the *Ratio*, and the Sum be given; that is, ω . R . Z. and in the former Instance 32768 . 4 . & 43690 . to find 2. *3. Data. ω. R. Z.*

Then because the second and fifth Principals are given, take the greatest Common Divisor between, as before in the next precedent Proposition. *Rules*

Or, (2.) multiply the second Principal by the Fourth, divide the Product by the Fourth, lacking an Unit: From the Quotient take the Fifth, and multiply the Remain by the Divisor: So shall 32768 be multiplied by 4, and the

Product 131072 be divided by 3 from the Quotient $43690\frac{2}{3}$; if 43690 be taken, and the Remain $\frac{2}{3}$ multiplied by 3, produceth 2.

Example. $\frac{\omega R}{R-1} - Z.\times R - 1 = \alpha$

$$32768 \times 4 = 131072; \quad 3)\,131072\,(43690\tfrac{2}{3} - 43690 = \tfrac{2}{3}\times 3 = 2$$

4. Data. T.R.Z. 4. If the Number of Terms, the *Ratio*, and the Sum be given; that is, T . R . Z . and in the former Instance 8 . 4 . & 43690 . to find 2.

Rule. Then from the fourth Principal multiplied into it self, according to the Units in the Third, substract 1; multiply the Remain by 1 less than the Fourth, sometime called the *Antecedent of the Ratio*, and multiply 1 less than the Fourth by himself, marked in Species sometime by *D*, and called the *Difference of the Terms of the Ratio:* Then as the Product of the Remain, by the Antecedent of the *Ratio*, is to the Product of that Antecedent into himself; or as the fourth Principal figurate to T—1, is to the Fourth lacking 1; so is the fifth Principal given to the First required.

Example. R figurate to T.—1 in R — 1. D :: Z . α

Or,

R figurate to T. — 1. R — 1 :: Z . α

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8
4.16.64.256.1024.4096.16384.65536
(65535×3) (3×3)
As 196605 . 9 :: 43690 . 2
Or, 65535 . 3 :: 43690 . 2

$$65536 - 1 = 65535$$

To find the last Term.

2. *To find the last Term* (or ω) *of a Geometrical Progression.*

1. Data. α. T.R. 1. If the first Term, the Number of Terms, and the *Ratio* be given; that is, α . T . R . and in the former Instance, 2 . 8 . and 4 . to find 32768.

Rules. Then either, (1.) multiply the first Principal figurately by the Fourth, according to the Units in the Third lacking 1; and so shall 2 be multiplied into 4, till the Index be 7, that is T—1.

Or, (2.) figurate the fourth Principal to the Third lacking 1, and multiply by the First.

Examples. α in R to T — 1 = ω

1 . 2 . 3 . 4 . 5 . 6 . 7
2.8.32.128.512.2048.8192.32768.

R figurate to T—1×α=ω

1 . 2 . 3 . 4 . 5 . 6 . 7
4.16.64.256.1024.4096.16384

$$16384 \times 2 = 32768$$

2. Data. α. T. Z. 2. If the first Term, the Number of Terms, and the Sum be given; that is, α . T . Z . and in the former Instance. 2 . 8 . and 43690, to find 32768.

Rule. Then divide the fifth Principal by the First, and from the Quotient take the greatest Figural Number, whose Index is 1 less than the Third, and multiply this Figural Number by the First: So shall 43690, divided by 2, give in the Quotient 21845; out of which the greatest second Sursolid, whose Index is 7, (that is, 1 less than 8) that can be taken, is 16384; which multiplied by 2, produceth 32768 the second Principal desired.

Example. $\frac{Z}{\alpha}$ — □ $\left.\begin{matrix}\text{Power}\\ \text{Index}\end{matrix}\right\}$ T—1

Ergo, Power × α = ω

$$\frac{43690}{2}\Big(\,21845 \quad 16384 \times 2 = 32768$$

qqc
7 = 8 — 1

3. Data. α. R. Z. 3. If the first Term, the *Ratio*, and the Sum be given; that is, α . R . Z . and in the former Instance, 2 . 4 . and 43690, to find 32768.

Rule. Then multiply the fifth Principal by the Fourth lacking an Unit, to the Product add the First, and divide the Total by the Fourth. So shall 43690, multiplied by 3, and 2 added, make the Total 131072, which divided by 4, gives in the Quotient 32768.

Example. $\frac{ZR - 1 + \alpha}{R} = \omega$

$$43690 \times 3 = 131070 + 2 = \frac{131072}{4}\Big(\,32768$$

4. Data. T.R.Z. 4. If the Number of Terms, the *Ratio*, and the Sum be given; that is, T.R.Z. and in the former Instance 8 . 4 . and 43690, to find 32768. Then

Then by the *Data* find the first Principal, according to the 4*th* Proposition of the 1*st* above-mentioned; and then take the Figurate Number of the Fourth Principal, according to the Units in the Third, lacking 1, and the *Analogy* will be; As 1 to this Figurate *Ratio*; so shall the first Principal found, to the Second required. Rule.

R figurate to T.—1 in R—1. D :: Z.α 1.2.3.4.5.6.7.8 Example.
Or R figurate to T.— 1. R — 1 :: Z.α 4.16.64.256.1024.4096.16384.65536

$$\frac{1}{65535}$$

As 196605 . 9 :: 43690.2.
Or, 65535 . 3 :: 43690.2.

1. R figurate to T — 1 :: α . ω As 1 . 16384 :: 2.32768.

3. *To find the Number of Terms* (or T) *of a Geometrical Progression.* *To find the Number of Terms.*

1. If the first Term, the last Term, and the *Ratio* be given; that is, α . ω . R. and in the former Instance, 2.32768, & 4. to find 8. 1. Data. α.ω.R.

Then multiply the first Principal by the Fourth successively, till the Second be produced, and to the Number of the several Multiplications add 1: So 2 multiplied by 4, shall in 7 Multiplications produce 32768; therefore to 7 add 1, and 8 is obtained. Rule.

$$\left.\begin{matrix}\text{N of Mult. of}\\ \alpha\text{ R to }\omega+1\end{matrix}\right\}=\text{T} \qquad \begin{matrix}2\,.\,8\,.\,32\,.\,128\,.\,512\,.\,2048\,.\,8192\,.\,32768\\ 1\,.\,2\,.\,3\,.\,4\,.\,5\,.\,6\,.\,7+1=8\end{matrix}$$ Example.

Or if the second Principal be divided by the First, and the Fourth be figurate to that Quotient, the Index of this figurate *Ratio* shall be the Number of Terms lacking 1. Rule.

α . 1 :: ω . R figurate to T—1 Example.

2) 32768 (16384
4.16.64.256.1024.4096.16384
1.2.3.4.5.6.7+1=8

2. If the first Term, the last Term, and the Sum be given; that is, α . ω . Z. and in the former Instance, 2.32768. & 43690, to find 8. 2. Data. α.ω.Z.

Then without respect to the fifth Principal; as the first Principal to 1, so shall the second Principal be to the *Ratio* multiplied into it self, according to the Distance of the Term given from the first Term. From hence therefore, if a Root be extracted of the highest Power therein, the Index of that Power and 1 more, shall be the Number sought: So shall 2 to 1, be as 32768 to 16384; therefore 16384 being the second Sursolid of 4, hath the Index 7, to which 1 added is 8. Rule.

α . 1 :: ω . R figurate to T—1. As 2 . 1 :: 32768 . 16384. Example.

Ergo, $\sqrt{}$ qqc . 16384 (4
7 + 1 = 8

Or divide the fifth Principal by the second, and by the Remain of that Division, the Remain of the first taken from the Second; then by the Sum of both the Quotients multiply the First till the Second be produced, and to the Number of the several Multiplications add an Unit. So shall 43690 divided by 32768 give 1 in the Quotient, and 10922 Remain, which shall be Divisor to 32766, that is the Remain after 2 is taken from 32768: this last Division giveth 3 in the Quotient, to which 1 added is 4, this multiplied into 2 the first 7 times, produceth 32768; therefore 7 and 1 make 8 the Number desired. Rule.

$$\frac{Z}{\omega}+\frac{\omega-\alpha}{\text{Remain}}\times\left.\begin{matrix}\text{N. of Mult. of}\\ \alpha\text{ to }\omega+1\end{matrix}\right\}=\text{T} \qquad \begin{matrix}10922\\ \hline \cancel{43690}\\ \cancel{32768}\end{matrix}\Big(1 \qquad \begin{matrix}32768\\ 2\\ \hline 32766\end{matrix} \qquad \begin{matrix}\cancel{32766}\\ \hline \cancel{10922}\end{matrix}\Big(3$$ Example.

1+3=4×2.8.32.128.512.2048.8192.32768
1.2.3.4.5.6.7+1=8

3. If the first Term, the *Ratio*, and the Sum be given; that is, α . R . Z . and in the former Instance 2, 4, and 43690, to find 8. 3. Data. α.R.Z.

Then multiply the first Principal by the Fourth, till a Number be produced next greater than the Fifth, and the Number of Multiplications shall be the Third. So 2 by 4 multiplied till 131072 be produced, the Number of the several Multiplications will be 8, the Number of Terms sought. Rule.

N

Example. N of Mult. of α R to $\square$ — Z $\Big\}$ = T $2\times4 = 8.32.128.512.2048.8192.32768.131072$
1 . 2 . 3 . 4 . 5 . 6 . 7 . 8

Rule. Or by Analogy, the first Principal to the Fifth, shall be as the Difference of the Terms of the *Ratio* to the *Ratio* figurate according to the Units in the Third lacking 1, and the Remain multiplied by the Antecedent of the *Ratio*; or as the *Ratio* lacking 1, to the *Ratio* figurate to T lacking 1; therefore if from the Quotient of this Number and 1 added, divided by the same Antecedent, if the upper Way be taken, or the Quotient and 1 of the lower way a Root be extracted of the highest Power therein, the Index of this Root shall be the Number of Terms.

Example. α . Z :: D . R figurate to T. — 1 in R — 1. $4-1=3\times3=9$
Or, α . Z :: R—1 . R figurate to T.—1.
As 2 . 43690 :: 9 . 196605.
Or 2 . 43690 :: 3 . 65535.
Ergo 3) 196605 (65535 + 1 = 65536.
And $\sqrt{}$qcc 65536 (4 . Index 8.

4. Data. ω.R.Z. 4. If the last Term, the *Ratio*, and the Sum be given; that is, ω . R . Z. and in the former Instance 32768 . 4 . and 43690 . to find 8.

Rule. Then multiply the greatest common Divisor between the second and fifth Principals by the Fourth till the Second be produced; and to the Number of the several Multiplications add an Unit. So 2 the common Divisor between 32768 and 43690, shall be multiplied by 4 the *Ratio* 7 times, which will produce 32768: therefore 1 added to 7 gives 8, the Number of Terms in this Example.

Example. M $\Big)\frac{\omega}{Z} = \alpha$ 2 $\Big)\frac{32768}{43690}\Big(\frac{16384}{21845}$

Ergo, N of Mult. of α R to ω + 1 $\Big\}$ = T $2\times4=8.32.128.512.2048.8192.32768$
1 . 2 . 3 . 4 . 5 . 6 . 7+1=8

Otherwise. Or, because this Common Divisor is the first Term, any of the Ways in the second and third Propositions precedent by α . ω . Z . or α . R . Z . may be used.

To find the Ratio. 4. *To find the* Ratio (or R) *of a Geometrical Progression.*

1. Data. α.ω.T. 1. If the first Term, the last Term, and the Number of Terms be given; that is, α . ω . T . and in the former Instance, 2 . 32768 . and 8. to find 4.

Rule. Then divide the second Principal by the First, and from the Quotient extract a Root, whose Index shall be less than the Third by an Unit: For the *Analogy* is, As the First to 1; so is the Second to the *Ratio* figurate to the Distance of the Term given from the First. Wherefore 16384, the Quotient of 32768 divided by 2, shall be a second Sursolid, which hath 7 for the Index, that is 1 less than 8, and 4 for the Root the *Ratio* desired.

Example. $\sqrt{\frac{\omega}{\alpha}}$ Index T—1=R $\frac{32768}{2}\Big($ 16384 $\Big($ $\sqrt{}$qqc . 4 / Index . 7=8—1

α.1 :: ω . R figurate to T—1   4.16.64.256.1024.4096.16384.
2.1 :: 32768 . 16384   1 . 2 . 3 . 4 . 5 . 6 . 7

2. Data. α.ω.Z. 2. If the first Term, the last Term, and the Sum be given; that is, α . ω . Z. and in the former Instance, 2.32768. and 43690, to find 4.

Rule. Then divide the fifth Principal by the Second, and by the Remain of that Division, divide the Remain of the First taken from the Second, and add both the Quotients together: So shall 1, the Quotient of 43690, divided by 32768, and 3 the Quotient of 32766, (that is, $\omega-\alpha$) divided by 10922, (the Remain of the first Division) make together 4 the *Ratio* sought.

Example. $\frac{Z}{\omega} + \frac{\omega-\alpha}{\text{Remain.}} = R$ $\frac{\overset{10922}{\cancel{43690}}}{\cancel{32768}}\Big(1$ $\frac{32768 - 2}{32766}$ $\frac{\cancel{32766}}{\cancel{10922}}\Big(3$ 1+3=4

3. Data. α. T.Z. 3. If the first Term, the Number of Terms, and the Sum be given; that is, α . T . Z . and in the former Instance, 2.8. and 43690, to find 4.

Rule. Then divide the fifth Principal by the First, and from the Quotient take the greatest figural Number, whose Index is 1 less than the Third, the Root of this figural Number shall be the *Ratio*. So 43690 divided by 2, giveth 21845; out of which the greatest second Sursolid, whose Index is 7, (that is 1 less than 8) that can be taken is 16384, the Root whereof is 4.

Z

$\frac{Z}{\alpha} - \square -$ Power / Index $\Big\}$ T − 1 Example.

Ergo, $\sqrt{}$ Power = R

$\frac{43690}{2}\Big($ 21845 / 16384 $\Big\{$ $\sqrt{}$qqc . 4 / Index . 7 = 8 − 1

4.16.64.256.1024.4096.16384.
1 . 2 . 3 . 4 . 5 . 6 . 7 .

4. If the last Term, the Number of Terms, and the Sum be given; that is, ω . T . Z . and in the former Instance, 32768 . 8 . and 43690, to find 4. 4.Data. ω.T.Z.

Then by the greatest common Divisor, between the second and fifth Principals, divide the Second: And from the Quotient extract a Root, whose Index shall be less than the Third by an Unit. So 2 the common Divisor between 32768 and 43690, shall divide 32768, and the Quotient 16384 shall be a second Sursolid, whose Index is 7, that is 1 less than 8, and 4 the Root is the desired *Ratio*. Rule.

$M\Big)\frac{\omega}{Z} = \alpha$ $\qquad 2\Big)\frac{32768}{43690}\Big(\frac{16384}{21845}$ Example.

Ergo $\sqrt{\frac{\omega}{\alpha}}$ Index T − 1 = R $\qquad$ 16384 $\Big\{$ $\sqrt{}$qqc . 4 / Index . 7 = 8 − 1

5. *To find the Sum* (or Z) *of a Geometrical Progression.* *To find the Sum.*

1. If the first Term, the last Term, and the Number of Terms be given; that is, α . ω . T . and in the former Instance 2.32768, and 8, to find 43690. 1.Data. α.ω. T.

Then divide the second Principal by the First; from the Quotient extract a Root, whose Index is less than the Third by an Unit; and by an Unit less than this Root divide the Remain of the First taken from the Second, and to this Quotient add the Second; so shall 32768, divided by 2, give 16384: the Index less than 8 by 1 is 7; the second sursolid Root therefore of 16384 is 4, from which 1 taken, leaves 3 for Divisor to 32766, the Remain of 32768; when 2 the First is substracted, the Quotient of this Division 10922 and 32768, make up 43690 the desired Sum. Rule.

$$\frac{\omega - \alpha}{\sqrt{\frac{\omega}{\alpha}}\ \text{Index}-1\ \text{T}-1} + \omega = Z \qquad \frac{32768}{2}\Big(16384\ \sqrt{}\text{qqc}\ 4 - 1 = 3\Big)32766\Big(\begin{array}{r}10922\\ \underline{32768}\\ \underline{43690}\end{array}$$
$$32768 - 2 = 32766$$
Example.

2. If the first Term, the last Term, and the *Ratio* be given; that is, α . ω . R . and in the former Instance, 2.32768 and 4, to find 43690. 2. Data. α.ω.R.

Then multiply the second Principal by the Fourth; from the Product take the First, and divide the Remain by 1 less than the Fourth. Rules.

Or, substract the First from the Second, and divide the Remain by 1 less than the Fourth, and to the Quotient add the Second.

So by the first of these shall 32768 be multiplied by 4; and 2 taken from the Product, leaves 131070 to be divided by 3: And by the latter way 32766 shall be divided by 3; and to the Quotient 10922 shall be added 32768, and both ways bring forth 43690.

$$\frac{\omega R - \alpha}{R - 1} = Z \qquad 32768 \times 4 = 131072 - 2 = \frac{131070}{4 - 1 = 3}\Big(43690$$
Examples.

$$\frac{\omega - \alpha}{R - 1} + \omega = Z \qquad 32768 - 2 = \frac{32766}{4 - 1 = 3}\Big(10922 + 32768 = 43690$$

And by way of *Analogy*: As the Difference of Terms of the *Ratio*, to the *Ratio* multiplied into it self according to the Units in the Third lacking 1, and the Remain multiplied by the Antecedent of the *Ratio*; or the *Ratio* lacking 1 to the *Ratio* figurate to T−1: So shall the First be to the Fifth: Or the same *Ratio* so figurate and multiplied as aforesaid, multiplied into the First, and divided by the said Difference, shall be equal to the Fifth. *Otherwise.*

α . 1 :: ω . R figurate to T−1.

For seeing, As 2 . 1 :: 32768 . 16384 $\qquad$ 4−1=3×3=9 Example.

Ergo, D . R figurate to T.−1 in R−1 :: α . Z .
9 . 196605 :: 2 . 43690.

Or, R—1 . R figurate to T.—1 :: α . Z .
3 . 65535 :: 2 . 43690 .

3. Data. α. T. R. 3. If the first Term, the Number of Terms, and the *Ratio* be given; that is, α . T . R . and in the former Instance, 2 . 8 . and 4, to find 43690.

Rule. Then multiply the first Principal by the Fourth continually, so many Times as there be Units in the Third; and from the last Product take the First, and divide the Remain by 1 less than the Fourth. So 2 by 4 shall be multiplied 8 times; and taking 2 from 131072 the last Product, and the Remain 131070 divided by 3, giveth 43690 as desired.

Example.

$$\frac{\alpha \text{ in R to T}-\alpha}{R-1}=Z$$

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8
2x4=8.32.128.512.2048.8192.32768.131072
2
4—1=3)131070(43690

Rule. Or if the Fourth be multiplied into himself, according to the Units in the Third lacking 1, these Products multiplied by the First, shall be equal to the Fifth lacking the First.

Example.

$$\left.\begin{array}{l}\text{Sum of}\\ \text{R figurate to T}-1\times\alpha+\alpha\end{array}\right\}=Z$$

4+16+64+256+1024+4096+16384=21844
2
Ergo, 43688+2=43690 43688

4. Data. ω. T. R. 4. If the last Term, the Number of Terms, and the *Ratio* be given; that is, ω . T . R . and in the former Instance 32768, 8 and 4, to find 43690.

Rule. Then continue the Division of the second Principal by the Fourth, so many times as there be Units in Third lacking 1: and take this last Quotient from the Product of the Fourth multiplied into the Second; this Remain divide by 1 less than the Fourth, so 32768 divided 7 times by 4 bringeth 2 in the Quotient, which taken from 131072, and the Remain divided by 3, giveth the Sum desired 43690.

Example.

$$\frac{\omega R-\frac{\omega}{R}\text{ to T}-1}{R-1}=Z$$

1 . 2 . 3 . 4 . 5 . 6 . 7 .
4) 32768 (8192 (2048 (512 (128 (32 (8 (2.
32768x4=131072—2=131070 (43690
4—1=3

Data *and Quesita in every Question.* Here also, as before in *Arithmetical Progression*, is to be observed, that by every 3 of these Principals both the other 2 may be found; and also that in all Questions duly propounded, 3 of the 5 are given to find sometime the one, sometime both the other: For though the common Sort of Questions, as the selling of an Horse by the Nails in his Shoes, a Coat by the Buttons, or an House by the Doors, &c. be all alike to the former Example first mentioned; yet may one Question be varied in the propounding 20 several Ways, according to these precedent Propositions, and by them accordingly may Resolution be had. But all these Calculations are for whole Numbers; if Fractions or Decimals be used, Alterations must be made according to their Nature and Use.

Common Sort of Questions.

Every Question may be varied 20 ways.

Alterations of Fractions and Decimals.

3ly. To find any middle Term, 3 Cases. §. 3. Next after the finding of the 5 Principals, it is necessary to know how to find any of the middle Terms or Means that lie between the two Extreams, in effecting of which there are 3 Cases considerable.

1. If the least Extream be an Unit. *Case* 1. If the least Extream of the *Progression* be an Unit, then square the Term given, and the Product shall be the Double of the given Term lacking one: as if I square the third Term, I get thereby the Fifth; So if the fourth Term be squared, the seventh is produced.

If two of the middle Terms be given to find another, then add the Number of the Terms given, and the Sum lacking 1 shall be the Number of the Term produced by multiplying the Terms one into another: As to find the 8*th* Term, the fourth and fifth Terms shall be multiplied one into another, for 4 and 5 makes 9, that is one more than 8.

Examples.

Example.

	1	2	3	4	5	6	7	8
Example.	1	4	16	64	256	1024	4096	16384.

$$\begin{array}{r} 16\times16 \\ \hline 96 \\ 16 \\ \hline 256 \end{array} \quad 3+3=6 \quad 5=3+3-1 \qquad \begin{array}{r} 64\times256 \\ \hline 1024 \\ 1536 \\ \hline 16384 \end{array} \quad 4+5=9 \quad 8=4+5-1$$

Case 2. If the least Term be not an Unit, and yet by the *Ratio* of the *Progression* may be divided to the Unit; then any one Term squared produceth just the Double of the given Term; and two of the Terms multiplied one into another, shall produce the Term the Number of their Terms added point at. *2. If the least Term be no Unit, yet may be divided to 1.*

As to find the 8*th* Term, multiply the 4*th* Term into it self: and to get the 7*th* Term, let the Third and Fourth be multiplied, for 3 and 4 make 7. Examples.

	1	2	3	4	5	6	7	8
Example.	3	9	27	81	243	729	2187	6561.

$$\begin{array}{r} 81\times81 \\ \hline 81 \\ 648 \\ \hline 6561 \end{array} \quad 4+4=8 \quad 8=4+4 \qquad \begin{array}{r} 27\times81 \\ \hline 567 \\ 162 \\ \hline 2187 \end{array} \quad 3+4=7 \quad 7=3+4$$

Case 3. If the least Term be not an Unit, nor by the *Ratio* of the *Progression* can be brought to an Unit; then the best way to avoid multiplicity of Rules, is to proceed by the following Analogy, which indeed will serve in all Cases, that is, As an Unit to the *Ratio* multiplied into it self, according to the Distance of the Term sought from the first Term; so is the first Term, to the Term sought. *3. If the least Term be no Unit, nor can be divided to 1.*

As to know the 8*th* Term of a *Geometrical Progression*, whose least Extream is 2, and *Ratio* 3, the Distance between the first Term and the 8*th* Term is 7, the *Ratio* figurate to 7 is 2187, which multiplied by 2 is 4374 the 8*th* Term desired. Example.

1 . R figurate to T—1 :: α . ω
1 . 2187 :: 2 . 4374.

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 .
2 . 6 . 18 . 54 . 162 . 486 . 1458 . 4374.
3 . 9 . 27 . 81 . 243 . 729 . 2187 .

Here among these *Geometrical Proportions* may have Place the *Lemma*'s set down by Mr. *Briggs* in his Latin Piece, that demonstrate the Original of *Logarithms*, making the Index of Unity 0, though they might have been placed before among Figural Numbers or *Logarithms*. Lemma's of *Mr.* Briggs.

Lemma 1. If in a Rank or Series of Numbers continually Proportional from an Unit, (as those following in *A*) any two of them be taken, as *C* 8 and *D* 32, with their Indices 3 and 5; two other Series of Numbers continually Proportional from an Unit, being made from the 2 Numbers taken out of the first Series, *viz.* from 8 and 32, as at *E* and *F*, the first Numbers in each Series next to the Unit being the former Numbers marked with *C* and *D*; and both Series being continued to 3 (the Index of the Number *C* in the Column *A*) shall be the Index of a Proportional Number in the Column *F*; and 5 in the like manner (the Index of the Number *D*) and the Index of a Number in the Column *E*; the last Proportional Number in the Columns *E* and *F* shall be equal to the Number found in the first Series marked with *A*, whose Index is 15, the Product of the 2 Indices to the last Numbers in the Columns *E* and *F*. 1. Lemma.

Q

Q		*A*	*B*	*P*	*E*	*M*
0		1	0	0	1	0
		2	1		8	1
		4	2		64	2
	C	8	3	1	512	3
		16	4		4096	4
1	*D*	32	5		32768	5
		64	6	2		
		128	7		*F*	*N*
		256	8		1	0
		512	9	3	32	1
2		1024	10		1024	2
		2048	11		32768	3
		4096	12	4		
		8192	13			
		16384	14			
3		32768	15	5		

A The first Series of Proportional Numbers.
B The Indices of that Series of Numbers.
E The second Series of Numbers given.
M The Indices of that Series, to which those in *P* are equal.
F The third Series, whose Indices are equal to those in *Q*.

2. Lemma. *Lemma* 2. If in a Series of Numbers continually proportional from an Unit, any one of them be divided continually by his Side or Root as often as it can, the Number of Divisions shall be the Index of the Number divided, shewing the Distance of that Number from the Unit, or the Number of Intervals between Unity and the Dividend.

As of 729 by the Root 3, the Index is 6.

Indices 0 . 1 . 2 . 3 . 4 . 5 . 6 .
Proportionals 1 . 3 . 9 . 27 . 81 . 243 . 729. 3)729(243(81(27(9(3(1
1 . 2 . 3 . 4 . 5 . 6

3. Lemma. *Lemma* 3. In a Series of Numbers continually proportional, any two of them being given, with the Index of one of them, to find the Index of the other, or its distance from Unity; as the given Numbers 8 and 32 in the Series above marked with *A* in the first *Lemma*, and the Index of the greater 5; Let then another Series of Numbers continually proportional be made by the Multiplication of 8, whose Index is sought by it self, and of the Product by it self continually, till you come to a Number whose Index is equal to 5, the Index given at *E*; The last Product by the first *Lemma*, shall be the same with the Product made by the continual Multiplication of 32, till the Index of the Product in this last Series be the Index sought: And therefore if the Number 32768 be divided by 32, the Side in the third Series, according to the Directions of the second *Lemma*, the Quotient will be 3, *viz*. 1024 . 32 . 1 . And so 3 shall be the Index, as well of this Product 32768 in this third Series, as of 8 in the first Series, from whence the 2 given Numbers were taken.

4. Lemma. *Lemma* 4. In a Series of Numbers continually proportional from Unity; If one Number multiply another, the Product will be one of the Numbers in that Series continued, and the Index thereof will be the Sum of the Indices given: As in the Series at *A*, if 4 multiply 256, the Product will be 1024, and the Index thereof 10, *viz*. the Sum of 2 & 8, the Indices annexed to 4 and 256.

5. Lemma. *Lemma* 5. If one Number be multiplied by another, the Number of Places in the Product shall be equal to the Number of Figures in both the Factors, unless the Product made of the first Figures toward the left Hand, in both the Numbers given, may be expressed by one Digit; as often as this shall happen, the Number of Places in the Product will be less by 1 than the Number of Figures in both the Factors: As if 68 be multiplied by 26, the Product 1768 is expressed with 4 Figures; but if 68 be multiplied by 14, the Product 952 is expressed by 3 Figures.

Thus

Thus came to be discovered the Number of Places, or Decimal Indices in every Figural Number which are the Logarithms, as was said before in the first Chapter of *Logarithms*. *Logarithms discovered.*

§. 4. After finding out the middle Terms in any *Geometrical Progression*, is convenient to know how to find one or more mean Proportionals between two Numbers given: In finding which are constituted 2 *Cases*. *4ly. To find out Means. Two Cases.*

Case 1. If one Mean Proportional be desired between two given Numbers. *1. If one Mean be sought.*

Then multiply them together, and take the Square Root of the Product. *Common Way.*

As to find a Mean Proportional between 4 and 9, they produce, being multiplied, 36; therefore 6 the Square Root thereof shall be the Proportional Mean between 4 and 9.

So between 6 and 24, is 12 found to be the Mean Proportional.

$$4\times 9 = 36(6\surd \qquad 6\times 24 = 144(12\surd$$
$$4 . 6 . 9 \qquad 6 . 12 . 24$$

Example.

Because Extraction of Roots is very easy by Logarithms, as for the Square Root to divide by 2, for the Cube by 3, &c. and oftentimes Fractions are given, or happen in the Work of Mean Proportionals: It is best for the Artist to work by Logarithms. Wherefore, *How by Logarithms.*

1. When the Logarithms of the Numbers propounded are of the same Kind, add them together, and half the Sum shall be the Logarithm of the Mean Proportional required, and of the same Kind with the Logarithms of the Numbers given. *If the Data are alike.*

Examples.

Integers, The Mean Proportional between 4 and 25, is 10: because 10 is the Square Root of 100, the Product of 4 by 25. *Examples.*

Fractions, The Mean Proportional between $\frac{1}{4}$ and $\frac{4}{25}$ is $\frac{1}{5}$: for the Fractions multiplied are $\frac{4}{100}$, the $\surd 3$ whereof is $\frac{2}{10}$ or $\frac{1}{5}$,

Decimals, The Mean Proportional between 0,25 (that is $\frac{1}{4}$) and 0,16 (that is $\frac{4}{25}$) is 0,20 (that is $\frac{2}{10}$ or $\frac{1}{5}$) as before. Their Logarithms follow.

Integers.

0,60205,99913	. Log. of 4
1,39794,00087	. Log. of 25
2,00000,00000	. Log. of 100
2) 1,00000,00000	. Log. of 10

Fractions.

—0,60205,99913	. Log. of $\frac{1}{4}$
—0,79588,00174	. Log. of $\frac{4}{25}$
—1,39794,00087	. Log. of $\frac{4}{100}$
2) —0,69897,00043	. Log. of $\frac{1}{5}$

Decimals.

—1,39794,00087	. Log. of 0,25″
—1,20411,99827	. Log. of 0,16″
—2,60205,99914	. Log. of 0,04
2) —1,30102,99957	. Log. of 0,2

2. When the Logarithms of the Numbers propounded are of divers Kinds, and the defective Log. the Log. of a common Fraction; then take the lesser Log. out of the Greater, and one half of the Remain shall be the Logarithm of the Mean Proportional, and of the same kind with the greater Logarithm. *If the Data are unlike.*

As the Mean Proportional between $\frac{3}{4}$ and 12 shall be 3, because the Product $\frac{36}{4}$ hath the Square Root $\frac{6}{2}$ or 3. *Examples.*

And the Mean Proportional between $\frac{1}{25}$ and 4, shall be $\frac{2}{5}$; for so is the Square Root of their Product $\frac{4}{25}$.

1,07918,12460	. Log. of 12
—0,12493,87366	. Log. of $\frac{3}{4}$
0,95424,25094	. Log. of $\frac{36}{4}$
2) 0,47712,12547	. Log. of 3

—1,39794,00084	. Log. of $\frac{1}{25}$
0,60205,99913	. Log. of 4
—0,79588,00171	. Log. of $\frac{4}{25}$
2) —0,39794,00085	. Log. of $\frac{2}{5}$

But if the Defective Log. be the Log. of a Decimal Fraction, then add the Logarithms together, and divide the Sum by 2, with the Addition and Division proper thereto, set forth at large in the Part treating of Logarithms.

As the Mean Proportional between 4 and $\frac{1}{25}$ turned into a Decimal, (that is, 0,04″) is 4′, equal to the former $\frac{2}{5}$.

0,60205,99913	Log. of 4.
—2,60205,99913	Log. of 0,04''
2) —1,20411,99826	Log. of 0,16''
—1,60205,99913	Log. of 0,4

Obſervations. Mean Proportionals under this firſt Caſe thus found, make evident,

1. What the third Proportional is equal to. How noted in Species. 1. That among Numbers geometrically continually Proportional, every third Proportional ſhall be equal to the Square of the Second divided by the Firſt; which is the reaſon of their being marked in Species as before, by $\alpha . \beta . \frac{\beta q}{\alpha}$. As 4 . 12 . 36 . for the Square of 12 is 144; which divided by 4, giveth 36.

2. If the Firſt be a Square, ſo the Third. 2. That if the Firſt be a ſquare Number, ſo ſhall the Third, becauſe the Product of the Extreams ſhall be equal to the Product of the Mean multiplied into himſelf; as 4 and 36 be both Squares, and 4 multiplied into 36, ſhall be equal to 12 x 12, that is 144.

3. Complement between ſingle Squares is the Mean. 3. That the Complement between ſingle Squares, is the Mean Proportional: As if the Segments of the Side 12 be 10 and 2, then 20 the Complement is the Mean Proportional between 100 and 4: For $100 + {}^{20}_{20} + 4$, the Squares of the Segments and the two Complements are equal to the Square of the whole Line 144.

2. If more Means than 1 be ſought. To get 2. *Caſe* 2. If two or more mean Proportionals be deſired between 2 Numbers given.

Then for 2 Means, multiply the greater Extream by the Square of the Leſſer, and the Cube Root of the Product is the leſſer of the two Means ſought: contrariwiſe, multiply the Square of the greater Extream by the Leſſer, and extract the Cube Root of the Product for the greater Proportional Mean. Or if the greater Extream be divided by the Leſſer, the Cube Root of the Quotient ſhall be the *Ratio* to multiply the leaſt Extream, *&c.*

Example. As if between 2 and 54, two proportional Means be ſought, the Leſſer will be 6 and the Greater 18: For 54 multiplied by 4, the Square of 2, produceth 216, whoſe Cube Root is 6 the leſſer Mean: and 2916 the Square of 54 multiplied by 2 produceth 5832, whoſe Cube Root is 18 the greater Mean; ſo if 2 divide 54 the Cube Root of 27, the Quotient is 3, which multiplying 2 is 6, and 6 is 18.

Leaſt Extream . 2 . Square . 4.	54 x 4 = 216 (6 . Leaſt Mean.
Greateſt Extream. 54 . Square . 2916.	2916 x 2 = 5832 (18. Greateſt Mean.
And 2) 54 (27 √ ϕ 3 x 2 = 6	3 x 6 = 18

Proportionals 2 . 6 . 18 : 54 . Triple Ratio.

To get 3. To get 3 Means between 2 Numbers given, proceed by the firſt Caſe to get the middle Proportional between the 2 given Extreams; then between that middle Proportional and the leaſt Extream, a mean Proportional is to be found in like manner, and alſo between the greateſt Extream and that middle Proportional.

Example. As between 2 and 512 to get 3 Means Pproportional, the middle Proportional is found to be 32, by multiplying 2 in 512 and taking the ſquare Root of the Product: Likewiſe between 2 and 32, the Mean is found to be 8, and between 32 and 512 the Mean is 128.

Extreams { 2	2 x 512 = 1024	(32 √ ʒ middle Proportional.
{ 512	2 x 32 = 64	(8 √ ʒ Proportional between 2 and 32.
	32 x 512 = 16384	(128 √ ʒ Proportional between 32 & 512.

Proportionals 2 . 8 . 32 . 128 . 512 . Quadruple Ratio.

To get many. When many Means are deſired to avoid multiplicity of Rules, the beſt way is to get the *Ratio:* for ſeeing the two Extreams and Number of Terms are given, the *Ratio* is obtained by the firſt Propoſition for finding the fourth Principal before ſpoken to; and when the *Ratio* is gotten, the leaſt Extream multiplied continually thereby, produceth the Means deſired.

Example in 4, the common Way. As to get 4 proportional Means between 2 and 2048, the Extreams given are the firſt and laſt Terms of a *Geometrical Progreſſion*, between which if 4 Means be gotten, the whole Number of Terms will be 6; therefore the Quotient of 2048 divided by 2, which is 1024, ſhall be a Surſolid Number whoſe Index is 5, that is 1 leſs than 6: the Number of Terms and the Surſolid Root of 1024 which is 4,

ſhall

ſhall be the *Ratio* ſought, by which 2 ſucceſſively multiplied ſhall produce the Means deſired.

$\sqrt{\frac{\omega}{\alpha}}$ Index T—1=R   2)2048(1024 √qc. 4. Index 5=6—1

Proportionals. 2. 8. 32. 128. 512. 2048. Ratio 4.

To avoid the Fractions that may ariſe in this Caſe as in the Firſt, the Practice may be by *Logarithms*, which thoſe expert therein will find eaſy and commodious. Wherefore, *How by Logarithms.*

1. When the Logarithms of the Numbers given are both of the ſame kind, and the Numbers Integers, take the Log. of the leſſer Number out of the Log. of the Greater, and the third Part of the Remainder added to the Log. of the Leſſer, ſhall give the Log. of the leſſer mean Proportional required; and if to this Log. the third Part be added, the Sum ſhall be the Log. of the other Mean Proportional required. *If the Data are alike, both Integers.*

Example. 2 Mean Proportionals between 2 and 54, are 6 and 18, as before was found. *Example.*

	1,73239,37598	. Log. of 54
	0,30102,99957	. Log. of 2 (27 √ɸ. 3.
3)	1,43136,37641	. Log. of 27
	0,47712,12547	. Third part of the Difference, or √ɸ.
	0,77815,12504	. Log. of 6, The leſſer Mean.
	1,25527,25051	. Log. of 18, The greater Mean.

But when the Logarithms are both of common Fractions, take the Log. of the greater Fraction from the Log. of the Leſſer: and the third Part of the Remainer added to the Log. of the greater Fraction, ſhall be the Log. of the greater Mean; and being added to this, ſhall make the Log. of the leſſer Mean. *Both Common Fractions.*

As to find 2 Means between $\frac{4}{45}$ and $\frac{3}{10}$, the Log. of $\frac{3}{10}$ ſhall be taken from the Log. of $\frac{4}{45}$, and the third Part of the Difference added to the Log. of $\frac{3}{10}$, ſhall be the Log. of $\frac{1}{5}$ the greater Mean; and the ſame third Part added to the Log. of $\frac{1}{5}$, ſhall be the Log. of $\frac{2}{15}$ the leſſer Mean. *Example.*

Fractions	—1,05115,25225	. Log. of $\frac{4}{45}$ the leſſer Fraction.
	—0,52287,87453	. Log. of $\frac{3}{10}$ the greater Fraction.
3)	—0,52827,37772	. Log. of $\frac{8}{27}$
	—0,17609,12591	. Third part of the Difference.
	—0,69897,00044	. Log. of $\frac{1}{5}$ the greater Mean.
	—0,87506,12635	. Log. of $\frac{2}{15}$ the leſſer Mean.

Proportionals $\frac{3}{10}$. $\frac{1}{5}$. $\frac{2}{15}$. $\frac{4}{45}$. *Ratio* $\frac{2}{3}$.

Thus alſo working with the *Logarithms* of Decimals will the Effects be anſwerable, always uſing the Subſtraction and Diviſion proper to Decimal Logarithms. *Both Decimals.*

As to find 2 Means between 0,048 and 0,75, the Log. of 0,75 taken from the other, leaves the Log. of 0,064, whoſe third Part or Cube Root is 4′, the Log. of which added to the Log. of 0,75, gives the Log. of 0,3 for the greater Mean, and again, added to this Log. gives the Log. of 0,12 for the leſſer Mean. *Example.*

Decimals	—2,68124,12374	. Log. of 0,048‴ . The leſſer Decimal.
	—1,87506,12634	. Log. of 0,75″ . The greater Decimal.
3)	—2,80617,99740	. Log. of 0,064‴ .
	—1,60205,99913	. Third part of the Difference.
	—1,47712,12547	. Log. of 0,3′ . The greater Mean.
	—1,07918,12460	. Log. of 0,12″ . The leſſer Mean.

Proportionals 0,75″. 0,3′. 0,12″. 0,048‴. *Ratio* 0, 4′.

2. When the Logarithms of the Numbers given are of divers Kinds, and the defective Log. is the Log. of a common Fraction, then add them together; and if out of the third Part of the Sum you may ſubſtract the Log. of the Fraction given, the Remain of this Subſtraction ſhall be the abundant Log. of the leſſer Mean ſought, to which add the third Part aforeſaid, and the Sum ſhall be the abundant Log. of the other Mean required. *If the Data are unlike, and one a common Fraction.*

But

But if when the given Logarithms are added, you may ſubſtract the third Part of the Sum out of the Log. of the given Fraction; then the Remainder of this Subſtraction ſhall be the Defective Log. of the leſſer Mean ſought: Out of which Log. if again you may ſubſtract the ſaid third Part, the Remain thereof ſhall be the Defective Log. of the other Mean required.

And if you may ſubſtract the Log. of the firſt Proportional found as above, out of the ſaid third Part, the Remainder thereof ſhall be the abundant Log. of the latter Proportional required.

Examples.

As between $\frac{3}{4}$ and 48 will be found 3 the leſſer Proportional, and 12 the Greater. And betwen $\frac{1}{4}$ and 2 are found 2 Means, $\frac{1}{2}$ the Leſſer, and 1 the Greater.

—0,12493,87366	Log. of $\frac{3}{4}$	—0,60205,99913	Log. of $\frac{1}{4}$
1,68124,12374	Log. of 48	0,30102,99957	Log. of 2
1,80617,99740	Log. of 64	0,90308,99870	Log. of 8
3) 0,60205,99913	3*d* Part of the Sum.	3) 0,30102,99956	3*d* Part of the Sum.
0,47712,12547	Log. of 3.	—0,30102,99956	Log. of $\frac{1}{2}$.
1,07918,12460	Log. of 12.	0,00000,00000	Log. of 1.
Prop. $\frac{3}{4}$. 3 . 12 . 48 .	Ratio 4.	Prop. $\frac{1}{4}$. $\frac{1}{2}$. 1 . 2 .	*Ratio* 2.

One a Decimal.

When the Defective Log. given is the Log. of a Decimal, then ſubſtract the Log. of the Decimal from the Log. of the Integer, and take the third Part of the Difference from the Log. of the Integer, the Remain ſhall be the greater Mean ſought. Take again the third Part out of the Log. of the Mean thus found, and this Remain ſhall be the Log. of the leſſer Mean required, always uſing the Subſtraction proper to Decimal Logarithms.

As in the laſt Examples, and two others that follow.

Examples.

1,68124,12374	Log. of 48	0,30102,99957	Log. of 2.
—1,87506,12634	Log. of 0,75″	—1,39794,00087	Log. of 0,25″
1,80617,99740	Log. of 64	0,90308,99870	Log. of 8.
3) 0,60205,99913	Third Part	3) 0,30102,99957	Third Part.
1,07918,12461	Log. of 12	0,00000,00000	Log. of 1.
0,47712,12548	Log. of 3	—1,69897,00043	Log. of 0,5′
Prop. 0,75. 3.12.48.	*Ratio* 4.	Prop. 0,25. 0,5. 1.2.	*Ratio* 2.

0,82930,37728	Log. of 6,75″	0,00000,00000	Log. of 1.
—1,39794,00087	Log. of 0,25″	—1,09691,00130	Log. of 0,125‴
1,43136,37641	Log. of 27.	0,90308,99870	Log. of 8.
3) 0,47712,12547	Third Part.	3) 0,30102,99956	Third Part.
0,35218,25181	Log. of 2,25″	—1,69897,00044	Log. of 0,5′
—1,87506,12634	Log. of 0,75″	—1,39794,00088	Log. of 0,25″
Prop. 0,25. 0,75. 2,25. 6,75.	*Ratio* 3.	Prop. 0,125. 0,25. 0,5.1.	*Ratio* 2.

Many Means gotten in like manner.

In like manner obſerving the Nature of the given Logarithms, whether of *Integers*, *Fractions*, or *Decimals*, and accordingly taking their Sum or Difference, may as many Means be gotten as ſhall be deſired: Thus,

Divide this Sum or Difference by 1 more than the intermediate Numbers ſought; this Quotient, which will always be the Log. of the *Ratio*, multiply ſeverally by 2, 3, 4, *&c.* till you come to the Diviſor, and theſe Products ſeverally muſt be added to, or ſubſtracted from one of the Logarithms of the given Numbers, or one of them therefrom as the Caſe requires, and the Remains ſhall be the Logarithms of the Means required.

Example in 5 Means.

As to get 5 intermediate Numbers between $\frac{3}{4}$ and 48 the Sum; but if $\frac{3}{4}$ be turned into a Decimal, then the Difference of their Logarithms is to be divided by 6, that is 1 more than the Means ſought: the Quotient of this Diviſion taken from the Log. of 48, ſhall leave the Log. of 24; double the Quotient taken away, ſhall leave the Log. of 12; and ſo for the reſt as below appeareth: And the like will be effected, if the Log. of $\frac{3}{4}$ be taken ſeverally from theſe Products.

And other Proportionals.

And if A, the ſixth Part of the Logarithms of the Extreams, be multiplied by 4, and the Product 4 A, or P, be added to the Log. of 48, the Sum ſhall be the 4*th* Propor-

Proportionl above 48. But if the Log. of $\frac{3}{4}$ be added to P, the Sum shall be the 4th Proportional below $\frac{3}{4}$, and so Numbers above and under each Extream are found.

Extreams	B	—0,12493,87366	. Log. of $\frac{3}{4}$.	I
	H	1,68124,12374	. Log. of 48.	K
	6 A 6)	1,80617,99740	. Log. of 64 the Sum	L
	1 A	0,30102,99957	. Log. of 2.	M
	2 A	0,60205,99914	. Log. of 4.	N
	3 A	0,90308,99871	. Log. of 8.	O
	4 A	1,20411,99828	. Log. of 16.	P
	5 A	1,50514,99785	. Log. of 32.	Q
Means	C	0,17609,12589	. Log. of $1\frac{1}{2}$.	R
	D	0,47712,12546	. Log. of 3.	S
	E	0,77815,12503	. Log. of 6.	T
	F	1,07918,12460	. Log. of 12.	V
	G	1,38021,12417	. Log. of 24.	W

α I . R . S . T . V . W . K ω

$\frac{3}{64} . \frac{3}{32} . \frac{3}{16} . \frac{3}{8} . \frac{3}{4} : 1\frac{1}{2} . 3 . 6 . 12 . 24 . 48 . 96 . 192 . 384 . 768.$

For $4A+B=\frac{I}{P}=\alpha$ Log.—1,32905,87194 $\frac{1}{16}) \frac{3}{4} (\frac{3}{64}$

And $4A+H=KP=\omega$ Log. 2,88536,12202 $48 \times 16 = 768$

The finding of Mean Proportionals under this second Case in the manner aforesaid, makes evident, *Observations.*

1. That the greater Complement between the Cubes of the Segments of any Line is the greater Mean, and the lesser Complement the lesser Mean. As if the Segments of 12 be 10 and 2, their Cubes are 1000 and 8, the greater Base 100, the Lesser 4, the Altitude of the Greater 10, of the Lesser 2, the greater Complement is made of 100 by 2, the Lesser of 4 by 10: So are the Cubes of the Segments with the 3 greater and 3 lesser Complements, equal to the Cube of the whole Line 1728. *1. Complements between Cubes, the 2 Means.*

Greater Cube.		*Greater Compl.*		*Lesser Compl.*		*Lesser Cube.*	
1000	.	200	.	40	.	8	
		200		40			
		200		40			
1000	+	600	+	120	+	8	=1728

2. That the Product of 3 *Geometrical Proportionals* continued, is the Cube of the Mean: So 8 multiplied into 40, and the Product 320 into 200, shall be 64000 the Cube of 40. *2. Product of 3 Proportionals, the Cube of the Mean.*

3. That if 4 Numbers be Geometrically continued in Proportion the Product of the Extreams by their Alternate Squares, are the Cubes of the Means, the Greater of the Greater, and the Lesser of the Lesser: As 1000 multiplied into 64 (the Square of 8) is 64000 the Cube of 40, the lesser Mean: And 8 multiplied into 1000000 (the Square of 1000) is 8000000, the Cube of 200 the greater Mean. *3. Product of the Extreams by the Squares alternate, are the Cubes of the Means.*

4. That in 4 such Proportionals, as the First to the Fourth; so is the Cube of the First to the Cube of the Second. *4. Analogy in 4 Proportionals of this sort.*

As 8 . 1000 :: 512 . 64000.

5. That if the Extreams of such 4 Proportionals are like Solids, being reduced to their least Terms they will be Cubes. As 16 . 24 . 36 . 54 . here 16 and 54 are like Solids, because made of 3 sides Proportional, (as $2 \times 4 \times 2 = 16$, $3 \times 6 \times 3 = 54$) reduced to their least Terms, are 8, 12, 18, 27, where 8 and 27 are Cubes. *5. Such Proportionals like Solids, their least Terms are Cubes.*

§. 5. In *Geometrical Progression*, as before in other Places hath been warned, the Artist must carefully consider the State of the Question, and upon what point the Resolution depends, and when Decimals or Fractions are given or happen in the Work, to work accordingly; and if one Question be included in another, to resolve them orderly: Also when some one of the 3 Principals are not very obvious in the Question propounded, but given under some Occult Phrases; the skill of the Artist is concerned to discover it, and sometime to dispose the first Person in *5ly. The State of the Question duly considered. Operation as the Data requires. Principal occult to be discovered.*

7 G

Persons to be orderly placed. in the Question to the last Place in the Progression, or such other Term as the State of the Question requireth.

Q. Of a Sum divided to 6 Men. *Example* 1. A certain Sum of Money was to be divided among 6 Men, so as *A* was to have $\frac{1}{3}$ of the whole, *B* $\frac{1}{3}$ of the remainder, and so to every one of the rest, and at last there was left 4 *l.* The Question is, what each Man received, and what was the Sum divided amongst them?

Answer. To which for Answer, upon perusal of the Question there appeareth to be clearly given the third Principal 6, and occultly the least Extream and *Ratio*, (*viz.* the first and fourth Principals) to find the fifth and each Particular Term: Wherefore to make the least Extream plain, and accordingly the *Ratio*, I consider that to the 4 *l.* remaining must be put 2, that it being divided into 3 Parts, and one third Part taken away, no more than 4 may be left, and this third Part of 6 being 2, must be the sixth Man's Portion, and accordingly the least Extream of the *Progression*; which 6 is also soon perceived to be the Remainder before, and consequently but $\frac{2}{3}$ Parts of a former Remain, that of necessity must be 9, and so the fifth Man's Portion found to be 3 the $\frac{1}{3}$ of 9; and 3 in comparison to 2 the sixth Man's Portion, is in the *Ratio Sesquialter*: Wherefore multiplying by the *Ratio* $1\frac{1}{2}$ or otherwise proceeding by the former Propositions, the several Terms and Sum with the Remain added, may be obtained for a full Resolution.

Remains. l. : s. : d.	*Terms.*	*Persons.*	*Portions.* l. : s. : d.
4 : 00 : 00	1	6 . *F*	2 : 00 : 00
6 : 00 : 00	2	5 . *E*	3 : 00 : 00
9 : 00 : 00	3	4 . *D*	4 : 10 : 00
13 : 10 : 00	4	3 . *C*	6 : 15 : 00
20 : 05 : 00	5	2 . *B*	10 : 02 : 06
30 : 07 : 06	6	1 . *A*	15 : 03 : 09

Sum of the Progression	41 : 11 : 03	Money Received.
Remain added	4 : 00 : 00	
Sum to be divided	45 : 11 : 03	

Q. Of Alms to 4 Beggars. *Example* 2. A Beggar meeting a Gentleman on the Road, requesteth an Alms: to whom the Gentleman replies, he hath but little Money about him, but if the Beggar will give him as much as he hath, he will give him 6 Pence: whereunto the Beggar at first unwilling, being afterward assured he should thereby be a Gainer did consent, and received his Alms. The Gentleman afterward meeting with the second, third and fourth Beggars, did likewise, and then had left himself no Money at all, having given the last Beggar but 4 Pence: The Question is, what Money he had at first, and what were the Alms he gave?

Answer. *Answ.* He had at first $5\frac{1}{2}$ *d.* so the first Beggar received one half Penny for an Alms, the second a Penny, the third two Pence, and the last as much.

In the stating this Question, as in the former, the third Principal 4 is given plain, the least Extream and *Ratio* more occult; yet the last Beggar's Alms being ascertained to be 4 *d*, it is sufficient to declare them, seeing by the Form of the Question it appeareth to have been doubled, therefore the half of Necessity must be the Remain after the third Beggar had his 6 *d* deducted, which 6 *d.* and 2 *d.* make 8 *d.* that had been doubled of the former Remain. The like is to be understood of the Rest, whereby the *Ratio* of the Alms is found to be double except the last, which is distracted on purpose to make the Question intricate and uncouth: and sometimes such Questions as these receive an easier Resolution by the Rule of false *Position* than by these Rules of *Progression*, though they seem hence to take their Foundation.

Terms of the *Progression* ————	1 . 2 . 3 . 4
Beggars in their Order ————	4 . 3 . 2 . 1
Money received by the Beggars ————	4 . 6 . 6 . 6
Money in the Almoner's Purse, and doubled by the Beggars—	2 . 4 . 5 . $5\frac{1}{2}$
Proper Alms received by the Beggars ————	2 . 2 . 1 . $0\frac{1}{2}$

For $5\frac{1}{2} \times 2 = 11 - 6 = 5 \times 2 = 10 - 6 = 4 \times 2 = 8 - 6 = 2 \times 2 = 4$

6ly. Mean Proportionals helpful to discover the distinct Quantities in mixt Bodies.

§. 6. The finding of Mean Proportionals besides many other Uses, is necessary to the discovery of the true and distinct Quantities of each sort in mixed Bodies, and

and resolveth Questions of that Nature, placed by some under *Alligation*, but cannot thereby be well resolved without borrowing help from hence.

Example 1. Let an Ingot of the finest Silver, from which a Mark being cut off, be melted with a Mark of Copper instead thereof; and afterward a Mark of that mixt Metal cut off, and the Remainder melted with another Mark of Copper: Again, the third time a Mark of this Mixture being cut off, and the Remainder melted with another Mark of Copper; and finding by the Saie, that this last Mixture is $8\frac{5}{128}$ Ounces fine: The Question is, how many Marks the Ingot weighed? *Q. Of the Weight of Silver mixt.*

Answ. 8 Marks; for the finest Silver (as the Ingot was before Allay) being 12 Ounces fine, two Mean Proportionals are to be sought between the same and $8\frac{5}{128}$ (the fineness of the Ingot after Allay), which to do, first $8\frac{5}{128}$ and 12 must be reduced to one Denominator, and then leaving the Denominators, the Proportions will respect their Numerators only. And as the Differences between the Proportionals respect the several Allays, so shall the Difference between the 2 greatest Proportionals, respect the greatest Fineness and least Allay; wherefore the Analogy is, as that Difference to 1, so is the greatest Proportional to the Weight desired. *Answer.*

$8\frac{5}{128}$ and $\frac{12}{1}$ reduced, are $\frac{1029}{128}$ and $\frac{1536}{128}$.

Proportionals . 1029 . 1176 . 1344 . 1536.

$1536 - 1344 = 192$. Then as 192 . 1 :: 1536 . 8 Marks.

Example 2. A Merchant hath a Piece of Wine of 128 Gallons, out of which he draweth certain Gallons, and filleth up the Vessel again with Water; the second Time he draweth out as much as he did at the First, and filleth it up again with Water; and the like he doth the third and fourth Time, and in the end findeth that there was left in the Vessel $75\frac{1}{32}$ Gallons of Wine, besides the Water that was put in: how many Gallons was drawn out at a Time? *Q. Of Wine mixt with Water, how much drawn out at a time.*

Answ. 16 Gallons: Here the Draught being repeated 4 times, 3 Mean Proportionals are to be gotten, (for they are always to be 1 less than the Mixtures) 128 and $75\frac{1}{32}$ then reduced as before, shall have their Numerators 2401 and 4096, between which the 3 Proportionals are 2744 . 3136 . 3584. Then because the Inquiry here, is not as in the last Question for the Quantity at first, but for the Draught out of that Quantity, the Analogy is, As the Denominator to 1, so is the Difference between the highest Proportionals to the Number sought. *Answer.*

$75\frac{1}{32}$ and $\frac{128}{1}$ reduced, are $\frac{2401}{32}$ and $\frac{4096}{32}$

Proportionals 2401 . 2744 . 3136 . 3584 . 4096.

$4096 - 3584 = 512$. Then as 32 . 1 :: 512 . 16 Gallons.

And in like manner, if the Quantity of Wine drawn out every Time in the 16 Gallons were desired: by an orderly deduction of the lesser Proportional out of the next Greater, and dividing the Difference by the common Divisor 32, the Quotients will determine the Desire. *Of the Wine drawn out every time.*

		Gall.	
So	$3584 - 3136 = 448$	Therefore 32) 448 (14 .	2 Draught.
And	$3136 - 2744 = 392$	Therefore 32) 392 ($12\frac{1}{4}$.	3 Draught.
And	$2744 - 2401 = 343$	Therefore 32) 343 ($10\frac{23}{32}$.	4 Draught.

And if the Prices of the Wine before and after Mixture had been given, to find the Quantity: (as suppose a Gallon before mixture were worth 5 *s.* and after Mixture 2 *s.* $\frac{3813}{4096}$ to know how many Gallons in the whole Piece): then after the Mean Proportionals are gotten between the 2 Prices, the Analogy is, As the Difference between the 2 highest Prices to the first Draught, so is the highest Price to the whole Quantity. *If the Price of the Wine be given, to find the Quantity.*

$2\frac{3813}{4096}$ and $\frac{5}{1}$ reduced, are $\frac{12005}{4096}$ and $\frac{20480}{4096}$

Proportionals . 12005 . 13720 . 15680 . 17920 . 20480.

Prices . $2\frac{3813}{4096}$. $3\frac{179}{512}$. $3\frac{53}{64}$. $4\frac{3}{8}$. 5. and $5 - 4\frac{3}{8} = \frac{5}{8}$

Then as $\frac{5}{8}$. 16 :: 5 . 128 . Gallons in the whole.

The ordinary Proof of *Geometrical Progression*, is according to the first Example of this Chapter, to place every Term in order, and collect the Sum by common *Addition*; what other Operations the Sections include, carry Evidence of their truth in their Operation, or may be examined one by another, where there are varieties of Resolution. *Proof of Geometrical Progression.*

CHAP.

CHAP. VI. *Transmutation.*

Transmutation whence, and what it serves for.

GEometrical *Progression* begat *Transmutation*, which serveth to shew what Number of Changes may be made by any Number of Persons or other things in their Places or Positions: As to know what Number of Changes in the Sound may be made by 5,6,7,8, or more Bells, or different Notes upon 4, 6, 8, or 10 stringed Instruments, and how often the Gammaut may be varied, *&c.*

What it is like.

Transmutation, sometime like *Multiplication*, regardeth the Place of the Numbers, or Numbers themselves, to increase the one by the other; and sometime the Place in which the Person or Numbers stands, is not so much respected as the Variety and Multiplicity of Changes.

Changes, the Sorts.

Changes are Simple or Intermixt.

Simple Changes, what.

Simple Changes are (without relation to any Consort) the Number of the several Positions or Places that a Company of Persons, or other things, may be set or placed in order one beside another; and every time some one or other of the Persons or Things changing their Station, is removed into the Place of another, and all of them never found standing alike.

To find such.

To get such a Number, multiply every Number from the Unit successively into each other's Product unto the Number assigned.

Q. Of the Sittings of 7 Scholars.

Example. Seven Scholars taken out of a Free-School to be sent to the University, were to be entertained at some College at Commons for a certain Sum of Money, with two Meals a Day, so long and no longer than that sitting altogether on a Form at every Meal, they might sit diversly, and never the whole 7 to be alike in Situation: The Question is, how long they were to stay there, or how many several Positions or Sittings there might be made by them in an unlike Positure?

Answer.

Answ. 5040 Sittings, which at 2 Meals a Day, amounts to almost 7 Years, fully to 6 Years 330 Days; for 1 multiplied by 2 is 2, and 2 by 3 is 6, and 6 by 4 is 24, and 24 by 5 is 120, and 120 by 6 is 720, and 720 by 7 is 5040.

$$\text{And } \frac{\overset{\textit{Sittings.}}{5040}}{\underset{\textit{Meals.}}{2}} \left(\frac{\overset{(330}{2520}}{365} \right(6 \text{ Years.}$$

A prodigious Number of Changes by the 24 Letters.

Hence it is no marvel that by 24 Letters there ariseth and is made such variety of Languages in the World, and such a prodigious Number of Words in each Language; seeing the Diversity of Syllables produceth that Effect, by the interchangable placing of Consonants single, double, treble, *&c.* with and among the Vowels. The Number of simple Changes in 24 by multiplying every Product successively into the next Number, is found to be 620448401733239439360000.

Intermixt Changes, what.

Changes intermixt, or Counterchanges, are the Number of Varieties which a Company may make amongst themselves, part with part, or some of them to match the rest in Consort, and regard their Companions, and not the Places they stand in as the simple Changes did, which changed none of their Company, but the Place they stood in.

To find such.

To get the Number of such a Variety, let the whole Number given be broken into as many Parts as there be Units therein and under; or beside those Parts, set the Complement of the Units that make up the whole Number; then get the Varieties that any of those Parts make to match with one of the Complements, and multiply this Number by the whole Number given.

Varieties like the Unciæ in the Table for Extraction of Roots.

The Varieties that the Parts make with 1 of the Complements, are like the Numbers in the Table for Extraction of Roots; for 1 makes no Changes, 2 makes but 2 simple Changes and none intermixt; 3 therefore begins the Dance: And of 3, if any 2 be given to match with the other 1, the Changes will be but 3, because 3 times 1 is 3: But if 1 of 3 be to match with any of the other 2, there will be 6 Changes; for 1 may be joined with the Second, or with the Third, and so the Second with the First or Third, and the Third with the First or Second.

Thus

Thus any Number lacking 1, in opposition to any 1 thereof, can make but so many intermixt Changes as there be Units in the whole Number given: but in opposition to 2, 3, or more, order their Changes accordingly, by multiplying the whole Number into those Tabellary Numbers, only as the given Number lacks 1, take the Numbers in the Table to 1 less than the Number given: As for 4, they must be 3, 3; for 5 the next Numbers, 4, 6, 4; and for 6 the next, 5, 10, 10, 5, *&c.* that is so many of the Tabellary Numbers as will serve for the Number of Parts.

Example. Suppose in 5 Numbers any 4 of them will find out the other 1, or any 3 of them the other 2, or any 2 of them the other 3, or any 1 of them the other 4: how many intermixt Changes there must be in any or all these Cases is the Question? Example.

For Resolution, 5 disposed with his Parts and Complements, stand thus:

Parts	4 . 3 . 2 . 1
Complements	1 . 2 . 3 . 4

Then 4 to 1 cannot change; therefore 1 multiplied by 5, the Number makes 5, the Variety of Rules or Operations that must be in that Case.

But if any 3 of 5 be given, to find any of the other 2; then any 4 Numbers consorting but 3 together at a time, making 4 Changes, this Tabellary Number 4 is to be multiplied by 5 the whole Number: So shall 20 be the Number of Changes or Rules necessary for the finding any of the other 2 desired; as was before evident both in the Chapters of *Arithmetical* and *Geometrical Progression*.

If 2 of 5 be given, to find any of the other 3: Then because any 2 of 4 associating together will make 6 Changes, this Tabellary Number 6 shall be multiplied by 5 the whole Number, and the Product 30 shall in this Case be the Number of Changes or Rules requisite.

Lastly, If any 1 of 5 will find out the rest, then doth 1 match himself 4 times: This Tabellary Number 4 therefore is to be multiplied by 5, and the Product 20 is the Number of Changes desired; and the whole Process appears thus:

	5
Parts of 5, the whole Number given————	4 . 3 . 2 . 1
Complements of the Parts to 5————	1 . 2 . 3 . 4
Varieties of the Parts to 1, or Tabellary Numbers——	1 . 4 . 6 . 4
Products of the Varieties by the whole Number ——	5 . 20 . 30 . 20

Hence notice came to be taken, how many Weights were necessary to weigh any aliquot Number of Pounds by: As to weigh any even Number of Pounds between 1 lb and 40 lb; take 4 Weights in triple Proportion continued, as 1, 3, 9, 27, which make up the Total 40; and to weigh thereby 21 lb, put 9 to the thing weighed, and 3 and 27 to the Counter-poise: For $3+27=30-9=21$, *&c.* *What Weights will weigh certain even Number of Pounds.*

So to weigh any aliquot Number of Pounds, between 1 lb and 121 lb, take 5 Weights in like Proportion, as 1, 3, 9, 27, 81, which make together 121.

The best Demonstration to prove the Truth of both Sorts of Changes, is to take Letters or Species, and place them orderly in their Places or Counter-places as the Case requires. So the simple Changes in 4 is seen to be 24, and the Intermixt 4, 12, 12, as hereafter followeth. *Proof of Transmutation.*

Simple Changes in 4.

	a . b . c . d	b . a . c . d	c . a . b . d	d . a . b . c
	a . b . d . c	b . a . d . c	c . a . d . b	d . a . c . b
1 . 2 . 3 . 4	a . c . b . d	b . c . a . d	c . b . a . d	d . b . a . c
1 . 2 . 6 . 24	a . c . d . b	b . c . d . a	c . b . d . a	d . b . c . a
	a . d . b . c	b . d . a . c	c . d . a . b	d . c . a . b
	a . d . c . b	b . d . c . a	c . d . b . a	d . c . b . a

Intermixt Changes in 4.

a . b . c . d
a . b . d . c
a . c . d . b
b . c . d . a

		4	
Parts ——	3 .	2 .	1
Complem's.	1 .	2 .	3
Varieties –	1 .	3 .	3
Products –	4 .	12 .	12

a . b .			b
a . c .	d	a .	c
b . c .			d
a . b .			a
a . d .	c	b .	c
b . d .			d
a . c .			a
a . d .	b	c .	b
c . d .			d
b . c .			a
b . d .	a	d .	b
c . d .			c

CHAP. VII. *Anatocism.*

Anatocism.

TO close up this Part of continued Proportions, comes in the latter Brood of *Geometrical Progression*, a Child more like the Father than the former of *Transmutation*.

Whence derived. How taken. Includes both Interest and Annuities.

Anatocism, derived from ἀνὰ and τικτω, importeth a bringing forth, renewing or increasing, is here taken for the annual Increase of Interest or Usury; and under the Title of *Interest* and *Annuities* commonly passes in most Authors, because Propositions of both have their Resolutions near of kin to each other, and are proper enough to be joined together.

Interest what. Principal what.

Interest is the Sum reckoned for the Loan or Forbearance of some Principal Sum lent for a certain Time, according to some certain Rate, and therefore called *Principal*, because it is the Sum that procreates the Interest, or from which the Interest is reckoned.

Interest the sorts.

Interest is twofold, *Simple* and *Compound.*

Simple Interest what, and how found.

Simple Interest is counted from the Principal only; and so the *Rule of Three*, or *Rule of five Numbers* before seen, being sufficient to find out at any Rate, and for any Time whatsoever, any Portion of *Simple Interest* required, no further inquiry is to be made thereabouts here.

Compound Interest, what.

Compound Interest is that which is counted from the Principal and Simple Interest forborn, called also *Interest upon Interest.*

Annuities, what. Why the Cases thereof differ from Interest.

Annuities, are yearly Paiments, or Annual Rents, paiable Half-yearly or Quarterly: And their Cases differ somewhat from those of Interest, because if an Annuity be forborn, the Paiments increase as well as the Interest; but if a Sum of Money be lent, that Principal only is to be restored with the Interest.

What to be noted in both.

In the Questions and Cases concerning *Interest* and *Annuities*, is to be noted;

1. The 5 Principals of a Geom. Prog. to be well known.

1*st*. That the Learner be well acquainted with the five Principals of *Geometrical Progression*, and the way to find any 2 of them by the other 3, as before to be seen in the 5*th* Chapter of this Part.

2. How to constitute a Progression by any Ratio.

2*ly*. How by any sort of *Ratio*, as well as Multiplex exampled in there, a Progression may be constituted, which is to be done by this *Analogy*: As the Antecedent of the *Ratio*, is to the Consequent thereof; so is the first Term of a *Progression Geometrical*, to the Second; and by consequence so the Second to the Third, the Third to the Fourth, *&c.*

Examples.

As if the *Ratio* were 2 to 5, beginning at 8: Then as 2 . 5 :: 8 . 20 the second Term. And as 2 . 5 :: 20 . 50, the third Term, *&c.*

So if the *Ratio* be at 4, 5, 6, 7, or 8 *per Cent.* As suppose 8, and the first Term of the Progression be 5; Then as 100 . 108 :: 5 . 5,4. And if of both Examples Progressions be instituted, seven Terms thereof will be, as hereunder follow.

1	2 . 5 :: 8 . 20	1	100 . 108 :: 5 . 5,4.
2	20	2	5,4
3	50	3	5,832
4	125	4	6,29856
5	312,5	5	6,8024448
6	781,25	6	7,346640384
7	1953,125	7	7,93437161472

3*ly*. The *Ratio* of any former Term in a Series of such continued Proportionals, unto any of the Terms following, is equal to the *Ratio* of the first Term to the Second, multiplied into it self, according to the Distance of that latter Term from the former. 3. *What the Ratio of one Term to another is.*

As the *Ratio* of 5,4 to 6,8024448, which is the Third from it, is as the *Ratio* of 100 to 108 triplicated, or as the Cube of 100 to the Cube of 108. Example.

(C.100) (C.108)
1000000 . 1259712 :: 5,4 . 6,8024448.

4*ly*, It is most convenient, first to reduce the *Ratio* of Interest, so that the Antecedent may be 100 or 1; and to use Decimals for common Fractions and Logarithms for the rest of the Work, because otherwise many laborious Extractions of Roots are required. 4. Ratio *of the Interest, how best to be reduced.*

As if the *Ratio* be of Pence $\overset{d.}{14,4}$. or of Shillings $\overset{s.}{1,2}$. then for $\overset{l.}{1}$ the *Ratio* will be as 100 to 106, commonly called *6 per Cent.* or 6 on the 100: For $\overset{d.}{240}$, (the Pence in a Pound Sterling) to $\overset{d.}{254,4}$. (the Pence a Pound, and the Interest of that Pound in a Year at the same Rate): Or $\overset{s.}{20}$, to $\overset{s.}{21,2}$. is as 100, to 106; and 100 . 106 :: 1 . 1,06 . So is 1 . 1,06 α . β . and β is procreated of the Pension or Paiment α whatsoever it be at that Rate in one whole Year. Example.

5*ly*, If the Paiment be half Yearly or Quarterly, that is, in Days 182,5 . or 91,25 . For β, otherwise *R*, commonly called the Rate when the first Term is an Unit: let the Log. of the Yearly Procreat be multiplied accordingly by $\frac{1}{2}$ or $\frac{1}{4}$ or $\frac{182,5}{365}$ or $\frac{91,25}{365}$: So the Quotient dividing the Numerator by the Denominator, shall be the Log. of the *Ratio* for half Years or Quarters: or in Decimals take $\sqrt{q}$ 1,06. or $\sqrt{qq}$ 1,06 instead of β. For it is vulgarly taken amiss, when half the yearly Interest is taken for the half Year, and the Quarter thereof for a Quarter of a Year: And though *Simple Interest* be commonly so taken, yet in *Compound Interest* it is not so, because the Increase is continual from the first beginning to the end of 3 Months, and so to 6 Months, and 12 Months. Then if the Increase in 3 or 6 Months should come to the Quarter or Half of the Whole, it would come to more than the whole at the Years end. 5. *To get the Log. of the Rate for the Paiment Yearly, Half-yearly, &c.* *Vulgar Error.*

As let the *Ratio* be as 100 to 106, the Log. of 100 is 2,00000,00000; the Log. of 106 is 2,02530,58653, the Difference between them is 0,02530,58653, and this is the Log. of the Rate for 1 Year: And in like manner the Difference between the Log. of any Principal Money, and the Log. of that Principal, and the Increase or Decrease added together, shall be the Log. of the Rate for 1 Year. And accordingly this Difference multiplied or divided as the Case may require by the Time or Term, be it Years, Months, Weeks or Days, may not unfitly be termed the Log. for the Time, seeing it performs the Office thereof: so for 2 Years it must be multiplied by 2, for 3 Years by 3; if for $\frac{1}{2}$ a Year by $\frac{1}{2}$, for $\frac{1}{4}$ by $\frac{1}{4}$, for a Month by $\frac{1}{12}$, for a Week by $\frac{7}{365}$, for a Day by $\frac{1}{365}$, according to the Time required. Example.

Rate at 6 l. per Centum.

2,00000,00000 .	Log. of 100 *l.* Principal.
2,02530,58653 .	Log. of 106 . Principal and Interest.
0,02530,58653 .	Difference or Log. of the Rate for 1 Year.
0,01265,29326 .	Log. of the Rate for $\frac{1}{2}$ Year.
0,00632,64663 .	Log. of the Rate for $\frac{1}{4}$ Year.
0,00210,88221 .	Log. of the Rate for a Month.
0,00048,53179 .	Log. of the Rate for a Week.
0,00006,93311 .	Log. of the Rate for a Day.

6*ly*,

6. The Number of Ratio's, and Number of Terms to be well observed.

6*ly*, Because in these *Progressions*, the Number of *Ratio*'s is less by 1 than T, the Number of Terms or Paiments, the Number of *Ratio*'s shall be T—1. And if α be the first Paiment and 1, then the Log. of β multiplied by T—1 shall be the Log. of ω the last Term; and the Log. of β multiplied by T, shall be the Log. of $\beta\omega$, that is, of β multiplied into himself according to the Number of Paiments, and β—1 shall be the Interest of the first Paiment: But if α be no Paiment but the Principal Stock put out to Interest, then shall $\beta\omega$ be the last Paiment for the Increase of α forborn so long time.

Example.

As suppose a *Progression*, whose *Ratio* as 1, to 1,06, were instituted thus:

α	. 1.
β	. 1,06
βq	. 1,1236
βc	. 1,191016
βqq	. 1,26247696
Z	. 5,63709296

Then if β be the first Paiment, the same Number shall be both of the Rates and Paiments, of which the Fourth shall be 1,26247696 for ω, and accordingly T shall be 4: but if α be the first Paiment, the same Number shall be both of the Terms and Paiments, of which the Fourth shall be 1,191016, for ω, less than the other by one Degree, which being multiplied by β, shall give the Sum 1,26247696 for $\beta\omega$. and so shall T. be 5.

And by Logarithms, if α be the first Payment.

0,02530,58653	. Log. of β = 1,06
4	. T—1
0,10122,34612	. Log. of ω = 1,26247696
0,02530,58653	. Log. of β = 1,06
5	. T
0,12652,93265	. Log. of $\beta\omega$ = 1,3382255776.
	The 5*th* Term if β be the First.

7. Whence arise the 2 first Theorems.

7*ly*, Wherefore $\beta\omega$ being procreated of α the Pension, or 1 *l.* let out at Interest according to T, there arise hence these two Theorems, and by marking *Quantamlibet Summam librarum*, or any Sum of Money imployed with Qt̃b. may briefly be expressed in Species thus:

Species Qt̃b. for what use.

	l.	*Let out.*
Theorem 1.	*Theor.* 1. 1. $\beta\omega$ :: Qt̃b	. Qt̃b with the Gain according to T.
Theorem 2.	*Theor.* 2. $\beta\omega$. 1 *l.* :: Qt̃b and Gain according to T	. To the present Value.

Example.

For in the foregoing Instance, because 1 to 1,26247696 is as 1 to the 1 *l.* let out, and 0,26247696 the Gain let run for 4 Years: It shall be on the contrary, that 1,26247696 to 1. shall be as 1 *l.* let out, and the Interest let run for 4 Years, to 1 *l.* the present Value.

Otherwise.

But if β be the first Paiment, then shall $\beta\omega$ accordingly be ω the last Paiment, α being none, or β multiplied into the last Paiment where α is the First; and then the Theorems shall be thus:

		Let out.
Theorem 1.	*Theor.* 1. 1 *l.* ω :: Qt̃b	. Qt̃b with the Gain according to T.
Theorem 2.	*Theor.* 2. ω. 1 *l.* :: Qt̃b and Gain according to T	. To the present Value.

8. Whence two other Theorems arise.

8*ly*, Forasmuch as $\frac{\beta\omega-\alpha q}{\beta-\alpha}$ that is, $\frac{\beta\omega-1}{\beta-1}$ = Z the Sum of all the Terms of the *Progression* (whose last is ω), and is therefore the Procreat of 1 *l.* Paiment let run according to the Number of Terms T. Hence arise 2 other Theorems.

Theorem 3. *Theor.* 3. β—1 . $\beta\omega$—1 :: Qt̃b. Pension let run according to T.
To the Pensions with the Interest to be paid in the End.

Theorem 4. *Theor.* 4. $\beta\omega$—1 . β—1 :: Qt̃b to come according to T.
To the equivalent Pension to be paid according to T.

Example.

For in the Instance above, seeing (β—1 that is) 0,06 dividing ($\beta\omega$—1 that is) 0,3382255776, the Quotient shall be (Z that is) 5,63709296, the Sum of the Progression. And because 0,06 to 0,3382255776, is as 1 *l.* Pension let run 5 Years to 5,63709296, the Pensions and Interest for forbearance then due: It

shall

ſhall be on the contrary, that 0,3382255776, to 0,06, ſhall be as 5,63709296, (that is 1 *l.* future according to T) to the equivalent Penſion, *viz.* 1 *l.*

And according to the firſt Paiment ſo is ω, and $\beta\omega$ to be accompted, and theſe Theorems to be underſtood and altered, as was above obſerved in the two firſt Theorems.

9thly, Seeing $\beta\omega$ is procreate of 1 *l.* let out according to T: And $\frac{\beta\omega-1}{\beta-1}$ is the Procreate of the Penſion of 1 *l.* let run according to T; which in ready Money equalleth the Price of the Penſion: *9. Whence ariſe two other Theorems.*

Say $\beta\omega \,.\, 1\,l. :: \frac{\beta\omega-1}{\beta-1} \,.\, \frac{\beta\omega-1}{\beta-1 \text{ in } \beta\omega}$. Whence therefore according to T, ſhall $\frac{\beta\omega-1}{\beta-1}$ the Price of the Penſion be procreated. Hence alſo ariſe 2 Theorems.

Theor. 5. $\beta-1$ in $\beta\omega \,.\, \beta\omega-1 ::$ Qtb Penſion according to T. Theorem 5.
To the Price of the ſame in ready Money.

Theor. 6. $\beta\omega-1 \,.\, \beta-1$ in $\beta\omega ::$ Qtb Preſent. Theorem 6.
To the Penſion to be bought according to T.

For in the former Inſtance, $\beta-1$ (being 0,06) multiplied in $\beta\omega$, (which is 1,3382255776) makes 0,080293534656. And becauſe 0,080293534656 to 0,3382255776 (that is $\beta\omega-1$) is as 1 *l.* Penſion in 5 Years to 4,212364— (the Price thereof in ready Money); it ſhall be on the contrary, that 0,3382255776, to 0,080293534656, ſhall be as 4,212364— paid preſent for the Annuity of 1 *l.* bought for 5 Years. Example.

And in theſe, as in the 2 Theorems laſt before, let ω and $\beta\omega$ be underſtood according to the firſt Paiment: And ſeeing 1 *l.* is taken for Qtb in every of the ſix Theorems, and Explanation thereof, for Annuities of greater Value, multiply the *4th* Number, or Number found, by the *Analogy*, by the Number of Pounds employed: Or let the Third in the *Analogy* be the ſame Number. And for Example-ſake, further followeth the Work of a Penſion for 10 Years, payable Half-yearly, at the Rate of 6 *per Cent.* that is 1 to 1,06, and T. the Number of Paiments 20. *Farther Example.*

Log. of 1,06, is . 0,02530,58653 .
in ½
0,01265,29326½. Log. of β = 1,0295, &c.
20 . T
0,25305,86530 . Log. of $\beta\omega$ = 1,7908, &c.
—2,47129,17111 . Log. of $\beta-1$ = 0,0295, &c.
—2,72435,03641 . Log. of $\beta-1$ in $\beta\omega$.
— 1,89812,15755 . Log. of $\beta\omega-1$ = 0,7908, &c.

It is therefore —1,89812,15755 .
—2,72435,17111 . *l.*
1,17376,98644 . Log. of the Price 14,922 + for 1 *l.* Penſion.

And —2,72435,17111
—1,89812,15755 . *l.*
—2,82623,01356 . Log. of the Penſion 0,067023, for 1 *l.* Price.

To theſe found Logarithms, add the Logarithm of Qtb.
Or multiply thoſe Values found by Qtb.

10thly, Queſtions both of Intereſt and Annuities, as others in *Geometrical Progreſſion*, (upon whom their Reſolution depends) may be varied many ways: So that the finding of the five Principals of a *Geometrical Progreſſion*, before ſeen in the Chapter thereof, and the true knowledg of the ſix Theorems laſt above-mentioned, are ſufficient to reſolve any Propoſitions concerning them; the Principal Varieties whereof follow, firſt diſtinctly, and afterward intermixt; and in both are chiefly wrought by Logarithms, all other Ways for any conſiderable Time being tedious and troubleſome. *10. Queſtions may be varied many ways. On what the Work depends. Beſt done by Logarithms.*

Touching Compound-Interest distinctly.

Of Compound Interest, what to be noted.

Compound-Interest respecteth four Things: By any three of which given, the other may be found out.

1. The Sum to be received. What it agreeth and answereth to, and how called.

1. The Sum or Amountment of any principal Sum lent or forborn, for a certain Time or Term, to be received with Interest upon Interest, after a certain Rate: This answereth to ω, or the Second of the five Principals in a *Geometrical Progression*, and agreeth with the first Theorem, and is sometime called the *Profit* or *Loss*.

2. The Sum lent. What it agreeth and answereth to, and how called.

2. The principal Sum lent, or ready Money to be paid for a Sum of Money to be forborn a certain Time or Term, with Interest upon Interest, after a certain Rate: This answereth to α, or the first of the five Principals in a *Geometrical Progression*, and agreeth with the second Theorem, and is sometime called *Abatement* or *Rebatement*, and sometime *Discount*.

3. The Rate. What it answereth to.

3. The Rate, according to which the principal Sum lent or forborn increaseth or decreaseth, as the Question concerneth Profit or Loss. This answereth to R, or the Fourth of the five Principals in a *Geometrical Progression*.

4. The Time or Term. What it answereth to.

4. The Time or Term in which the Increase or Decrease is to be, whether Years, Quarters, Months, Days, or what else. This marked by T, or N, answereth to the Third of the five Principals in a *Geometrical Progression*.

Prop. 1. To find the Sum to be paid.

Proposition 1. To know what Sum ought to be paid for a Sum forborn a certain Time, or Term, at any Rate, with Compound Interest.

Q. Of 200 *l.* forborn 7 Years, what it comes to.

As to know what Sum shall be paid for 200 *l.* forborn 7 Years, with Interest upon Interest, after the Rate of 6 *l. per Cent. per Annum:* Or what comes 200 *l.* to in 7 Years, by Interest upon Interest, at the Rate of 6 on the 100?

Answer.

Answ. l. 300,726, and somewhat more: For seeing here is given α. T. R. to find ω. that is α. 200. T. 7. and R. as 1 to 1,06, the 300,726, *&c.* may be found by the former Rules in *Progression*, only there the *Ratio* was figurate to T—1, but here to T, as by the Decimals appeareth.

By Decimals.

Thus	1,06	R
	1,1236	
	1,191016	
	1,26247696	
	1,3382255776	
	1,418519112256	
	1,50363025899136	
	200	α
	300,72605179827200	ω

By the first Theorem.

And by the first Theorem.

1 . 1,50363025899136 :: 200 . 300,72605179827200.

Or

Or thus the common Way,

By the Common Way.

	2 00 1,06	α R
	1200 2000	
	212,00 1,06	1
	127200 212000	
	224,7200 1,06	2
	13483200 22472000	
	238,203200 1,06	3
	1429219200 2382032000	
	252,49539200 1,06	4 Γ
	151497235200 252495392000	
	267,6451155200 1,06	5
	16058706931200 267645115520000	
	283,703822451200 1,06	6
	1702222934707200 28370382245120000	
ω	300,72605179827200	7

The Work by Logarithms is both easy and speedy thus:

To the Product of the Log. of the Rate (which as aforesaid is the Difference between the Log. of the Principal Money, and the Log. of the Principal and Simple Interest in 1 Year) multiplied by the Term or Time (called the *Log. for the Time*) add the Log. of the Sum so lent or forborn. *By Logarithms.*

	2,00000,00000	. Log. of 100
	2,02530,58653	. Log. of 106
Difference	0,02530,58653	. Log. of 1,06 . R. or the Rate for 1 *l.* for 1 Year.
	7	. T.
Product	0,17714,10571	. Log. of 1,5036, *&c.* or Log. for the Time.
	2,30102,99957	. Log. of 200 . α
Sum	2,47817,10528	. Log. of 300,726, *&c.* ω.

If the same Sum of 200 *l.* were forborn only for half a Year, then half the Difference or Log. of the Rate shall be the Log. for the Time, which added to the Log. of 200, shall be the Log. of 205,91, *&c.* and not full 206 *l.* as is commonly reckoned at *Simple Interest.* *Example for half a Year.*

0,01265,29326	. Log. for $\frac{1}{2}$ Year.
2,30102,99957	. Log. of 200
2,31368,29283	. Log. of 205,91, *&c.*

If the Sum of the former Example be forborn 3 Years 5 Months and 4 Days: then first the Log. of the Rate is to be multiplied for the 3 Years by 3, and then by $\frac{1}{12}$, or the Log. of the Month by 5, and then by $\frac{1}{365}$, or the Log. of the Day by 4; and to these is to be added the Log. of 200, as before. *Example for 3 Years, 5 Months and 4 Days.*

0,02530,58653	. Log. of the Rate in 1 Year.
3	. Years.
0,07591,75959	. Log. for 3 Years.
0,01054,41105	. Log. for 5 Months.
0,00027,73244	. Log. for 4 Days.
0,08673,90308	. Log. for the Time.
2,30102,99957	. Log. of 200.
2,38776,90265	. Log. of 244,213, &c.

If the Rate be unusual, or the Sum uneven.

If occasion be to accompt the Interest at an unusual Rate, or the Sum for which the Interest be reckoned be an uneven Sum; there is no Difference in the Method of proceeding from the former, but alike easy by Logarithms.

Example.

As if 56 *l.* 13 *s.* 6 *d.* be forborn 4 Years and 8 Months, and Compound Interest be reckoned for the forbearance after the Rate of 5 *l.* 10 *s.* *per Cent.* the Sum to be paid will be *l.* 72,7617, &c.

2,02325,24596	. Log. of 105,5. or 105 *l.* 10 *s.*
2,00000,00000	. Log. of 100.
0,02325,24596	. Log. of 1,055. the Rate.
0,09300,98384	. Log. for 4 Years.
0,01550,16397	. Log. for 8 Months, or $\frac{2}{3}$ of a Year.
0,10851,14781	. Log. for the Time.
1,75339,15288	. Log. of 56,675 . or 56 *l.* 13 *s.* 6 *d.*
1,86190,30069	. Log. of 72,7617, &c.

Example in a more unusual Rate.

If the Rate be more unusual, as at 5 *l.* 10 *s.* for 56 *l.* 13 *s.* 6 *d.* *per Annum*, and it be desired to know what 98 *l.* 10 *s.* 6 *d.* would amount to after that Rate in 3 Years an half and 10 Days; the Operation will be in like manner, but otherwise than by the use of Logarithms extraordinary Difficult.

56 : 13 : 6	1,79361,57939	. Log. of *l.* 62,175, or 62 *l.* 03 *s.* 6 *d.*
5 : 10 : 0	1,75339,15288	. Log. of 56,675, or 56. 13. 6.
62 : 03 : 6	0,04022,42651	. Log. of the Rate for 1 Year.
	0,12067,27953	. Log. for 3 Years.
	0,02011,21325	. Log. for $\frac{1}{2}$ Year.
	0,00110,20346	. Log. for 10 Days.
	0,14188,69624	. Log. for the Time.
	1,99354,64435	. Log. of 98,525 . or 98 *l.* 10 *s.* 6 *d.*
	2,13543,34059	. Log. of 136,5945, &c.

Prop. 2. To find the ready Money.

Prop. 2. To know what Sum in ready Money ought to be paid for a Sum of Money to be received after a certain Time or Term, by *Compound Interest.*

Q. Of l. 300,726 &c. payable 7 Years hence, what now worth.

As to know what *l.* 300,726, &c. to be paid 7 Years hence is worth ready Money, at the Rate of 6 *l.* *per Cent. per Annum.* Or what Sum in 7 Years did amount to *l.* 300,726, &c. by Interest upon Interest, after the Rate of 6 on the 100.

Answer. *By Progression. By the second Theorem. By Logarithms.*

Answ. 200 *l.* For this being the Converse of the 1*st Prop.* here is ω . T . R . given to find α, that is, ω . 300,726, &c. T . 7 . and R . as 1 to 1,06, and so as before noted *Prop.* 1. only figurating the *Ratio* to T, may be found by the former Rules in *Progression*, or the second Theorem: For 1,5036302589913б . 1 :: 300,7260517982 72 . 200. But the work as aforesaid, being most easy by Logarithms, the Rule thereof is thus:

Substract the Log. for the Time out of the Log. of the Sum to be paid.

2,02530,58653	. Log. of 106.
2,00000,00000	. Log. of 100.
0,02530,58653	. Log. of 1,06, the Rate for 1 *l.* in 1 Year.
7	. Years T.
0,17714,10571	. Log. for the Time, or 1,50363, &c.
2,47817,10528	. Log. of 300,726, &c. ω.
2,30102,99957	. Log. of 200 . α.

For other Rates or Times.

Thus whatever Rate or Time of forbearance it be, when the Log. for the Time is gotten, let it be taken from the Log. of the Sum to be paid, if Increase be accompted.

As

As *l.* 244,213, *&c.* will be payable at 3 Years 5 Months and 4 Days hence; yet it may be received presently, if allowance be made for the Time or Term, after the Rate of *6 per Cent. per Annum:* what will be payable in ready Money? *Example for 3 Years, 5 Months 4 Days.*

Answ. 200 *l.* For the Log. for the Time 0,08673,90308, taken from the Log. of 244,213, *&c.* will leave the Log. of 200.

0,02530,58653	.	Log. of the Rate in 1 Year.
3	.	Years.
0,07591,75959	.	Log. for 3 Years.
0,01054,41105	.	Log. for 5 Months.
0,00027,73244	.	Log. for 4 Days.
0,08673,90308	.	Log. for the Time.
2,38776,90265	.	Log. of 244,213, &c.
2,30102,99957	.	Log. of 200.

But in all the Questions found in other Authors under this Proposition, if there have been a Loss or Decrease, the Log. for the Time shall be added to the Log. of the Sum of Money not lost. *Example II. Loss.*

As 200 *l.* is remaining at the end of 7 Years, by Loss or Discount, after the Rate of *6 per Cent. per Annum:* what was the Sum at the beginning of the 7 Years?

Answ. l. 300,726, &c.

0,02530,58653	.	Log. of the Rate in 1 Year.
7	.	Years.
0,17714,10571	.	Log. for the Time.
2,30102,99957	.	Log. of 200.
2,47817,10528	.	Log. of 300,726, &c.

And because this Proposition is converse to the First, (as before noted) the Resolution of this last Question is done by the first Proposition. And if under the first Proposition such a Question as this had been set, *viz. l.* 300,726, &c. is to be received at the end of 7 Years; but losing the Compound Interest at 6 *per Cent.* it will be paid presently: what shall be paid for the same at present? The Work by this second Proposition, as above, will appear to be 200 *l.* So as Questions like this last are proper to this Proposition, and those like the other to the first Proposition, however transposed in other Books. *This second Proposition converse to the first. What Questions are proper to both.*

Prop. 3. To know after what Rate any Sum hath increased or decreased by Compound Interest in a certain Time or Term. *Prop. 3. To find the Rate.*

As to know after what Rate 200 *l.* in 7 Years will increase to *l.* 300,726, &c. Or suppose 200 *l.* by Interest upon Interest in 7 Years did amount to *l.* 300,726, &c. what was the *Ratio* compared to 100? *Q. Of the Rate by which 200 l. in 7 Years increased to 300,726 &c. Answer.*

Answ. 6 *l. per Cent. per Annum.* And so by the former Rules in *Progression* may be found, here being given α . ω . T . to find R . that is α . 200 . ω . 300,726, &c. and T . 7 . For 200 . 1 :: 300,726, &c. 1,503630, &c. differing from the way there only in this, that here the Number found by the *Analogy*, is the *Ratio* figurate to T, thereto T—1, as before noted in the two precedent Propositions. *By Progression.*

By Logarithms the Work is thus. Divide the Difference of the Logarithms of the two Sums propounded, by the Term or Time given; to the Quotient add the Log. of the intended or supposed Antecedent of the Rate, and the Total shall be the Log. of the Antecedent and Consequent: Then deducting the absolute Number of the Antecedent, from the absolute Number of the other, the Remain shall be the Rate desired. *By Logarithms.*

	2,47817,10528	.	Log. of *l.* 300,726, &c. ω.
	2,30102,99957	.	Log. of 200. α.
7)	0,17714,10571	.	Difference, or Log. for the Time.
	0,02530,58653	.	Quotient, or Log. of the Rate.
	2,00000,00000	.	Log. of 100. Antecedent of the Rate.
	2,02530,58653	.	Log. of 106, Antecedent and Consequent.
			6. on 100 the Rate.

If the Time be an Half-year, Quarter, &c.

If the Term or Time given, besides the whole Number of Years, be some odd Parts of a Year, as half a Year, a Quarter, Months, Days, or such-like, then reduce them all into one Denomination; and as a Fraction, with 365 (the Days in a Year) *&c.* abbreviate them to their least Terms; and thereby, or rather by the Decimal thereof, divide the Log. of the Difference as above.

Example.

As if it were desired to know after what Rate 200 *l.* in 3 Years, 5 Months, and 4 Days, will increase to *l.* 244,213, *&c.* Because the Time reduced into Days will not be abbreviated, but both Terms thereof large, the Decimal is rather to be chosen, which is 3,42762556; by which the Difference divided maketh the Quotient, and so the Rate as above.

	2,38776,90265	.	Log. of *l.* 244,213, &c.
	2,30102,99957	.	Log. of 200.
3,42762556)	0,08673,90308	.	Difference.
	0,02530,58653	.	Quotient.
	2,00000,00000	.	Log. of 100, Antecedent.
	2,02530,58653	.	Log. of 106, Antecedent and Consequent.
			6. Rate.

Prop. 4. To find the Time or Term.

Prop. 4. To know in what Time or Term a Sum of Money will increase or decrease by Compound Interest, to a Sum propounded according to a certain Rate.

Q. Of the Time in which 200 l. increased to 300,726, &c.

As to know in what Time 200 *l.* after the Rate of 6 *per Cent. per Annum*, by Interest upon Interest, will increase to *l.* 300,726, *&c.* or when will *l.* 300,726, *&c.* be payable for 200 *l.* lent, after the Rate of 6 *l.* on the 100 for a Year?

Answer. *By Progression.*

Answ. At the end of 7 Years after the Loan, as may be found by the former Rules in *Progression*, with a little Difference, for here is given $\alpha . \omega . R.$ to find T. that is, α . 200 . ω . 300,726, *&c.* and R. 1,06. So 200 multiplied by 1,06 till 300,726, *&c.* be produced, the Number of Multiplications 7, is the Number desired; For 1 needs not be added as in *Progression*. The Work is short by Logarithms, thus.

By Logarithms.

Divide the Difference of the Logarithms of both Sums given by the Log. of the *Rate*; and the Quotient shall be the Number desired.

	2,47817,10528	.	Log. of *l.* 300,726, *&c.* ω
	2,30102,99957	.	Log. of 200. α
	0,17714,10571	.	Difference, or Log. for the Time.
Log. of the Rate	0,02530,58653	(	7 Years T.

If any Remain be on Division.

If after the Division any Thing remain, divide that Remain by the Log. of the Rate for a Month: And if upon that Division any thing remain, divide it by the Log. of the Rate for a Day.

Example.

As 200 *l.* hath increased to *l.* 244,213, *&c.* and I desire to know in what Time it hath so increased by Compound Interest, after the Rate of 6 *per Centum per Annum*.

Answer.

Answ. In 3 Years, 5 Months, and 4 Days: For after the First Division, which bringeth 3 Years in the Quotient, there is left remaining 1082,14349; this divided by 210,88221, the Log. of the Rate for a Month, bringeth 5 in the Quotient, and leaveth remaining 27,73244; which divided by 6,93311, the Log. of the Rate for a Day, giveth 4 in the Quotient.

2,38776,90265	.	Log. of *l.* 244,213, &c.
2,30102,99957	.	Log. of 200.
0,08673,90308	.	Difference.

(1082 14349 (27 73244

$\cancel{8673,90308}$ / $\cancel{2530,58653}$ (3 Years . $\cancel{1082,14349}$ / $\cancel{210,88221}$ (5 Months . $\cancel{27,73244}$ / $\cancel{6,93311}$ (4 Days.

Touching

Touching Annuities distinctly.

Of Annuities, what to be noted.

Annuities respect 5 Things, by any 3 whereof the other may be found as before in *Progression.*

1. The Arrerages. *What it agreeth and answereth to.*

1. The Sum, or Arrerages of the Rent of a Lease, or Annuity, or Pension forborn a certain Term or Time, to be paid with Interest upon Interest after a certain Rate. This answereth to Z, or the fifth of the 5 Principals in a *Geometrical Progression,* when the Annuity is proposed as the first; otherwise it answereth to ω or the second Principal, when the ready Money or Price is given for the First, and agreeth with the third of the 6 Theorems in this Chapter.

2. The Price. *What it agreeth and answereth to.*

2. The Sum or Price, which in ready Money will buy or purchase an Annuity for a certain Time or Term, after a certain Rate by Interest upon Interest; this answereth to α, or the first of the 5 Principals in a *Geometrical Progression,* where the Arrerages are proposed as the Second, and agreeth with the 5*th* of the six Theorems abovementioned.

3. The Annuity or Pension. What it agreeth and answereth to.

3. The Annuity or Pension that any Sum of ready Money will buy or purchase for a certain Time, after a certain Rate with Interest upon Interest. This answereth to α, or the first of the 5 Principals in a *Geometrical Progression,* where the Arrerages are proposed as the Fifth, and agreeth both with the 4*th* and last of the aforesaid 6 Theorems.

4. The Time or Term. What it answereth to.

4. The Time or Term any Annuity or Pension, after a certain Rate by Compound Interest, may be detained to amount to a Sum propounded; This marked by T or N, answereth to the Third of the 5 Principals of a *Geometrical Progression.*

5. The Rate. What it answereth to.

5. The Rate by which any Annuity or Pension by Compound Interest did increase to a Sum propounded; this answereth to R. or the fourth of the 5 Principals of a *Geometrical Progression.*

And according to the different State of the Question, under one or other of these Classes, is Resolution attainable by some or other of the Rules proper thereto: For as before in *Progression,* one of these 5 being sought, the Question may be varied 4 several Ways; and where the *Data* are all of those 5 Principals, the Rules there may be used with what alteration is necessary for Fractions and Decimals: otherwise because the Second of these is none of the former, some new Rules will be necessary, wherefore all the Varieties shall be exampled in:

Prop. 1. To find the Arrerages.

Proposition 1. To know what Sum of Money ought to be paid for the Arrerages of Rent reserved upon a Lease, or for an Annuity or Pension forborn a certain Time or Term, after a certain Rate by Compound Interest.

Data under the first Prop.

The *Data* for the Resolution of Questions under this Classis, will be either,

2 . 3 . 4 . The Price, Annuity, and Term.
2 . 3 . 5 . The Price, Annuity, and Rate.
2 . 4 . 5 . The Price, Term, and Rate.
or 3 . 4 . 5 . The Annuity, Term, and Rate.

Of these 4, the last easiest.

The last being easier than the first, shall be first in Example, and the rest in their retrograde Order.

Variety 4. Q. Of 5 l. per Annum, forborn 7 Years, what it comes to.

Variety 4. If an Annuity of 5 *l. per Annum* be detained 7 Years: what will the Amount thereof be by Interest upon Interest, after the Rate of 6 *l. per Cent. per Annum?* or what are the Arrerages of an Annuity of 5 *l.* a Year with the Compound Interest, after the Rate of 6 on the 100 for a Year forborn 7 Years?

Answer. By Progression.

Answ. l. 41,96918824928o: for seeing here is given the Annuity, Term and Rate, or α . T . R . 3 of the 5 Principals of a *Geometrical Progression,* to find Z. that is, α . 5 . T . 7 . and R . 6 on the 100, which is as 1 to 1,06 by the Rules in the third Proposition for finding the 5*th* Principal there, 41,969, *&c.* is found thus:

5

	5 α 1,06 R	1
	30 50	
1	5,30 1,06	2
	3180 5300	
2	5,6180 1,06	3
	337080 561800	
3	5,955080 1,06	4
	35730480 59550800	
4	6,31238480 1,06	5
	3787430880 6312384800	
5	6,6911278880 1,06	6
	401467673280 669112788800	
6	7,092595561280 1,06	7 T
	42555573367680 709259556128000	
7	7,51815129495680	8

Collection of the Terms.

5,
5,30
5,6180
5,955080
6,31238480
6,6911278880
7,092595561280

Z 41,969188249280

,06) 5 α
2,51815129495680 (41,969188249280.

Otherwise.

Otherwise by figurating the *Ratio* as in *Progression* also mentioned, but different herein, that here an Unit the Antecedent of the *Ratio* is to be added to the Sum, as,

1,
1,06
1,1236
1,191016
1,26247696
1,3382255776
1,418519112256
1,50363025899136

Sum of the first 7 Terms with 1, 8,393837649856
5 α Annuity.
41,969188249280 Z Arrerages.

By the third Theorem.

And the third Theorem thus;

0,06 . 0,50363025899136 :: 1 . 8,393837649856
5 α
41,969188249280 Z

By Logarithms.

But as before in Interest, so in Annuities, the Work is easy and speedy in Logarithms, thus;

To

To the Log. for the Time, add the Log. of so much Principal Money as at Simple Interest in 1 Year will raise such an Annuity, and the Sum will be the Log. of that Principal Money, and the Arrerages; then substract the Principal out thereof, and the Residue is the Sum desired.

The Principal Money that at Simple Interest, after any Rate, will raise the Annuity propounded, is gotten either by the *Golden Rule direct* without, or rather with Logarithms, as in the former Instance. If 6 *l.* require 100 *l.* Principal to raise the same, then shall 5 *l.* require 83 *l.* 6 *s.* 8 *d.*

As 6 . 100 :: 5 . 83 : 6 : 8.
Or, 2,00000,00000 . Log. of 100
0,69897,00043 . Log. of 5
2,69897,00043 . Log. of 500
0,77815,12504 . Log. of 6
1,92081,87539 . Log. of 83,33333, &c.

Then the rest of the Resolution is thus:

2,02530,58653 . Log. of 106
2,00000,00000 . Log. of 100
0,02530,58653 . Log. of 1,06 . Rate . R or ß
7 . Years . T
0,17714,10571 . Log. of 1,50363, &c. Time or ßω
1,92081,87539 . Log. of 83,33333, &c. Principal in 1 Year, to raise 5 *l.* at Simple Interest.
2,09795,98110 . Log. of 125,30251, &c. Principal & Arrerages.
41,96918, &c. Z . Arrerages.

Or if the Log. of the Rate lacking 1, be taken from the Log. for the Time lacking 1, and to the Remain the Log. of the Annuity be added, the Sum will be the Log. of the Arrerages desired. *Otherwise.*

—1,70211,18155 . Log. of 0,50363, &c. Time—1. or ßω—1
—2,77815,12504 . Log. of 0,06 Rate—1. or ß—1
0,92396,05651 . Log. of 8,39383, &c.
0,69897,00043 . Log. of 5 α . Annuity.
1,62293,05694 . Log. of 41,96918, &c. Z . Arrerages.

If the Rents or Annuities be payable Half-yearly or Quarterly, as most commonly they are; then after the Principal Money that will raise such a Paiment is gotten, the rest of the Work differs nothing from the former. *If the Paiments by Half-yearly or Quarterly.*

To get this Principal Money, let the Log. of the Rate be halfed or quartered, according as the Rent or Annuity becomes payable; and for this Half or Quarter, get the *Geodetical* or Decimal as of another Log. Or commonly thus, enter the Table of Logarithms next after the Antecedent of the *Ratio* (neglecting the Index) with the halfed or quartered Log. till the next lesser Log. to that propounded be found; and there by the Difference of the next Lesser to the next Greater, and the next Lesser to the propounded Log. the *Geodeticals* or Decimals sought may be found, as in the Chapter of the *Reduction of Logarithms* was taught. And to the left Hand of this *Geodetical* or Decimal thus found out, is to be prefixed the absolute Number of the lesser Log. in the Table exceeding the Antecedent of the *Ratio* lacking the same Antecedent: And this shall be the first Number of the *Rule of Three*, to find the Principal Money required, to which the Antecedent shall be the Second, and the Paiment propounded the Third: Or if Operation be made by Logarithms, the Log. of that *Geodetical* or Decimal, with the absolute Number prefixed as aforesaid, shall be the Log. to be substracted from the Sum of the other two Logarithms. *How to get the Principal Mony.*

Example. The Rent of a Lease of 5 *l. per Annum*, payable Half-yearly, is in Arrear 7 Years: what do the Arrerages thereof amount to, after the Rate of 6 *l. per Cent. per Annum* by Compound Interest? *Q. Of 5 l. per Ann. payable Half-yearly, what the Arrears in 7 Years.*

Answ. *l.* 42,5873, &c. For here the Log. of the Rate as before being 0,02530,58653 (because the Paiments are Half-yearly) is to be halfed; and with this half, which is 0,01265,29326, if the Table of Logarithms be entred next *Answer.*

next after 100 (the Antecedent of the *Ratio*) it will be found to fall between the Logarithms of 102 and 103, and to be more than the Log. of 102, the Index neglected 0,00405,27608: If therefore Ciphers be adjoined as many as shall be necessary, and Division made by 423,70529 (the Difference between the Log. of 102 and 103) the Decimal 9565, *&c.* will be gotten to be placed on the right Hand of 2, the Absolute Number exceeding the Antecedent of the *Ratio*; and having gotten the Decimal, the Shillings, Pence, *&c.* therein, are soon found as before in *Reduction of Decimals.* If otherwise, the Learner be more expert at Common Fractions, then the Difference between the lesser Log. found as aforesaid, and the Log. propounded, multiplied by 20, and divided by the Difference between the lesser and greater Logarithms found in the Table, giveth the Quotient in Shillings; the Remain of this Division, if any, multiplied by 12, and divided by the former Divisor, giveth the Quotient in Pence, *&c.*

```
   2,02530,58653 . Log. of 106
   2,00000,00000 . Log. of 100
  )0,02530,58653 . Log. of   1,06.          Difference.
 2)0,01265,29326 . Log. of   1,029565, &c. Half.
   2,00860,01718 . Log. of 102
   2,01283,72247 . Log. of 103
   0,00405,27608 . Difference of the Half and Log. of 102.
   0,00423,70529 . Difference between the Log. of 102 and 103.
```

```
      (1970                                   2 prefixed is l. 2,9565,&c.
    213822(1                                                   20
   2756054 7(1                                 s.    19 | 1300
  2394131 956(5                                              12
405,27608,0000,&c.(,9565,&c.                                | 2600
 423,7052 9999                                           1 | 300
  423,705 222                                  d.        1 | 5600
   423,70 55                                                   4
    423,70                                     q.        2 | 2400
```

Or for the *Geodetical* thus:

```
                  (551210
 405,27608       3868 4687(9                 5512109       (23774779
        20       8105,52160(19s.                  12       661 45308(1 d.
 8105,52160      423,705299                  66145308      423,70529
                  423,7052
```

This Principal Money found (omitting the Remain of the first Division as inconsiderable) the rest of the Resolution to the last foregoing Question, is as followeth.

```
   As 2,9565 . 100 :: 2,5 . 84,5594, &c. by Decimals.
 Thus 2,00000,00000 . Log. of 100.        by Logarithms.
      0,39794,00087 . Log. of 2,5.
      2,39794,00087 . Log. of 250.
      0,47077,78833 . Log. of 2,9565. or 2 l. 19 s. 1 1/2 d.
      1,92716,21254 . Log. of 84,5594, &c.
      0,17714,10571 . Log. for the Time 14 half Years, or 7 Years.
      2,10430,31825 . Log. of 127,1461, &c.
                                   42,586  &c. Z.

 Or thus —1,70211,18155 . Log. of 0,50363, &c. Time—1 . or βω—1.
         —2,47077,78833 . Log. of 0,029565, &c. Rate—1 . or β—1.
          1,23133,39322 . Log. of 17,03467, &c.
          0,39794,00087 . Log. of 2,5           α .
          1,62927,39409 . Log. of 42,586, &c.   Z .
```

As well in Annuities as before in Interest, the Log. of the Rate if unusual, and the Log. for the Time if odd Months, Days, *&c.* once gotten, the rest of the Work is in like manner, so that Examples thereof need not be added here. *Unusual Rates and uneven Times, as in Interest.*

Variety 3. If an Annuity to endure 7 Years be sold for *l.* 27,911907198138 63, *&c.* ready Money: what will the Arrerages thereof being forborn all the Term amount to, at the Rate of 6 *l. per Cent. per Annum*, by compound Interest? Variety 3. *Q. Of an Annuity sold, what the Arrears.*

Answ. l. 41,969188249280. Here being given the Price, the Term and Rate, because the Price or ready Money propounded, is by the first Proposition of Interest as α and Z, the Arrerages sought as ω to α, Resolution may be had by the first Theorem, seeing the Amountment of a Sum of Money, purchasing an Annuity in any Number of Years by Interest upon Interest, shall equal the Arrerages of that Annuity so long detained: And because this variety gives α. T. R. to find ω. by the Rules in the first Proposition, for finding the second Principal of a *Geometrical Progression*, the said 41,969, *&c.* may thereby be obtained, only differing in figurating the Rate as before noted in Interest, but briefly by Logarithms thus. Answer. *By the first Theorem. By Progression. By Logarithms.*

Add the Log. for the Time, to the Log. of the Price or ready Money, and the Sum shall be the Log. of the Arrerages desired.

1,44578,95123	. Log. of 27,9119, *&c.*	Price.
0,17714,10571	. Log. of 1,5036, *&c.*	Time as before.
1,62293,05694	. Log. of 41,9691, *&c.*	Arrerages.

Variety 2. The Arrerages of an Annuity of 5 *l. per Annum*, after the Rate of 6 *l. per Cent. per Annum* by Compound Interest, are valued to be worth *l.* 27,9119, *&c.* in ready Money: how much do those Arrerages amount to? Variety 2. *Q. Of Arrerages of 5 l. per Ann. valued ready Money.*

Answ. l. 41,9691, *&c.* as before: here are given the Price, the Annuity, and the Rate; so the Work without Logarithms is thus. Answer.

Get (as before under the fourth *Variety*) the Principal Money, that at Simple Interest after the Rate propounded, will raise such an Annuity; from this Number take the given Price, and by the Remainder divide the Number so gotten as aforesaid, then multiply this Quotient by the Price, and the Product is the desired Arrerages. *By Decimals.*

But to avoid those long and tedious Multiplications and Divisions that are in Decimals, Logarithms are used thus.

After the Principal Money, that at Simple Interest will raise the Yearly Annuity propounded, is gotten, taken there from the given Price, and the Log. of the Remainder out of the Log. of that Principal, then to the remaining Log. add the Log. of the Price; and the Sum is the Log. of the Arrerages. *By Logarithms.*

1,92081,87539	. Log. of	*l.* 83,33333, *&c.*	Principal, as before.
		27,91190, *&c.*	Price.
1,74367,76968	. Log. of	55,4214, *&c.*	Remainer.
0,17714,10571	. Log. of	1,5036, *&c.*	Time.
1,44578,95123	. Log. of	27,9119, *&c.*	Price.
1,62293,05694	. Log. of	41,9691, *&c.*	Arrerages.

Variety 1. An Annuity of 5 *l. per Annum* for 7 Years, was purchased for *l.* 27,9119, *&c.* ready Money: how much was the Arrerages reckoned to amount to by Compound Interest, if all had been let alone till the end of the Term? Variety 1. *Q. Of 5 l. per Ann. for 7 Years bought, what the Arrears.*

Answ. l. 41,9691, *&c.* as before: The Rate being a principal Key in this Work, and none of the *Data* in this Variety, but the Price, Annuity, and Term; the Log. for the Time (being as aforesaid, the Log. of the Rate multiplied by the Term) cannot well be had. And because the Price is to be multiplied by the Rate figurate according to the Term, or their Logarithms added together; it is a most difficult Proposition for a young Scholar to resolve such a Question by the former Rules. But it is to be considered, seeing 5 is the first Term of one *Progression*, and 27,9119, *&c.* the first Term of another, which shall have the *Ratio* and Number of Terms alike to the first *Progression* that beginneth with 5, and the Number of Terms in both must be 7, and the Arrerages sought must be the Sum of all the Terms beginning with 5, and the last Term only of the other; by necessary Consequence 27,9119, *&c.* must be multiplied by the seventh Power of some Root, which shall make the seventh Term of that Progression equal to the Total of all the 7 Terms of the Progression that beginneth with 5: And because in this Example no other Number can do this but 1,503630258991 36, which is the Answer. *A difficult Proposition. By Progression.*

the *7th* Power of the Root 1,06, therefore 27,91190, *&c.* multiplied thereby, or their Logarithms added together, the Product of their Multiplications answering to the Sum of their Logarithms will be 41,96918824928, which Number shall be Z, if 5 be α, and shall be ω if 27,911907198, *&c.* be α the Terms of both 7, and the *Ratio* 1,06. This may a little illustrate the Matter, but the best way is to proceed as above in the second *Variety*.

By Logarithms.

1,44578,95123	.	Log. of 27,91190, *&c.* Price.
0,17714,10571	.	Log. of 1,50363, *&c.* Time.
1,62293,05694	.	Log. of 41,96918, *&c.* Arrerages.

Prop. 2. To find the Price.

Proposition 2. To know what an Annuity or Pension for a certain Time or Term, after a certain Rate by Compound Interest is worth, to buy or sell in ready Money.

Data under the second Prop.

The *Data* for the Resolution of Questions under this Classis will be either,

1 . 3 . 4 . The Arrerages, Annuity and Term.
1 . 3 . 5 . The Arrerages, Annuity and Rate.
1 . 4 . 5 . The Arrerages, Term and Rate.
Or 3 . 4 . 5 . The Annuity, Term and Rate.

These set as the former.

The Varieties here are set as the former in their retrograde Order.

Variety 4. Q. Of 5 l. per An. for 7 Years, what worth. By the 5th Theorem.

Variety 4. What is an Annuity of 5 *l.* a Year for 7 Years, worth in ready Money at 6 *l. per Cent. per Annum* by Compound Interest?

Answ. l. 27,91190, *&c.* For the Annuity, Term and Rate being given, and seeing in this Example $\beta\omega$ is 1,5036302589913б and $\beta-1$ is 0,06, according to the 5th Theorem it shall be, That,

$\beta-1$ in $\beta\omega$	$\beta\omega-1$	*Ann.*	*Price.*
0,090217815539481б .	0,5036302589913б ::	5 .	27,91190, *&c.*

By Progression.

And because by the *Data* α . 5 . T . 7 . and R . 1,06 . Z may be found according to the third Proposition for finding the *5th* Principal of a *Geometrical Progression*, and the *4th* Variety of the first Proposition here; then Z which will be 41,9691, *&c.* being divided by R. figurate to T. which is 1,50363, *&c.* giveth in the Quotient 27,91190, *&c.* as before.

By Logarithms.

But the work is most expeditious by Logarithms thus: Out of the Log. of the Principal Money, which at Simple Interest will in 1 Year raise such an Annuity after the Rate propounded, substract the Log. for the Time, and the absolute Number of the remaining Log. out of that Principal.

1,92081,87539	.	Log. of *l.* 83,33333, *&c.* as before *Prop.* 1.
0,17714,10571	.	Log. for the Time there also.
1,74367,76968	.	Log. of 55,4214, *&c.*
		27,9119, *&c.* Price.

Otherwise.

Or if from the Log. of the Annuity, added to the Log. for the Time lacking 1, be taken the Log. of the Rate lacking 1, added to the Log. for the Time; the Remain shall be the Log. of the desired Number, agreeable to the Work of the *5th* Theorem above-mentioned.

0,69897,00043	.	Log. of 5 . Annuity α.
—1,70211,18155	.	Log. of 0,50363, *&c.* Time—1 . or $\beta\omega-1$.
0,40108,18198	.	Sum . $\beta\omega-1$ in α.
—2,77815,12504	.	Log. of 0,06 . Rate—1 . or $\beta-1$.
0,17714,10571	.	Log. of 1,50363, *&c.* Time or $\beta\omega$.
—2,95529,23075	.	Sum $\beta-1$ in $\beta\omega$.
1,44578,95123	.	Log. of 27,91190, *&c.* Price.

Otherwise.

Also if the Arrerages be first found, the Log. for the Time taken out thereof, shall leave the Log. of the Price remaining.

1,62293,05694	.	Log. of 41,9691, *&c.* Arrerages, as before *Prop.* 1.
0,17714,10571	.	Log. for the Time.
1,44578,95123	.	Log. of 27,9119, *&c.* Price.

If the Paiment be Half-yearly. **Example.**

If this Annuity had been payable half Yearly, then the Principal Money, as appeareth under the former Proposition, being *l.* 84,5594, *&c.* the Price will be augmented to *l.* 28,32258, *&c.* as by the first work of Logarithms (most used because the shortest) here appeareth.

1,

1,92716,21254 . Log. of *l.* 84,55944, *&c.* Principal.
0,17714,10571 . Log. for the Time.
1,75002,10683 . Log. of 56,23686, *&c.*
28,32258, *&c.* Price.

And if the Paiment be Quarterly, when the Principal Money is had that will raise such a Quarterly Paiment at Simple Interest, the rest of the Work is alike. *If Quarterly.*

Variety 3. The Arrears of an Annuity detained 7 Years after the Rate of 6 *l. per. Cent. per Annum* by Compound Interest, do amount to 41,969188, *&c.* what is it worth in ready Money? Variety 3. Q. *Of Arrears in 7 Years, what worth.*

Answ. l. 27,91190, *&c.* The *Data* here being the Arrerages, Term and Rate; and because the Arrerages or Z may be accompted ω, or the last Term of a *Geometrical Progression*, when the Price which is here the Number sought shall be α or the first Term; and so by the Rules in the first Proposition, for finding the first Principal of a *Geometrical Progression*, the *Ratio* being figurate to T as aforesaid, may 27,91190, *&c.* be found by ω . 41,969, *&c.* T . 7 . and R . 1,06 . the *Data*: And this is also agreeable to the second Theorem. Answer. *By Progression.* *By the second Theorem.*

Wherefore by Logarithms, if the Log. for the Time be taken from the Log. of the Arrerages; the Remain shall be the Log. of the ready Money or Price desired. *By Logarithms.*

1,62293,05694 . Log. of 41,9691, *&c.* Arrerages.
0,17714,10571 . Log. for the Time.
1,44578,95123 . Log. of 27,9119, *&c.* Price.

Variety 2. If the Arrerages of an Annuity of 5 *l. per. Annum*, after the Rate of 6 *l. per. Cent. per Annum*, come to *l.* 41,96918, *&c.* what ready Money will purchase the same? Variety 2. Q. *Of Arrears of* 5 *l.* per Ann. *what worth.*

Answ. l. 27,9119, *&c.* The *Data* here are the Arrerages, Annuity and Rate; so the Work without Logarithms is thus: Answer.

Get (as before) the Principal Money, that at Simple Interest after the Rate propounded will raise such an Annuity, to this add the Arrerages, and divide the Sum by the Principal, then by the Quotient divide the Arrerages, and this last Quotient is the Price desired. *By Decimals.*

Accordingly the Rule of working by Logarithms is framed for avoiding the tedious Multiplications and Divisions thus: *By Logarithms.*

After the Principal Money, that at Simple Interest after the Rate propounded will raise the Annuity, is found and added to the Arrerages, from the Log. of the Sum take the Log. of that Principal Money, and take the remaining Log. from the Log. of the Arrerages.

1,92081,87539 . Log. of *l.* 83,3333, *&c.* Principal.
41,9691, *&c.* Arrerages.
2,09795,98110 . Log. of 125,3024, *&c.* Sum.
0,17714,10571 . Log. for the Time.
1,62293,05694 . Log. of 41,9691, *&c.* Arrerages.
1,44578,95123 . Log. of 27,9119, *&c.* Price.

Variety 1. How much ready Money will purchase an Annuity of 5 Pounds *per Annum* for seven Years, whose Arrerages forborn so long will amount to *l.* 41,96918, *&c.* Variety 1. Q. *Of Arrears of* 5 *l.* per Ann. *7 Years, what worth.*

Answ. l. 27,91190, *&c.* Here are given the Arrerages, Annuity and Term: And because the Arrerages are as Z in a *Geometrical Progression*, when the Annuity is α, by α . 5 . T . 7 . & Z . 41,9691, *&c.* the Rate may be found according to the third Rule in *Progression Geometrical*, for finding the 4*th* Principal; but with some Variation, because of the Decimal in the *Data* here, (the former being fitted for Integers): For by reason of the Retrogression of Figurate Decimals towards the right Hand, contrary to Integers which increase to the Left, it is very difficult to find out the highest Figurate Decimal (according to that Rule) to be taken out of the Quotient of Z divided by α, seeing greater Numbers must be left than those taken away. Yet inasmuch as the Quotient of Z divided by α, both here and there, is the Sum of a *Progression*, whose first Term or α is 1, and so consequently is 1, and the Sum of all the Terms of the *Ratio*, multiplied into it self figurately according to the Number of Terms lacking 1, the Contrariety lies only Answer. *By Progression.* *A difficult Proposition.*

only in this, That in Integers from the Quotient muſt be taken the higheſt Power contained therein, whoſe Index was T—1 : in Decimals to the Quotient muſt be added ſuch a Number as will make the Quotient a Power according to T, lacking 1, (in the left-hand Place) ; and inſtead there of *Addition*, let this Number found in the Quotient be multiplied by one or more of the Digits, till the Power deſired be produced ; and the Number ſo multiplying, ſhall be the Decimal Root to be added to the Antecedent of the *Ratio*, and the Antecedent ſhall always be the Root of 1, and ſo many Ciphers as there be Decimals.

Wherefore dividing 41,969188249280 (or Z) by 5 (that is α) the Quotient is 8,393837649856 : So ſhall this Number be the Sum of a Progreſſion beginning at 1, and the Arrears of 1 *l.* Annuity let run 7 Years, and contain therein a Zenzicube Number, whoſe Index is 6, (or T—1) but muſt be exalted to T, (that is, to a Power whoſe Index is 7) lacking only the Left-hand Unit. And becauſe the Decimals in the Quotient are 12, and the Number of Terms of that Power but 6, it is eaſy to diſcern the Root was Seconds, and that the 7*th* Power in Decimal Seconds with 1 Integer, cannot have leſs than 15 Places. Now if 8,393837649856 be multiplied by 2″ . 3″ . 4″ . 5″ . or any other Digit except 6″, and 1 prefixed to the left Hand, the Product will be no qqc, or Power of the 7*th* Quantity, but multiplied by 6″, produceth ,503630258991 36, which with 1 prefixed, and no other, is the Number ſought, that is, 1,503630258991 36, the ſecond Surſolid, or 7*th* Power of the Root 1,06, to which the Antecedent is 1,00, the √qqc of 1,00000000000000, or which is all one therewith, the √cc of 1,000000000000.

By Logarithms.

Thus then having found the *Ratio* thereby, with the other *Data*, the Price may be found by any of the other 3 Varieties for this ſecond Propoſition. And ſeeing this Figural Power found out, is always the Rate multiplied into it ſelf according to T, if Z be divided thereby, the Quotient will be the Price : And accordingly the Log. thereof taken from the Log. of Z, will leave remaining the Log. of the ready Money, or Price deſired. So as upon the whole Matter it is moſt eaſy to proceed, as in the ſecond Variety above.

1,62293,05694	. Log.	of	*l.* 41,969188, *&c.*	Z.
0,17714,10571	. Log.	of	1,503630, *&c.*	Time.
1,44578,95123	. Log.	of	27,911907, *&c.*	Price.

Prop. 3. *To find the Annuity.*

Propoſition 3. To know what Annuity or Penſion any Sum of ready Money will buy, for a certain Time or Term, after a certain Rate, by Compound Intereſt ?

Data under the third Prop.

The *Data* for the Reſolution of Queſtions under this Claſſis, will be either,

1 . 2 . 4 . The Arrerages, Price and Term.
1 . 2 . 5 . The Arrerages, Price and Rate.
1 . 4 . 5 . The Arrerages, Term and Rate.
Or 2 . 4 . 5 . The Price, Term and Rate.

How ſet.

The Varieties of this Claſſis being alike eaſy, are ſet in their proper Order.

Variety 1. *Q. Of Arrears for 7 Years, and Price, what the Annuity.*

Variety 1. An Annuity run in Arrears for 7 Years, will come to *l.* 41,96918, *&c.* and may be bought for *l.* 27,91190, *&c.* what was the Annuity ?

Anſwer.

Anſw. 5 *l.* a Year : For ſeeing the Arrerages of every Annuity, divided by the Price or ready Money, giveth in the Quotient the *Ratio* figurate according to the Number of Terms ; and here being given the Arrerages, Price and Term, from the Quotient may be extracted the Septupled Root : And then by the *Analogy*, according to the 4*th* or 6*th* Theorems, 5 may be found. For the Quotient of 41,96918, *&c.* divided by 27,91190, *&c.* being 1,503630258991 36, and the Septupled Root thereof 1,06, it ſhall by the

By the 4th and 6th Theorems.

	$\beta\omega$—1		β—1		*Arrerages.*		*Annuity.*	
4th Theor.	0,503630258991 36	.	0,06	::	41,969188249280	.	5	.

	$\beta\omega$—1		β—1 in $\beta\omega$		*Price.*		*Annuity.*	
6th Theor.	0,503630258991 36	.	0,090217815539 4816	::	27,9119, *&c*	.	5	.

By Logarithms.

And accordingly by Logarithms, take the Log. of the Price from the Log. of the Arrerages, the Remainder ſhall be the Log. for the Time ; that is, the Log. of the Rate multiplied by the Term, which divided by the Term, ſhall give in the Quotient the Log. of the Rate. And then the Log. of the Rate lacking 1, added to the Log. of the Arrears ; and from thence the Log. for the Time lacking 1 ſubſtracted, leaves remaining the Log. of the Annuity : Or elſe the Log. of the

the Rate lacking 1, and the Log. for the Time added to the Log. of the ready Money; and from thence the Log. for the Time lacking 1 substracted, shall leave remaining the Log. of the Annuity.

1,62293,05694 . Log. of 41,96918, *&c.* Arrerages.
1,44578,95123 . Log. of 27,91190, *&c.* Price.
0,17714,10571 . Log. for the Time 1,50363, *&c.*
7) 0,02530,58653 . Log. of the Rate 1,06.

The former way —1,62293,05694 . Log. of 41,96918, *&c.* Arrerages.
—2,77815,12504 . Log. of 0,06 R—1.
0,40108,18198 . Sum.
—1,70211,18155 . Log. of 0,50363, *&c.* Time—1.
0,69897,00043 . Log. of 5 Annuity.

The latter way —2,77815,12504 . Log. of 0,06 R—1.
0,17714,10571 . Log. of 1,50363, *&c.* Time.
1,44578,95123 . Log. of 27,91190, *&c.* Price.
0,40108,18198 . Sum.
—1,70211,18155 . Log. of 0,50363, *&c.* Time—1.
0,69897,00043 . Log. of 5. Annuity.

Variety 2. The Arrears of an Annuity to be forborn, at the Rate of 6 *l. per Cent. per Annum*, by Compound Interest, amounting to *l.* 41,969188, *&c.* are sold for *l.* 27,91190, *&c.* what was the Annuity? *Variety 2. Q. Of Arrears sold, what the Annuity.*

Answ. 5 *l.* a Year as before: Here being given the Arrerages, Price and Rate, the Arrerages 41,9691, *&c.* divided by 27,9119, *&c.* as in the last Variety, giveth the Power of the Root figurate according to T, as aforesaid in the Quotient, that is 1,50363, *&c.* and then Resolution may be had, as in the first Variety of this third Proposition, according to the Theorems, or by Logarithms. *Answer.* *By the Theorems and Logarithms.*

1,62293,05694 . Log. of 41,9691, *&c.* Arrerages.
1,44578,95123 . Log. of 27,9119, *&c.* Price.
0,17714,10571 . Log. of 1,5036, *&c.* Time.

The former way 1,62293,05694 . Log. of 41,9691, *&c.* Arrerages.
—2,77815,12504 . Log. of 0,06 R—1.
0,40108,18198 . Sum.
—1,70211,18155 . Log. of 0,5036, *&c.* Time—1.
0,69897,00043 . Log. of 5. Annuity.

The latter way —2,77815,12504 . Log. of 0,06. R—1.
0,17714,10571 . Log. of 1,5036, *&c.* Time.
1,44578,95123 . Log. of 27,9119, *&c.* Price.
0,40108,18198 . Sum.
—1,70211,18155 . Log. of 0,5036, *&c.* Time—1.
0,69897,00043 . Log. of 5. Annuity.

Variety 3. An Annuity detained 7 Years, with the Compound Interest, after the Rate of 6 *l. per Cent. per Annum*, did amount to *l.* 41,9691, *&c.* how much was the Annuity? *Variety 3. Q. Of Arrears in 7 Years, what the Annuity.*

Answ. 5 *l.* a Year. Seeing the *Data* here being the Arrerages, Term and Rate answering to T . R . and Z. of a *Geometrical Progression*, and the Annuity sought, as α the first Term, that is T . 7 . R . 1,06, and Z . 41,9691, *&c.* the Rules in the 4*th* Proposition for finding the first Principal there, will give Resolution here. And seeing the Rate septupled according to T, is 1,50363025899136, by the 4*th* Theorem; *Answer.* *By Progression.* *By the 4th Theorem.*

ρω—1		ρ—1		*Arrerages.*		*Annuity.*
0,50363025899136	.	0,06	::	41,9691882492 80	.	5

And accordingly by Logarithms: If from the Log. of the Arrerages added to the Log. of the Rate lacking 1, be taken the Log. for the Time lacking 1, the Remainder shall be the Log. of the Annuity. *By Logarithms.*

1,

1,62293,05694	. Log. of	41,96918, &c.	Arrerages.
—2,77815,12504	. Log. of	0,06	R—1.
0,40108,18198	. Sum.		
—1,70211,18155	. Log. of	0,50363, &c.	Time—1.
0,69897,00043	. Log. of	5.	Annuity.

Otherwise.

And because the Arrears of any Annuity, divided by the Annuity it self, always giveth in the Quotient the Arrears of 1 *l.* Annuity let run according to T: And contrary, the Arrears of 1 *l.* Annuity for the Term propounded, dividing the Arrears given, shall be the Annuity, and the Difference of their Logarithms accordingly: If then by the first Proposition be gotten the Arrears of 1 *l.* Annuity forborn for the Term given, let the Log. thereof be taken from the Log. of the Arrerages propounded, and the Remainder is the Log. of the Annuity desired.

The Principal Money to raise 1 *l.* Annuity at Simple Interest, after the Rate propounded by the 4*th* Variety of the first Proposition, is found to be 16 *l.* 13 *s.* 4 *d.* and in Decimals 16,6666, &c. the Residue of the Work follows.

1,22184,87496	. Log. of *l.* 16,66666, &c. or 16 *l.* 13 *s.* 4 *d.* Principal, to raise 1 *l.* Annuity at Simp. Interest
0,17714,10571	. Log. for the Time.
1,39898,98067	. Log. of 25,06050, &c.
	Arrerages 8,39383, &c. of 1 *l.* Annuity 7 Years.
1,62293,05694	. Log. of 41,96918, &c. Arrerages given.
0,92396,05651	. Log. of 8,39383, &c. Arrerages of 1 *l.* Annuit.
0,69897,00043	. Log. of 5. Annuity.

Variety 4. *Q. Of the Price to buy: what Annuity.*

Variety 4. What yearly Annuity will *l.* 27,91190, &c. in ready Money purchase for 7 Years, after the Rate of 6 *l.* on the 100 for a Year, by Compound Interest?

Answer. *By the sixth Theorem.*

Answ. 5 *l.* a Year: For here are given the Price, Term and Rate. So by the 6*th* Theorem, If the Price be multiplied by the Product of the Rate lacking 1, led into the Rate figurate according to the Term, and this Product be divided by the said Rate so figurate lacking 1, the Quotient shall be the Annuity desired, as before under the first Variety of this Proposition.

	ßω—1		ß—1 in ßω		*Price.*		*Annuity.*
As	0,50363025899136	.	0,09021781553948 16	::	27,9119, &c.	.	5 .

By Logarithms.

And by Logarithms, the Log. for the Time lacking 1, shall be taken out of the Log. for the Time added to the Log. of the Rate lacking 1, and the Log. of the Price.

0,17714,10571	. Log. of 1,50363, &c. Time or ßω.
—2,77815,12504	. Log. of 0,06 R—1 . or ß—1.
1,44578,95123	. Log. of 27,91190, &c. Price.
0,40108,18198	. Sum.
—1,70211,18155	. Log. of 0,50363, &c. Time—1 . or ßω—1.
0,69897,00043	. Log. of 5. Annuity.

Otherwise.

Or if by the second Proposition be gotten the ready Money, that after the Rate propounded will buy 1 *l.* Annuity yearly for the Term given, then substract the Log. thereof out of the Log. of the ready Money given, and you have the Log. of the Annuity; because the Price propounded, divided by the Annuity, will always bring in the Quotient the Price of 1 *l.* Annuity to continue according to T. And contrary, the Price propounded divided by the Price of 1 *l.* Annuity according to T, shall give the Annuity; and accordingly the Difference of their Logarithms.

The Principal Money to raise 1 *l.* Annuity at Simple Interest, after the Rate propounded, being found as before to be 16 *l.* 13 *s.* 4 *d.* the Work appeareth thus:

1,

1,22184,87496 . Log. of 16,66666, &c. or 16 *l.* 13 *s.* 4 *d.* Principal in 1 Year, to raise 1 *l.* Annuity.
0,17714,10571 . Log. for the Time.
1,04470,76925 . Log. of 11,08428, &c.
Ready Money 5,58238, &c. to buy 1 *l.* Annuity 7 Years.

1,44578,95123 . Log. of 27,9119, &c. Price given.
0,74681,95080 . Log. of 5,5823, &c. Price of 1 *l.* Annuity.
0,69897,00043 . Log. of 5, Annuity.

And in all the Varieties of this Classis, the like may be done for Annuities after other Rates, and for Half-yearly and quarterly Paiments if occasion be. *For Half-yearly and quarterly Paiments.*

Proposition 4. To know in what Time or Term a Sum of Money propounded will be paid by an Annuity according to a certain Rate given: or how long an Annuity was detained to increase to a Sum propounded, after a certain Rate, by Compound Interest. Prop. 4. *To find the Time or Term.*

The *Data* for the Resolution of Questions under this Classis, will be either, Data *under the 4th Prop.*

1 . 2 . 3 . The Arrerages, Price and Annuity.
1 . 2 . 5 . The Arrerages, Price and Rate.
1 . 3 . 5 . The Arrerages, Annuity and Rate.
Or, 2 . 3 . 5 . The Price, Annuity and Rate.

The Varieties here, as the last Precedent, follow in their proper order. *How placed.*

Variety 1. An Annuity of 5 *l.* a Year, if the Arrerages by Compound Interest be forborn a certain Time or Term, will amount to *l.* 41,96918, &c. and may be bought for *l.* 27,91190, &c. ready Money: what is the Time or Term of that Annuity? Variety 1. Q. *Of* 5 *l.* per Ann. *such Arrears and Price, what the Time.*

Answ. 7 Years. The *Data* here, are the Arrerages, Price and Annuity: And because the Arrerages divided by the Price, give in the Quotient always the Rate multiplied into it self, according to the Term, and the Difference of their Logarithms is the Log. thereof; it follows, that the Index of the Highest Power of the Quotient shall be the Term desired. So the Quotient of their Division being 1,50363025899136, is a Power of the 7*th* Quantity arising from the Root 1,06. Answer. *By Decimals.*

And accordingly the Log. of the Annuity added to the Log. of the said Power lacking 1, and from the Sum the Log. of the Arrerages substracted, leaveth the Log. of the Rate lacking 1: wherefore the Difference between the Log. of the Rate, and the Antecedent thereof, dividing the Difference between the Logarithms of the Arrerages and Price, shall give the Term desired. *By Logarithms.*

1,62293,05694 . Log. of 41,9691, &c. Arrerages.
1,44578,95123 . Log. of 27,9119, &c. Price.
0,17714,10571 . Log. of 1,5036, &c. Time. $\beta\omega$.
—1,70211,18155 . Log. of 0,5036, &c. $\beta\omega—1$.
0,69897,00043 . Log. of 5. Annuity.
0,40108,18198 . Sum.
1,62293,05694 . Log. of 41,9691, &c. Arrerages.
—2,77815,12504 . Log. of 0,06. R—1 or $\beta—1$.
0,02530,58653 . Log. of 1,06. R.
0,00000,00000 . Log. of 1,00. Antecedent.

$\dfrac{,17714,10571}{2530,58653}$ (*Years.* 7 . Term.

Variety 2. The Arrerages of an Annuity, accompted by Compound Interest, after 6 *l. per Cent. per Ann.* were *l.* 41,9691, &c. and purchased for *l.* 27,9119, &c. For what Term of Years was the Annuity to endure? Variety 2. Q. *Of Arrears and Price, what the Time at such Rate.*

Answ. 7 Years. In this Variety the *Data* being the Arrerages, Price and Rate, and the Price being as α, the Arrerages as ω, and the Rate R of a *Geometrical Progression* to find T, the Question may be resolved by the Rules in the first Answer. *By Progression.*

Proposition for finding the third Principal of such a Progression, observing only that the Number found out here will be the Rate figurate according to T, but there lack 1.

α . 1 . ω . R figurate to T.
For 27,9119, *&c* . 1 :: 41,9691, *&c* . 1,50363025899136.

And so this Number being a Power, (as was noted in the last Variety above) the Index of the highest Power therein shall be the Term desired, and the Rate being the Root thereof, dividing it till the Quotient be an Unit, shall shew the Index by the Number of Divisions, to which it will always be equal.

By Logarithms. But shortning the Work by Logarithms, the Process is thus; take the Log. of the Price from the Log. of the Arrerages, and divide the Log. remaining by the Log. of the Rate.

1,62293,05694	. Log. of 41,9691, *&c.*	Arrerages.
1,44578,95123	. Log. of 27,9119, *&c.*	Price.
0,17714,10571	. Log. of 1,5036, *&c.*	Time.
0,02530,58653	. Log. of 1,06.	Rate.

$\frac{,17714,10571}{2530,58653}$ (*Years.* 7 . Term.

Variety 3. Q. Of 5 l. per Ann. and Arrerages, in what Time. *Variety* 3. If an Annuity of 5 *l.* yearly detained, with the Compound Interest, after the Rate of 6 *l. per Cent. per Annum*, increase to *l.* 41,9691, *&c.* how long was it detained?

Answer. *Answ.* 7 Years: Here the *Data* are the Arrerages, Annuity and Rate answering to α . R . Z of a *Geometrical Progression* to find T. that is α . 5 . R . 1,06 . and Z . 41,96918, *&c.* So by the Rules in the third Proposition, for finding the third Principal of such a Progression, the Question will be answered. *By Progression.*

For 5 . 41,9691, *&c.* :: 0,06 . 0,50363025899136.

And this Number found being the Ratio figurate to T—1, answering to the Log. for the Time lacking an Unit; if 1 be added, it shall be a Power, the Index whereof shall be the desired Term.

By Logarithms. And so to the absolute Number of the remaining Log. when the Log. of the Annuity is taken from the Sum of the Log. of the Arrerages, and the Rate lacking 1, add 1, and divide the Log. thereof by the Log. of the Rate.

1,62293,05694	. Log. of	41,9691, *&c.*	Arrerages Z.
—2,77815,12504	. Log. of	0,06	R—1 or ꞵ—1
0,40108,18198	. Sum.		
0,69897,00043	. Log. of	5.	Annuity α.
—1,70211,18155	. Log. of	0,5036, *&c.*	Time—1, or ꞵω—1.
0,17714,10571	. Log. of	1,5036, *&c.*	Time, or ꞵω.

Rate . 0,02530,58653 (*Years.* 7 . Term.

Variety 4. Q. Of 5 l. per Ann. bought for l. 27,9119, &c. for what Time. *Variety* 4. If *l.* 27,9119, *&c.* were to be paid by 5 *l. per Annum*: in what Time will it be paid, with allowance of 6 *l. per Cent. per Annum*?

Answer. *Answ.* In 7 Years: For here being given the Price, Annuity and Rate, the Work, without Logarithms, is to get the Principal Money, which at Simple Interest, after the Rate propounded, will raise such an Annuity, and take the Price out thereof, and by the Remainder divide the same Principal, this being a Figural Number, whereof the Rate is the Root, divided by the Root, till the Root be brought in the Quotient, the Number of the several Divisions shall be the desired Term. *By Decimals.*

By Logarithms. And much sooner by Logarithms, for after finding the Principal Money, which at Simple Interest in one Year, after the Rate propounded, will raise the Annuity; from the same Principal take the given Price, and the Log. of the Remain from the Log. of that Principal, and divide the remaining Log. by the Log. of the Rate.

1,

1,92081,87539 . Log. of 83,3333, *&c.* Principal to raise 5 *l.* a Year.
27,9119, *&c.* Price.
1,74367,76968 . Log. of 55,4214, *&c.*
0,17714,10571 . Log. of 1,5036, *&c.* Time, or $\beta\omega$.
Years.
Rate. 0,02530,58653 7 . Term T.

In like manner may the Term be found for Half-yearly Paiments, Quarterly, or others, and at other Rates as the Case may require, under all the Varieties of this Classis.

For other Rates, and half-yearly and quarterly Paiments.

Proposition 5. To know after what Rate, by Compound Interest, any Annuity did increase to a Sum propounded; or the Rate the Compound Interest is reckoned at by the ready Money the Annuity is sold for to endure a certain Time or Term.

Prop. 5. To find the Rate.

The *Data* for the Resolution of Questions under this Classis, will be either

Data under the fifth Prop.

1 . 2 . 3 . The Arrerages, Price and Annuity.
1 . 2 . 4 . The Arrerages, Price and Term.
1 . 3 . 4 . The Arrerages, Annuity and Term.
or 2 . 3 . 4 . The Price, Annuity and Term.

The first two of these Varieties agree in Facility, as the two latter in Difficulty, with some of the former.

Set in order, but the latter 2 harder than the first.

Variety 1. The Arrears of an Annuity of 5 *l.* a Year to endure a certain Time, with Interest upon Interest, were reckoned to amount to *l.* 41,96918, *&c.* and may be bought for *l.* 27,9119, *&c.* after what Rate was the Interest reckoned?

Variety 1. Q. Of 5 l. per Ann. such Arrears and Price, what the Rate.

Answ. After 6 *l. per Cent. per Annum.* The *Data* here, *viz.* the Arrerages, Price and Annuity, being the same as in the first Variety of the 4*th* Proposition, the Work is the same as there, till the last Division to find the Term.

By Logarithms.

1,62293,05694 . Log. of 41,96918, *&c.* Arrerages.
1,44578,95123 . Log. of 27,91190, *&c.* Price.
0,17714,10571 . Log. of 1,50363, *&c.* Time. $\beta\omega$.
—1,70211,18155 . Log. of 0,50363, *&c.* $\beta\omega - 1$.
0,69897,00043 . Log. of 5. Annuity.
0,40108,18198 . Sum.
1,62293,05694 . Log. of 41,96918, *&c.* Arrerages.
—2,77815,12504 . Log. of 0,06. R—1, or $\beta - 1$.
1,00. Antecedent added.
1,06. Rate, or β.

Variety 2. For *l.* 27,9119, *&c.* ready Money, one may buy 7 Years Arrerages of an Annuity; which cast up by Compound Interest, amounts to *l.* 41,9691, *&c.* after what Rate is the Interest accompted?

Variety 2. Q. Of an Annuity for 7 Years, such Arrears and Price, what the Rate.

Answ. After 6 *l.* on the 100 for a Year. Here the *Data* being the Arrerages, Price and Term, the same as in the first Variety of the third Proposition, the Work shall be as the former Part thereof, seeing the Rate was there found to get the Annuity thereby.

By Logarithms.

1,62293,05694 . Log. of 41,96918, *&c.* Arrerages.
1,44578,95123 . Log. of 27,91190, *&c.* Price.
0,17714,10571 . Log. for the Time 1,50363, *&c.*
7) 0,02530,58653 . Log. of the Rate 1,06.

And this agreeth with the Rules in the first Proposition for finding the 4*th* Principal of a *Geometrical Progression*, seeing the Price is as α, the Arrerages as ω. So by $\alpha . \omega .$ T. may R be found, with this only Difference, that here the Index of the Quotient is T, but there T—1.

By Progression.

Variety 3. An Annuity of 5 *l.* a Year, with the Compound Interest thereof in 7 Years, amounted to *l.* 41,96918, *&c.* after what Rate on the 100 did it increase?

Variety 3. Q. Of 5 l. per Annum 7 Years such Arrears, what the Rate.

Answ.

Answer.

Answ. As before, after 6 *l.* on the 100 for a Year: The Arrerages, Annuity and Term here are given, and are the same *Data* as in the first Variety of the second Proposition: And because there the *Ratio* was gotten before the Price could be found, the Work here will be the same for the *Ratio*, and by Logarithms thus after this way.

By Logarithms.

Take the Log. of α, the Annuity from the Log. of Z the Arrerages, and the Remainder is the Log. of the Arrerages of 1 *l.* let run according to T; to which a Log. being added, that will make the Sum the Log. of a Power lacking 1, whose Index shall be T, which in this Example will be done by the Log. of 0,06 the *Ratio* lacking 1, and the Sum will be the Log. of 0,5036302589913 6, which is $\rho\omega$—1. And then without farther Work, add 1 the Antecedent to the Absolute Number of the Log. so added, and you have the Rate 1,06. See more at the end of the next Variety.

1,62293,05694	. Log. of 41,96918, *&c.*	Arrerages.	
0,69897,00043	. Log. of 5.	Annuity.	
0,92396,05651	. Log. of 8,39383, *&c.*	Arrerages of 1 *l.* in 7 Years.	
—2,77815,12504	. Log. of 0,06	R—1,	or ρ—1.
—1,70211,18155	. Log. of 0,50363, *&c.*	Time—1,	or $\rho\omega$—1.

Ergo, to	0,06	
add	1,00	Antecedent.
	1,06	Rate.

Variety 4. Q. Of 5 l. per Ann. 7 Years, such Price, what the Rate. Answer.

Variety 4. If *l.* 27,91190, *&c.* in ready Money, purchase an Annuity of 5 *l.* a Year, to endure for 7 Years: after what Rate on the 100 is the Compound Interest thereof reckoned?

Answ. After 6 *l. per Cent. per Annum.* For seeing the *Data* here, *viz.* the Price, Annuity and Term, agree with the *Data* of the first Variety of the first Proposition, and the *Ratio* there was found thereby before the Arrerages could be found, the Work to get the same here will be the same, and so needs no Repetition, but is a most difficult Proposition to resolve, as there was observed, because it is hard to find a Number to multiply the ready Money by, that shall produce the Arrerages of the Annuity forborn to T. which in this Example must be 1,503630258991 36, and no other; and accordingly the Log. thereof added to the Log. of the Price, maketh the Sum the Log. of 41,96918, *&c.* the Arrerages, or Z of a Progression, where 5 is α; and the last Term of a Progression or ω, where 27,9119, *&c.* is α. But both in this and the last Variety, it is best to divide by the Time the Log. for the Time, which may be gotten by the first or second Varieties of the first and second Propositions of Annuities.

By Logarithms.

1,44578,95123	. Log. of 27,9119, *&c.*	Price.
0,17714,10571	. Log. of 1,5036, *&c.*	Time.
1,62293,05694	. Log. of 41,9691, *&c.*	Arrerages.
&c.		

Of Interest and Annuities mixt.

Touching Interest and Annuities intermixt.

Propositions many.

Because the Principal Propositions about Interest are 4, and about Annuities are 5, that is 9 in all as aforesaid, such a vast Number of Propositions concerning them, as well amongst themselves as mixed one with another, according to the foregoing Chapter of *Transmutation*, might be thence deduced, as would weary the Reader: But the most useful are such as follow, and relate more especially to the second and third Propositions of Annuities, or the first or second of Interest already handled.

Those most useful.

Prop. 1. *To put a Value on Wares sold for Time.*

Prop. 1. To put a Value, present or future, on Commodities sold according to the Time given for Paiment.

For the Future.

By the first Proposition of Interest, add the Log. for the Time, to the Log. of the Value in ready Money, and the Sum is the Log. of the future Value.

Q. Calico what worth at 9 Months.

Example. Calico is worth 25 *d.* the Yard ready Money: what is it worth to be paid at 9 Months end, accompting 10 *per Cent. per Annum* by Compound Interest?

Answ.

Answ. $26\frac{3}{4}$ *d.* and somewhat more. Answer.

0,04139,26852 . Log. of 1,10. R.
0,03104,45139 . Log. for the Time $\frac{3}{4}$, or 9 Months.
1,39794,00087 . Log. of 25.
1,42898,45226 . Log. of 26,85, *&c.*

On the contrary, by the second Prop. of Interest, take the Log. for the Time from the Log. of the future Value, and the Difference is the Log. of the Present. *For the present.*

Wherefore the Log. 0,03104,45139, taken from the Log. of 26,85, *&c.* shall leave the Log. of 25.

Prop. 2. To ballance an Accompt present or future, of several Merchants, where divers Sums of Money are paid and received by each of them, to be reckoned with the Compound Interest. Prop. 2. *To ballance an Accompt.*

According to the Times of the several Receipts and Paiments by the first Proposition of Interest, the Charge and Discharge of every Person is to be gotten, by adding the Log. for the Time between the Accompt, and such Receipt or Paiment accordingly, to the same Receipt or Paiment for the future Ballance of the Accompt. *For the future.*

Example. A and *B* have paid divers Sums of Money one to and for another; and received of and for one another divers other Sums between the 25*th* Day of *March* 1611, and the 27*th* Day of *March* 1613, according to their particular Accompts following: they agree to clear Accompts, and allow each to other 10 *per Cent. per Annum*, Interest upon Interest; and that their Reckoning shall conclude upon the 27*th* Day of *March* 1613, and demand which of them is indebted to the other, and how much? Q. *Of divers Sums paid and received between* A *and* B, *which in debt.*

The Accompt of A. *Particular Accompt of* A.

		l.			l.
Mar. 27. 1611.	Received of *B*	200	*June* 27. 1611.	Paid to *B*	100
Sept. 27.	Received for Accompt partable in halves between them	260		More then paid for Accompt partable in halves between them	200
Dec. 27.	Received of *B*	300	*June* 27. 1612.	Paid to *B*	300
Mar. 27. 1612.	Received for Accompt partable in thirds, *viz.* $\frac{1}{3}$ for himself, and $\frac{2}{3}$ for *B*	210	*Sept.* 27.	Paid for Accompt, partable in Thirds, *viz.* $\frac{1}{3}$ for himself, and $\frac{2}{3}$ for *B*	300
Sept. 27.	Received of *B*	200	*Dec.* 27.	Paid to *B*	300

The Accompt of B. *Particular Accompt of* B.

		l.			l.
June 27. 1611.	Received of *A*	100	*Mar.* 27. 1611.	Paid to *A*	200
Sept. 27.	Rec. for Accompt partable in Thirds, *viz.* $\frac{1}{3}$ for *A* & $\frac{2}{3}$ for himself	450	*June* 27.	Paid for Accompt partable in halves between them	300
June 27. 1612.	Received of *A*	300	*Dec.* 27.	Paid to *A*	300
Sept. 27.	Received for Accompt partable in halves between them	300	*June* 27. 1612.	Paid for Accompt partable in Thirds, *viz.* $\frac{1}{3}$ for *A* & $\frac{2}{3}$ for himself	300
Dec. 27.	Received of *A*	300	*Sept.* 27.	Paid to *A*	200

Accompt of A *cast up.*

The Charge of *A*.	The Discharge of *A*.
0,08278,53704. *Log. of* R *in* 2 *Years.* 2,30102,99957. *Log. of* 200 *received.*	0,07243,71991. *Log. of* R *in* $1\frac{3}{4}$ *Year.* 2,00000,00000. *Log. of* 100 *paid.*
2,38381,53661. *Log. of* 242.	2,07243,71991. *Log. of* 118,150, &c.
0,06208,90278. *Log. of* R *in* $1\frac{1}{2}$ *Year.* 2,11394,33523. *Log. of* 130 *received.*	0,07243,71991. *Log. of* R *in* $1\frac{3}{4}$ *Year.* 2,00000,00000. *Log. of* 100 *paid.*
2,17603,23801. *Log. of* 149,979, &c.	2,07243,71991. *Log. of* 118,150, &c.
0,05174,08565. *Log. of* R *in* $1\frac{1}{4}$ *Year.* 2,47712,12547. *Log. of* 300 *received.*	0,03104,45139. *Log. of* R *in* $\frac{3}{4}$ *Year.* 2.47712,12547. *Log. of* 300 *paid.*
2,52886,21112. *Log. of* 337,957, &c.	2,50816,57686. *Log. of* 322,229, &c.
0,04139,26852. *Log. of* R *in* 1 *Year.* 2,14612,80357. *Log. of* 140 *received.*	0,02069,63426. *Log. of* R *in* $\frac{1}{2}$ *Year.* 2,30102,99957. *Log. of* 200 *paid.*
2,18752,07209. *Log. of* 154.	2,32172,63383. *Log. of* 209,761, &c.
0,02069,63426. *Log. of* R *in* $\frac{1}{2}$ *Year.* 2,30102,99957. *Log. of* 200 *received.*	0,01034,81713. *Log. of* R *in* $\frac{1}{4}$ *Year.* 2,47712,12547. *Log. of* 300 *paid.*
2,32172,63383. *Log. of* 209,761, &c.	2,48746,94260. *Log. of* 307,234, &c.
0,07243,71991. *Log. of* R *in* $1\frac{3}{4}$ *Year.* 2,17609,12591. *Log. of* 150 *paid by* B.	0,06208,90278. *Log. of* R *in* $1\frac{1}{2}$ *Year.* 2,17609,12591. *Log. of* 150 *received by* B.
2,24852,84582. *Log. of* 177,226, &c.	2,23818,02869. *Log. of* 173,053, &c.
0,03104,45139. *Log. of* R *in* $\frac{3}{4}$ *Year.* 2,00000,00000. *Log. of* 100 *paid by* B.	0,02069,63426. *Log. of* R *in* $\frac{1}{2}$ *Year.* 2,17609,12591. *Log. of* 150 *received by* B.
2,03104,45139. *Log. of* 107,409, &c.	2,19678,76017. *Log. of* 157,321, &c.

Collection of the Charge.	*Collection of the Discharge.*
242.	118,1509, *&c.*
149,9796, *&c.*	118,1509, *&c.*
337,9575, *&c.*	322,2298, *&c.*
154.	209,7617, *&c.*
209,7617, *&c.*	307,2341, *&c.*
177,2264, *&c.*	173,0534, *&c.*
107,4099, *&c.*	157,3213, *&c.*
1378,3351	1405,9021

Total Charge of *A*, *l.* 1378,3351.
Rest due to *A*, 27,5670.
1405,9021. Total Discharge of *A*.

Accompt of B *cast up.*

And by like Examination of the Accompt of *B*, there will be found in his Hands the Sum of *l.* 27,5670, *&c.* due to *A* as aforesaid, seeing the Charge of the one is the Discharge of the other.

The Charge of B.		*The Discharge of* B.	
Dr. $1\frac{3}{4}$ Year by 100 R of A.	118,1509	Cr. 2 Years to 200 Pd to A.	242.
$1\frac{1}{2}$ Year by 150 R for A.	173,0534	$1\frac{3}{4}$ Year to 150 Pd for A.	177,2264
$0\frac{3}{4}$ Year by 300 R of A.	322,2298	$1\frac{1}{4}$ Year to 300 Pd to A.	337,9575
$0\frac{1}{2}$ Year by 150 R for A.	157,3213	$0\frac{3}{4}$ Year to 100 Pd for A.	107,4099
$0\frac{1}{4}$ Year by 300 R of A.	307,2341	$0\frac{1}{2}$ Year to 200 Pd to A.	209,7617
$1\frac{3}{4}$ Year by 100 Pd by A.	118,1509	$1\frac{1}{2}$ Year to 130 R by A.	149,9796
$0\frac{1}{2}$ Year by 200 Pd by A.	209,7617	1 Year to 140 R by A.	154.
B Debitor	1405,9021	*B* Creditor	1378,3351

Total Discharge of *B*, *l.* 1378,3351.
Rest in the Hands of *B*, 27,5670
1405,9021 Total Charge of *B*.

So

So to ballance this Accompt, it appears that *B* is indebted to *A l.* 27,5670, or 27 *l.* 11 *s.* 4 *d.* &c. due on the Foot of his Accompt, to be paid *March* 27. 1613.

Ballance B *indebted to* A.

On the contrary, to ballance an Accompt at present, when the Paiments are to be made afterward, by the second Prop. of Interest take the Log. for the Time gotten, as aforesaid, from the Log. of the Monies so to be paid or received.

For the present.

Example. A owes to *B* 800 *l.* to be paid at 4 Years end: And *B* is indebted to *A* 900 *l.* to be paid in 6 Years, that is to say, at every 2 Years end 300 *l.* Now they agree to clear their Debts presently, allowing each other 8 *per Cent. per Annum*, Interest upon Interest: The Question is, which of them must pay Money to the other, and how much?

Q. *Of the Debts of* A *and* B *to be paid in Time, how to clear at present.*

The Charge of A.

2,90308,99870	Log. of 800.
0,13369,50220	Log. of R in 4 Years.
2,76939,49650	Log. of 588,023.

Collection of the Discharge.

257,201, *&c.*
220,509, *&c.*
189,051, *&c.*
666,761, *&c.*

The Discharge of A.

2,47712,12547	Log. of 300.
0,06684,75110	Log. of R in 2 Years.
2,41027,37437	Log. of 257,201.
2,47712,12547	Log. of 300.
0,13369,50220	Log. of R in 4 Years.
2,34342,62327	Log. of 220,509.
2,47712,12547	Log. of 300.
0,20054,25330	Log. of R in 6 Years.
2,27657,87217	Log. of 189,051.

Total Charge of *A*	*l.* 588,023, *&c.*	
Rest due to *A*	78,738, *&c.*	
	666,761, *&c.*	Total Discharge of *A*.

And by like examination of *B* his Accompt, so much over-ballance will be found in his Hands as is due to *A* on the Foot of his Accompt; and therefore *B* must pay to *A*, to ballance the Accompt, *l.* 78,738, *&c.* or 78 *l.* 14 *s.* 9 *d.* *&c.* in ready Money.

Ballance B *indebted to* A.

Prop. 3. To discover what ready Money is to be paid for a Lease, when certain Years of the Term are of greater Value than the Rest, and so consequently to bring several Annuities issuing out of the same Lands into one Paiment.

Prop 3. *To find the ready Money paid for a Lease, when part of the Term of more Value than the rest: Or to bring divers Annuities into one.*

Suppose the greatest Annuity or Annual Profit, to begin and continue for the whole Term or Number of Years propounded: Then according to the 4*th Variety* of the Second *Proposition* of Annuities, find what such an Annuity or Rent is worth in ready Money. And also in like manner having substracted the lesser Rent or Annuity from the Greater, find what the Difference of the Rents or Annuities to continue for the Term of the least Annuity is worth in ready Money: Then abate this ready Money from the former, and the remainder is the ready Money that ought to be paid for the Lease, or both Annuities; and with this ready Money, according to the 4*th* Variety of the third Proposition of Annuities, is to be found the Annuity or Rent to continue for the whole Term in lieu of the other.

Example. Certain Lands stand charged with the Paiment of 40 *s. per Annum* for 13 Years, and afterwards for paiment of 10 *l. per Annum* for 17 Years: Or suppose a Lease for 30 Years to come, whereof the present yearly Profit all Out-Rents paid be 2 *l.* but after 13 Years expired, it will be worth 10 *l.* Yearly, all out-Rents paid. If this Lease be offered to sale, what may be given for it, reckoning $6\frac{1}{4}$ *per Cent. per Annum*, Interest upon Interest? Or if the Owner of the Lands agree with the Party to whom the aforesaid Annuities are to be paid, to reduce both into one for the whole 30 Years; the Question is, what that new Annuity ought to be at the Rate aforesaid by Compound Interest?

Q. *Of Lands charged with* 2 *Annuities, what worth to be sold for the Term of a Lease,* &c.

Answ. The Purchase of the Lease will be found worth in ready Money, *l.* 64,245, *&c.* which will buy an Annuity of 4 *l.* 15 *s.* 10 *d.* and something above, for 30 Years.

Answer.

O,

0,02632,53434 . Log. of R. $6\frac{1}{4}$, or 1,0625.
30 . T.
0,78976,03020 . Log. for the whole Time.
2,20411,99827 . Log. of 160 Principal, to raise 10 *l.* the greatest Annu.
1,41435,96807 . Log. of 25,963, *&c.*
Ready Money 134,037—to buy 10 *l. per Annum* 30 Years.

Now seeing for 13 Years of these 30, the Annuity or Annual Profit is but 2 *l. per Annum*, this is to be taken from 10 *l. per Annum*, and there remaineth 8 *l. per Annum*; and the 134,037—before found being too much, so much as 8 *l. per Annum* is worth in ready Money for 13 Years, therefore in like manner the Price thereof is found.

0,02632,53434 . Log. of R as aforesaid.
13 . T.
0,07897,60302
0,26325,3434
0,34222,94642 . Log. for 13 Years.
2,10720,99696 . Log. of 128 Principal. to raise 8 *l.* the Difference of Ann.
1,76498,05054 . Log. of 58,208, *&c.*
Ready Money 69,792— to buy 8 *l. per Annum* 13 Years.

This *l.* 69,792—taken from *l.* 134,037—leaves *l.* 64,245, *&c.* for the Value of the Lease, which will buy an Annuity of *l.* 4,7931, *&c.* or 4 *l.* 15 *s.* 10 *d*, &c. *per Annum* for 30 Years.

1,20411,99827 . Log. of 16 Principal, to raise 1 *l.* Annuity at Simple Interest, after the Rate of $6\frac{1}{4}$ *per Cent.*
0,78976,03020 . Log. for the Time of 30 Years.
0,41435,96807 . Log. of 2,5963, *&c.*
Ready Money 13,4037—to buy 1 *l.* Annuity for 30 Years.

1,80783,93335 . Log. of 64,245, *&c.* Price given.
1,12722,46987 . Log. of 13,4037, *&c.* Price of 1 *l.* Annuity.
0,68061,46348 . Log. of 4,7931, *&c.* New Annuity to continue 30 Years.

When the several Annuities are payable together.

If the several Annuities be issuant together, then get the Price of them severally, and add them together for the Sum to buy the Annuity desired, and the Total taken from the Price of the Whole, shews the Price of the Purchase so charged with the Annuities.

Q. Of a Farm to be sold, charged with 2 Annuities, what the Purchaser may keep in hand to satisfy for them.

Example. A Farm is to be sold for 680 *l.* which stands charged with the Paiment of 30 *l. per Annum*, viz. 7 *l.* 10 *s. per* Quarter for 2 Years: And also 9 *l. per Annum*, viz. 2 *l.* 5 *s. per* Quarter for 7 Years: how much Money may the Purchaser retain in his Hands of the 680 *l.* to satisfy the Paiments abovementioned, reckoning after $6\frac{1}{4}$ *per Cent. per Annum* by Compound Interest?

Answer.

Answ. l. 107,0339, *&c.* that is, for the first Annuity *l.* 56,0804, *&c.* and for the other *l.* 50,9535, *&c.* So this *l.* 107,0339 taken from 680 *l.* leaves *l.* 572,9661 to be paid for the Purchase as charged with the said Annuities. And if the Question were, what new Annuity might have been granted in lieu of the other two, this *l.* 107,0339 being found to be the ready Money both the said Annuities are worth, the Work will be resolved as before by the 4*th* Variety of the third Prop. of Annuities.

0,02632,53434 . Log. of 1,0625 . R . as before.
2 . T.
0,05265,06868 . Log. for the Time 2 Years or 8 Quarters.
2,69125,06683 . Log. of 491,1913, *&c.* Principal to raise 7 *l.* 10 *s.* in 1 Quart.
2,63859,99815 . Log. of 435,1109, *&c.*
Ready Money 56,0804, *&c.* to buy 30 *l. per Annum* for 2 Years payable quarterly.

0,02632,53434 . Log. of 1,0625 . R . as before.
7 . T.

0,18427,74038 . Log. for Time 7 Years, or 28 Quarters.
2,16837,19230 . Log. of 147,3573, &c. Principal to raise 2 *l.* 5 *s.* in 1 Quarter.

1,98409,45192 . Log. of 96,4038, &c.
Ready Money 50,9535, &c. to buy 9 *l.* *per Annum* for 7 Years, payable quarterly.

Prop. 4. To exchange one Annuity for another, with or without Money to boot.

Prop. 4. *To exchange one Annuity for another.* *With Money to boot.*

With Money is, when the Terms of both Annuities are equal, but the Annuities are one greater than the other, then abate the lesser Annuity from the Greater; and as aforesaid, by the 4*th* Variety of the second Proposition of Annuities, find what the Remainder esteemed as an Annuity for the Time given is worth in ready Money, and so much is to be given with the lesser Annuity for the Purchase of the Greater.

Example. *A* hath an Annuity of 70 *l.* *per Annum*, and *B* another of 150 *l.* both to endure 15 Years: They agree to exchange, and that *A* shall pay to *B* so much ready Money as will countervail the Difference of the Annuities: how much ready Money shall *B* receive of *A*, after the Rate of 6 *per Cent.* *per Annum* by Compound Interest?

Q. *Of* 70 *l.* per Ann. *exchanged for* 150 *l.* per Ann. *what Money to be given.*

Answ. *l.* 776,9799, &c. For 70 taken from 150, leaves 80; and 80 *l.* *per Annum* for 15 Years is worth so much ready Money.

Answer.

0,02530,58653 . Log. of 1,06 . R.
15 . T.

0,37958,79795 . Log. for the Time.
3,12493,87366 . Log. of 1333,3333, &c. Principal to raise 80 *l.* *per An.*

2,74535,07571 . Log. of 556,3534, &c.
Ready Money 776,9799, &c. to buy 80 *l.* *per Annum* for 15 Years.

Exchange of one Annuity for another without Money, must ballance the Inequality of the Annuities with more or less Time: Wherefore as before, get the ready Money that will buy each Annuity, and then by the 4*th* Variety of the 4*th* Proposition of Annuities, see how many yearly, half-yearly, or quarterly Paiments accordingly the ready Money of one Annuity will buy of the other.

Without Money to boot.

Example. *A* hath an Annuity of 20 *l.* *per Annum* for 21 Years, yearly to be paid, and will exchange with *B* for an Annuity of 30 *l.* *per Annum*: how long shall the Annuity of 30 *l.* *per Annum* be paid to *A*, that neither of them lose, reckoning 6 *l.* *per Cent.* *per Annum*, Interest upon Interest?

Q. *Of* 20 *l.* per Ann. *exchanged for* 30 *l.* per Ann. *what Term.*

Answ. 10 Years *A* shall receive 30 *l.* *per Annum*, but for the 11*th* Year no more than 27 *l.* 7 *s.* 4¾ *d.*

Answer.

0,02530,58653 . Log. of 1,06 . R.
21 . T.

0,53142,31713 . Log. for the Time.
2,52287,87453 . Log. of 333,3333, &c. Principal to raise 20 *l.* *per Ann.*

1,99145,55740 . Log. of 98,0518, &c.
Ready Money 235,2815, &c. to buy 20 *l.* *per Ann.* for 21 Years.

2,69897,00043 . Log. of 500 Principal, to raise 30 *l.* *per Annum.*
235,2815, &c.

2,42278,42932 . Log. of 264,7185, &c.
0,27618,57111

(23127058
27618,57111|1 (Years. 10
2530,58653

Then if 2530,58653 answer to 1 Year, or 365 Days, 2312,70581 the Remain shall answer to 333 Days, and part of a Day. And if 365 Days pay 30 *l.* then 333 Days shall pay *l.* 27,3698, &c. or 27 *l.* 7 *s.* 4¾ *d.*

In like sorts Debts by a new Agreement to be paid at other Times.

Hence also after the manner of Annuities, a Debt to be paid at certain Times, and afterwards agreed to be paid at other Times, with Allowance or Discount of Compound Interest, the Paiments are found.

Q. Of 5000 l. payable in 5 Years, what to be paid in every 3 Months.

Example. One oweth 5000 *l.* to be paid in 5 Years, *viz.* every Year 1000 *l.* but afterwards agreeth with his Creditor to pay the Whole in 20 equal Paiments, *viz.* every 3 Months one Paiment: The Question is, what each Paiment shall be, with Allowance of Interest upon Interest, after the Rate of 10 *l. per Cent. per Annum?*

Answer.

Answ. l. 241,2561, *&c.* every Quarter during the 5 Years: For 1000 *l.* a Year for 5 Years being found to be worth, in ready Money, *l.* 3790,7868, *&c.* that ready Money will buy such a quarterly Paiment as aforesaid, by the 4*th* Variety of the third Proposition of Annuities.

1,61752,36773 . Log. of 41,4499, *&c.* Principal to raise 1 *l. per* Quarter.
0,20696,34260 . Log. for the Time 5 Years, or 20 Quarters.
1,41056,02513 . Log. of 25,7371, *&c.*
Ready Money 15,7127, *&c.* to buy 1 *l.* Annuity for 5 Years.

3,57872,93597 . Log. of 3790,7868, *&c.* Price given.
1,19625,10297 . Log. of 15,7127, *&c.* Price of 1 *l.* Annuity.
2,38247,83300 . Log. of 241,2561, *&c.* New Annuity.

If the Debt be payable at once.

But supposing by the new Agreement, the whole Debt had been agreed to be discharged in one Paiment, as at the end of 3 Years, with Allowance of Interest as aforesaid: Then after the ready Money found, as aforesaid, by the 4*th* Variety of the second Proposition of Annuities, the rest of the Resolution is wrought by the first Proposition of Interest, and that one Paiment found to be *l.* 5045,5372, *&c.*

Example.

0,04139,26852 . Log. of 1,10 . R.
3 . T.
0,12417,80556 . Log. for the Time.
3,57872,93597 . Log. of 3790,7868, *&c.* ready Money.
3,70290,74153 . Log. of 5045,5372, *&c.* ω.

Prop. 5. To lessen the Rent or Fine.

Prop. 5. To lessen the yearly Rent of an House or Land by increasing the Fine, or to lessen the Fine by increasing the Rent.

The Rent.

For lessening the Rent by the 4*th* Variety of the second Proposition of Annuities, find what the Sum diminished, esteemed as an Annuity, is worth ready Money, and add this ready Money to the Fine, and the Aggregate is the new Fine.

Q. *Of* 10 *l.* per Ann. *lessened to* 4 *l. what Fine.*

Example. A Lease is to be sold for 30 Years, whereof the Fine is 100 *l.* and the Rent 10 *l.* yearly. The Lessee is desirous to pay less Rent, and to increase the Fine or Income: If therefore the yearly Rent be decreased to 4 *l. per Annum*, what Fine shall he pay after 10 *l. per Cent. per Annum* by Compound Interest.

Answer.

Answ. l. 156,5614, *&c.* For seeing the 10 *l.* yearly Rent decreased to 4 *l.* is diminished 6 *l. per Annum*, the ready Money that will buy an Annuity of 6 *l. per Annum*, at the Rate and for the Time aforesaid, is found to be *l.* 56,5614, *&c.* which added to 100 *l.* makes *l.* 156,5614, *&c.* as before.

0,04139,26852 . Log. of 1,10. R.
30 . T.
1,24178,05560 . Log. for the Time.
1,77815,12504 . Log. of 60 Principal to raise 6 *l. per Annum.*
0,53637,06944 . Log. of 3,4385, *&c.*
Ready Money 56,5614, *&c.* to buy 6 *l.* Annuity for 30 Years.
Fine 100.
Total 156,5614, *&c.* Fine to be paid.

For

For lessening the Fine by the *4th* Variety of the third Proposition of Annuities, find what Annuity may be bought for that Quantity of the Fine desired to be abated, and add that Annuity to the Rent, and the Aggregate is the new Rent. *The Fine.*

Example. If a Lease of an House for 21 Years be worth 100 *l.* Fine, and 10 *l.* *per Annum* Rent: of how much yearly Rent ought it to be, to bring the Fine down to 60 *l.* reckoning 10 *l.* *per Cent. per Annum*, Interest upon Interest? *Q. Of 100 l. Fine lessened to 60 l. what Rent.*

Answ. *l.* 14,6250, *&c.* For seeing the Fine of 100 *l.* decreased to 60 *l.* the Lessee abateth 40 *l.* of his Fine. And this 40 *l.* at the Rate, and for the Term aforesaid, will buy an Annuity of *l.* 4,6250, *&c.* which added to 10 *l.* makes the Rent *l.* 14,6250, *&c.* as before. *Answer.*

0,04139,26852 . Log. of 1,10 . R.
21 . T.
0,86924,63892 . Log. for the Time.
1,00000,00000 . Log. of 10 Principal to raise 1 *l.* Annuity.
0,13075,36108 . Log. of 1,3513, *&c.*
Ready Money 8,6486, *&c.* to buy 1 *l.* Annuity for 21 Years.

1,60205,99913 . Log. of 40. Price given.
0,93694,58113 . Log. of 8,6486, *&c.* Price of 1 *l.* Annuity.
0,66511,41800 . Log. of 4,6250, *&c.* New Annuity.
Rent 10.
Total 14,6250, *&c.* Rent to be paid yearly.

In like manner Exchanges of Fines for Rents, or Rents for Fines, or Fines and Rents for Rents or Fines, are performed: And the Annuity for the one, and the ready Money for the other found out accordingly.

Prop. 6. To find what Annuity for any Term one may buy for a Sum of Money, when the Annuity beginneth presently, and the Buyer hath Time for the Payment of his Money. *Prop. 6. To find what Annuity to begin presently, may be bought for a Sum paid afterward.*

This *Prop.* by the second *Prop.* of Interest, first findeth what the Purchaser's Money is worth to be paid presently; and then by the *4th Variety* of the third *Prop.* of *Annuities*, findeth what Annuity to endure for the Time propounded may be bought for that ready Money.

Example. What Annuity to endure for 10 Years, and begin presently, may I grant for 500 *l.* to be received at 4 Years end, reckoning Interest upon Interest, after the Rate of 10 *l.* *per Cent. per Annum?* *Q. Of an Annuity for 10 Years bought for 500 l. paid 4 Years after.*

Answ. *l.* 55,5786, *&c.* For 500 *l.* due at 4 Years end, being worth in ready Money *l.* 341,5067, *&c.* will buy at the Rate aforesaid an Annuity of *l.* 55,5786, *&c.* for 10 Years. *Answer.*

0,04139,26852 . Log. of 1,10 . R.
4 . T.
0,16557,07408 . Log. for the Time 4 Years.
2,69897,00043 . Log. of 500 : ω.
2,53339,92635 . Log. of 341,5067, *&c.* α.
0,41392,68520 . Log. for the Time 10 Years.
1,00000,00000 . Log. of 10 Principal to raise 1 *l.* Annuity.
0,58607,31480 . Log. of 3,8554, *&c.*
Ready Money 6,1445, *&c.* to buy 1 *l.* Annuity for 10 Years.
2,53339,92635 . Log. of 341,5067, *&c.* Price above found.
0,78849,14961 . Log. of 6,1445, *&c.* Price of 1 *l.* Annuity.
1,74490,77674 . Log. of 55,5786, *&c.* Annuity desired.

But if the present Value of a Lease be given, and the Term begin afterward, the present Worth is only to be valued according to the future beginning of the Term. *If the Term begin after.*

Example.

Q. Of the Reversion of a Lease, what worth.

Example. If a Lease for 30 Years to begin presently, be worth 1000 *l.* what is it worth in Reversion, to begin at 7 Years end, after the Rate of 10 *l. per Cent. per Annum*, Interest upon Interest?

Answer.

Answ. l. 513,1581, *&c.* Because the Purchaser pays his Money 7 Years before he enters upon the Land, the Drift is to know what 1000 *l.* due at 7 Years end, after the Rate aforesaid, is worth at present; which by the second *Prop.* of Interest, is discovered to be *l.* 513,1581, *&c.* and no more is the present Value of that Lease.

0,04139,26852	. Log. of 1,10 . R .
7	. T .
0,28974,87964	. Log. for the Time.
3,00000,00000	. Log. of 1000 . . ω .
2,71025,12036	. Log. of 513,1581, *&c.* α . .

Prop. 7. *To countervail Fines and Rents, and the Lease in Reversion.*

Prop. 7. To countervail a present Fine, with an Annual Rent, beginning and continuing with a Lease, when the Lease in Reversion is offered to sale for ready Money.

Here is to be found what the Fine will amount to, being put out at Interest for so many Years as are to come between the Bargain and the beginning of the new Lease, according to the first Proposition of Interest: and then find what Annuity to endure for the like Term, with the Lease, may be bought for that Fine so increased by the 4*th Variety* of the third Proposition of Annuities; and that same Annuity is the Annual Rent desired.

Q. Of the Rent upon renewing a Lease instead of a Fine.

Example. Suppose I have 10 Years to come of an old Lease, and would renew it for 21 Years after the Expiration of the Old; for which I am demanded 100 *l.* to be paid presently, and instead thereof I offer such a Rent during the new Lease, as shall countervail the said present Fine, after the Rate of 10 *per Cent. per Annum* by Compound Interest: what shall that Rent be?

Answer.

Answ. l. 29,9903, *&c.* For the 100 *l.* paid 10 Years before-hand at the Rate propounded, will produce *l.* 259,3742, *&c.* which Sum will at the same Rate, purchase a Rent of *l.* 29,9903, *&c.* for 21 Years.

2,00000,00000	. Log. of 100. α.
0,41392,68520	. Log. for the Time 10 Years, as before.
2,41392,68520	. Log. of 259,3742, *&c.* ω.
1,00000,00000	. Log. of 10 Principal to raise 1 *l.* Annuity.
0,86924,63892	. Log. for the Time 21 Years, as before.
0,13075,36108	. Log. of 1,3513, *&c.* as before.
Ready Money	8,6486, *&c.* to buy 1 *l.* Annuity for 21 Years.
2,41392,68520	. Log. of 259,3742, *&c.* Price as before.
0,93694,58113	. Log. of 8,6486, *&c.* Price of 1 *l.* Annuity.
1,47698,10407	. Log. of 29,9903, *&c.* Annuity or Rent required.

Prop. 8. *To find an Annuity answerable to another, and Money to begin presently.*

Prop. 8. To get the Annuity for a certain Term, which another Annuity for a lesser or greater Term with some Money will buy to begin presently.

This Proposition is almost like the 4*th* before: For first must be found, what the Buyer's Annuity is worth ready Money, and thereto add the Money to be paid with the lesser Annuity, or the greater Annuity with the lesser Time, as the Case requires; and then find what Annuity that Sum will buy for the Term propounded, the one by the 4*th* Variety of the second Proposition of Annuities, and the other by the 4*th* of the Third.

Q. Of an Annuity for 21 Years, bought for 200 l. and 50 l. per Annum.

Example. What Annuity to endure for 21 Years is worth 200 *l.* ready Money, and an Annuity of 50 *l. per Annum* for 7 Years, reckoning 10 *l. per Cent. per Annum*, Interest upon Interest?

Answer.

Answ. l. 51,2708, *&c. per Annum.* For such an Annuity for 21 Years, at the Rate of 10 on the 100, will be purchased for the Sum of *l.* 443,4209, *&c.* that is, 200 *l.* in present Money, and *l.* 243,4209, *&c.* the Value of the Annuity of 50 *l. per Annum* for 7 Years.

2,

2,69897,00043 . Log. of 500 Principal to raise 50 *l.* Annuity.
0,28974,87964 . Log. for the Time 7 Years as before.
2,40922,12079 . Log. of 256,5790, *&c.*

Ready Money 243,4209, *&c.* to buy 50 *l. per Ann.* for 7 Years.

2,64681,62080 . Log. of 443,4209, *&c.* Price as before.
0,93694,58113 . Log. of 8,6486, *&c.* Price of 1 *l.* Annuity.
1,70987,03967 . Log. of 51,2708, *&c.* Annuity desired.

Prop. 9. To know by the Price or Fine of a Lease to endure for a certain Term, what a greater or lesser Term of the same is worth, not altering the Rent.

Prop. 9. To find what a longer or shorter Term is worth, not altering the Rent.

When the Fine given is the Price of the lesser Term, and the Value of the Greater required; then first by the 4*th* Variety of the third Proposition of Annuities, find what Annuity to endure with the first Lease may be bought for the Fine: And then by the 4*th* Variety of the second Proposition of Annuities, find what this last found Annuity esteemed with the second Lease is worth in ready Money, and this ready Money is the Value of the Lease for the greater Term.

If the Fine be the Price of the lesser Term, and the greater be sought.

Example. If a Lease of an House for 10 Years be worth 100 *l.* Fine, and 10 *l.* a Year Rent: What Fine is a Lease of the same House worth for 20 Years, not altering the Rent, after the Rate of 10 *l. per Cent. per Annum* by Compound Interest?

Q. Of a Lease, with 100 l. Fine for 10 Years, what Fine for 20 Years.

Answ. l. 138,5504, *&c.* because at the Rate aforesaid 100 *l.* will buy an Annuity of *l.* 16,2741, *&c.* for 10 Years; and this Annuity in 20 Years is worth as before *l.* 138,5504, *&c.* ready Money.

Answer.

1,00000,00000 . Log. of 10 Principal to raise 1 *l.* Annuity.
0,41392,68520 . Log. for the Time 10 Years.
0,58607,31480 . Log. of 3,8554, *&c.*

Ready Money 6,1445, *&c.* to buy 1 *l.* Annuity for 10 Years.

2,00000,00000 . Log. of 100 Price given.
0,78849,14961 . Log. of 6,1445, *&c.* Price of 1 *l.* Annuity.
1,21150,85039 . Log. of 16,2741, *&c.* Annuity for 10 Years.

2,21150,85039 . Log. of 162,7415, *&c.* Principal to raise *l.* 16,2741, *&c.* Annuity.
0,82785,37040 . Log. for the Time 20 Years.
1,38365,47999 . Log. of 24,1910, *&c.*

Ready Money 138,5504, *&c.* to buy *l.* 16,2741, *&c.* for 20 Years.

On the contrary, when the Fine given is the Price of the greater Term, and the Value of the Lesser is required: Then first find, as above, what Annuity to endure with the greater Term may be bought for the Fine; and afterward find what this last found Annuity is worth in ready Money, according to the lesser Term, and this ready Money is the Value of the Lease for that lesser Term.

If the Fine be the Price of the greater Term, and the Lesser be sought.

Example. *A* for a Lease of 50 Years paid 360 *l.* and after 11 Years agreed with his Landlord to surrender his Lease, conditionally, that he might receive so much as the Residue of the Years in the Lease will come to, after the Rate of 5 *l. per Cent. per Annum*, and reckoning the Rent half-yearly: how much therefore shall the Landlord pay back of the 360 *l.*?

Q. Of 360 l. paid for a Lease of 50 Years, surrendred at 11 Years, how much to be paid back.

Answ. l. 335,5696, *&c.* For after finding the Principal Money, that at 5 *l. per Cent. per Annum* Simple Interest, will raise 1 *l.* Annuity in half a Year, to be *l.* 40,4740, *&c.* the Fine 360 *l.* working as before, is found to buy an Annuity of *l.* 9,74433, *&c.* half-yearly, which for 39 Years (that is the Residue of 50 after expiration of 11) is worth in ready Money *l.* 335,5696, *&c.* as abovesaid, and plainly appeareth by the Operations following.

Answer.

$\frac{1}{2}$) 0,02118,92991 . Log. of 1,05. R.		2,00000,00000 . Log. of 100.	
0,01059,46495$\frac{1}{2}$. Half.		0,39282,35307 . Log. of 2,4707, &c.	
		1,60717,64693 . Log. of 40,4740, &c.	

1,60717,64693 . Log. of 40,4740, &c. Principal to raise 1 *l.* Annuity at Simple Interest in half a Year.
1,05946,49550 . Log. for the Time 50 Years.
0,54771,15143 . Log. of 3,5294, &c.

Ready Money 36,9445, &c. to buy 1 *l.* Annuity for 50 Years, payable half-yearly.

2,55630,25008 . Log. of 360 Price given.
1,56755,03222 . Log. of 36,9445, &c. Price of 1 *l.* Annuity.
0,98875,21786 . Log. of 9,7443, &c. Half-yearly Annuity for 50 Years.

2,59592,86479 . Log. of 394,3925, &c. Principal to raise *l.* 9,7443, &c. Half-yearly.
0,82638,26649 . Log. for the Time 39 Years.
1,76954,59830 . Log. of 58,8228, &c.

Ready Money 335,5696, &c. to buy *l.* 9,7443, &c. Half-yearly for 39 Years.

Prop. 10. *To turn a Rate on the Hundred, to a Purchase by the Year,* &c.

Prop. 10. To reduce a Rate on the Hundred to a Purchase by the Year, or the contrary.

Analogies.

To the Resolution of these serve the following Analogies, *viz.*

For the former; As the Rate, to an Unit: so is an Hundred, to the Number of Years desired.

For the Rate on the Hundred.

For the latter; As the Number of Years propounded, to an Unit: so is an Hundred, to the Rate desired.

As if I would know what Rates on the Hundred Lands bought at 16 or 20 Years Purchase is equivalent to:

$$\left.\begin{array}{l}\text{Then as } 16 \;.\; 1 :: 100 \;.\; 6\tfrac{1}{4} \\ \text{or } 20 \;.\; 1 :: 100 \;.\; 5\end{array}\right\}\text{by the Latter.}$$

For the Years Purchase.

Or on the contrary; How many Years Purchase the Lands are bought for, when the Money is reckoned at $6\frac{1}{4}$ *l.* or 5 *l. per Cent. per Annum?*

$$\left.\begin{array}{l}\text{Then as } 6{,}25 \;.\; 1 :: 100 \;.\; 16 \\ \text{or } 5 \;.\; 1 :: 100 \;.\; 20\end{array}\right\}\text{by the Former.}$$

Whereby it appears that Land bought for 16 Years Purchase, is all one with $6\frac{1}{4}$ *per Cent. per Annum*; and Land bought for 20 Years Purchase, is alike as if the Purchase-Money were reckoned at 5 *l.* on the 100.

If the Enquiry be which is most.

When the Enquiry is, Which is most, so many Years Purchase, or such a Rate, both are to be gotten and compared together.

Q. Of 1000 *l. paid for* 70 *l.* per Ann. 7 *Years before any Profit, if more or less than* 20 *Years purchase.*

Example. September the 29*th*, 1643, A Merchant paid 1000 *l.* for certain Lands, which was out in Lease till *September* the 29*th*, 1650; during which Lease the Merchant shall receive but a Pepper-corn a Year, but afterward he will receive 70 *l. per Annum*: whether now doth this Merchant pay more or less than 20 Years Purchase?

Answer.

Answ. More than 20 Years Purchase, by 7 *l.* 2 *s.* +: For 20 Years Rent at 70 *l. per Annum*, is but 1400 *l.* and 1000 *l.* paid 7 Years before any Rent received, makes the Arrerages, with Interest upon Interest, after the Rate of 5 *l. per Cent. per Annum*, (that is all one with 20 Years Purchase) amount to the Sum of 1407 *l.* 2 *s.* + by the first Proposition of Interest.

0,02118,92991 . Log. of 1,05 . R.	70 *l.*
7 . T.	20
0,14832,50937 . Log. for the Time.	1400
3,00000,00000 . Log. of 1000. α.	
3,14832,50937 . Log. of 1407,1004, &c. ω.	

When

When the Enquiry is, Whether more or less be taken than a Rate propounded, or than so many Years Purchase, both being gotten and compared together, as last above-mentioned, the Difference will appear on which Side it is.

If the Enquiry be which is most of a Rate, or Year's Purchase.

Example. If a Man disburse 500 *l.* and at 12 Months end receive in part 80 *l.* and at the end of every Half-year afterward 40 *l.* till 10 Years in all be expired: doth he take more or less than 10 in the 100 Interest upon Interest?

Q. Of 500 l. received first, 80 l. and 40 l. half-yearly, &c. whether 10 l. in the 100 Interest.

Answ. He takes more than 10 in the 100 by *l.* 4,53977, *&c.* For seeing he is a Year before he receive any, the Year's Interest of 500 *l.* at 10 *l. per Cent. per Annum,* is 50 *l.* which added to 500 *l.* makes 550 *l.* out of which 80 *l.* taken, (which was then paid in) leaves 470 *l.* the Compound Interest whereof, at the Rate propounded for the other 9 Years, amounts with the Principal by the first Proposition of Interest, but to *l.* 1108,23567, *&c.* And the Rent of 40 *l.* every Half-year at the same Rate for nine Years, or eighteen Half-years, amounts to *l.* 1112,77544, *&c.* by the first Proposition of Annuities.

Answer.

0,04139,26852 . Log. of 1,10 R.
9 . T.
0,37253,41668 . Log. for the Time.
2,67209,78579 . Log. of 470.
3,04463,20247 . Log. of 1108,2356, *&c.*

0,37253,41668 . Log. for the Time.
2,91352,44915 . Log. of 819,4538, *&c.*
3,28605,86583 . Log. of 1932,2292, *&c.*
1112,7754, *&c.*

When the Enquiry is, which is best, such a Sum or such an Annuity, the ready Money of the Annuity gotten is to be compared with the Sum propounded.

If the Enquiry be, whether a Sum or an Annuity be best.

Example. *A* offers *B* 50 *l.* for an Annuity of 10 *l. per Annum* for 8 Years: whether were *B* best to accept thereof, accompting the Compound Interest at 10 *l. per Cent. per Annum?*

Q. Of 10 l. per *Ann. for 8 Years, or 50 l. be best.*

Answ. No; because an Annuity of 10 *l.* yearly for 8 Years, at the Rate propounded, is worth in ready Money *l.* 53,349, *&c.* by the first Proposition of Annuities, which is *l.* 3,349, *&c.* more than 50 *l.*

Answer.

0,04139,26852 . Log. of 1,10. R.
8 . T.
0,33114,14816 . Log. for the Time.
2,00000,00000 . Log. of 100 Principal to raise 10 *l.* Annuity.
1,66885,85184 . Log. of 46,6507, *&c.*
53,3492, *&c.*

In like manner when the Difference between so many Years Purchase, and the Money paid, or to be paid for the Purchase, is sought; both are gotten, and the one substracted from the other.

If the Difference of the Years Purchase & the Price be sought.

Example. Upon *May*-Day, *Anno Regni Elizabethæ* 30, *A* sold to *B* certain Lands, that yielded 33 *l.* 16 *s.* 8 *d. per Annum* Rent: At which Sale *A* and *B* agreed thus, That *A* should pay to *B* 50 *l.* upon the first Day of *November* next following: And that then *B* should surrender the Lands back again to *A* (in nature of a Mortgage) for the Paiment of 100 Marks *per Annum* for eight Years, *viz.* at the end of every Year, still upon the first Day of *November*, 66 *l.* 13 *s.* 4 *d.*

Example in fulfilling an Order of the High Court of Chancery.

Before the Day appointed for the first Paiment *B* died, so as the Surrender was not made, nor any of the abovesaid Paiments paid: Wherefore upon Suit in *Chancery*, it was ordered by the Lord Keeper, that there should be an Accompt made, and the Questions in the Accompt to be, what the Land was worth at the Sale, after the Rate of 20 Years Purchase; and what the Paiments above-mentioned were worth in ready Money at the same Time, reckoning the Interest at 10 *per Centum per Annum*; And then look how much the Land was worth more than the Paiments, so much should the Heirs of *B* pay to *A*: how much therefore by this Order ought to be paid to *A*?

The Case.

Answ. *l.* 289,8833, *&c.* For such is the Difference between *l.* 676,6666, *&c.* the Worth of the Land at 20 Years Purchase, and *l.* 386,7833, *&c.* the Sum of the ready Money of the Annuity added to the 50 *l.* and taking from the Total one

What to be paid according to the Order.

one half Year's Interest, because by the Agreement the 50 *l.* was not to be paid, nor the Annuity to begin till one half Year after the Bargain was made.

l.	*s.*	*d.*	
33 :	16 :	08	. Yearly Rent of the Land.
		20	. Years.
676 :	13 :	04	. Sum of 20 Years Purchase.

0,04139,26852 . Log. of 1,10. R.
$8\frac{1}{2}$.

0,33114,14816
0,02069,63426

Marks

2,82390,87366 . Log. of 666,6666, &c. Principal to raise 100 Annu.
0,33114,14816 . Log. for the Time 8 Years.
2,49276,72550 . Log. of 311,0049, &c.

Marks

Ready Money 355,6617, &c. to buy an Annuity of 100 for 8 Years.
50 added.

2,60816,40598 . Log. of 405,6617, &c.
0,02069,63426 . Log. for the Time $\frac{1}{2}$ Year.
2,58746,77172 . Log. of 386,7833, &c.

676,6666, &c. — 386,7833, &c. = 289,8833, &c.

Proof of Anatocism. The Varieties of Questions are even numberless; but what hath been said in this Chapter, is sufficient for a through understanding of all *Anatocism:* And since both the way of Work by *Decimals*, and also by *Logarithms* hath been seen, and several of the Operations and Propositions converse one to another; the Truth of the Conclusions is sufficiently proved thereby, and to give farther Example here, will be altogether superfluous.

Partis tertiæ Libri quarti

FINIS.

The

The Fourth Part of the Fourth Book.

CHAP. I.

Of EQUATIONS.

Nothing remains now to finish the whole Work, but to overlook *Equations*, which in the Chapter of *Ratio's* before are called *Compound Proportions*, to distinguish them from others; and sometimes *Proportions of Equality*, because the Number or Numbers, Magnitude or Magnitudes, between which Comparison is made, are equal one to the other in Value, though of different Denominations.

Equations close *the Work. They are Compound Proportions, or Proportions of Equality.*

The whole Computation of *Equations*, consisteth in the Invention, Reduction and Resolution of an *Equation*; and is called (though by some, with reference only to the latter) the *Art of Equation*, *Rule of Equation*, *Rule of Coss*, *Rule of Quantity*, *Almucabula*, (signifying an hidden or secret Tradition) *Algeber*, or *Algeber's Rule*, another Arabick word taken by some for the Name of the Inventor, from the Article *Al* and *Geber* a Man; and by some that it was *Geber* the old Arabian: But others will have it only for a Name of singular Excellence, *quasi Ars magistralis*, it being of such Perfection, that it performs not only what may be done by other Rules of Proportion, *Alligation*, *False Position*, and the Rules of *Archindus*, and six Quantities of *Cataym*, (as *Digges* in his *Stratioticos* tells us) but with much Facility, Evidence and Demonstration.

Wherein their Computation consists. The several Names. Excellency thereof.

From this *Algeber* came the Name *Algebra*, now most common thereto: Nevertheless neither the one nor the other of them is the right Word, but *Algiebar*, as Dr. *Dee* in his Math. Preface, &c. proves from *Avicen*, translated by *Andreus Alpagus*, (one very skilful in the Arabick Tongue) who calls The Science of working *Algiebar* and *Almachabel*, The Science of finding an unknown Number, by adding of a Number, *Division* and *Equation*: Nor could *Geber* (however skilled therein, and other profound Knowledg) be the Inventor, since as that Learned Doctor saith, *Diophantus*, a Greek Philosopher and Mathematician, before the Time of *Geber*, wrote 13 Books thereof.

Algebra *from whence. How to be wrote. Geber not the Inventor.*

The Numbers or Quantities compared one to another, are set one on the one Side, and the other on the other, of a pair of Parallels or Gemowe Lines, called the *Sign of Equation*, and sometime Metonymically called *Equation* it self, there being no two things more alike or equal, than such two right Lines ═══ equally extended, whereby is saved the oft repetition of the words *equal to*: As if I would say, 1 *s.* is equal to 12 *d.* or 3 Crowns are equal to 15 *s.* they are set thus:

Parallels of what use. How called. What to be understood thereby.

s.		d.	△		s.
1	═══	12	3	═══	15

Example.

So shall one Shilling be understood to be equal in Value to twelve Pence, and 3 Crowns equivalent to 15 Shillings, though their Denominations differ, and neither 1 nor 12, nor yet 3 and 15 are alike.

Equations, in the Chapter of *Ratio's* before-mentioned, were there divided into two principal Sorts, *viz. Pure* and *Mixt*, as one Number or Magnitude compared to another, or many others to one or to many, and both Sorts there observed principally to converse with Contract Numbers: Not but that sometime Abstract Numbers are used, as well as and with those that are Contract; but if the Denominations in Contract, or the Abstract Numbers themselves on both Sides of the *Equation*, be alike, the *Equation* is either Nugatory or Impossible. For when the *Equation* shall be identical in Numbers and Denominations, it is nugatory or trifling; as to say, 3 ʒ = 3 ʒ, is all one as to say, 3 Squares is as much as 3 Squares. And when the Denominations are identical, and the Numbers annexed different, the *Equation* is impossible, for to say 2 ʒ = 3 ʒ, is as much as to say 2 = 3, which

The Sorts of Equations. *Abstract Numbers used how. Nugatory and Impossible* Equations, *what.*

which is impossible. So as if one Number or Magnitude be valued with another, their Denominations must be different; but one greater Abstract Number may be compared to divers lesser Numbers, and one Contract Number of higher Power or Demonination with divers lower.

One sort of Equation reduced to another.

Oftentimes by *Reduction*, (as afterwards in the Chapter thereof may be seen) *Equations* of the latter Sort are reduced from one Form to another: For some of the Numbers on the one Side may be abated, or else carried over to the other Side of the *Equation*; so as one Number may be left alone at one Side, which is therefore called, the *Solitary Side of the Equation*.

Solitary Side, what.

Equality of a Number to its Parts abstract, what helped thereby.

Although *Equations* are most frequent with Contract Numbers, yet the Equality considered between Abstract Numbers, (as lesser Numbers may be Parts of the Greater) hath not been without some Effect, and among others in giving Light to the Way of parting equally an even Magnitude by 2 odd Parts thereof: And hence this Problem gained Solution, *viz.*

To part one Vessel of Liquor by unequal Vessels.

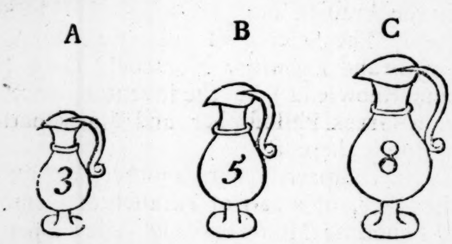
To part a Vessel full of Wine, or other Liquor, into two equal Parts, by other Vessels, each of them unequal Parts of the greater Vessel, and both together equal thereto.

What Parts to be taken.

Nevertheless all Parts of a Number which together are equal thereto, are not taken to make such a Division, lest the *Equation* should be nugatory: But those most apt for the *Equation*, and which have a seeming Difficulty, as one an Unit below, and the other an Unit above the Half: For suppose a Vessel full of Wine contain 8 Pints or Gallons, *&c.* propounded to be equally divided by two other Vessels unequal, containing together as much as the other, these two Vessels must be one 5, and the other 3 Pints or Gallons, *&c.* which are one above 4, the Half of 8, and the other 3, one below 4: For if the Parts were 1 and 7, or 2 and 6, there were no difficulty therein.

Example.

Let then the 3 Vessels be represented by *A*, *B*, *C*; of which *C* being full, and the other 2 empty, pour out of *C* into *B* till it be full, so will there be left 3 Pints in *C*; then fill *A* out of *B*, so will there be in *A* 3 Pints, in *B* 2, and in *C* 3: Then emptying *A* into *C*, it will have 6 Pints, and *B* 2, but *A* none. Furthermore, emptying the 2 Pints of *B* into *A*, and filling *B* out of *C*, there will be in *A* 2 Pints, in *B* 5, and in *C* but 1: But now it is evident, that if from *B* you fill *A*, yet will *B* retain 4 Pints; and if *A* be emptied into *C*, there will be 4 Pints also.



Equations chiefly are used in Contract Numbers, but not to be confounded therewith.

However the chief exercise of *Equations* be among Contract Numbers, and of them especially the 3 latter, *viz.* *Cossicks*, *Surds*, and *Species*; yet I see little reason to crowd all the Simple Elements thereof into the Accompt of *Equations*, and less to subject all Figural Numbers, and their Operations, under the Title of *Algebra*, as some have done, but confusedly.

Nor because *Equations* frequently deal with *Cossicks*, (whence the Rule for resolving an *Equation* came to be called the *Rule of Coss*) will it be reasonable to subjugate the whole Process of *Equations* with *Cossicks*, as is done by others; since *Equations* are as familiar to *Species* as to them.

How some sort Equations.

The late use of *Species* hath occasioned some to make three Sorts of *Equations*.

1. *Pure.*

The first Pure or Absolute, wherein is no Power on either Side of the *Equation*.

2. *Powerful.*

2. A powerful *Equation*, wherein is some Figural Number or other.

3. *Affected.* *Reckoned here only of 2 sorts, Pure and Affected.*

3. A mixt or affected *Equation*, made up of both, and of many Numbers: But seeing all *Equations* by Cossical Reduction may be brought to two Numbers or more, those brought thereby to 2 only, are here reckoned for pure *Equations*, though the one thereof be a Power, and all others are to be counted for mixt *Equations*.

As

As well pure *Equations* as those mixt, (now commonly known by the Name of *Affected*, and sometime *Adfected*) when made up with Powers, whether Cossical or others, come under farther observation. *Both sorts, how considered.*

1. When the Quantities are orderly different one from the other, without omission of any Quantity between them. *1 Quantities orderly distant, none omitted.* Examples.

Examples. *Pure.*	√	*Affected.*	√
1 ℨ = 3 N	3	1 ʒ = 2ℨ + 3N	3
2 ʒ = 4 ℨ	2	1 φ = 6ʒ − 5ℨ	5
4 φ = 12 ʒ	3	3 φ = 2ʒ + 10ℨ − 4 N	2

In all these Examples, one Denomination immediately succeeds the other; for between φ and ʒ. ʒ and ℨ. ℨ and N. no Quantity intervenes.

2. When the Quantities are orderly different one from the other, but between them are 1, 2, 3, or more Quantities omitted. *2. Quantities orderly distant, some omitted.* Examples.

Examples. *Pure.*	√	*Affected.*	√
1 ʒʒ = 4 ʒ	2	1 ʒʒ = 12 ʒ + 64 N	4
1 ſs = 16 ℨ	2	1 ʒφ = 108 ʒ − 3 ʒʒ	3
1 ʒφ = 32 ℨ	2	1 φφ = 7ʒφ + 9 φ − 8N	2

In the first Pure, between the Quantities ʒʒ and ʒ, is one Quantity, *viz.* φ omitted. In the next, between ſs and ℨ, three Quantities are omitted, *viz.* ʒʒ φ. and ʒ. And in the other Instance are omitted four Quantities, ſs. ʒʒ. φ. and ʒ. between ʒφ and ℨ.

In the first Affected is omitted only one Quantity, that is φ, between ʒʒ and ʒ, and ℨ between ʒ and N. In the next also one Quantity only is omitted, *viz.* ſs between ʒφ and ʒʒ, and φ between ʒʒ and ʒ. But in the last Instance two Quantities are omitted, that is ʒʒʒ and Bſs, between φφ and ʒφ, and ſs and ʒʒ between ʒφ and φ, and ʒ and ℨ between φ and N.

3. When the Quantities omitted in an *Affected Equation*, are between the Sides or Parts of the *Equation*, but on either Side none. *3. Quantities in Affected between the Parts omitted.* Examples.

Examples.

1 ʒφ = 30 ℨ + 4 N	√ 2	
2 ʒʒ + 3 φ = 82 ℨ − 3 N	√ 3	

Here between ʒφ on the solitary Side, and ℨ on the other Side in the first Example, are four Quantities omitted, but between ℨ and N none. And in the second Instance, between φ and ℨ is but one Quantity omitted, and none on either Side between ʒʒ and φ, or between ℨ and N.

4. When the Quantities omitted in an *Affected Equation* are on either Side, but none between the Sides or Parts. *4. Quantities in an Affected between the Parts none, but on either side omitted.* Examples.

Examples.

1 ʒφ = 1 ſs + 32 N	√ 2
4 φφ − 3 Bſs = 10 ʒφ + 1024 N	√ 2

Between ʒφ and ſs in the first Example, no Quantity is wanting, but ʒʒ. φ. ʒ. and ℨ. are omitted between ſs and N. And in the other Example Bſs at one Side the 7*th* Quantity, and ʒφ the 6*th* Quantity at the other Side, shew none omitted; but between φφ and Bſs one Quantity is omitted, and between ʒφ and N five Quantities are wanting.

5. When the Quantities omitted are disorderly between, and on both Sides of the *Equation*. *5. Quantities omitted disorderly.*

Examples.

1 φφ + 2ʒ = 20ſs − 4ℨ − 112 N	√ 2
3 ʒφ − 100ℨ = 16ʒʒ + 618N − 1φ	√ 3

The first Example wants 6 Quantities between φφ and ʒ of the one Side, and 3 Quantities between ſs and ℨ of the other Side. And ʒφ is distant from ℨ in the latter Example 4 Quantities on the one Side, and φ from N 2 Quantities on the other Side.

The Truth of these, and other *Equations*, may be tried, having the Root as *Cossicks* before noted, by taking Abstract Numbers. For in the last Example, seeing the Root is 3, the ʒφ thereof shall be 729, the ʒʒ 81, and the Cube 27: So shall 3 ʒφ be 2187; from which 100 ℨ, that is 300 taken, there shall remain *Proof of Equations.*

for

for the left Side of the *Equation* 1887. And so 16ʒʒ is 1296, to which 618 added, makes 1914; from whence 27 the Sum of 1 φ abated, leaves 1887 for the right Side of the *Equation* equal to the other.

$$2187 - 300 = 1296 + 618 - 27 = 1887.$$

CHAP. II. *Invention of Equations.*

Invention of Equations first to be done. First Equation what, and why so called. Invention, &c. how called. What the Steps thereto. 1. To be stored with Analytical Provision.

WHEN a Question is propounded to be resolved by an *Equation*, the *Equation* is first to be found, and such an one too as is apt and pertinent to the Question. This *Equation* found, (because many times altered by *Reduction*, as in the next Chapter) is sometime called the *First Equation* in resolving the Question; and the finding thereof, here called *Invention*, is by some called *Composition*.

He that would arrive at the end of *Equations*, will find it convenient, for the *Invention* of an *Equation*, to proceed gradually by these two following Steps.

1. Let him be well stored with Analytical Provision, to use as occasion serves: For as in examining a Proposition of 4 Proportionals, it is necessary to know, That the Rectangle of the Means is equal to that of the Extreams: So in a Work of *Equations*, it is as well beneficial as necessary, to know both how to deduce and draw an *Equation* out of a Proposition, and out of the first and most easy *Equations*, (which are nothing but the simple Affections or Expositions of the Terms) to draw forth, by meet and proper Consequences, other *Equations* equal to the First, but in other Terms.

Example by two Numbers given, what may be found thereby. 1. By A and E, what found.

For suppose of two Numbers given, A be the Greater, E the Lesser, Z the Sum, X the Difference, &c. as before shewed in *Species*, it will follow by *Oughtred*, Chap. 11. of his *Clavis*:

1. That by the *Species* for any two Numbers given, (whereof one the Greater, and the other the Lesser) may be found the 13 following Conclusions.

Data.	*Quesita.*	*Resolution.*
Greater A.	Sum.	Z=A+E.
	Difference.	X=A−E.
	Rectangle.	P=Æ, or AE.
	Sum of the Squares.	Ƶ=Aq+Eq.
	Difference of the Squares.	Ӿ=Aq−Eq.
	Sum of the Sum and Difference.	Z+X=2A.
	Difference of the Sum and Difference.	Z−X=2E.
	Rectangle of the Sum and Difference.	ZX=Aq−Eq, or Ӿ.
	Square of the Sum.	Zq=Aq+2Æ+Eq, or Ƶ+2Æ
	Square of the Difference.	Xq=Aq−2Æ+Eq, or Ƶ−2Æ
	Sum of the Squares of the Sum and Difference.	Zq+Xq=2Aq+2Eq, or 2Ƶ.
	Difference of the Squares of the Sum & Difference.	Zq−Xq=4Æ.
Lesser E.	Square of the Rectangle.	Pq=Æq, or AqEq.

2. By Z and A or E, what found.

2. That by the Species for the Sum of any two Numbers, (whereof one the Greater, and the other the Lesser) and one of those Numbers, may be found the other, the Difference, the Rectangle, the Sum of the Squares, and the Difference of the Squares.

Data.	*Quesita.*	*Resolution.*
Greater A	Lesser.	E = Z—A.
	Difference.	X = 2A—Z.
	Rectangle.	P = ZA—Aq.
	Sum of the Squares.	Ƶ = Zq—2ZA+2Aq.
	Difference of the Squares.	Ӿ = 2ZA—Zq.
Sum Z		
	Greater.	A = Z—E.
	Difference.	X = Z—2E.
	Rectangle.	P = ZE—Eq.
	Sum of the Squares.	Ƶ = Zq—2ZE+2Eq.
Lesser E	Difference of the Squares.	Ӿ = Zq—2ZE.

3. That

3. That by the Species for the Difference of any 2 Numbers, (whereof one the Greater and the other the Lesser) and one of those Numbers, may be found the other, the Sum, the Rectangle, the Sum of the Squares, and the Difference of the Squares. *3. By X & A or E, what found.*

Data.	*Quæsita.*	*Resolution.*
Greater A	Lesser.	$E = A - X.$
	Sum.	$Z = 2A - X.$
	Rectangle.	$P = Aq - XA.$
	Sum of the Squares.	$\mathcal{Z} = 2Aq - 2XA + Xq.$
	Difference of the Squares.	$\mathcal{X} = 2XA - Xq.$
Difference X.		
	Greater.	$A = E + X.$
	Sum.	$Z = 2E + X.$
	Rectangle.	$P = Eq + XE.$
	Sum of the Squares.	$\mathcal{Z} = 2Eq + 2XE + Xq.$
Lesser E	Difference of the Squares.	$\mathcal{X} = 2XE + Xq.$

4. That by the Species for the Rectangle of any 2 Numbers (whereof one the Greater and the other the Lesser) and one of those Numbers, may be found the other, the Sum, the Difference, the Sum of the Squares, and the Difference of the Squares. *4. By P & A or E, what found.*

Data.	*Quæsita.*	*Resolution.*
Greater A.	Lesser.	$E = \frac{P}{A}.$
	Sum.	$Z = \frac{Aq + P}{A}.$
	Difference.	$X = \frac{Aq - P}{A}.$
	Sum of the Squares.	$\mathcal{Z} = \frac{Aqq + Pq}{Aq}.$
	Difference of the Squares.	$\mathcal{X} = \frac{Aqq - Pq}{Aq}.$
Rectangle P.		
	Greater.	$A = \frac{P}{E}.$
	Sum.	$Z = \frac{P + Eq}{E}.$
	Difference.	$X = \frac{P - Eq}{E}.$
	Sum of the Squares.	$\mathcal{Z} = \frac{Pq + Eqq}{Eq}.$
Lesser E.	Difference of the Squares.	$\mathcal{X} = \frac{Pq - Eqq}{Eq}.$

5. That by the Species for the *Ratio* of any 2 Numbers (whereof one the Greater, and the other the Lesser) and one of those Numbers, may be found the other, the Sum, the Difference, the Rectangle, the Sum of the Squares, and the Difference of the Squares. *5. By RS & A or E, what found.*

Data.	*Quæsita.*	*Resolution.*
Greater A	Leſſer.	$E = \frac{SA}{R}$.
	Sum.	$Z = \frac{RA + SA}{R}$.
	Difference.	$X = \frac{RA - SA}{R}$.
	Rectangle.	$P = \frac{SAq}{R}$.
	Sum of the Squares.	$Z_{,} = \frac{RqAq + SqAq}{Rq}$.
	Difference of the Squares.	$X_{,} = \frac{RqAq - SqAq}{Rq}$.
Ratio R. S		
	Greater.	$A = \frac{RE}{S}$.
	Sum.	$Z = \frac{RE + SE}{S}$.
	Difference.	$X = \frac{RE - SE}{S}$.
	Rectangle.	$P = \frac{REq}{S}$.
	Sum of the Squares.	$Z_{,} = \frac{RqEq + SqEq}{Sq}$.
Leſſer E	Difference of the Squares.	$X_{,} = \frac{RqEq - SqEq}{Sq}$.

Many more may be deduced thence.

Value not altered by different Expreſſions.

And not only theſe, but from theſe a Multitude of other Propoſitions may be deduced by Conſequence, and yet the different Terms of Expreſſion alter nothing of the equal Value of ſuch Concluſions; as ſuppoſing according to the firſt of theſe Propoſitions A be 3, and E 2. then,

Examples.

$$\text{Becauſe}\left\{\begin{array}{l} Z+X=2A \\ 5+1=6 \\ Z-X=2E \\ 5-1=4 \\ Zq\text{-}Xq=4Æ \\ 25-1=24 \end{array}\right\}\text{ſhall}\left\{\begin{array}{l} \frac{1}{2}Z+\frac{1}{2}X=A \\ 2\frac{2}{1}+\frac{1}{2}=3 \\ \frac{1}{2}Z-\frac{1}{2}X=E \\ 2\frac{1}{2}-\frac{1}{2}=2 \\ \frac{1}{4}Zq-\frac{1}{4}Xq=Æ \\ \frac{25}{4}-\frac{1}{4}=6 \end{array}\right\}$$

$$\text{And}\left\{\begin{array}{l} Z=A+E=2A-X=2E+X=\frac{Aq+P}{A}=\frac{P+Eq}{E},\ \&c. \\ 5=3+2=6-1=4+1=\frac{9+6}{3}=\frac{6+4}{2},\ \&c. \\ X=A-E=2A-Z=Z-2E=\frac{Aq-P}{A}=\frac{P-Eq}{E},\ \&c. \\ 1=3-2=6-5=5-4=\frac{9-6}{3}=\frac{6\quad 4}{2},\ \&c. \end{array}\right.$$

Authors where Store of ſuch Proviſion is.

Deductions of this ſort being endleſs, to endeavour their recital here were an *Herculean* Task: He that deſires to ſee more, let him read *Bachetus* before *Diophantus*; *Billy* before his Continual Proportionals, Mr. *Thomas Harriot* his *Artis Analyticæ Praxis*, Mr. *Jonas Moore* in the *9th* and *10th* Chapters of his Second Book, Mr. *Richard Balam* the *16th* Chapter of his *Algebra*; and above all that profound Analyſt Mr. *William Oughtred* in the *18th* Chapter of his *Clavis Mathematicæ limata*, whom none hath outdone.

2. Step to find the Equation to ſuppoſe in Coſſicks 1 ℨ, *in* Species A *for that ſought. When a to be taken.*

2. Being well provided of ſuch Store as aforeſaid, and grounded in the common Principles of *Arithmetick* and *Geometry*, (becauſe ſometime the Uſe of ſome known Problem will much expedite and eaſe the Work) the next Step of the Artiſt is to ſuppoſe the Number or Magnitude ſought in any Queſtion to be 1 ℨ working in *Coſſicks*, or in *Species* A, and the given Magnitudes Conſonants. But if A be any of the *Data*, let the Capital Letter be changed for *a*, or ſome other not given, for diſtinction ſake.

And

How to proceed with the Supposition. And with this Supposition, proceed to examine the Question according to the Tenor thereof (as though this were given, and you were proving the Truth of it) till you bring it to an *Equation*.

Q. Of a Wager, how much offered to be laid. *Example.* Two Men in a Controversy are upon a Wager, one offers to lay 20 *l.* the other to lay as much Money as will make the Sum of both when added together as much as the Product, if one Number be multiplied by the other: how much then did he offer to lay on the Wager against the other 20 *l?*

Resolution. Here supposing he offered to lay 1 ʒ; this according to the Question must be added to 20, and multiplied by 20; when added, the Total is 1 ʒ + 20, when multiplied, the Product is 20 ʒ; and so I have gotten an *Equation* that is 1 ʒ + 20 = 20 ʒ. And this being the first *Equation*, and capable of *Reduction*, as in the next Chapter, by taking 1 ʒ at the one side from 20 ʒ at the other; so will there be left 20 = 19 ʒ another *Equation*, and be resolved by Consequence, if 19 ʒ be 20, that 1 ʒ shall be 1$\frac{1}{19}$ *l.*

What to be done if more be sought than one thing. Sometime for ease in working the Question, A may be joined with some Number, and sometime the Numbers or Magnitudes sought are more than one; and then oftentimes one being found, the other is known by Consequence. *If more than one Supposition.* But if it be necessary to suppose more than once, let the Suppositions be different Letters if the Work be in *Species*; and as many Letters as are used for Roots, so many several *Equations* to find out the Value of those Letters in Relation to A the first Supposition, which is to be kept distinct from others. And sometime A being supposed, those other sought Numbers or Magnitudes shall by Consequence be drawn from A before the first *Equation* be found: but till *Reduction* be learned, one Example more may suffice here.

Q. Of the Number of Bottles of Wine. *Example.* A Vintner sold 30 Bottles of Wine for 210 Pence, whereof some were White, others Claret; but he sold one Bottle of White for 5 *d.* and a Bottle of Claret for 8 *d:* how many Bottles of each sort were there?

Resolution in Species. *By supposing for the White.* Here the Numbers given being thus noted, *viz.* 30 *B*, 5 *C*, 8 *D*, 210 *F*, as in divers other Questions where 2 Things are sought, the Supposition may be for either. And if A be supposed for the White, the Value of A when found, shall be the Number of those Bottles: But if A be supposed for the Claret, the Number of those Bottles shall be denoted thereby; wherefore supposing A to be the Number of the Bottles of White-wine, then by consequence because there were 30 Bottles in all, the Number of the Bottels of Claret must be B—A; then each part multiplied by their Rates respectively, it will follow the White to be CA, and the Claret to be DB—DA, and both together to be equal to F. So will the first *Equation* be found, and stand thus, CA + DB — DA = F, which by *Reduction* as in the next Chapter may be brought to $A = \frac{DB - F}{D - C}$. So if from DB 240 be taken F 210, and the residue 30 divided by 3, that is, D—C, the Value of A is found to be 10 for the Number of Bottles of White-wine; and then by consequence there must be 20 Bottles of Claret, because the whole were 30; and 10 at 5 *d.* a Bottle, and 20 at 8 *d.* a Bottle make together 210 *d.*

By supposing for the Claret. In like manner, If A had been supposed for the Claret, then should the White be B—A, which severally multiplied by their Roots, makes the Product of the Claret to be DA, and the Product of the White to be CB—CA, and both together the first Equation DA + CB — CA = F: And by the Reduction of the next Chapter brought to $A = \frac{F - CB}{D - C}$. So from F 210, shall 150 the Product of C into B be taken, and the Remainder 60 divided by 3, that is, D—C, or 8—5, and the Quotient 20 shall be the Bottles of Claret.

Resolution in Cossicks. And those less accustomed to Species, may work with Cossicks thus: If 1 ʒ be supposed for the Number sought, either of White or Claret, then the other by Consequence shall be 30 — 1 ʒ; and accordingly each multiplied by the Prices respectively shall produce, if 1 ʒ be supposed for the White, 5 ʒ + 240 — 8 ʒ; but if 1 ʒ be supposed for the Claret, 8 ʒ + 150 — 5 ʒ; and either of them must be equal to 210: And when reduced, as taught in the next Chapter, the one will be 3 ʒ = 30, or 1 ʒ = 10 of the White; and the other will be 3 ʒ = 60, or 1 ʒ = 20 of the Claret, as before.

Supposition is always the true Number sought. From hence it appears, that this Supposition 1 ʒ or A, is always the true Number or Magnitude sought, and no such false Position, as before in the *Rule of Falshood* was taken: But it is observed, that as to those that know not the Cause, it

was

Difference herein from Falshood. was somewhat strange there, out of False Positions to procure True Conclusions, (seeing commonly Erroneous Premisses have such Conclusions); so here it seems much more strange and admirable, that a right Number shall be taken at first touch before any absolute Knowledg can be had what it is: And on this Accompt *Record* says, it may be called the *Rule of Dark* or *Strange Position*.

Rule of dark or strange Position.

Proof of the Invention of Equations. The Different work by Cossicks and Species, seeing both concur in the Conclusion, may serve for a Proof of the Truth of both Operations.

CHAP. III. *Reduction of Equations.*

Reduction of Equations. THE first *Equation* found as suggested, either by the Sense of the Question, or by some known Theorem pertinent thereto, according to the former Chapter; the next Work is to reduce or order this *Equation*, so that it may be fit for Resolution.

How called by Balam. *Balam* calls this *Reduction*, *Equative Inference*, Chap. 13. But seeing, according to the Nature of all *Reduction*, the Terms of the *Equation* are only altered thereby, but the same Equality always kept, it may very properly be called *Reduction*.

The sorts of Reduction. *Equations* are reduced several ways.

1. By the Simple Elements of Numbers; as First, By some of the Simple Elements of Numbers, as *Addition*, *Substraction*, *Multiplication* and *Division*: For according to the common Axiom, equal things added to or taken from, or multiplying or dividing equal things, shall make the Totals, Remains, Products or Quotients, accordingly equal.

Addition to take off Fractions, or increase the Data. *Addition* is used on both sides of an *Equation*, to clear the Species or Quantities of Fractions, or increase the *Data* with New Quantities.

Examples. *Examples.* 5ʒ + 2½ N = 6ȥ + 10½ N   √2.
5Aq + 2½ B = 6 A + 10½ B

In both, if ½ be added on either side, they stand thus:

5ʒ + 3N = 6ȥ + 11N.   5 Aq + 3B = 6A + 11B.

So because A + E = Z, if B be added, it shall be that A + E + B = Z + B.

Substraction to take away Fractions, and double Denominations. *Substraction* is used as well to clear the *Equation* of double Denominations, as of Fractions: And so not only ½ may be taken from either Side of the former Example, and the *Equations* stand thus:

5ʒ + 2N = 6ȥ + 10N.   5 Aq + 2B = 6A + 10B.

Examples. But because the 2½ and 10½, on both Sides of the *Equation* are of one Denomination, *viz.* Absolute Numbers, the lesser of them (seeing their Signs are alike, that is, both +) must be taken from the greater; and so 2½ taken from 10½, leave the *Equations* thus:

5ʒ = 6ȥ + 8N.   5 Aq = 6A + 8B.

If doubled at one Side. If any Denominations on the one Side of the *Equation* be doubled, they must be added or substracted to lessen the Terms.

Example. *Example.* 13ʒ = 2φ + 1ʒ + 2ȥ + 4N − 3ʒ   √2.

Because ʒ is found with + and − on one Side of the *Equation*, + 1ʒ shall be taken from − 3ʒ, and the Remain − 2ʒ, set down in the *Equation* thus:

13ʒ = 2φ + 2ȥ + 4N − 2ʒ.

To lessen the Value. *Substraction* is also used to lessen the Value of the first given Species by new Ones. And so if A + E = Z and B be taken away, then shall A + E − B = Z − B.

Multiplication to turn Fractions into Integers &c This called Conversion and Isomeria. *Multiplication* is used to turn Fractions, or Heterogeneal Numbers, into Integers or Homogeneal; this is sometime called *Conversion*; and by *Vieta*, *Isomeria*, and is thus done: Multiply cross-wise after the manner of Fractions, all the other Species or Magnitudes, into the Denominator of each other's Fraction except his own, and these Products shall make the New *Equation*. And seeing the Work is as *Reduction* of common Fractions, as occasion serves, the greatest Common Divisor may be used.

If

If one Side be an Integer, and the other a Fraction; then multiply the Integer by the Denominator of the Fraction, and set this Product against the Numerator of the Fraction for the new *Equation*.

Examples.

Examples. $\dfrac{3\phi+3ʒ}{3ℨ}=6N.$ $A+B=\dfrac{D}{C}.$

These converted by *Multiplication*, shall be thus:

$3\phi+3ʒ=18ℨ.$ $AC+BC=D.$

Thus $A-C=\dfrac{Aq+Bq}{D}+B+C$ converted, is $DA-DC=Aq+Bq+DB+DC.$

So where the Fractions are on both Sides of the *Equation*.

Examples. $\dfrac{1ℨ+2N}{3\phi}=\dfrac{1ʒ-2ℨ+3N}{3ʒ+6N}.$ $A+\dfrac{B}{DC}=B+\dfrac{5A}{G}.$

Converted by *Multiplication*, shew themselves thus:

$3ʃs+9\phi-6ʒʒ=3\phi+6ʒ+6ℨ+12N.$ $DCGA+BG=BGDC+5DCA.$

Division to reduce them to their least Terms, &c. *This called* Depression, Hypobibasmus, *and* Parabolismus.

Division is used, either to depress an *Equation* in Numbers or Quantities, or both, and so reduce them to their least Terms; and therefore is this often called *Depression*, and is the *Hypobibasmus* of *Vieta*: Or else when the Magnitude sought, or his highest Power in the *Equation* is joined with some other Species, then must the rest of the Species be divided by that Species so adjoined, and the higher Power cleared thereof: And this latter Sort is *Vieta* his *Parabolismus*.

Examples of Hypobibasmus.

Examples of the first sort are performed, in all respects, like *Reduction* of Compound Cossical Fractions and Species.

As $3ʃs=6ʒʒ+6ʒ+12ℨ-6\phi$ $\sqrt{2}.$

Depressed in Numbers. *Depressed in Quantities.*

$1ʃs=2ʒʒ+2ʒ+4ℨ-2\phi.$ $1ʒʒ=2\phi+2ℨ+4N-2ʒ.$

And thus the Cossical Examples last above-mentioned, after Conversion, may be also depressed; that is,

$3\phi+3ʒ=18ℨ.$ to $1ʒ+1ℨ=6N.$ $\sqrt{2}.$

And the other in Numbers only to

$1ʃs+3\phi-2ʒʒ=1\phi+2ʒ+2ℨ+4N.$

Examples in Species. $3Ac+3Aq=18A.$

Depressed in Numbers. *Depressed in Quantities.*

$Ac+Aq=6A.$ $Aq+A=6.$

And depressed in the Quantities only as,

$Ac+BAq=DA.$ to $Aq+BA=D.$

Examples of the latter sort, or Parabolism.

Examples of Parabolismus.

$AqB=D+C$, divided by B is, $Aq=\dfrac{D+C}{B}.$

And so $AqB-DA=R$, divided by B is, $Aq-\dfrac{D}{B}\times A=\dfrac{R}{B}.$

And $DAq+DqA=Zc$, divided by D is, $Aq+DA=\dfrac{Zc}{D}.$

2. Reduction by Transposition or Translation.

Secondly, *Equations* are reduced by Transposition or Translation, that is, removing some Species or Quantities from one Side of the *Equation* to the other; in which Removal the Signs shall be changed, for + at one Side of the *Equation* shall be − at the other. This sort of *Reduction* is used, when the same Denominations or Quantities are on both Sides of the *Equation*, to lessen the Terms, and when one Side is to be left solitary.

Example to lessen the Terms.

Thus in the first Example of the former Chapter, where the first *Equation* found was $1ℨ+20=20ℨ$, seeing ℨ on both Sides the *Equation* with +, the lesser is abated out of the greater; so 1ℨ taken from 20ℨ, the Remain 19ℨ answered at one Side to the 20N at the other Side.

But if it had been 22N — 1ℨ = 20ℨ, then the Signs being contrary, — 1ℨ at one Side ſhall be added to + 20ℨ at the other Side; and the *Equation* would have been by this Tranſpoſition, adding 1ℨ to each Side, 22N = 21ℨ.

Example to leave one Side ſolitary.

So to leave one Side ſolitary in Tranſpoſition of the next *Equation* there, the Signs are accordingly changed, for +C is made —C, and —D is made +D; and becauſe +DB was tranſpoſed with the ſame Sign he had before, the Sign of F remaining on that Side he was before, is changed from + to —.

$$CA + DB - DA = F \text{ tranſpoſed is, } A = \frac{DB - F}{D - C}.$$

In like manner all other Tranſpoſitions are performed.

As 1ſs + 3φ — 2ʒʒ = 1φ + 2ʒ + 2ℨ + 4N tranſpoſed, ſhall be
1ſs = 2ʒʒ + ʒ + 2ℨ + 4N — 2φ.

And DA—DC=Aq+Bq+DB+DC ſhall be DA=Aq+Bq+DB+2DC.

If one Side be left empty, what to be done.

Sometime in *Tranſpoſition*, one ſide of the *Equation* is left empty, and then the Quantities with + ſhall be equal with them that are —; wherefore the Defect is eaſily ſupplied, for it never happeneth but when the Quantities are joined with contrary Signs.

Example.

Example. 4ʒ=5ʒ+10N—7ℨ; here tranſpoſing becauſe ʒ is on both ſides, and taking 4ʒ from 5ʒ, the *Equation* will be left thus, 0=1ʒ+10N=7ℨ: but now becauſe 7ℨ are—, they are equal to 1ʒ+10N, and the *Equation* may be ſet thus, 7ℨ = 1ʒ + 10N, or by placing the higheſt Power therein Solitary, 1ʒ=7ℨ—10N.

3. Reduction by Exaltation, when one Side is a Power.

Thirdly. An *Equation* is ſometime reduced by *Exaltation*, which is uſed when ſome *Coſſick* or *Species* on the one ſide of the *Equation* is referred to the Power of a Number on the other ſide; for then muſt the plain Part of the *Equation* be exalted to that Power by ſquaring, cubing, &c. that plain Part, and cancelling the Coſſick Character on the other Part.

Example.

Example. If 1ℨ+4N=√ʒ36, then ſhall 1ℨ+4N be ſquared on the one ſide and ſet againſt 36 on the other Side, and √ʒ cancelled. And ſo 1ℨ+4N=√ʒ36 reduced, ſhall be 1ʒ+8ℨ+16N=36N, and tranſlated thus, 1ʒ+8ℨ=20N, or thus, 1ʒ=20N—8ℨ.

Examples in Species.

B + C = √A exalted, ſhall be Bq + 2BC + Cq = A.
And B = √AB — D tranſpoſed, is B + D = √AB.
And then exalted ſhall be Bq + 2BD + Dq = AB.
And if A be left Solitary $\frac{Bq + 2BD + Dq}{B} = A$.

Proof of Reduction of Equations.

That all theſe ſorts of *Reduction* keep the Numbers and Magnitudes ſtill in equality, may be proved by reducing the *Coſſicks* and *Species* into *Integers*; and then comparing them before *Reduction* with thoſe reduced, the Truth of the ſeveral Operations will appear.

CHAP. IV. *Reſolution of Equations.*

Reſolution of Equations.
What to be ſet by themſelves.

AFTER the Invention and Reduction of the *Equation*, as in the two Chapters laſt before, the *Equation* is to be reſolved, being ſo found and ordered that the known Magnitudes may be on the one ſide, and the unknown on the other, if *Species* be uſed, but if *Coſſicks*, the higheſt Quantity ſet Solitary.

Reſolution, what.
Root of an Equation improperly called ſo.

The Reſolution of an *Equation* is to find out the Value of the Root ſuppoſed, often called, but improperly, the Root it ſelf, becauſe the Root of every *Coſſick* Number muſt be a *Coſſick* Number, and ſuch as by Multiplication will make that Rooted Number, which this ſo called Root will not.

How gotten.

The Value of this Suppoſititious Root is to be procured or extracted as the *Equation* is Pure or Affected.

Touching Pure Equations.

Of pure Equations.

1. To get the Value of 1ℨ when the Quantities are orderly distant, and none omitted.

1. When the Quantities or Magnitudes compared one to another, are orderly different each from other without omission of any Quantity between them, then divide the Number of the lowest Denomination by the Number of the highest Denomination, and the Quotient shall be the Value of 1ℨ. But oftentimes the Reduction of the first found *Equation* brings the Numbers so low, that Division is needless, because 1ℨ is one part of the *Equation*.

Q. Of the Value of a Cloak.

Example 1. A Gentleman hireth a Servant to serve him a Year for 24*s.* and a Cloak; but at 8 Months end they fall at variance, and the Gentleman dischargeth his Servant, and giveth him 13*s.* and the Cloak: what was the Cloak rated at?

Resolution.

Answ. 9*s.* Here supposing the Value of the Cloak sought to be 1ℨ or A, or keeping the *Species* for Cloak; if 8 Months give 1 Cloak and 13*s.* then shall 12 Months give $1\frac{1}{2}$ Cloak and $19\frac{1}{2}$*s.* and this shall be equal to the whole Year's Wages, and being set thus $1\frac{1}{2}\text{Cl} + 19\frac{1}{2}s. = 1\text{Cl} + 24s.$ by Transposition of both Species will be reduced to $\frac{1}{2}\text{Cl} = 4\frac{1}{2}s.$ And so by consequence without Division it is easily perceived, if half a Cloak be worth 4*s.* 6*d.* the whole Cloak shall be valued at 9*s.* and such will be the Quotient if $4\frac{1}{2}$ be divided by $\frac{1}{2}$.

Q. Of the Age of Alexander, &c.

Example 2. *Alexander* being asked how old he was, answered, he was 2 Years elder than *Ephestio*; yea said *Ephestio*, and my Father is as old as we both and 4 Years more; and my Father, said *Alexander*, having all those Years, was 96 Years of Age: the Question is, how old each of them was?

Answer.

Here supposing the Age of *Ephestio* (who was the Youngest) to be 1ℨ, then *Alexander* being 2 Years Elder, must be 1ℨ + 2N. and then must the Father of *Ephestio* be 2ℨ + 6N (that is, 4 Years more than them both) all which added together make up the Age of *Alexander*'s Father, that is, 4ℨ + 8N, and is to be equal to 96.

This *Equation* 4ℨ + 8N = 96N reduced by *Transposition* to 4ℨ = 88N, if depressed, or the 88 divided by 4, makes 1ℨ = 22N for the Age of *Ephestio*.

Supposition.		*Proof.*
1ℨ	Age of *Ephestio*	22 Years.
1ℨ + 2	Age of *Alexander*	24
2ℨ + 6	Age of *Ephestio*'s Father	50
4ℨ + 8	Age of *Alexander*'s Father	96

2. To get the Value of 1ℨ when the Quantities are omitted.

2*ly*. When the Quantities or Magnitudes compared one to another are distant one from another by 1, 2, 3, or more Quantities or Powers, after *Reduction* of the *Equation* first found as before, and Division thereof (if occasion be) from the Quotient, extract a Root according to the Quantities omitted; that is, if one Quantity be omitted, extract the Square Root, if 2 the Cube Root, if 3 the squared Square Root, &c.

Q. Of the Length and Breadth of a Pavement.

Example 1. A Floor paved with square Bricks, is longer than it is broad by $\frac{1}{7}$; and the whole Pavement contained 3584 Bricks: what was the Length and Breadth thereof?

Resolution.

Answ. 56 Broad, and 64 Long: For supposing the Breadth which is the lesser Quantity to be 1ℨ, then shall the Length be $1\frac{1}{7}$ℨ: now multiplying the Breadth by the Length, (as in all such Forms to find the *Area*) there is produced $\frac{8}{7}$ʒ, which according to the Proposition must be equal to 3584, and by *Reduction* 8ʒ = 25088, and again, 1ʒ = 3136: and so if in this *Equation* 3584 be divided by $\frac{8}{7}$, will the Quotient be 3136; and because between ʒ and N, one Denomination is omitted, that is ℨ, the Square Root of 3136 is taken, which is 56 for the Breadth, to which 8 that is $\frac{1}{7}$ of 56 added, makes 64 for the Length.

And if in this Instance 1ℨ had been supposed for the Length, then shall the Breadth be $\frac{7}{8}$ℨ, and the Square $\frac{7}{8}$ʒ, and $\frac{7}{8}$ʒ = 3584, which divided by $\frac{7}{8}$ is 4096, whose Square Root is 64, as before.

Q. Of the Length, Breadth and Height of a Pile of Brick.

Example 2. A Pile of Brick, whose Length was to the Breadth as 7 to 2, that is, *Tripla-sesquialter*, and the Height was 5 times as much as the Length; being sold for 980 Crowns, the Owner received for every Yard so many Crowns as the Pile had Yards in Breadth: what then was the Length, Breadth, and Height of this Pile?

Resolution. Supposition for the Breadth.

Answ. Two Yards broad, 7 Yards long, and 35 Yards high. For here supposing the Breadth 1ℨ, then was the Length $3\frac{1}{2}$ℨ, and the Height $17\frac{1}{2}$ℨ; all which multiplied one into another, make $\frac{245}{4}$φ, and this shall be equal to all the Yards

in

in the whole Pile: And seeing every Yard cost as many Crowns as the Breadth contained Yards, if the Breadth be 1 ȥ, then did every Yard cost 1 ȥ of Crowns; and then by the *Golden Rule*,

	Yard.		*Crowns.*		*Whole Yards.*		*Crowns.*
As	1	.	1 ȥ	::	$\frac{245}{4}$ φ	.	$\frac{245}{4}$ ʒʒ.

So must $\frac{245}{4}$ ʒʒ $=$ 980 Crowns, and consequently 245 ʒʒ $=$ 3920 Crowns; wherefore dividing 3920 by 245, the Quotient is 16; from whence the *Zenzizenzike* Root taken, because three Quantities are omitted in this *Equation*, the Root 2 is the Breadth, therefore shall 7 be the Length, and 35 the Height, as before; each of which will be gotten in like manner, if Supposition be made for them.

For the Length and Height.

Supposition for the Length.

1 ȥ Length.
$\frac{2}{7}$ ȥ Breadth.
5 ȥ Height.
$\frac{10}{7}$ φ Pile.

y △ y △
1 . $\frac{2}{7}$ ȥ :: $\frac{10}{7}$ φ . $\frac{20}{49}$ ʒʒ.

$\frac{20}{49}$ ʒʒ $=$ 980 △

20 ʒʒ $=$ 48020 △

1 ʒʒ $=$ 2401 △ √ʒʒ 7. *Length.*

Supposition for the Height.

1 ȥ Height.
$\frac{1}{5}$ ȥ Length.
$\frac{2}{35}$ ȥ Breadth.
$\frac{2}{175}$ φ Pile.

y △ y △
1 . $\frac{2}{35}$ ȥ :: $\frac{2}{175}$ φ . $\frac{4}{30625}$ ʒʒ.

$\frac{4}{30625}$ ʒʒ $=$ 980 △.

4 ʒʒ $=$ 6002500 △.

1 ʒʒ $=$ 1500625 △ √ʒʒ 35. *Height.*

Proof thereof.

And that all these Varieties are true, appears, because if every Yard of the whole Pile cost 2 Crowns, that is as many as there were Yards in the Breadth, then shall the whole Pile cost 980 Crowns, because it contained 490 solid Yards.

	Breadth.		*Length.*		*Height.*		*Solid Yards.*		△		△
For	2	×	7	×	35	$=$	490	×	2	$=$	980

Falshood cannot do what Equations can.

By the Work of these two last Examples of the Pavement and Pile it appeareth, That though all the Operations of *Falshood* may be performed by *Equations*; yet such Questions as these, which concern Figural Forms, will not be wrought right by *Falshood*, as at leisure the Artist may make experiment.

Several other Questions resolved.

Under these two Sorts of *Resolution* fall very many of the Examples, Questions and Propositions propounded in most Books of *Arithmetick*: Some of which follow, resolved by way of *Cossicks* as well as *Species*, and may serve to ripen the Apprehensions of the Ingenious, as well in the Invention (the most difficult Part) of the *Equation*, as in the *Resolution* of such kind of Proposals.

Q. *Of a Number multiplied by 6, and added to 8.* **Resolution.**

1. What Number is that which multiplied by 6, and the Product added to 8, will be 48?

Answ. $6\frac{2}{3}$: For if multiplied by 6, and 8 be added, the Total is 48.

B 6. C 8. D 48. Then $A \times B + C = D$, or $AB + C = D$.

And $A = \frac{D-C}{B}$, or $\frac{48-8}{6} = \frac{40}{6} = 6\frac{2}{3}$.

1 ȥ × 6 $=$ 6 ȥ. And 6 ȥ $+$ 8 $=$ 48. Then will 6 ȥ $=$ 40
And 1 ȥ $= 6\frac{2}{3}$.

Q. *Of 2 Parts of* 100, &c.

2. Which are the two Parts of 100, that the Quotient of the greater Part divided by 3, added to the Quotient of the lesser Part divided by 5, shall together make 30?

Resolution.

Answ. 75 and 25: For 75 divided by 3 gives 25, and 25 divided by 5 gives 5, and 25 and 5 make together 30.

B 100. D 30. Greater Part $A = a$ Supposition.
Lesser Part $E = B - a$ Consequence.

Then $\frac{a}{3} + \frac{B-a}{5} = D$. And $5a + 3B - 3a = 15D$.

And $2a + 3B = 15D$. And $a = \frac{15D - 3B}{2}$ or $\frac{450-300}{2} = \frac{150}{2} = 75$.

If

If A be $1z$, then E $100-1z$.

$\frac{1z}{3}+\frac{100-1z}{5}=30$. Then $5z+300-3z=450$.

And $2z+300=450$. And $2z=150$. And $1z=75$.

3. Two Numbers, one Greater and the other Lesser, or A and E, the Greater is as much as the Lesser, and 4 more; and the Square of the Greater is equal to the Square of the Lesser, and 32 more: what are those Numbers? Q. *Of 2 Numbers, what they are.*

Answ. Greater 6, Lesser 2: For $6=2+4$; and $36=4+32$. Resolution.

B 4. D 32. Greater suppose *a*. Lesser consequently $a-B$.

Then $aq=aq-2aB+Bq+D$. And $2aB=Bq+D$.

And $a=\frac{Bq+D}{2B}$. Or $\frac{16+32}{8}=\frac{48}{8}=6$.

A E B

$1z$ $1z-4$ D

$1\mathfrak{z}=1\mathfrak{z}-8z+16+32$. Or $1\mathfrak{z}=1\mathfrak{z}-8z+48$.

And $8z=48$. And $1z=6$.

And thus both in this and the next precedent Example, may the lesser Number be found if the Supposition be therefore.

4. Two Numbers, or A and E, the Square of A lacking the Square of E is 45, and A lacking E is 5: what are those Numbers? Q. *Of 2 Numb. what they are.*

Answ. A 7, and E 2: For $49-4=45$. And $7-2=5$. Resolution.

B 45. X 5. Greater supposed *a*. Lesser consequently $a-X$.

Then $aq-aq-2aX+Xq=B$. And $2aX=B+Xq$.

And $a=\frac{B+Xq}{2X}$, Or $\frac{45+25}{10}=\frac{70}{10}=\overset{E}{7}$

A E

$1z$ $1z-5$

$1\mathfrak{z}-1\mathfrak{z}-10z+25=45$. And $10z-25=45$.

And $10z=70$. And $1z=7$.

5. A Number joined with 18, and taken from 100, the Sum and Remain are found to be in a *Subtriple Ratio*, that is, as 1 to 3: what is that Number? Q. *Of the Ratio of a Sum and Remain, what the Number.*

Answ. $11\frac{1}{2}$: For $29\frac{1}{2}$ to $88\frac{1}{2}$, is as 1 to 3. Resolution.

B 18. C 100. R 1. S 3. Supposition A.

Then $B+A \,.\, C-A :: R \,.\, S$. And because the Product of the Extreams is equal to the Product of the Means, it shall be that $RC-RA=BS+SA$.

And $A=\frac{RC-BS}{R+S}$, Or $\frac{100-54}{1+3}=\frac{46}{4}=11\frac{1}{2}$

$1z$. Then $1z+18 \,.\, 100-1z :: 1 \,.\, 3$.

And $100-1z=3z+54$. And $4z+54=100$.

And $4z=46$. And $1z=11\frac{1}{2}$.

6. There is a Number from which if $\frac{2}{5}$ be taken, the Number will be left as much under 100, as it was at first above: what is that Number? Q. *Of a Number, &c.*

Answ. 125, of which $\frac{2}{5}$ is 50. And $125-50=100-25$. Resolution.

$\frac{R}{S}$ $\frac{2}{5}$. B 100. Supposition A.

Then $A-\frac{R}{S}=\frac{SA-RA}{S}$. And $B-\frac{SA-RA}{S}=A-B$.

And $2B=\frac{2SA-RA}{S}$. And $2BS=2SA-RA$.

And $A=\frac{2BS}{2S-R}$. Or $\frac{200\times5}{2\times5-2}=\frac{1000}{8}=125$.

$1z$ greater than 100, so shall $1z-\frac{2}{5}z$ be lesser than 100.

And 100 being the Mean between them, $100=1z-\frac{1}{5}z$.

And $\frac{4}{5}z=100$. And $1z=125$.

Q. *Of the Stock of two Merchants.* Resolution.

7. Two Merchants trade till they gain 150 *l.* whereof *A* having 200 *l.* in Stock more than *B*, taketh 100 *l.* what Money had each in Stock?

Answ. *A* 400 *l.* *B* 200 *l.* For seeing *A* of the 150 took 100, that is 50 *l.* more than the 50 *l.* left for *B*; it must follow, That if 50 *l.* Gain require 200 *l.* Stock, then 100 *l.* Gain shall require 400 *l.* Stock.

C 150. D 200. F 100. Suppose Stock of *B* to be *a*. Then *A* is $a+D$.

And as $2a+D \,.\, C :: a \,.\, \frac{Ca}{2a+D}$. And $\frac{Ca}{2a+D}+F=C$.

And $Ca+2aF+DF=2aC+DC$. And $2aF-Ca=DC-DF$.

And $a=\frac{DC-DF}{2F-C}$. Or $\frac{30000-20000}{200-150}=\frac{10000}{50}=200$.

And by *Cossicks*, supposing the Stock of *B* to be 1 ℥: Then *A* is 1 ℥ + 200.

And as 2 ℥ + 200 . 150 :: 1 ℥ . $\frac{150 ℥}{2 ℥+200}$.

And $\frac{150 ℥}{2 ℥+200}+100=150$. And 150 ℥ + 200 ℥ + 20000 = 300 ℥ + 30000.

And 50 ℥ = 10000. And 1 ℥ = 200.

Q. *Of Travel.* Resolution.

8. A Man travelling 9 Miles a Day, is followed by another from the same Place, that sets forth 10 Days after, and went 14 Miles a Day: in how many Days will the latter overtake the former?

Answ. In 18 Days: For 5 Miles (that is 14 — 9) Gain in a Day will reach 90 Miles (that is 10×9) in 18 Days.

B 9. C 90. D 14. Supposition A.

Then $BA+C=DA$. And $C=DA-BA$.

And $A=\frac{C}{D-B}$. Or $\frac{90}{14-9}=\frac{90}{5}=18$.

1 ℥. Then shall 9 ℥ + 90 = 14 ℥. And 5 ℥ = 90.
And 1 ℥ = 18.

Q. *Of the Days a Servant worked and played.* Resolution.

9. A Gentleman hired a Servant for a Year, upon Condition that for every Day he laboured, he should have 1 *s.* and for every Day he was idle, he should lose or discount 8 *d.* Now at the Year's end the Master was by this Agreement to give the Servant nothing, nor was the Servant in the Master's Debt: what Days did the Servant work, and how many Days did he play?

Answ. 146 he wrought, and 219 he plaid: For 146 Shillings and 219 Eight-pences are equal.

B 365. C 12. D 8. Greater supposed A. Lesser then B—A.

Then $CA=BD-DA$. And $CA+DA=BD$.

And $A=\frac{BD}{C+D}$. Or $\frac{365\times 8}{12+8}=\frac{2920}{20}=146$.

1 ℥ Greater, 365 — 1 ℥ Lesser. Then 12 ℥ = 2920 — 8 ℥.

And 20 ℥ = 2920. And 1 ℥ = 146.

Q. *Of two Sorts of Monies.* Resolution.

10. There are two Sorts of Monies, in Number 1000 Pieces, worth 80 *l.* whereof 10 of the first Kind, and 20 of the other are worth 1 *l.* what Number were there of both these Sorts severally?

Answ. 600 of the one, and 400 of the other: For the Quotients of 600 divided by 10, and 400 by 20, make together 80.

B 1000. C 10. D 20. F 80. Greater supposed A. Lesser then B—A.

Then $\frac{A}{C}+\frac{B-A}{D}=F$. And $DA+BC-CA=DCF$.

And $A=\frac{DCF-BC}{D-C}$. Or $\frac{20\times 10\times 80-1000\times 10}{20-10}=\frac{16000-10000}{10}$

And $\frac{6000}{10}=600$ of the Greater.

$1z$ Greater. Then $1000-1z$ the Lesser.

Then $\frac{1z}{10}+\frac{1000-1z}{20}=80$. And $20z+10000-10z=16000$.

And $10z+10000=16000$. And $10z=6000$. And $1z=600$.

11. Suppose *Berwick* and *London* are distant 228 Miles, out of which 2 Foot-posts take their Journeys, and meet the 12*th* Day; but the one went each day one Mile farther than the other: how many Miles did each of them go every day? *Q. Of two Posts, how many Miles they travel.*

Answ. 10 the one, and 9 the other. For the difference between them being 1, yet each of them severally multiplied by 12, will make together 228. Resolution.

X 1. B 12. C 228. Greater supposed A. Lesser consequently A—X.

Then $BA+BA-BX=C$. And $A=\frac{C+BX}{2B}$.

Or $\frac{228+12}{24}=\frac{240}{24}=10$ Greater. 9 Lesser.

$1z$ Greater. $1z-1$ Lesser.

Then as 1 Day . $1z$ :: 12 Days . $12z$.

And as 1 . $1z-1$ :: 12 . $12z-12$.

And $24z-12=228$.

And $24z=240$. And $1z=10$ Greater. 9 Lesser.

12. Suppose the same Distance as above, and the one Post travel 10 Miles a Day from the one Place, and the other 9 from the other: when shall they meet? *Q. Of two Posts, when they meet.*

Answ. The 12*th* Day: For 10 multiplied by 12, and 9 by 12, make together 228. Resolution.

B 10. C 9. D 228. Supposition A.

Then $BA+CA=D$. And $A=\frac{D}{B+C}$. Or $\frac{228}{19}=12$.

$1z$. Then $10z+9z=228$. And $19z=228$. And $1z=12$.

13. Three are in Company: The Stock of *A* and *B* together is 238 *l.* of *B* and *C* 470 *l.* of *A* and *C* 568 *l.* what Stock had each of them severally? *Q. Of the Stock of 3 Merchants.*

Answ. *A* 168 *l.* *B* 70 *l.* *C* 400 *l.* For $168+70=238$. And $70+400=470$. And $168+400=568$. Resolution.

Persons *A*, *B*, *C*. Stocks supposed a, b, c. D 238. F 470. G 568.

Then $a=D-b$. And $a=G-c$. Therefore $D-b=G-c$.

And $c=G+b-D$. And because by the *Data* it appears that $c=F-b$. Therefore $G+b-D=F-b$. And $2b+G=F+D$.

And $2b=F+D-G$. And $b=\frac{F+D-G}{2}$. Or $\frac{470+238-568}{2}=\frac{140}{2}=70$.

In *Cossicks*: If b be supposed $1z$, then a is $238-1z$. And c is $330+1z$.

And $330+2z=470$. And $2z=140$. And $1z=70$.

14. One buying 100 Yards of Cloth, being demanded what a Yard cost, answered, For how much less than 80 *s.* I bought 40 Yards, by so much less than 95 *s.* I bought 50 Yards: what was the Price of a Yard? *Q. Of the Price of a Yard of Cloth.*

Answ. 1 *s.* 6 *d.* For 40 Yards at that Rate is 60 *s.* which is 20 less than 80; and 50 Yards at that Rate is 75 *s.* less than 95 by 20 also. Resolution.

B 40. C 50. D 80. F 95. Supposition A.

Then $BA-D=CA-F$. And $F-D=CA-BA$.

And $A=\frac{F-D}{C-B}$. Or $\frac{95-80}{50-40}=\frac{15}{10}=1\frac{1}{2}$.

$1z$. Then $40z-80=50z-95$. And $40z=50z-15$.

And $10z=15$. And $1z=1\frac{1}{2}$.

15. A General marshaling his Men in *Battalia*, intended a Square Form; and to that purpose, when he had placed a Number at random in the Front, found *Q. Of the Number of Souldiers, and the Fronts.*

100

100 Men over; and thinking to amend it by placing one more in the Front, found 201 Men wanting: how many Souldiers had he, and what were the Fronts?

Resolution. *Answ.* The whole Number of Souldiers were 22600, the first Front 150, the other 151; for the Square of 150 is 22500, to which 100 added, makes 22600; and the Square of 151 is 22801, which is 201 above 22600.

B 100. C 201. First Front supposed A. Then the Second A + 1.
Square thereof Aq. Square thereof Aq + 2A + 1.
Then Aq + B = Aq + 2A + 1 − C. And B = 2A + 1 − C.

$$\text{And } 2A = B + C - 1. \quad \text{And } A = \frac{B + C - 1}{2}. \quad \text{Or } \frac{100 + 201 - 1}{2} = \frac{300}{2} = 150.$$

$1z$ First Front. Then the Second shall be $1z + 1$.
Square of the First $1\mathfrak{Z}$. Square of the Second $1\mathfrak{Z} + 2z + 1$.
Then $1\mathfrak{Z} + 100 = 1\mathfrak{Z} + 2z - 200$. And $100 = 2z - 200$.
And $2z = 300$. And $1z = 150$.

Likewise if Supposition be made for the second Front $1z$; Then the first Front shall be $1z - 1$, and their Squares $1\mathfrak{Z}$, and $1\mathfrak{Z} + 1 - 2z$; and the Body of Men by the one shall be $1\mathfrak{Z} - 201$, and by the other $1\mathfrak{Z} + 101 - 2z$; which *Equation* transposed $2z = 302$; and $1z = 151$.

Q. *Of Legacies to 4 Children.* 16. A poor Man dying which had 4 Children, bequeathed 72 Crowns to his 4 Children; so that the Second and Third should have together 7 times as much as the First; and the Portions of the Third and Fourth should be five Times so much as the Second's Part, and the First and Fourth should have twice so much as the Third: how were the 72 Crowns to be divided?

Resolution. *Answ.* A $4\frac{1}{2}$. B $11\frac{1}{4}$. C $20\frac{1}{4}$. D 36. which together make 72. And $11\frac{1}{4} + 20\frac{1}{4} = 31\frac{1}{2}$, that is 7 times $4\frac{1}{2}$. And $20\frac{1}{4} + 36 = 56\frac{1}{4}$, that is 5 times $11\frac{1}{4}$. And $4\frac{1}{2} + 36 = 40\frac{1}{2}$, that is twice $20\frac{1}{4}$.

Persons, A 1. B 2. C 3. D 4. Crowns b. Supposition for A if a. Then by Consequence must B + C be $7a$; and D the Residue be $b - 8a$. And because $A + D = 2C$, therefore $C = \frac{b}{2} - 3\frac{1}{2}a$; and because $C + D = 5B$, therefore $5B = 1\frac{1}{2}b - 11\frac{1}{2}a$; or $5B = \frac{3}{2}b - \frac{23}{2}a$. And $B = \frac{3}{10}b - 2\frac{3}{10}a$, and the Total shall be equal to b.

$$\begin{array}{rl} \text{That is A} = & a \\ B = & \frac{3}{10}b - 2\frac{3}{10}a. \\ C = & \frac{b}{2} - 3\frac{1}{2}a. \\ D = & b - 8a. \\ \hline \text{Total} & 1\frac{4}{5}b - 12\frac{4}{5}a = b. \quad \text{And } \frac{4}{5}b = \frac{64}{5}a. \quad \text{And } 4b = 64a. \end{array}$$

And $b = 16a$. And $a = \frac{b}{16}$, or $\frac{72}{16} = 4\frac{1}{2}$.

$1z$ supposed for A. Then $B + C = 7z$. And D the Rest $72 - 8z$.
And the half of A and D is $36 - 3\frac{1}{2}z$ for C. So shall the Part of B be $\frac{1}{5}$ of $108 - 11\frac{1}{2}z$, that is $21\frac{3}{5} - 2\frac{3}{10}z$. And the Total collected thus,

$$\begin{array}{rl} A = & 1z \\ B = & 21\frac{3}{5} - 2\frac{3}{10}z \\ C = & 36 - 3\frac{1}{2}z \\ D = & 72 - 8z \\ \hline \text{Total} & 129\frac{3}{5} - 12\frac{4}{5}z = 72. \quad \text{And } 12\frac{4}{5}z = 57\frac{3}{5}. \end{array}$$

And $64z = 288$. And $1z = 4\frac{1}{2}$.

Q. *Of keeping Sheep.* 17. A Farmer agreeth with a Shepherd to keep 80 Sheep for a Year, and at 3 Months end delivereth the Shepherd 30 Sheep more; and 4 Months after that 3 Months bringeth to the Shepherd 30 Sheep more, saying to him, Keep me all these Sheep so long, till such time as the Money I promised you at first be all earned: how long must he keep these 140 Sheep?

Resolution. *Answ.* Nine Months the 80 Sheep, six Months the first 30, and two Months the last 30; which together are equivalent to the keeping of 80 Sheep twelve Months.

B 80. C 12. D 30. F 3. G 7. Supposing the Time of keeping 80 be A, then must the Time of keeping the first 30 be A − F; and of the latter be A − G.

And

And then $BA+DA-DF+DA-DG=BC$. And by transposition,

$BA+2DA=BC+DF+DG$. And $A=\frac{BC+DF+DG}{B+2D}$

Or $\frac{960+90+210}{80+60}=\frac{1260}{140}=9$.

For the 80 suppose the Time $1z$. Then must $1z-3$ be for the first 30.
And $1z-7$ for the last.
Then $1z\times80$ is $80z$; and $1z-3\times30$ is $30z-90$; and $1z-7\times30$ is $30z-210$.
And these 3 Products together make $140z-300$, which are equal to 80×12, that is $140z-300=960$; and $140z=1260$; and $1z=9$.

18. A Merchant hath laden two Ships with Wine, in the one 20 Tuns, and in the other 30 Tuns; and coming to the *Custom-House*, payeth for the Custom of the former Cargo a Tun of Wine, and receiveth of the Officers in Money 60 *s.* and for the latter Cargo a Tun of Wine lacking 15 *s.* The Question is, how the Wine was rated a Tun, and so consequently how much he paid for the Custom of a Tun of Wine? Q. *Of the Rate and Custom of a Tun of Wine.*

Answ. The Tun of Wine was valued at 7 *l.* 10 *s.* so must two Tuns be 15 *l.* From which 3 *l.* 15 *s.* abated, leaves 11 *l.* 5 *s.* to be paid for Custom of the whole 50 Tuns; and so for 1 Tun is 4 *s.* 6 *d.* Resolution.

B 20. C 30. D 60. F 15. Supposition *A* for the Value of 1 Tun.

Then if B cost $A-D$, shall C cost $\frac{CA-CD}{B}$. And this shall be equal to $A-F$.

And $CA-CD=BA-BF$. And then shall $BA-CA=CD-BF$.

And $A=\frac{CD-BF}{C-B}$. Or $\frac{1800-300}{30-20}=\frac{1500}{10}=150$ *s.* or 7 *l.* 10 *s.*

$1z$. Then as $20 . 1z-60 :: 30 . 1\frac{1}{2}z-90$. And
$1\frac{1}{2}z-90=1z-15$; and $\frac{1}{2}z=75$; and $1z=150$.

19. A Post sets out from *Amsterdam* to carry Letters to *Danske*; and at the same time the Merchant to whom the Letters were sent, taketh Horse in *Danske* to ride to *Amsterdam*, and they meet together on the Way. The Post delivereth his Letters, and the Merchant asking what he must have? he answereth, I have gone as much one Day as another, and should have made my Journey in 20 Days, for which my Hire was 30 *s.* conditionally, that if I met you in the Way, that then proportionally for the Way you had travelled, I should be cut off from my Wages: To which the Merchant replied, that he had rode every Day alike, and should have been at his Journey's End in 16 Days: how much then shall the Merchant pay the Post? Q. *Of a Post carrying Letters.*

Answ. $13\frac{1}{3}$ *s.* For having found the Days of the Post's Travel to be $8\frac{8}{9}$, such a Proportion of 30 *s.* will belong to him. Resolution.

B 20. C 16. Supposition for the Post's Days A. The whole Way D. F. 30.

Then as $C . D :: A . \frac{DA}{C}$. And as $B . D :: A . \frac{DA}{B}$

And $\frac{DA}{C}+\frac{DA}{B}=D$. And $\frac{BDA+CDA}{BC}=D$.

And $BDA+CDA=BCD$. And $A=\frac{BC}{B+C}$. Or $\frac{320}{36}=8\frac{8}{9}$ Days.

Then as $B . F :: A . \frac{FA}{B}$. Or as $20 . 30 :: 8\frac{8}{9} . 13\frac{1}{3}$ Shillings.

$1z$ supposed for the Post's Days.
Then as $16 . 1 :: 1z . \frac{1}{16}z$.
And as $20 . 1 :: 1z . \frac{1}{20}z$.

Total $\frac{36}{320}z. = 1$ whole Journey.

And $36z=320$. And $1z=8\frac{8}{9}$.
And if 20 Days Hire be 30 *s.* what $8\frac{8}{9}$ Days? *Answ.* $13\frac{1}{3}$ *s.*

Q. Of two Ships sailing.

20. A Ship saileth out of the *Texel* to *Spain*, with such a Wind that he might perform his Voyage in 15 days; but when 6 days were past, the Wind changing, the Ship sailed backwards as much in 4 days, as it had done forward in one day. At the beginning of the second Wind there departed another Ship from *Spain* towards the *Texel*, (being light loaden) sailed forwards to the *Texel* as often 7 Leagues, as the other Ship backward from his Port 2 Leagues: in how much Time after the first 6 days, and how far from the *Texel* shall the Ships meet? and in what Time may the Ship from *Spain* arrive at the *Texel*, supposing the Parts 300 Leagues asunder?

Resolution.

Answ. In $14\frac{2}{5}$ days they meet; and the Ship from *Spain* had then sailed 252 Leagues, which is 48 from the *Texel*; and in 17 days $\frac{1}{7}$ may arrive there: discovered thus,

B 15. C 6. D 1. F 4. G 2. H 7. K 300. Supposition for the meeting A.

Then as $B \,.\, K :: C \,.\, \frac{KC}{B}$. Leagues that the Ship coming from the *Texel* had made toward *Spain* with the first Wind.

And $K - \frac{KC}{B}$. ——— Leagues which that Ship had to sail when the Wind changed.

Then as $B \,.\, K :: \frac{D}{F} \,.\, \frac{KD}{BF}$. Leagues which that Ship sailed backward every day.

Then as $G \,.\, H :: \frac{KD}{BF} \,.\, \frac{HKD}{GBF}$. Leagues that the Ship coming from *Spain* made every day towards the *Texel*.

Then as $D \,.\, \frac{KD}{BF} :: A \,.\, \frac{KA}{BF}$. Leagues that the first Ship had gone backward at the day of meeting.

Then as $D \,.\, \frac{HKD}{GBF} :: A \,.\, \frac{HKA}{GBF}$. Leagues that the second Ship had sailed forward at the day of meeting.

And $\frac{HKA}{GBF} - \frac{KA}{BF} = K - \frac{KC}{B}$. And $\frac{HKA}{GF} - \frac{KA}{F} = KB - KC$.

And $\frac{HA}{GF} - \frac{A}{F} = B - C$. And $\frac{HA}{F} - \frac{AG}{F} = BG - CG$.

And $A = \frac{BFG - CFG}{H - G}$. Or $\frac{120 - 48}{7 - 2} = \frac{72}{5} = 14\frac{2}{5}$ days.

1℞ Supposition. As $15 \,.\, 300 :: 6 \,.\, 120$. $\frac{KC}{B}$ as before. And $300 - 120 = 180$.

And as $15 \,.\, 300 :: \frac{1}{4} \,.\, 5 \,.\, \frac{KD}{BF}$. And as $2 \,.\, 7 :: 5 \,.\, 17\frac{1}{2} \,.\, \frac{HKD}{GBF}$, as before.

Then as 1 . 5 :: 1℞ . 5℞. So 5℞ + 180 distance of the first Ship from *Spain*.
And 1 . $17\frac{1}{2}$ :: 1℞ . $17\frac{1}{2}$℞, Run of the second Ship towards the *Texel*.
Then 5℞ + 180 = $17\frac{1}{2}$℞; and $12\frac{1}{2}$℞ = 180; and 1℞ = $14\frac{2}{5}$.

Wherefore by Consequence if they meet in $14\frac{2}{5}$ days, then had the Ship from *Spain* sailed 252 Leagues, because she sailed $17\frac{1}{2}$ Leagues in a day; and such a Course will run 300 Leagues in $17\frac{1}{7}$ days.

Q. Of digging a Well.

21. A Gentleman hired a Workman to dig a Well 12 Feet deep for 12 *s.* When the Workman had digged 8 Feet, they fall at Variance, and the Gentleman will pay him off: what must he give the Workman for his Work, considering the Labour at the Bottom is worth more than that at the Top?

Resolution.

Answ. $5\frac{7}{13}$ *s.* For such a Proportion of the 12 *s.* is the Sum of an *Arithmetical Progression*, beginning with $\frac{2}{13}$, and continued to 8 Terms; which by an increase of $\frac{2}{13}$ to every Term, and continued to 12 Terms, will make the Sum thereof 12 *s.*

Here working in Numbers without Species, a *Progression Arithmetical* is framed of 12 Terms from an Unit, whose Sum being 78, and the Sum of 8 Terms thereof but 36, the *Analogy* is, As the Sum of the greater Number of Terms, is to the whole Price: So is the Sum of the lesser Number of Terms, to the Price desired.

$$1+2+3+4+5+6+7+8+9+10+11+12 = 78$$
$$1+2+3+4+5+6+7+8 = 36$$

Where-

Wherefore as $\overset{Z}{78} . \overset{s.}{12} :: \overset{Z}{36} . \overset{s.}{5\frac{7}{13}}$ price of the 8 Feet.

Admit then in *Species* alike *Progression* be framed, whose first Term be α: Then shall the Sum of 12 Terms be $78\,\alpha$, and the Sum of 8 Terms $36\,\alpha$: And let 12 *s.* the whole Price be B, and A supposed for the Number sought, or Sum of 8 Terms. Then because the Product of the Means is equal to the Product of the Extreams;

$$78\,\alpha\,A = 36\,\alpha\,B. \qquad \text{And } 78\,A = 36\,B.$$

$$\text{And } 13\,A = 6\,B. \quad \text{Or } \frac{6 \times 12}{13} = \frac{72}{13} = 5\tfrac{7}{13}.$$

So by *Cossicks* supposing $1\,z$ for the Sum of 8 Terms:

$78\,z = 36 \times 12$. Or $78\,z = 432$. And $1\,z = 5\frac{7}{13}$.

22. A Workman is agreed with to make a Well of 12 Feet deep for 12 *s.* and after digging 8 Feet thereof, fell sick, and desired Money for what he had done: but because the digging the lower Part is more labour than the upper, Allowance was agreed on, that every Foot should be a Penny more than the other: what then shall the Workman receive for digging the first 8 Feet? Q. *Of digging a Well.*

Answ. 6 *s.* 8 *d.* For such is the Sum of 8 Terms of a *Progression*, the Sum of 12 Terms whereof is 12 *s.* and the Excess 1 *d.* Resolution.

The *Data* here being T 12. X 1. Z 12 *s.* or 144 *d.* by the former Rules in *Progression* may be found α the first Term, or ω the last Term; and thereby the Sum of 8 Terms of such a *Progression* to be 6 *s.* 8 *d.* as before.

$$\text{For } \frac{2Z}{2T} - \frac{TX}{2} + \frac{X}{2} = \underset{6\frac{1}{2}\,d.}{\alpha}. \quad \text{And } \frac{2Z}{2T} + \frac{TX}{2} - \frac{X}{2} = \underset{17\frac{1}{2}\,d.}{\omega}.$$

Otherwise let a *Progression* be framed to 12 Terms, whose first Term may be supposed α, and the Excess 1. So shall the second Term be $\alpha+1$, and the Third $\alpha+2$, *&c.* Then will the eighth Term be $\alpha+7$, and the 12*th* Term $\alpha+11$; and the Sum of 12 Terms $12\alpha+66$, and the Sum of 8 Terms thereof $8\alpha+28$. Now because $12\alpha+66=144$. And $12\alpha=78$. And $1\alpha=6\frac{1}{2}$; therefore $8\alpha+28=80$, that is $52+28$.

Likewise by *Cossicks* $12\,z+66=144$. And $1\,z=6\frac{1}{2}$.
Wherefore $8\,z+28=80$. For $8 \times 6\frac{1}{2}=52$. And $52+28=80$.

23. Four Men drink together, and play at Tables for the Wine, every Game 1 *d.* and after they had plaid a while, they found *A* had lost most; whereupon he payeth a Pint of Wine of 5 *d.* upon the Reckoning; and besides found his Loss three times as much more as *B* who had lost least of all; and *C* had lost 2 *d.* more than *B*, and *D* was 4 *d.* less indebted than *A*. Whereupon at last they plaid again who should pay all, and it happened upon *B*: So besides the 5 *d.* *A* had paid, *B* paid 27 Pence: how much should each of them have paid at the first? Q. *Of paying a Reckoning.*

Answ. *A* 14 *d.* *B* 3 *d.* *C* 5 *d.* *D* 10 *d.* For taking 5 from 14, the Remain 9 is 3 times 3; and $3+2=5$; and $14-4=10$. Resolution.

b 5. *d* 2. *e* 4. *f* 27. Suppose the Debt of *A* in all to be *a*.

Then substracting the 5 *d.* paid, the Remain shall be $a-b$. And this being 3 times so much as *B*, he should therefore pay $\frac{a-b}{3}$, to which *d* added, makes the Debt of *C* to $\frac{a-b}{3}+d$. And because *D* was 4 *d.* less indebted than *A*, therefore the Score of *D* was $a-e$. And then these added together must be equal to 27.

$$\left.\begin{array}{l} A\,.\; a-b \\ B\,.\; \frac{a-b}{3}. \\ C\,.\; \frac{a-b}{3}+d. \\ D\,.\; a-e \end{array}\right\} \begin{array}{l} \textit{Total of their Debts.} \\ 2\tfrac{2}{3}\,a - 1\tfrac{2}{3}\,b + d - e = f. \end{array}$$

Then

Then $8a-5b+3d-3e=3f$. And $8a=3f+5b+3e-3d$.

And $a=\dfrac{3f+5b+3e-3d}{8}$. Or $\dfrac{81+25+12-6}{8}=\dfrac{112}{8}=14$.

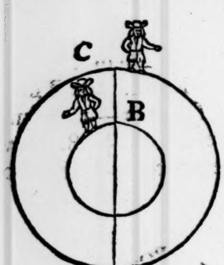
Also let the Debt of *A* be $1z$. Then taking 5 *d.* thence, the rest $1z-5$ must be divided by 3. So is the Debt of *B* $\dfrac{1z-5}{3}$. And *C* being 2 *d.* more, must be $\dfrac{1z-5}{3}+2$. And *D* is indebted $1z-4$. The Total of which 4 Sums is $2\frac{2}{3}z-10\frac{1}{3}$; and equal to 27.
Then if $2\frac{2}{3}z-10\frac{1}{3}=27$, shall $2\frac{2}{3}z=37\frac{1}{3}$.
And $8z=112$; and $1z=14$, the whole Loss of *A*.

Q. Of compassing Circles.

24. Let there be two Circles one within another, described upon the same Center (as below); and suppose one upon the outermost Circle in C goeth on the right Hand, and compasseth it about in 12 Hours: And another within upon the innermost Circle in B, at the same Time and in the same Proportion as the first, goeth on the left Hand, and compasseth it about in 3 Hours: when shall they come to be one under the other, or as near together as when they first began their Journies?

Resolution.

Answ. In $2\frac{2}{5}$ Hours: For if the greatest be in compass 8 Miles, then shall he at C in that time go $1\frac{3}{5}$ Mile. And so must the smallest Circle at B be 2 Miles, (because if he in the greatest Circle go in 12 Hours 8 Miles, then he in the smallest must go in 3 Hours 2 Miles): and then if in 3 Hours he goeth within a Compass of 8 Miles, in $2\frac{2}{5}$ Hours he shall go within the Compass of $6\frac{2}{5}$ Miles; which added to $1\frac{3}{5}$ Mile, shall make together 8 Miles.



b 12. d 3. Supposition for the Time sought A.

Then as $b \,.\, 1 :: A \,.\, \dfrac{A}{b}$. And as $d \,.\, 1 :: A \,.\, \dfrac{A}{d}$.

Both which Quotients added together, make $\dfrac{A}{b}+\dfrac{A}{d}$.

Equal to 1 Hour. And then reduced

$\dfrac{Ad+Ab}{bd}=1$. And $Ad+Ab=bd$. And $A=\dfrac{bd}{d+b}$.

Or $\dfrac{36}{12+3}=\dfrac{36}{15}=2\frac{2}{5}$.

The Time desired $1z$. Then as $12 \,.\, 1 :: 1z \,.\, \frac{1}{12}z$.
And as $3 \,.\, 1 :: 1z \,.\, \frac{1}{3}z$. And $\frac{1}{12}z+\frac{1}{3}z=\frac{5}{12}z$.
And $\frac{5}{12}z=1$. And then $5z=12$. And $1z=2\frac{2}{5}$.

Q. Of compassing Circles.

25. If in the Circles aforesaid the two Men go both one Way, and keep going in like Pace and Proportion: when shall they come to the same Posture they were in at first?

Resolution.

Answ. In 4 Hours: For if the greatest Circle be 8 Miles, then shall the least be 2 Miles: and if 12 Hours give 8 Miles, then 4 Hours shall give $2\frac{2}{3}$ Miles for him in C: and if 3 Hours give 2 Miles, then 4 Hours shall give $2\frac{2}{3}$ Miles for B, thereby shewing the one in his Circle hath gone as many Miles as the other in his Circle.

Here one of the Quotients is to be taken from the other, that is, $\dfrac{A}{b}$ from $\dfrac{A}{d}$; and then $\dfrac{A}{d}-\dfrac{A}{b}$ reduced, shall be $\dfrac{Ab-Ad}{db}=1$.

And $Ab-Ad=db$. And $A=\dfrac{db}{b-d}$. Or $\dfrac{36}{12-3}=\dfrac{36}{9}=4$.

So by *Cossicks* $\frac{1}{3}z-\frac{1}{12}z=\frac{1}{4}z$. And $\frac{1}{4}z=1$. And $1z=4$.

Q. Of two Servants, what Cloth they received of their Master.

26. A Merchant hath 2 Servants, to whom he delivereth together the Value of 300 *l.* in Linen Cloth; and one of them selleth his Part, and loseth therein with his Charges $\frac{1}{8}$ Part of that he received of his Master, and with the rest he buyeth Spices, and gaineth by them 42 *l.* The Second gaineth by his Cloth $\frac{1}{4}$ Part as much as he had received of his Master, out of which he spendeth 12 *l.*

And

And when they came home, they both pay their Master 330 *l.* how much Money in Cloth did each of them receive of their Master?

Answ. *A* 200 *l.* and *B* 100 *l.* For 200 with 25 (that is $\frac{1}{8}$) loss, is 175, to which 42 (Gain) added, is 217. And 100 with 25 (that is $\frac{1}{4}$) Gain, is 125; from which 12 (spent) being taken away, the Remain is 113. And $217+113=330$. Resolution.

C 300. D 42. F 12. G 330. Suppose *A* received *a*.

Then taking away $\frac{1}{8}$ leaves $\frac{7}{8}a$; to which adding D, makes $D+\frac{7}{8}a$, that *A* must bring his Master. Now if *A* received of his Master *a*, then must *B* receive $C-a$; to which $\frac{1}{4}$ added, and his Expences deducted, makes him bring his Master home $1\frac{1}{4}C-1\frac{1}{4}a-F$, both which must be equal to G; wherefore shall $1\frac{1}{4}C+D-\frac{3}{8}a-F=G$; and $1\frac{1}{4}C-\frac{3}{8}a=G+F-D$; and $\frac{10C-3a}{8}=G+F-D$. And $10C-3a=8G+8F-8D$; and $3a=10C+8D-8G-8F$. And consequently $a=\frac{10C+8D-8G-8F}{3}$. Or $\frac{3000+336-2640-96}{3}=\frac{3336-2736}{3}=\frac{600}{3}=200$.

In like sort by *Cossicks*, if *A* receive $1\mathfrak{z}$, then $\frac{1}{8}$ taken away, and 42 added, makes him bring home $\frac{7}{8}\mathfrak{z}+42$. And if *A* receive $1\mathfrak{z}$, then *B* receives $300-1\mathfrak{z}$; to which $\frac{1}{4}$ added, and 12 taken away, makes him bring home $363-1\frac{1}{4}\mathfrak{z}$: both Sums of *A* and *B* added, makes $405-\frac{3}{8}\mathfrak{z}=330$. And $\frac{3}{8}\mathfrak{z}=75$; and $3\mathfrak{z}=600$; and $1\mathfrak{z}=200$.

27. If a Merchant buy Rie for 36 *s.* the Quarter: for how much shall he sell it again that he may gain by 120 *s.* imployed therein, as much as he received for a Quarter? *Q. Of Rie, how sold by the Quarter.*

Answ. For $51\frac{3}{7}$ *s.* the Quarter: Seeing if 36 gain $15\frac{3}{7}$, 120 shall gain $51\frac{3}{7}$. Resolution.

B 36. C 120. Supposition A.

Then as $B \,.\, A :: C \,.\, \frac{AC}{B}$. And $\frac{AC}{B}=C+A$.

And $AC=BC+BA$. And $AC-BA=BC$.

And $A=\frac{BC}{C-B}$. Or $\frac{4320}{120-36}=\frac{4320}{84}=51\frac{3}{7}$.

By *Cossicks*, supposing $1\mathfrak{z}$: Then as $36 \,.\, 1\mathfrak{z} :: 120 \,.\, 3\frac{1}{3}\mathfrak{z}$.
And $3\frac{1}{3}\mathfrak{z}=120+1\mathfrak{z}$. And $2\frac{1}{3}\mathfrak{z}=120$; and $7\mathfrak{z}=360$.
And $1\mathfrak{z}=51\frac{3}{7}$.

28. A Merchant hath bought 7 Yards of Cloth, 8 Ells of Damask, and 9 Yards of Sattin together, for 74 *l.* Flemish: And of the same selleth 5 Yards of Cloth, 4 Ells of Damask, and 6 Yards of Sattin for 47 *l.* The Yard of Cloth cost 4 *l.* what cost the Ell of Damask, and Yard of Sattin severally? *Q. Of Damask, Sattin & Cloth, the Price.*

Answ. The Ell of Damask $2\frac{3}{4}$ *l.* and the Yard of Sattin $2\frac{2}{3}$ *l.* For $7\times4=28$. And $8\times2\frac{3}{4}=22$; and $9\times2\frac{2}{3}=24$; and $28+22+24=74$. Resolution.

B 7. C 8. D 9. F 74. G 5. H 4. K 6. M 47.

Suppose the Ell of Damask cost A.

Then Cloth cost	BH.	Sold G for	GH.
Damask	CA.	H for	HA.
Sattin	F–BH–CA.	K for	$\frac{KF-KBH-KCA}{D}$.

Because as $D \,.\, F-BH-CA :: K \,.\, \frac{KF-KBH-KCA}{D}$.

And then $GH+HA\frac{KF-KBH-KCA}{D}=M$.

And by Reduction $GHD+HAD+KF-KBH-KCA=MD$.

And likewise $HAD-KCA=MD+KBH-KF-GHD$.

And then $A=\frac{MD+KBH-KF-GHD}{HD-KC}$. Or $\frac{423+168-444-180}{36-48}=\frac{33}{12}=2\frac{3}{4}$.

In *Cossicks*, supposing the Ell of Damask cost $1\mathfrak{z}$.

Then Cloth cost	28 *l.*	Sold 5 Yards for	20 *l.*
Damask	$8\mathfrak{z}$.	4 Ells for	$4\mathfrak{z}$.
Sattin the rest	$46\,l.-8\mathfrak{z}$.	6 Yards for	$30\frac{2}{3}l.-5\frac{1}{3}\mathfrak{z}$.

 Because

Because as $9 \,.\, 46\,l. - 8\,\text{℞} :: 6 \,.\, 30\tfrac{2}{3}\,l. - 5\tfrac{1}{3}\,\text{℞}$.
Then $20 + 4\,\text{℞} + 30\tfrac{2}{3} - 5\tfrac{1}{3}\,\text{℞}$, that is $50\tfrac{2}{3} - 1\tfrac{1}{3}\,\text{℞} = 47$.
And $1\tfrac{1}{3}\,\text{℞} = 3\tfrac{2}{3}$; and $4\,\text{℞} = 11$; and $1\,\text{℞} = 2\tfrac{3}{4}$.

Q. Of Sugar the Hundred Netto.

29. A Grocer buyeth 21 C. 98 ℔ of Sugar for 600 *l.* Flemish; and being allowed Tare upon the Hundred 10 ℔, the Hundred *Netto* costeth him 30 *l.* Flemish: how many Pound is the Hundred *Netto* accompted?

Resolution.

Answ. 102 ℔: For if 30 *l.* buy 112 ℔, then will 600 *l.* buy 2240 ℔; which at 102 for 1 C. makes 21 C. 98 ℔.

B 21. D 98. F 600. G 10. H 30. Supposition A.

Then as $H \,.\, A+G :: F \,.\, \frac{AF+GF}{H}$. And $\frac{AF+GF}{H} = B+D$.

And $AF+GF = BH+DH$. And $AF = BH+DH-GF$.

And $A = \frac{BH+DH}{F} - G$. Or $\frac{630}{600}\,C. + \frac{2940}{600}\,℔ - 10\,℔ = 1\frac{1}{20}\,C. + 4\frac{18}{20}\,℔ - 10\,℔$.

Or $1\frac{1}{20}\,C - 5\frac{2}{20}\,℔$. And $\frac{1}{20}\,C = \frac{102}{20}\,℔$; and $1\,C = 102\,℔$.

Supposing 1 ℞ in *Cossicks*; Then as $30\,l. \,.\, 1C + 10\,℔ :: 600\,l. \,.\, 20C + 200\,℔$.

And $21C + 98\,℔ = 20C + 200\,℔$; and $21C = 20C + 102\,℔$; and $1C = 102\,℔$.

Q. Of Pitch, how much sold.

30. Two Merchants sold Pitch for 504 *l.* whereof *B* sold 3 Tuns more than *A.* Whereupon *A* said to *B*, I would have sold all your Pitch for 288 *l.* And *B* said to *A*, I would have sold all your Pitch for 216 *l.* how many Tuns of Pitch did each of them sell?

Resolution.

Answ. *A* sold 9 Tuns, and *B* 12 Tuns; which at the Rate of 504 *l.* for the whole 21 Tuns, is 24 *l.* a Tun, and makes 9 amount to 216 *l.* for *A*, and 288 *l.* for *B*.

C 504. D 3. F 288. G 216. Suppose *A* sold *a*; then *B* sells $a+3$.

And as $a \,.\, G :: a+D \,.\, \frac{Ga+DG}{a}$. And as $a+D \,.\, F :: a \,.\, \frac{Fa}{a+D}$.

Then both Quotients added are $\frac{Faq+Gaq+2DGa+DqG}{aq+Da} = C$.

And $Faq+Gaq+2DGa+DqG = Caq+CDa$. And because by the *Data* it appears C was equal to F and G; therefore their Squares are also equal, and so omitted. And then $2DGa+DqG = CDa$; and $CDa - 2DGa = DqG$.

And $a = \frac{DqG}{CD-2DG}$. Or $\frac{1944}{1512-1296} = \frac{1944}{216} = 9$.

In *Cossicks*, supposing 1 ℞ for *A*; Then must *B* sell $1\,\text{℞}+3$.

And $1\,\text{℞} \,.\, 216 :: 1\,\text{℞}+3 \,.\, \frac{216\,\text{℞}+648}{1\,\text{℞}}$. And as $1\,\text{℞}+3 \,.\, 288 :: 1\,\text{℞} \,.\, \frac{288\,\text{℞}}{1\,\text{℞}+3}$.

Total $\frac{504\,\text{ʒ} + 1296\,\text{℞} + 1944}{1\,\text{ʒ} + 3\,\text{℞}} = 504$. And by Reduction

$504\,\text{ʒ} + 1296\,\text{℞} + 1944 = 504\,\text{ʒ} + 1512\,\text{℞}$. And $216\,\text{℞} = 1944$. And $1\,\text{℞} = 9$.

Q. Of Cloth shrunk.

31. Eight Ells of Cloth being $2\frac{1}{4}$ broad, after shrinking $3\frac{3}{4}$ Ells thereof make but $3\frac{3}{5}$ in length, and $2\frac{1}{2}$ Ells in breadth, make but $2\frac{1}{3}$. A second sort of Cloth being $1\frac{3}{4}$ Ell broad; when it is wet is no broader than $1\frac{1}{2}$, and the length of 6 Ells but $5\frac{1}{4}$: how much of this second sort of Cloth will serve to line the first 8 Ells after the shrinking.

Resolution.

Answ. $12\frac{756}{2625}$ Ells; which will appear, if the length and breadth of each dry, or the length and breadth of each wet and shrunk, be multiplied one by the other, because the Product of the one will be equal to the Product of the other.

As $\frac{8}{1} \times \frac{9}{4}$. Or $10\frac{2}{7} \times 1\frac{3}{4} = \frac{72}{4} = 18$.

Omitting the *Species*, for brevity-sake, the Work by *Cossicks* is thus:

As $3\frac{3}{4}$ dry . $3\frac{3}{5}$ wet :: 8 dry . $7\frac{17}{25}$ wet and shrunk in length.

As $2\frac{1}{2}$ dry . $2\frac{1}{3}$ wet :: $2\frac{1}{4}$ dry . $2\frac{1}{10}$ wet and shrunk in breadth.

$7\frac{17}{25}$ and $2\frac{1}{10}$, or $\frac{192}{25} \times \frac{21}{10} = \frac{4032}{250}$ Square Content.

Then

Then for the length of the second Cloth, suppose 1 ℨ.
As 6 dry . $5\frac{1}{4}$ wet :: 1 ℨ dry . $\frac{7}{8}$ ℨ, wet and shrunk in length.
The breadth given $1\frac{1}{2}$ multiplied by $\frac{7}{8}$ ℨ $= \frac{21}{16}$ ℨ, Square Content.
Then $\frac{21}{16}$ ℨ $= \frac{4032}{250}$. And 5250 ℨ $=$ 64512.
And 1 ℨ $= 12\frac{756}{2625}$.

32. If 6 Ells of Black, 5 Ells of Red and 7 *l.* be worth as much as 9 Ells of Black, and $3\frac{1}{3}$ Ells of Red: And if at the same Price 8 Ells of Black, and 7 of Red lacking 5 *l.* cost as much as 6 Ells of Black, 5 Ells of Red and 9 *l.* how much shall 1 Ell of Black and 1 Ell of Red cost severally? *Q. Of the Price of Black and Red.*

Answ. One Red 3 *l.* and 1 Black 4 *l.* Resolution.

Here being two *Equations* given, there needs no Supposition.

Equations given.	*Reduced.*
$6B + 5R + 7 = 9B + 3\frac{1}{3}R$	$1\frac{2}{3}R + 7 = 3B.$
$8B + 7R - 5 = 6B + 5R + 9$	$1B + 1R = 7.$
Both reduced *Equations* added	$1B + 2\frac{2}{3}R + 7 = 3B + 7.$
This last *Equation* reduced is	$1\frac{1}{3}R = 1B.$
The second reduced *Equation* added	$1B + 1R = 7.$
Total	$1B + 2\frac{1}{3}R = 1B + 7.$

Then $2\frac{1}{3}R = 7$. And $7R = 21$. And $1R = 3$.
And if 1R be 3, then 1B must be 4, because by the second reduced *Equation* both were equal to 7.

33. A King lying with a great Army before an Enemy, the Adversary endeavoureth to corrupt one of the Heralds to declare the Strength of the Army: who willing to receive the Reward, dealeth subtilly, and deviseth this Answer, (nevertheless true) Look, saith he, how many Dukes there are, and for each of them there are twice so many Earls, and under every Earl there are 4 times so many Souldiers as there be Dukes in the Field: And when the Muster of the Souldiers was taken, the 200 Part of them was 9 times so many as the Number of the Dukes: how many then were there of each Sort? *Q. Of an Army, how many therein.*

Answ. 15 Dukes, 450 Earls, (that is 15 times 15 twice); and the Number of Souldiers (being 4 times 15 multiplied by 450) is 27000, the 200 Part whereof (being 135) is 9 times 15 the Number of the Dukes. Resolution.

B 200. Supposition for the Dukes A.
Then the Earls 2Aq; that is $A \times A + A \times A$.
And the Souldiers 8Ac, that is $2Aq \times 4A$.

Then $\frac{8Ac}{B} = 9A$. And $8Ac = 9BA$; and $\frac{Ac}{A} = \frac{9B}{8}$.

And $Aq = \frac{9B}{8}$. Or $\sqrt{q}\,\frac{1800}{8} = \sqrt{q}\,225\,($ 15.

Dukes suppose 1 ℨ. Then Earls 2 ʒ. And Souldiers 8 ȼ.

Then $\frac{8\text{ȼ}}{200} = 9$ ℨ. And 8 ȼ $=$ 1800 ℨ. And 8 ʒ $=$ 1800N.

And 1 ʒ $=$ 225N; whose Square Root is 15.

34. Among 4 Walls, the 2 longest are in proportion to the shortest, as 5 to 3; and to the Height they be double Sesquialter: now multiplying the longest by the shortest, and the Total by the Height, there will arise 39930 Feet: what then is the Length and Height of each Wall? *Q. Of the length and height of four Walls.*

Answ. The Height of the shortest is 22 Feet, and the longest being double *Sesquialter* (that is $2\frac{1}{2}$ as much) must be 55, and the Length of the shortest 33, that is as 5 to 3. Resolution.

B 39930. Supposition for the Height of the shortest A.
Then the longest must be $2\frac{1}{2}$ A. And because the longest to the shortest are as 5 to 3, the shortest must be $1\frac{1}{2}$ A. For $5 . 3 :: 2\frac{1}{2} . 1\frac{1}{2}$.
And $A \times 2\frac{1}{2}A \times 1\frac{1}{2}A = \frac{15}{4}Ac$. And $\frac{15}{4}Ac = B$.

Wherefore $Ac = \frac{4B}{15}$. or $\frac{159720}{15} = 10648$. And $\sqrt{c}\,10648 = 22$.

In *Cossicks* 1ƶ. Then 1ƶ × $2\frac{1}{2}$ƶ × $1\frac{1}{2}$ƶ = $\frac{15}{4}$φ.
And $\frac{15}{4}$φ = 39930. And 15φ = 159720. And 1φ = 10648.
And √φ 10648 = 22.

Q. Of Stock traded with.

35. Traffique is made at *Danske*, and Gain thereupon in 100 *l.* as many Pounds as there were in Stock at first; after which Traffique with the Gain only is made at *Hamburgh*, where the Gain of 100 *l.* is as much as the Gain at *Danske*, and the Gain at *Hamburgh* being found to be $16\frac{1}{5}$ *s.* the Question is, what was the Stock at first?

Resolution.

Answ. 30 *l.* For if 100 *l.* gain 30, then 30 shall gain at *Danske* 9 *l.* And if 100 *l.* gain 9, then 9 shall gain $16\frac{1}{5}$ *s.*

Here to make the Denominations agree, $16\frac{1}{5}$ *s.* is to be turned into a part of a Pound, and is $\frac{81}{100}$ *l.*

$$B\,100 . \frac{R}{B}\ \frac{81}{100}.\ \text{Suppose the Stock A.}$$

$$\text{Then } B . A :: A . \frac{Aq}{B}.\ \text{Gain at } \textit{Danske}.$$

$$\text{And } B . \frac{Aq}{B} :: \frac{Aq}{B} . \frac{Aqq}{Bc}.\ \text{Gain at } \textit{Hamburgh}.$$

$$\text{Then } \frac{Aqq}{Bc} = \frac{R}{B}.\ \text{And } Aqq = \frac{BcR}{B}.\ \text{or } \frac{81000000}{100} = 810000.$$

And √qq 810000 = 30.

Supposing the Stock in *Cossicks* 1ƶ.

$$\text{Then } 100 . 1ƶ :: 1ƶ . \frac{1ʒ}{100}.\ \text{Gain at } \textit{Danske}.$$

$$\text{And } 100 . \frac{1ʒ}{100} :: \frac{1ʒ}{100} . \frac{1ʒʒ}{1000000}.\ \text{Gain at } \textit{Hamburgh}.$$

And $\frac{1ʒʒ}{1000000} = \frac{81}{100}$. And 1ʒʒ = 810000; and the Squared square Root of 810000 is 30. 810000 (900 (30.

Q. Of the Number of Merchants and their Gain.

36. Certain Merchants make a Company, and every one putteth in Stock 150 times so many Pounds as there are Merchants, and they gain as much on the Hundred as the Number of Merchants are; and if $9\frac{1}{2}$ be taken out of the Gain, and $9\frac{1}{2}$ be added to the Gain, and the Remain and Total be multiplied together, the Product will be 1550 *l.* how many Merchants were there, and how much did they gain?

Resolution.

Answ. 3 Merchants; 450 *l.* each Man's Stock (that is 3 times 150) and the Gain $40\frac{1}{2}$ *l.* For $40\frac{1}{2} - 9\frac{1}{2} = 31$; and $40\frac{1}{2} + 9\frac{1}{2} = 50$; and $50 \times 31 = 1550$.

Suppose the Number of Merchants in *Cossicks* 1ƶ, (omitting the Work in *Species*) so is every Man's Stock 150ƶ; and the whole Stock 150ʒ.

And as 100 . 1ƶ :: 150ʒ . $\frac{150}{100}$φ. the whole Gain.

Then $\frac{3}{2}$φ − $\frac{19}{2}$ × $\frac{3}{2}$φ + $\frac{19}{2}$ = 1550. that is, $\frac{9ʒφ - 361}{4} = 1550$.

And 9ʒφ − 361 = 6200. And 9ʒφ = 6561; and 1ʒφ = 729, whose Zenzicube Root is 3.

Of Affected Equations.

Touching Mixt or Affected Equations.

It is to be noted, that the *Equation* first found may be reduced to 3 sorts of Numbers or more.

Brought to 3 Numbers.

An *Equation* consisting of 3 Numbers or Magnitudes, must have one a Power, another a Root, and the third an Absolute Number; and of these by the Signs + and − there are 3 sorts, in every of which the greatest Denomination is set Solitary as equal to the other two.

As ʒ = ƶ + N. ʒ = N − ƶ. ʒ = ƶ − N.

And accordingly with the Antients, the Extraction, or rather the finding the Value of the Root, is of 3 Varieties; but of late both the Extraction of the Roots of these and others consisting of more than three Magnitudes, comprehended under

der one and the ſame Method, is moſt ingeniouſly taught by the aforeſaid Mr. *Oughtred*, as ſhall be ſhewn hereafter. Notwithſtanding becauſe very many Propoſitions of ordinary uſe are reduced to *Equations* of 3 Numbers, and the antient Way of finding the Value of the Root thereof is by ſome accompted the moſt eaſy, that ſhall be firſt examined.

Reſolution the old Way.

Reſolution of ſome Affected Equations *the antient Way.*

In 3 Numbers.

When an *Equation* conſiſts of 3 Numbers or Magnitudes, either they are in their Natural Order, as aforeſaid, without omiſſion of any Denomination between them, or they are not.

1. If in a natural Order, and none omitted, the firſt Sort.

1ſt. If they are in their Natural Order, and without ſuch Omiſſion, and the *Equation* be of the firſt ſort that ʒ = ȥ + N. or ʒ = N + ȥ which is all one: The Rule is, multiply half the middle Quantity ſquarely; to the Product add the Abſolute Number, out of the Total take the Square Root, and thereto add the former half of the middle Quantity, and this laſt Total ſhall be the Value of one Root of that *Equation*.

Examples.

As ſuppoſe 1ʒ = 4ȥ + 21N. or 1ʒ = 90N + 9ȥ.

In the firſt, the half of 4 (belonging to ȥ the middle Quantity) is 2, which ſquared is 4, to which 21 added makes 25, whoſe Square Root is 5, to which 2 the half added, makes the Total 7, for the Value of the Root ſought.

For 1ʒ = 4ȥ + 21 N. The Root being 7, ſhall make 49 = 28 + 21.

In the other Inſtance $4\frac{1}{2}$ȥ or $\frac{9}{2}$ȥ, the half of 9ȥ ſquared, produceth $\frac{81}{4}$; this added to 90 or $\frac{360}{4}$, makes $\frac{441}{4}$, the Square Root whereof is $\frac{21}{2}$, to which $\frac{9}{2}$ added makes $\frac{30}{2}$ or 15 for the Value of 1ȥ, and appears true.

Seeing if 15 be 1ȥ, then 9ȥ is 135, whereunto 90 being added makes the Total 225, and ſo much is the Square of 15.

If ſome Quantity omitted.

But when the *Equation* of this ſort omits ſome Denomination orderly between the Quantities, then after the Root found in like manner as above, from this ſo found Root or Value, extract a Root according to the Number of Quantities omitted, *viz.* if 1 the Square Root, if 2 the Cube Root, *&c.*

Examples.

As ſuppoſe 1ʒʒ = 2ʒ + 8N. Or 1ʒφ = 7φ + 8 N.

In the firſt the Value of the Root gotten as above, will be 4, which becauſe one Denomination is omitted muſt be a Square Number, and the Root thereof 2 the Number ſought.

Thus $\frac{1}{2}$ of 2 is 1, and 1 ſquared is 1, to which 8 added is 9, whoſe Root is 3, to which 1 the half added makes 4, and √ʒ 4 = 2.

And 1ʒʒ = 2ʒ + 8N. The Root being 2 ſhall make 16 = 8 + 8.

In the other Inſtance, the Root gotten as aforeſaid will be 8, out of which the Cube Root muſt be extracted, becauſe 2 Quantities were orderly omitted in the *Equation*; and this Cube Root 2 ſhall be the Value of the Root ſought, whereof the *Equation* is framed.

Becauſe $\frac{1}{2}$ of 7φ the Middle Quantity is $\frac{7}{2}$, which ſquared is $\frac{49}{4}$, to which 8 added that is $\frac{32}{4}$, makes $\frac{81}{4}$, the Square Root whereof is $\frac{9}{2}$, to which the half $\frac{7}{2}$ added, makes $\frac{16}{2}$ or 8, and √φ8 = 2.

And if 1ȥ be 2, then the Cube will be 8, and 7φ are 56, which with 8 N makes 64, a Zenzicube Number, and hath 2 for the Root.

2. If in a natural Order, and none omitted, the ſecond Sort.

2ly. Equations affected of the ſecond Sort, (that is) conſiſting of 3 Numbers in their natural Order, without omiſſion of any Denominations between them, and have their Middle Quantity joined with the Sign, as ʒ = N − ȥ, have the Rule to find the Value of the Root in all reſpects, ſave one like the firſt Sort of *Affected Equations*; which Difference is only inſtead of adding half the Middle Quantity to the Root: here it is to be ſubſtracted, and the Remainder ſhall be the Value of the Root deſired.

Examples.

As ſuppoſe 1ʒ = 60N − 7ȥ. Or 1ʒ = 153N − 8ȥ.

In the firſt, the half of 7, which is $3\frac{1}{2}$ or $\frac{7}{2}$ ſquared, is $\frac{49}{4}$; whereto 60, or $\frac{240}{4}$ added, makes $\frac{289}{4}$ the Square Root, whereof is $\frac{17}{2}$; from whence $\frac{7}{2}$ taken, leaves $\frac{10}{2}$ or 5 for the Value of the Root deſired.

For 1ʒ = 60 N − 7ȥ: The Root being 5, makes 25 = 60 − 35.

In the other Instance, half 8 is 4, the Square 16; to which 153 added, is 169, the Square Root whereof is 13; from whence 4 taken, leaves 9 for the Value of 1 ℨ.

So shall the Square be 81, and so much is 153 lacking 72 that is 8 ℨ.

If some Quantity omitted.

But when in *Equations* of this sort some Quantities are orderly omitted between the Quantities propounded; then after the Root or Value rather found as before, out thereof let there be a Root extracted, according to the Number of Quantities omitted, as before noted in the former sort of *Equations*.

Examples.

As suppose 1 ʒʒ = 117N — 4ʒ. Or 1 ʒϕ = 810N — 3ϕ.

In the first of these, squaring 2 the Half of 4, there ariseth 4, which added to 117 maketh 121, the Square Root of which is 11; whence if 2 the first Half be substracted, there resteth 9; which because one Denomination is omitted, the Square Root thereof 3 is the Number desired.

In the other $\frac{3}{2}$ squared is $\frac{9}{4}$, which added to 810, makes $\frac{3249}{4}$, the Square Root of which is $\frac{57}{2}$; if then $\frac{3}{2}$ be abated, there remaineth $\frac{54}{2}$, or 27, which is a Cube Number, and the Cube Root thereof 3 is the Value desired.

	1 ʒʒ = 117N — 4ʒ		81 = 117 — 36.
So the Root being 3		is as	
	1 ʒϕ = 810N — 3ϕ		729 = 810 — 81.

3. If in a natural Order, and none omitted, the third sort.

3*ly*. *Affected Equations* of the third Sort, to wit, consisting of 3 Quantities in their natural Order, without omission of any Denominations, and having their least Quantity joined with —, as ʒ = ℨ — N, have 2 Roots or Values, and therefore are called *Ambiguous Equations*: For finding both which, the Rule is thus; Square half the middle Quantity as before, from thence substract the smallest Quantity, extract the Square Root of the Residue, and add this Root to the Half first taken for the Value of one, and take it away from the Half for the Value of the other Root: And either of these Numbers shall be as the Root of the *Equation*, and one of them shall serve to resolve the *Equation*, as occasion shall require; and which of the two is necessary, by the Frame of the Question is directed.

For though no Number can have 2 Square Roots, or 2 Cube Roots, &c. yet one Number may have both a Square Root and a Cube Root. Yea, as before hath been seen, one Number may have several Roots of different Denominations, as 64 hath a Square Root 8, a Cube Root 4, and a Zenzicube Root 2. And so in *Equations* of this kind, 2 Roots or a double Value may be had, unless the Half of the Middle Quantity be equal to the Absolute Number, for then is the Moiety or half of the Middle Quantity the Number sought. And these 2 Roots so called, must be such, as being added together, will make the Number of the Middle Quantity, and multiplied together, will make the Number of the least Quantity, and so may be found without farther Operation.

Examples.

As suppose 1 ʒ = 12 ℨ — 32N.

The Parts of 32 examined, are found to have no more to his Composition likely to this purpose, than 2, 4, 8, 16; but if 2 and 16 be taken, their addition 18 will be greater than 12: wherefore 4 and 8 making 12 by *Addition*, and 32 by *Multiplication*, shall be the 2 desired Numbers.

And this is agreeable to the Rule: For the half of 12 is 6, whose Square is 36, from which is to be taken 32 of the Residue; 4 the Square Root is 2, which either taken from 6 leaves 4 for the one, or added to 6 makes 8 for the other Root or Value required.

So 1 ʒ = 12 ℨ — 32N, let the Root be $\frac{4}{8}$ and 16 = 48 — 32.
64 = 96 — 32.

If some Quantity omitted.

But when the *Equation* of this sort omits some Quantities orderly, there happeneth oftentimes but one Root in *Integers*, and not two: From which Root or Value, as in the others before, must be extracted a Root according to the Denominations omitted.

Examples.

As suppose 1 ʒʒ = 24ʒ — 135N. Or 1 ʒϕ = 12ϕ — 32N.

In the first 12 squared is 144; from which if 135 be taken, the Square Root of the remaining 9 is 3; to which if 12 be added it makes 15, which is no Square Number;

Number; but if 3 be abated from 12, the Remainder 9 is a Square Number, and hath 3 for his Root.

In the other 6 times 6 is 36, whence 32 abated leaves 4, whose Square Root is 2; to which if 6 be added ariseth 8, the Cube Root whereof is 2 for the Number desired; but if 2 be abated from 6, there remaineth 4, which is no Cube Number, nor hath a Cube Root in whole Numbers.

Thus the Root being
3 . $1\text{ʒʒ} = 24\text{ʒ} - 135N$ is as $81 = 216 - 135$.
2 . $1\text{ʒɸ} = 12\text{ɸ} - 32N$ is as $64 = 96 - 32$.

Because in every Operation where an *Equation* ariseth, it is to be noted, That according to the Supposition for Resolution of the Proposal, so will the *Equation* be produced of the one sort or other; and when *Equations* of this sort happen, wherein is a double Valuation of the Root, the form of the Question sheweth which of the 2 Roots is to be taken, unless the Proposition may be truly resolved by both, as sometimes it cometh to pass.

Questions wherein the first sort of Affected Equations *come to be resolved.*

Ex. Where the first Sort are resolved.

1. A Merchant buyeth 45 *C.* and 88 ℔. of Pepper, the Tare is 12 ℔. *per Cent.* the Hundred cost $6\frac{3}{5}$ *l.* and the whole 274 *l.* how many Pound is the Hundred accompted?

Q. Of the Pounds in an Hundred of Pepper.

Answ. 132 ℔.

Resolution.

Here supposing the Hundred to be 1ℨ, the Tare abated makes it $1\text{ℨ} - 12$.

Then as $1\text{ℨ} \,.\, 1\text{ℨ} - 12 :: 45\text{ℨ} + 88 \,.\, \frac{45\text{ʒ} - 452\text{ℨ} - 1056}{1\text{ℨ}}$.

And then as $1\text{ℨ} \,.\, 6\frac{3}{5} :: \frac{45\text{ʒ} - 452\text{ℨ} - 1056}{1\text{ℨ}} \,.\, \frac{297\text{ʒ} - 2983\frac{1}{5}\text{ℨ} - 6969\frac{3}{5}}{1\text{ʒ}}$.

Wherefore $\frac{297\text{ʒ} - 2983\frac{1}{5}\text{ℨ} - 6969\frac{3}{5}}{1\text{ʒ}} = 274$.

And $274\text{ʒ} = 297\text{ʒ} - 2983\frac{1}{5}\text{ℨ} - 6969\frac{3}{5}$. And $23\text{ʒ} = 2983\frac{1}{5}\text{ℨ} - 6969\frac{3}{5}$.
And $1\text{ʒ} = 129\frac{81}{115}\text{ℨ} + 303\frac{3}{115}$. And $1\text{ℨ} = 132$ ℔.

For $\frac{1}{2}$ of $129\frac{81}{115}$ is $64\frac{98}{115}$ or $\frac{7458}{115}$ squared is $\frac{55621764}{13225}$. and $303\frac{3}{115}$ or $\frac{4007520}{13225}$ added is $\frac{59629284}{13225}$ the $\sqrt{\text{ʒ}}$ whereof is $\frac{7722}{115}$ whereto $\frac{7458}{115}$ added makes the Total $\frac{15180}{115}$ or 132.

Proof.

Proof.

If 132 ℔. abate 12 ℔. for Tare, then 1 *C.* shall be but 120 ℔.
And as 1 *C* . 120 ℔ :: 45 *C* + 88 ℔ . 5480 ℔.
And as 132 ℔ . $6\frac{3}{5}$ *l* :: 5480 ℔ . 274 *l.*

2. Two Men have Silks to sell, *viz.* *A* hath 40 Ells, and *B* 90; and the Silk of *A* being not so fine as the Silk of *B*, he selleth in every Angel more by $\frac{1}{3}$ of an Ell than *B* doth, and both their Monies made 42 Angels: now how much did each of them sell for an Angel?

Q. Of Silks, how much sold for an Angel.

Answ. *A* sold $3\frac{1}{3}$ Ells, *B* sold 3 Ells for an Angel.

Resolution.

Here supposing the least Quantity which is *B*, to be 1ℨ;
Then must *A* sell $1\text{ℨ} + \frac{1}{3}$ Ell: and each whole Parcel divided thereby,

is for *B* $\frac{90}{1\text{ℨ}}$ for *A* $\frac{40}{1\text{ℨ} + \frac{1}{3}N}$ or reduced $\frac{120}{3\text{ℨ} + 1N}$

And these two Numbers declaring the Angels each Man received;

It follows that $\frac{120}{3\text{ℨ} + 1N} + \frac{90}{1\text{ℨ}} = 42$. And reduced $\frac{390\text{ℨ} + 90}{3\text{ʒ} + 1\text{ℨ}} = 42$.

And $390\text{ℨ} + 90N = 126\text{ʒ} + 42\text{ℨ}$. And $126\text{ʒ} = 348\text{ℨ} + 90N$.
And $21\text{ʒ} = 58\text{ℨ} + 15N$. And $1\text{ʒ} = \frac{58}{21}\text{ℨ} + \frac{15}{21}N$.
And $1\text{ℨ} = 3$. For $\frac{1}{2}$ of $\frac{58}{21}$ is $\frac{29}{21}$, squared is $\frac{841}{441}$; whereto $\frac{15}{21}$ or $\frac{315}{441}$ added, is $\frac{1156}{441}$, the $\sqrt{\text{ʒ}}$ whereof is $\frac{34}{21}$, to which $\frac{29}{21}$ added makes $\frac{63}{21}$ or 3.

A

Proof.

Proof.

A $3\frac{1}{3}$ or $\frac{10}{3}$) 40 ($\frac{120}{10}$ (12 Angels. B 3) 90 (30 Angels. (*Ells* … *Ells*)

And $12 + 30 = 42$.

Ex. *Where the second Sort are resolved.*

Questions wherein the second sort of Affected Equations *come to be resolved.*

Q. *Of the Stock and Gain of 3 Merchants.*

1. Three traffique together, *A* putteth in Stock 14 *l.* less than *B*, and *B* and *C* together put in 148 *l.* they gain 42 *l.* more than their Stock, of which *A* taketh $60\frac{20}{33}$ *l.* what was each Man's Stock and Part of the Gain?

Resolution.

Answ. The Stock of *A* 50 *l.* *B* 64 *l.* *C* 84 *l.* Gains in all 240 *l.* of which to *A* $60\frac{20}{33}$ *l.* *B* $77\frac{19}{33}$ *l.* *C* $100\frac{9}{11}$ *l.*

Here supposing the Stock of *A* to be $1\mathfrak{z}$; then must the Stock of *B* be $1\mathfrak{z} + 14$; and *B* and *C* making up 148, the Stock of *C* must be $134 - 1\mathfrak{z}$; and all these three added, make $1\mathfrak{z} + 148$ the whole Stock, which with 42 more, that is $1\mathfrak{z} + 190$, is all the Gains.

Then as $1\mathfrak{z} + 148 \,.\, 1\mathfrak{z} + 190 :: 1\mathfrak{z} \,.\, \frac{1\mathfrak{Z} + 190\mathfrak{z}}{1\mathfrak{z} + 148}$. Gain of *A*

And then $\frac{1\mathfrak{Z} + 190\mathfrak{z}}{1\mathfrak{z} + 148} = 60\frac{20}{33}$. And $33\mathfrak{Z} + 6270\mathfrak{z} = 2000\mathfrak{z} + 296000$.

And $33\mathfrak{Z} = 296000 - 4270\mathfrak{z}$. And $1\mathfrak{Z} = \frac{296000}{33} - \frac{4270}{33}\mathfrak{z}$.

And $1\mathfrak{z} = 50$

For $\frac{1}{2}$ of $\frac{4270}{33}$ is $\frac{2135}{33}$, squared is $\frac{4558225}{1089}$, to which $\frac{296000}{33}$, that is $\frac{9768000}{1089}$, added is $\frac{14326225}{1089}$. the $\surd\mathfrak{Z}$ whereof is $\frac{3785}{33}$, from which $\frac{2135}{33}$ taken, the Remainder is $\frac{1650}{33}$ or 50.

Proof.

Proof.

The Stock of *A* less by 14 than *B*; if therefore it be 50, that of *B* must be 64, that is $50 + 14$. And because *B* and *C* made 148, if *B* be 64, then must *C* be 84, for $84 + 64 = 148$: and the Gains being 42 more than the Stock, must be 240, for $50 + 64 + 84 + 42 = 240$; and then by the *Rules of Fellowship*,

If 198 gain 240, the Gain of $\left\{\begin{array}{l} 50 \\ 64 \\ 84 \end{array}\right.$ is $\begin{array}{ll} 60\frac{20}{33} & A. \\ 77\frac{19}{33} & B. \\ 101\frac{9}{11} & C. \end{array}$

Q. *Of the Days in which a Journey is performed.*

2. A Traveller hath a Journey to go of 2955 Miles, and the first Day he goeth $1\frac{1}{2}$ Mile, and every Day afterwards increaseth his Journey by $\frac{1}{6}$ of a Mile as in an *Arithmetical Progression*: in how many Days shall he finish his Journey?

Resolution.

Answ. In 180 Days.

Here are given the first Term of the *Progression* $1\frac{1}{2}$, the Excess $\frac{1}{6}$ and the Sum 2955, to find the Number of Terms; for which supposing $1\mathfrak{z}$, then shall all the Excesses or Number of Spaces be $1\mathfrak{z} - 1N$, by which the Excess multiplied, the Product is $\frac{1\mathfrak{z} - 1N}{6}$ the Sum of all the Excesses; to which the first Term added which is $1\frac{1}{2}$, or reduced to like Denomination $\frac{9}{6}$, so will the Total be $\frac{1\mathfrak{z} + 8N}{6}$ the last Term of that *Progression*; which gotten, because $\alpha + \omega$ in $\frac{1}{2}T = Z$, the first $\frac{9}{6}$ is added to the last Term $\frac{1\mathfrak{z} + 8N}{6}$ and the Total $\frac{1\mathfrak{z} + 17N}{6}$ multiplied by $\frac{1}{2}\mathfrak{z}$, (seeing the whole Number of Places is supposed to be $1\mathfrak{z}$) the Product is $\frac{1\mathfrak{Z} + 17\mathfrak{z}}{12}$, the Sum of the *Progression*.

So is $\frac{1\mathfrak{Z} + 17\mathfrak{z}}{12} = 2955$. And $1\mathfrak{Z} + 17\mathfrak{z} = 35460$.

And

And 1ʒ = 35460 — 17ℨ. And 1ℨ = 180.

For $\frac{1}{2}$ of 17 is $8\frac{1}{2}$, or $\frac{17}{2}$ squared $\frac{289}{4}$, to which $\frac{141840}{4}$ added, makes $\frac{142129}{4}$ whose Square Root is $\frac{377}{2}$, from whence $\frac{17}{2}$ taken, there remaineth $\frac{360}{2}$, or 180.

Proof. Proof.

Seeing α $1\frac{1}{2}$. X $\frac{1}{6}$. Z 2955, are given thereby, the Rules before in *Progression* direct to the finding of T the Number here sought.

For $\sqrt{:}\ \dfrac{\alpha q - \alpha X + \frac{1}{4}Xq + 2ZX}{Xq} : -\alpha + \frac{1}{2}X = T$. That is in Numbers

$$\sqrt{:}\ \dfrac{\frac{9}{4} - \frac{1}{4} + \frac{1}{144} + \frac{5910}{6}}{\frac{1}{36}} : -\tfrac{3}{2} + \tfrac{1}{12} = 180.$$

$$\text{Or } \sqrt{:}\ \tfrac{1}{36}\Big)\dfrac{324 - 36 + 1 + 141840}{144} : -\tfrac{3}{2} + \tfrac{1}{12} = 180.$$

$$\begin{array}{r} 141840 \\ 1 \\ 324 \\ \hline 142165 \\ -36 \end{array}$$

$$\tfrac{1}{36}\Big)\dfrac{142129}{144}\Big(\sqrt{q}\ \dfrac{\dot{5}1\dot{1}6\dot{6}4\dot{4}}{144}\Big(\dfrac{2262}{12}. \quad \text{And } \dfrac{2262 - \frac{3}{2} + \frac{1}{12}}{12} = 180.$$

$$\text{And } \dfrac{2262 - \frac{36}{2} + \frac{12}{12}}{12} = 180. \quad \text{And } \dfrac{377 - 18 + 1}{2} = \dfrac{360}{2} = 180.$$

Questions, wherein the third Sort of Affected Equations *come to be resolved.* *Ex. Where the third Sort are resolved.*

1. Two Numbers added together make 8, but added severally, with their respective Squares and Cubes, make 194: what are those Numbers? *Q. Of 2 Numbers, what they are.*

Answ. 3 and 5.

Here supposing one of the two Numbers to be 1ℨ, then must the other be 8—1ℨ. These with their Squares and Cubes are thus: Resolution.

1ℨ	Numbers	8 — 1ℨ.
1ʒ	Squares	1ʒ + 64 — 16ℨ.
1φ	Cubes	24ʒ + 512 — 1φ — 192ℨ.

1φ + 1ʒ + 1ℨ.
—1φ + 25ʒ — 209ℨ + 584.

Total of both 26ʒ — 208ℨ + 584.

And then 26ʒ — 208ℨ + 584 = 194. And 26ʒ = 208ℨ — 390.
And 1ʒ = 8ℨ — 15. And 1ℨ = 3 or 5.

For $\frac{1}{2}$ of 8ℨ is 4, squared is 16; from whence 15 substracted, there is left 1, whose Square Root is 1, which added to 4 makes 5, or taken from 4 leaves 3, for the Value of either Root; both which are necessary to the Solution of this Question.

Proof. Proof.

$$\begin{array}{l} \ \ ℨ \quad\ ʒ \quad\ \ φ \\ 3 + 9 + 27 = 39. \end{array} \quad \text{And} \quad \begin{array}{l} \ \ ℨ \quad\ \ ʒ \quad\ \ \ φ \\ 5 + 25 + 125 = 155. \end{array}$$

And 39 + 155 = 194.

2. A Man owed a Sum of Money, which might be divided into two such Parts, that being multiplied would make 24, and their Cubes added together 280: what was the Debt? *Q. Of a Debt, what it was.*

Answ. 10 *l.* Resolution.

Here supposing one Part of the two to be 1ℨ, then must the other be $\frac{24}{1ℨ}$.

The Cubes of both which are,

$$\overset{\surd}{1\mathfrak{Z}}.\ \overset{q}{1\text{ʒ}}.\ \overset{c}{1\phi}.\quad \overset{\surd}{\frac{24}{1\mathfrak{Z}}}.\ \overset{q}{\frac{576}{1\text{ʒ}}}.\ \overset{c}{\frac{13824}{1\phi}}.\quad \text{And } \frac{1\phi}{1} + \frac{13824}{1\phi} \text{ added are } \frac{1\text{ʒ}\phi+13824}{1\phi}$$

And this equal to 280. Wherefore $1\text{ʒ}\phi = 280\phi - 13824$.
And $1\mathfrak{Z} = 4$ or 6.

For $\frac{1}{2}$ of 280 is 140, squared is 19600; out of which 13824 taken, the Remain is 5776, whose Square Root is 76; which added to 140, maketh 216, and abated from it, leaveth 64; out of both which the Cube Root extracted, because there are 2 Quantities omitted in the *Equation*, the Roots desired are 6 and 4.

Proof.

Proof.

Parts. *Debt.*		*Cubes.* *Sum.*
$6+4=10$.	$6\times4=24$.	$216+64=280$.

Ex. *Where the Equation is according to the Supposition.*

Questions wherein according to the Supposition, so the Equation *happens to be of one sort or other.*

Q. *Of a Number, what it was.*

1. A Number thought upon hath two Parts, of which the one is 4, and the other multiplied into it self; and then also with 4 the two Products will be 117: what was that Number?

Resolution.

Answ. 13.

Here supposing the Number to be $1\mathfrak{Z}$, then 1 Part being 4, the other Part must be $1\mathfrak{Z}-4$: This Part squared is $1\text{ʒ}-8\mathfrak{Z}+16$. And the said unknown Part (that is $1\mathfrak{Z}-4$) multiplied by 4, is $4\mathfrak{Z}-16$; which added to the Square, makes $1\text{ʒ}-4\mathfrak{Z}$; which being equal to 117, makes an *Equation* of the first Sort thus:

$1\text{ʒ}=4\mathfrak{Z}+117$. And $1\mathfrak{Z}=13$.

For $\frac{1}{2}$ of 4 is 2, squared is 4, to which 117 added is 121, the Square Root of which is 11, and thereto 2 added is 13, the Number desired.

But if Supposition be made for the unknown Part, then if that Part be $1\mathfrak{Z}$, the whole Number shall be $1\mathfrak{Z}+4$: And then multiplying this Part by it self, and also by 4, the known Part there will arise 1ʒ and $4\mathfrak{Z}$; which being equal to 117, makes an *Equation* of the second Sort thus: $1\text{ʒ}=117-4\mathfrak{Z}$. And $1\mathfrak{Z}=9$, the unknown Part.

For $\frac{1}{2}$ of $4\mathfrak{Z}$ is 2, which squared is 4, this added to 117 is 121, whose Square Root is 11; from whence 2 taken, there remaineth 9 for the Part unknown.

Proof.

Proof.

Parts. Whole Number.
$4+9=13$.

One Part led into the other	$9\times4=$	36
One Part squared	$9\times9=$	81
		117 Total.

Q. *Of 3 Numbers, what the Least and Mean.*

2. There are 3 Numbers in *Geometrical Proportion*, the greatest Extream is $20\frac{1}{4}$, the least Extream with the double of the middle Term makes 22: what is the least Extream, and the middle Term?

Resolution.

Answ. The least Extream is 4, and the middle Term 9.

Here supposing the least Extream to be $1\mathfrak{Z}$: And because the least with the double of the middle Term must make 22, the middle Term shall be $11-\frac{1}{2}\mathfrak{Z}$, for his double is $22-1\mathfrak{Z}$, which with $1\mathfrak{Z}$ is 22. Then because the Product of the Mean multiplied into it self is equal to the Product of the Extreams, the two Extreams multiplied, which are $1\mathfrak{Z}$ and $20\frac{1}{4}$, produce $\frac{81}{4}\mathfrak{Z}$. And the Mean that is $11-\frac{1}{2}\mathfrak{Z}$, squared is $121+\frac{1}{4}\text{ʒ}-11\mathfrak{Z}$; which being equal to the other, the *Equation* stands thus, $121+\frac{1}{4}\text{ʒ}-11\mathfrak{Z}=\frac{81}{4}\mathfrak{Z}$. And by Reduction $484+1\text{ʒ}-44\mathfrak{Z}=81\mathfrak{Z}$. And again $1\text{ʒ}=125\mathfrak{Z}-484$, an *Equation* of the third Sort. And $1\mathfrak{Z}=4$.

For $\frac{1}{2}$ of 125 is $\frac{125}{2}$, squared is $\frac{15625}{4}$; from which 484 that is $\frac{1936}{4}$ abated, the Remain is $\frac{13689}{4}$, whose Square Root is $\frac{117}{2}$; this abated from $\frac{125}{2}$, leaves $\frac{8}{2}$ or 4 for the Root desired. And although by Addition of the Square Root to the Half, as in other *Equations* of the third Sort, that is $\frac{117}{2}$ to $\frac{125}{2}$, another Root will be obtained, to wit $\frac{242}{2}$, or 121; yet cannot this be

be the Root intended by the Propoſition, becauſe the leaſt Extream with the double of the middle Term was bounded to make but 22, when this Root 121 is much more of it ſelf. But if the middle Term be ſuppoſed 1 ℨ, then becauſe the double thereof with the leaſt Term muſt make 22, that leaſt Term muſt be 22—2 ℨ. And ſeeing the Square of the Mean 1 ℨ, that is 1ʒ, muſt be equal to the Product of the two Extreams, that is 22 — 2ℨ × 20$\frac{1}{4}$, which is 445$\frac{1}{2}$—40$\frac{1}{2}$ℨ, there ariſeth an *Equation* of the ſecond Sort, that is 1ʒ = 445$\frac{1}{2}$ — 40$\frac{1}{2}$ℨ. And 1 ℨ = 9.

For $\frac{1}{2}$ of 40$\frac{1}{2}$ is $\frac{81}{4}$, whoſe Square is $\frac{6561}{16}$, to which adding 445$\frac{1}{2}$, or $\frac{7128}{16}$, the Total is $\frac{13689}{16}$ the Square of $\frac{117}{4}$; from which $\frac{81}{4}$ abated, there remaineth $\frac{36}{4}$, or 9 for the middle Term.

Proof. Proof.

Proportionals 4 . 9 . 20$\frac{1}{4}$. And 4 + 9 + 9 = 22.
Extreams 4 × 20$\frac{1}{4}$ = 81. Mean 9 × 9 = 81.

Reſolution of Affected Equations, *by Mr.* Oughtred's *Way.*

Reſolution of Affected Equations by Mr. Oughtred.

Mr. *Oughtred* in the 16*th* Chapter of his *Clavis*, hath delivered two Rules for the Reſolution of every *Equation*, wherein are 3 Species or Quantities orderly aſcending, ſuch as thoſe herein already ſpoken to; and directs, that the Abſolute Number (being one of the three) ſhall be reckoned the Rectangle (or Product) of the two Magnitudes ſought, whether Root, Square, or Cube, *&c.* that is to ſay, ſuch as is the Power of the middle *Species*. And in the middle *Species*, if the higheſt be Negative, the Coefficient ſhall be counted the Sum of the ſought Magnitudes, and be manifeſt of both. But if the higheſt *Species* be Affirmative, the Coefficient ſhall be the Difference of the Magnitudes ſought; and the ſame *Species* ſhall be ſhewn of the Greater denied, or of the Leſſer affirmed.

And ſeeing (by the ſecond Chapter of this 4*th* Part in the *Invention of Equations* it appeareth) that the Sum and Rectangle of two Magnitudes given, the Difference is given; or the Difference and Rectangle given, the Sum is given; and by the Sum and Difference the Magnitudes themſelves are given. So as Z and X given, A and E may be found by the ſaid Rules, thus by him ſymbolized.

Two Rules in Species by Mr. Oughtred *for the former* 3.

$$1.\quad \tfrac{1}{2}Z \pm \sqrt{:}\,\tfrac{1}{4}Zq - \text{Æ} : (\tfrac{1}{2}X) = \begin{matrix}A.\\E.\end{matrix}$$

$$2.\quad \sqrt{:}\,\tfrac{1}{4}Xq + \text{Æ} : (\tfrac{1}{2}Z) \pm \tfrac{1}{2}X = \begin{matrix}A.\\E.\end{matrix}$$

Theſe two Rules being the ſame in effect with thoſe before handled in the Antient Coſſical way, for Reſolution of orderly *Affected Equations*, will need no Explanation here; and the rather, for that the ſame Author afterward, in a Tract of 28 *Sections*, or *Precepts*, hath taught the Inveſtigation of the Root of all ſorts of *Affected Equations*, as well diſordered as others, in one Method; which to avoid multiplicity of Rules, is chiefly to be choſen and followed.

Difference in the Reſolution from Coſſicks.

In this joint way of Work, contrary to that by *Coſſicks*, the Abſolute Number or Magnitude is ſet ſolitary, and not the higheſt; ſo as the three ſorts of *Equations* before

Set thus	ʒ = ℨ + N.	ʒ = N — ℨ.	ʒ = ℨ — N.
are here	Aq — A = N.	Aq + A = N.	A — Aq = N.
And ſo	Aq — XA = Æ.	Eq + XE = Æ.	ZA — Aq = Æ.

The Number known to be ſet alone againſt all the other.

The Reaſon whereof is, becauſe every *Equation* may, by Reduction, be brought to an Abſolute Number or Magnitude, which being known, is certain and without Fiction or Figuration, and ſo may be ſet againſt all the other unknown Parts of the *Equation*. For it is evident in this *Equation*,
8ʒφ = 10ſs + 20ʒʒ + 400φ + 31250N. Or in *Species* tranſlated,
8Acc — 10Aqc — 20Aqq — 400Ac = 31250, that the eighth Part of
10ſs + 20ʒʒ + 400φ + 31250N muſt be a Zenzicube Number. And that 20ʒʒ + 400φ + 31250N contain a certain Number of Surſolids: And alſo that 400φ + 31250N, contain certain Zenzizenzikes. And ſo conſequently 31250 ſhall contain therein certain Cubes, as may be proved, the Root being 5.

The

28 Precepts of Mr. Oughtred of Affected Equations.

The Substance of those 28 *Sections*, or *Precepts*, (save what concerns the use of *Logarithms*, already learned) follows in order as the Author hath left them; the Translation whereof, with the following Examples, will be sufficient, without Additional Illustration.

1. To constitute an Affected Equation.

1. The manner of constituting an *Affected Equation*; Let there be taken at pleasure, for B, 3; for Cq, 16; for Dc, 125; for Fqq, 1296, *&c.* Neither is it material, whether the Numbers are truly figurate or not. And let there be constituted of these Coefficients, a *Sursolid Equation*, according to the manner of the *Analytical Table*, (in Chap. 11. *Figuration of Rational Species*) *viz.*

Example.

$Lqc - 5BLqq + 10CqLc - 10DcLq + FqqL = Gqc$; which in Numbers appointing L (the Root) 47, shall be $1qc - 15qq + 160c - 1250q + 6480l = 170304782$: or omitting the Distinction of the *Unciæ*; For 15qq, say BLqq; for 160c, say CqLc; for 1250q, say DcLq; and for 6480l, say FqqL. For if L be 47, then shall $Lq = 2209$, and $Lc = 103823$, and $Lqq = 4879681$, and $Lqc = 229345007$.

The Practice of this Constitution.

BLqq	229345007	Lqc.
15×4879681	—73195215	
CqLc	156149792	
160×103823	+16611680	
DcLq	172761472	
1250×2209	—2761250	
FqqL	170000222	
6480×47	+ 304560	
	170304782	Gqc.

2. Every one to be counted as the former Example.

2. Count every propounded *Equation* as this now found.

$1qc - 15qq + 160c - 1250q + 6480l = 170304782$.

Or the Numbers changed into *Species*.

$Lqc - BLqq + CqLc - DcLq + FqqL = Gqc$.

How to express more Affections.

And if there were more *Species* of the Affections, consequently they might be expressed by Hcc, Kqqc, Mqcc, Nccc, and so further.

3. To search out the two Parts.

Example.

3. Of the Root L there shall by these be two Parts searched out, to wit, A the first Side, and E the second Side, or whatsoever is subsequent; wherefore $L = A + E$, and all the Powers of L equally to the like Powers of $A + E$. As $Lq = Aq + 2AE + Eq$. And $Lc = Ac + 3AqE + 3AEq + Ec$, *&c.*

4. Heterogeneals not to be added or substracted, &c.

4. In the propounded *Equation*, the Power to be resolved 170304782, or Gqc, is a Sursolid, of which kind also are the several Species of the Affections; for *Heterogeneals* cannot be added nor substracted among themselves.

5. Two things to be considered. (1) The Degree of Affection. (2) The Coefficient.

5. Wherefore in the several Affections, 2 things are to be considered, The Degree of Affection, and the Coefficent; as in 15qq, the Degree of Affection is squared Square, and the Coefficient 15 Root: In 160c, the Degree of Affection is Cube, and the Coefficient 160 Square: In 1250q, the Degree of Affection is Square, and the Coefficient 1250 Cube: Lastly, in 6480l the Degree of Affection is Root, and the Coefficient 6480 squared Square: and hence arise 2 Consectaries for Extraction of the singular Sides.

6. Consectary from thence.

Example.

6. The first Consectary is, if the Root of the Coefficient according to his own kind multiplied into the Degree of Affection, shall multiply the same Coefficient, the Product shall be of the same kind with the Power to be resolved: As in the precedent *Equation*, if the Side 15 multiplied squared-squaredly, be multiplied into 15; and if $\sqrt{q}\,160$ cubed be multiplied into 160 squared; and if $\sqrt{c}\,1250$ squared be multiplied into the Cube of 1250; Lastly if $\sqrt{qq}\,6480$ be multiplied into the squared Square of 6480: of all these several Multiplications shall arise a Sursolid Number. And this Analytical Multiplication is the manner of reducing every Coefficient to the Species of the Power to be resolved, most used in Extraction of every first Side.

7. Another Consectary from the first.

7. From whence also most clearly appeareth, That if the Number arising of the Coefficients, in this manner reduced and compared, be less than the Power to be resolved, the Side thereof also is less than the side of the Power to be resolved; but if Greater, it is Greater; and if Equal, Equal. Therefore in this *Equation*

Equation 1qc — 15qq + 160c — 1250q + 6480l = 170304782; or 170304782 + 15 qq — 160 c + 1250q — 6480 l = 1 qc. If then the lateral Coefficient 15, and √q160, and √c1250, and √qq 6480 be made Surfolids, they shall produce four Homogeneal Species of Affections, to wit, 7593.., 3238.., 1450.., 0581.., which by Logarithms is most easily done, and sufficiently exact for the purpose. Example.

Logarithms.		*Coefficient Numbers.*
1) 2) 3) 4) are the Dimensions in the Coefficient.		
1) 5 × 1,17609,12591		15 qq
5,88045,62955		+7593 ..
2) 2,20411,99827		160 c
5 × 1,10205,99913	12,65	
5,51029,99565		—3238 ..
3) 3,09691,00130		1250q
5 × 1,03230,33376	10,8 —	
5,16151,66880		+1450 ..
4) 3,81157,50059		6480 l
5 × 0,95289,37514	8,97	
4,76446,87570		—0581 ..

The Species being gathered into one Sum, according to the Order of their Signs among those propounded in the *Equation*, it shall be, that

170304700 + 759300 — 323800 + 145000 — 058100 = 1qc = 170827100.

which also in other *Equations* may likewise be done.

8. The second Consectary is, if the Power to be resolved be divided by the Coefficient, the Quotient shall be referred to the same Degree of Affection; that is, the Quotient shall be the Side if the Affection be under the Side, or the Square if under the Square, and so of other Degrees: as in the former *Equation*, if 170304782 be divided by 15, the Quotient shall be squaredly Quadratical; if by 160, the Quotient shall be Cubical; if by 1250, the Quotient shall be Quadratical; and if by 6480, the Quotient shall be Lateral; wherefore not always the Quotient it self, but for the most part the Root thereof, according to the Degree of Affection, shall be the singular side to be extracted. *8. The second Consectary.* Example.

9. In searching out the second Figure of the Root, this ought to be remembred, that according to the Number of Figures in the Quotient, the Degree thereof shall be very nearly reckoned; as if the Quotient consist in one Figure only, it may be a Side; if in 2, a Square; if in 3, a Cube, *&c.* And if the Quotient exceed 5, or 50, or 500, *&c.* it may be extended perhaps to the Degree following, especially in the greater Affections: And these are the Laws of *Analytical Division.* *9. What to be noted in finding the second Figure of the Root.*

10. Neither in this sort of *Multiplication* or *Division*, shall there be need to run through the whole Power to be resolved, with the whole Coefficient, but only to the next Point agreeing thereto. *10. Proceed only to the next Point.*

11. For in the Resolution of *Affected Equations*, all the Pointings of the Degrees ought to be made in the Power to be resolved, as in other Figural Numbers; those of the highest Degrees above, and of the Residue beneath. Also the Coefficients, every one according to his own Kind, are to be pointed. The Points of the former Examples shall be thus: *11. How the Numbers are to be pricked.*

$$1qc - \dot{1}5qq + 1\dot{6}0c - \dot{1}25\dot{0}q + 0\dot{6}48\dot{0}l = \dot{1}70\dot{3}04\dot{7}82$$

Example.

12. And regularly (especially if the Coefficient be Negative) the Number of Points in all ought to be equal. Wherefore if the Power to be resolved have more or fewer Points upon it self than the Coefficient, so many Ciphers shall be set before that which is deficient, that the Points to both may be equal. And in getting the several Sides, the Point of the Coefficient proper to that Side, is to be accommodated to the like Point above of the Power to be resolved: which shall be *12. Points to be equal and void, Places supplied with Ciphers.*

be done, if the Unit's Place in the Coefficient be removed in order to the lower Points of the Power agreeable to his Degree.

13. If the Coefficient be a Fraction or Surd.

13. If any Coefficient be a Fraction, or Surd Side, let it be reduced to Integers with Decimal Parts.

14. Root to be extracted with Decimals.

14. And if need be, to pursue the Extraction of the Root in the Decimal Parts, adjoin as many Ciphers as shall be meet after the Separatrix, and mark them above and below with Points in like sort.

15. The Table for the Divisors and Gnomons.

Affirmative and Negative.

15. The following Table shews as well the Divisors as the Gnomons, for finding the several Sides in *Affected Equations*; collected and continued out of the *Analytical Table*, before mentioned. And it is to be noted, That all the *Species* of every Coefficient are Affirmative, if the same be Affirmative; but Negative, if Negative.

For the first Side.	*For the several following Sides to compleat the Gnomon.*					
Aq	2AE .	Eq }				= Cq
BA	BE .					
Ac	3AqE.	3AEq .	Ec }			= Dc
BAq	B2AE.	BEq .				
CqA	CqE.					
Aqq	4AcE.	6AqEq .	4AEc .	Eqq }		= Fqq
BAc	B3AqE.	B3AEq .	BEc .			
CqAq	Cq2AE.	CqEq .				
DcA	DcE.					
Aqc	5AqqE.	10AcEq .	10AqEc.	5AEqq.	Eqc }	= Gqc
BAqq	B4AcE.	B6AqEq .	B4AEc.	BEqq.		
CqAc	Cq3AqE.	Cq3AEq .	CqEc.			
DcAq	Dc2AE.	DcEq .				
FqqA	FqqE.				&c.	

16. Divisors to be orderly collected and added.

16. The Divisors every where are taken of those, which are had in the Measure given, disposed and gathered together in due order, according to their Signs.

17. Ambiguous Equations.

17. If the highest Power of any *Equation* be Negative, that *Equation* is ambiguous.

18. The first of the Side whence.

18. The first singular Side is drawn out of these Rules, taken from the 2 Consectaries in *Sect. 6.* and 8.

(1) When the Coefficient may be neglected.

(1.) If the Coefficient so far depart to the latter Part, that it scarce reach to the first Point of the Power to be resolved, neither (also Analytically reduced) make any great change in it; it may altogether be neglected, in Extraction of the first singular Side.

(2) When devolved into the Consequent Points.

(2.) If the Coefficient break forth in the fore part, and be Affirmative, it is to be devolved into the Consequent Points, until a Place be made for Division: By which Division the Quotient found shall be referred to the Degree of Affection; which also in extracting the lesser Root of an Ambiguous *Equation* ought to be understood.

(3) When Negative and of many Points.

(3.) But if it be Negative, and consist of more Points than the Power to be resolved, the deficient Places may be supplied with Cyphers prefixed; and for the first singular Side, the Root of the Coefficient it self may be taken according to his kind.

(4) When the Points to both equal.

(4.) If the Points to both are equal, and the Numbers differ not much in the first Point, both of the Coefficient and of the Power to be resolved; the Coefficient by his Root extracted according to the Species with which he is pointed, under the Point agreeable thereto, reduced to the Species of the Power (by Analytical Multiplication) may be added to the Power to be resolved, if it be Negative, or taken away if it be Affirmative. For if $Ac + CqA = Dc$, then shall $Ac = Dc \mp CqA$; but if the greater Side of an Ambiguous *Equation* be sought, the Power to be resolved may be taken away from the Coefficient reduced: for if $CqA - Ac = Dc$, then shall $Ac = CqA - Dc$; then the Root of the Sum or Difference shall be the first Side to be extracted. And note that the greater Side

of

of an Ambiguous *Equation* may be found sometime by Division, sometime by Extraction of the Root of the Coefficient; but for the most part by Reduction of the Coefficient.

19. And by these Precepts diligently weighed at last, the first true singular Side shall be that, which first of all sheweth such a Diagonal; which together with the Coefficients (as the Condition of the *Equation* requireth) multiplied according to the precedent Table, and all gathered together into one Sum, (diligent respect every where had as well to the Signs as Places) bringeth forth a Number not greater than the Power to be resolved, from whence it is to be substracted. And it is to be noted, that every Negative Number is less than any Affirmative, and than any lesser Negative, as — 4 is less than 1, and than — 1; also that Substraction changeth the Sign of the Subtrahend; as from 4 take away 6, there resteth 4 — 6, that is — 2; and from — 4 take away — 6, there resteth — 4 + 6, that is 2; again from 4 take away — 6, there resteth 4 + 6, that is 10; wherefore in Extraction of the first singular Side, it is so often to be tried until the true Side be found, which by the next greater shall certainly be known. *19. To find the first of the Side.*

20. In constituting the Divisor for finding the second Side, the Place of the Coefficient multiplied into every Degree, ought to be ordered according to the pointing of his own Degree, that is, the Place of the Coefficient under the Side shall be distant towards the left Hand, one Place from the Point or Place of the same Coefficient; the Place of the Coefficient under the Square, two Places; under the Cube, three, &c. And to avoid Confusion, it will be profitable, in the Residue of the Power to be resolved, to distinguish those Points alone which serve to the present Root to be extracted. *20. To get the Divisor for the Second of the Side.*

21. Then the second singular Side shall be thus gotten; Let the Divisors of every Kind, out of the precedent Table, be gathered together into one Sum, and disposed in due order, and the Residue of the Power to be resolved divided by all that Divisor. For the Quotient, according to the Laws of Analytical Division (if need require it) weighed, shall give the second singular Side to be gotten. But in this search oftentimes great Difficulty happeneth, especially if the Aggregate of the Negative Magnitudes dividing, be almost equal to the Aggregate of the Affirmatives, (so that the Divisor may be less than the Residue of the Power to be resolved); which difficulty notwithstanding the sagacious Analyst will easily avoid. *21. To find the Second of the Side.*

22. Let this Rule therefore be perpetual; That the true singular second Side is that which first of all sheweth such a Gnomon, consisting of the Complements of every Kind, and multiplied Coefficients, as the Condition of the *Equation* requireth, according to the precedent Table, and all gathered together into one Sum, diligent respect had every where, as well to the Signs as Places; which Gnomon may not be greater than the Power to be resolved, from whence it is to be substracted: Wherefore it is often to be tried, until the true Side be found; which also by the next greater will most certainly be known. *22. To know the Second of the Side.*

23. All the singular Sides after the Second, by Simple Division, are most easily obtained. *23. To get the other Figures after the Second.*

24. If the Affections are compounded of Affirmatives and Negatives, the Antecedent Precepts are to be mixt with Discretion and Judgment: And in the Sides to be valued, always the greater Affection shall be considered before the Lesser. *24. If the Affections are mixt.*

25. But because oftentimes above it is said, it will be needful to try, which in many-fold Affections, and where the Degrees are lofty, will be exceeding laborious, I will add here for a Close, two Manners of easing these Trials: One by Depression, another by the Canon of Logarithms. But in both, if the *Equation* shall be ambiguous, all the Signs thereof shall be changed. Here also is to be noted, that every Negative Number is less than any Affirmative, and than any Lesser Negative. *25. Two ways. By Depression. By Logarithms.*

26. The Invention of the Singular Sides by Depression: If the first Side be sought, all the Points after the First, in the several *Species* of the given *Equation*, may be cut off by the Separatrix: Afterwards all the *Species* may be applied to the Side, that is, depressed by one Degree. *26. Invention by Depression.*

Example

Example 1.

Example 1. $1qq - 72c + 2386001 = 8725815$. This by Depression shall be made $1c + 238{,}6 - 7{,}2q = L)\ 872{,}5$.

Let A be 4, then shall 4) 872,5 (218,1 the just Number.

And $+ 64 + 238{,}6 - 115{,}2 = 187{,}4$ less than the Just.

Let A be 5, then shall 5) 872,5 (174,5 the just Number.

And $+ 125 + 238{,}6 - 180{,}0 = 183{,}6$ greater than the Just.

The true Side therefore $A = 5 - 1$, that is 4.

Example 2.

Example 2. Of the *Ambiguous Equation*, $1c - 3257l = - 45744$

This by Depression shall be made $1q - 32{,}5 = L) - 45{,}7$.

Let A be 4, then shall 4) — 45,7 (—11,4 the just Number.

And $+ 16 - 32{,}5 = - 16{,}5$ less than the Just.

Let A be 5, then shall 5) — 45,7 (— 9,1 the just Number.

And $+ 25 - 32{,}5 = - 7{,}5$ greater than the Just.

The true Side therefore $A = 5 - 1$, that is 4.

For the second of the Side.

If the second Side be sought, all the Points after the second may be cut off in the several *Species*; Afterwards all the *Species* may be applied to the Square, that is depressed by two Degrees. As in the first Example.

$1qq - 72c + 2386001 = 8725815$. This by Depression shall be made

$1q + L)\ 238600 - 72l = Q)\ 8725815$.

Let A be 47, then shall 2209) 8725815 (3949 the just Number.

And $2209 + 5077 - 3384 = 3896$ less than the Just.

Let A be 48, then shall 2304) 8725815 (3787 the just Number.

And $2304 + 4971 - 3456 = 3819$ greater than the Just.

The true Side therefore is $48 - 1$, that is 47.

27. Omitted.

27. This being wholly about the Use of Logarithms, is omitted here.

28. Examples of Logarithms.

28. This contains nothing but Examples, wherein trial is made by Logarithms. In which Examples all the Points after the two first are cut off by the Seperatrix.

Example 1. $1qq - 72c + 2386001 = 8725815$ the just Number.

Let the two first singular Sides be sought.

47 . 1,67209,78579	|—72|	|+238600|
C . 5,01629,35737	1,85733,24964	5,37767,04393
QQ . 6,68839,14316	5,01629,35737	1,67209,78579
	6,87362,60701	7,04976,82972
+4879...	—7475...	+11214...

And $+ 4870\ldots + 11214\ldots - 7475\ldots = + 8618\ldots$ less than the Just.

48 . 1,68124,12374	1,85733,24964	5,37767,04393
C . 5,04372,37122	5,04372,37122	1,68124,12374
QQ . 6,72496,49496	6,90105,62086	7,05891,16767
+5308...	—7962...	+11453...

And $+ 5308\ldots + 11453\ldots - 7962\ldots = + 8799\ldots$ greater than the Just.

The true Root therefore shall be $48 - 1$, that is 47.

Example

Example 2. 1c — 3257l = — 45744 the juſt Number.

Let the two firſt Singular Sides be ſought.

48 .	1,68124,12374	— 3257 .	3,51281,77586	
C .	5,04372,37122		1,68124,12374	
	+ 1106	— 1563 .	5,19405,89960	

And + 1106 — 1563 = — 457. Leſs than the Juſt (at leaſt not Greater).

49 .	1,69019,60800	— 3257 .	3,51281,77586	
C .	5,07058,82400		1,69019,60800	
	+ 1176	— 1596 .	5,20301,38386	

And + 1176 — 1596 = — 420. Greater than the Juſt.

The true Root therefore ſhall be 49 — 1, that is 48.

The ſecond Side may alſo be found by Logarithms, Depreſſion preceding.

As in Example, 1qq — 1246,00q = 08972,6256.

This depreſſed ſquaredly, ſhall be made 1q — 1246 = Q) 8972,6.

The two firſt Singular Sides may be ſuppoſed.

34 .	1,53147,89170	8972,6	3,95291,83073	
Q .	3,06295,78340		3,06295,78340	
	+ 1156	Value 7,76	0,88996,04733	The Juſt.

And + 1156 — 1246 = — 90. Leſs than the Juſt.

36 .	1,55630,25008	8972,6	3,95291,83073	
Q .	3,11260,50016		3,11260,50016	
	+ 1296	Value 6,92	0,84031,33057	The Juſt.

And + 1296 — 1246 = + 50. Greater than the Juſt.

Upon theſe Examples, in the 26*th* and 28*th* Sections, the ſame Author afterwards hath added ſome Notes of Explanation; the Subſtance whereof is thus: *Notes of the Author.*

That is called *The juſt Number*, which ariſeth of the Application of the Power to be reſolved to the Degree of the ſuppoſed Side, by which Depreſſion is made: For this is the Meaſure to which all the other *Species* duly gathered together, ought to be equal. As in the firſt Example of the 26*th* Section, 1c+238,6—7,2q=L) 872,5. If for the firſt Side be ſuppoſed 5, it muſt be that C:5:+238,6—7,2Q:5:=872,5 divided by 5; that is 125+238,6—(7,2x25) 180, to wit 183,6 to be equal to 174,5 the Juſt. But it is greater, and therefore the true Side is leſs than 5; therefore let 4 be again ſuppoſed, and make trial whether C:4:+238,6—7,2 Q:4: be equal to 872,5 divided by 4. *What called the juſt Number.*

But leſt in theſe Examples, as alſo in the following, theſe Trials be taken up by chance, it muſt be admoniſhed, **Monitions.**

1. If the Homogeneal Power of the Extracted Root, exceed the Power to be reſolved; or if the Magnitudes increaſing the Power to be reſolved, exceed them which they leſſen; The true Side A (for the moſt part) ſhall be leſs than the Side extracted, but otherwiſe greater: As in this *Equation.* **1. Monition.**

1c+26,00c0l=180931713

180,9 (4 The Side A.

26,0 Cq.

√q26 is 5, in 26 is made 130, taken from 180, there reſteth 50,C:3+: which is leſs than 180; wherefore the true Side A is greater than 3.

2. Monition. 2*ly*. If the Divisors under the same Sign with the Residue of the Power to be resolved, exceed them which are under a diverse Sign, the true Side E (for the most part) shall be less than the Quotient; but otherwise greater: As in this *Equation*.

15681 — 1c = 21952.

The same also happeneth in *Ambiguous Equations*, when the Residue of the Power to be resolved is Affirmative: As in this *Equation*.

67681 — 1c = 214273.

The Work of both.

21	952	(28 The two first Sides.
15	68	Cq
—8		Ac
+31	36	CqA
+23	36	*Subtrahend.*
R—1	408	
1	2	—3Aq
	6	—3A
—1	26	
+1	568	Cq
+	308	*Divisor.*

The Sign R is —; But —1,26 is less than +1,568. Wherefore the true Side E is greater than the Quotient 4.

214	273	(47 The 2 first Sides.
67	68	Cq
—64		—Ac
+270	72	CqA
+206	72	*Subtrahend.*
R+ 7	553	
4	8	—3Aq
	12	—3A
— 4	92	
+ 6	768	Cq
+ 1	848	*Divisor.*

The Sign is +; But the Divisor out of the Degrees of the Side A Negative, is less than the Coefficient Affirmative Divisor; that is —4,92, is less than +6,768. Wherefore the true Side E shall be greater than the Quotient 4.

3. Monition. 3*ly*. If after these Monitions some Doubt remain, trial by 5 shall be most fit to be begun; and from thence Inquiry to be continued by odd Numbers: Or the same may be done by Depression, or by Logarithms.

16 Examples of the same Author, and his Notes thereupon.

The other Examples of the same Author, with his Notes thereupon.

Ex. 1. 1qc — 15qq + 160c — 1250q + 064801 = 170304782
That is Lqc — BLqq + CqLc — DcLq + FqqL = Gqc.

1703	04782	(47
1	5	—B
1	250	—Dc
1	60	Cq
	6480	Fqq
1024		Aqc
102	40	CqAc
2	5920	FqqA
+1128	9920	
384	0	—BAqq
20	000	—DcAq
—404	000	
724	9920	*Subtrahend.*
R 978	05582	
128	0	5Aqq
6	40	10Ac
	160	10Aq
	20	5A
7	680	Cq3Aq
	1920	Cq3A
	160	Cq
	6480	Fqq
+142	50040	
38	40	—B4Ac
1	440	—B6Aq
	240	—B4A
	15	—B
1	0000	—Dc2A
	1250	—Dc
—40	87665	
+101	62375	*Divisor.*
896	0	5AqqE
313	60	10AcEq
54	880	10AqEc
4	8020	5AEqq
	16807	Eqc
53	760	Cq3AqE
9	4080	Cq3AEq
	54880	CqEc
	45360	FqqE
+1333	62047	
268	80	—B4AcE
70	560	—B6AqEq
8	2320	—B4AEc
	36015	—BEqq
7	0000	—DC2AE
	61250	—DcEq
—355	56465	
978	05582	*Subtrahend.*

In this Example √qc1703 is 4+, by *Sect.* 18. *Rule* 1. For as it appeareth out of *Sect* 7. there is not made any notable Change in the first Point, by the Coefficients analytically reduced; wherefore the true Side A shall be 4.

The true Side E is less than the Quotient 9; because the Divisors under the Sign +, (which is the Sign of the Residue) exceed them which are under the Sign —. *Exam-*

Example 2.

1c + 4200001 = 247651713

That is Lc + CqL == Dc.

247	651	713	(417
42	000	0	Cq
64			Ac
168	000	0	CqA
232	000	0	*Subtrahend.*
R 15	651	713	
4	8		3Aq
	12		3A
4	200	00	Cq
9	120	00	*Divisor.*
4	8		3AqE
	12		3AEq
	1		Ec
4	200	00	CqE
9	121	00	*Subtrahend.*
R 6	530	713	
	504	3	3Aq
	1	23	3A
	420	000	Cq
	925	530	*Divisor.*
3	530	1	3AqE
	60	27	3AEq
		343	Ec
2	940	000	CqE
6	530	713	*Subtrahend.*

In this Example

42) 247 (6—, by *Sect.* 18. *Rule* 2. For 42 reduced Analytically by *Sect.* 6 and 8, it is made 252, greater than 247: And the true Side A is less than 6, because C:6—: exceeds 247,6.

Example 3.

1c + 1007q = 247617936

That is Lc + BLq == Dc.

247	617	936	(417
10	07		B
64			Ac
161	12		BAq
225	12		*Subtrahend.*
R 22	497	936	
4	8		3Aq
	12		3A
8	056		B2A
	100	7	B
13	076	7	*Divisor.*
4	8		3AqE
	12		3AEq
	1		Ec
8	056		B2AE
	100	7	BEq
13	077	7	*Subtrahend.*
R 9	420	236	
	504	3	3Aq
	1	23	3A
	825	74	B2A
	1	007	B
1	332	277	*Divisor.*
3	530	1	3AqE
	60	27	3AEq
		343	Ec
5	780	18	B2AE
	49	343	BEq
9	420	236	*Subtrahend.*

In this Example

10) 247 (24 + = Q:5—: by *Sect.* 18. *Rule* 2. But 10 Q:5: = 250 ⊏ 247,6 by *Monition* 1.

Example

Example 4.

1qq — 442990051 = 022252086

That is Lqq — DcL = Fqq.

0	2225	2086	(354
—44	2990	05	—Dc
+81			Aqq
—132	8970	15	—DcA
—51	8970	15	*Subtrahend.*
R 52	1195	3586	
10	8		4Ac
	54		6Aq
	12		4A
+11	352		
— 4	4299	005	—Dc
+ 6	9220	995	*Divisor.*
54	0		4AcE
13	50		6AqEq
1	500		4AEc
	625		Eqq
+69	0625		
—22	1495	025	—DcE
+46	9129	975	*Subtrahend.*
R 5	2065	3836	
1	7150	0	4Ac
	73	50	6Aq
		140	4A
+ 1	7223	640	
—	4429	9005	—Dc
1	2793	7395	*Divisor.*
6	8600	0	4AcE
	1176	00	6AqEq
	8	960	4AEc
		256	Eqq
+ 6	9784	9856	
— 1	7719	6020	—DcE
+ 5	2065	3836	*Subtrahend.*

In this Example,
√c44,3 is 3+, by *Sect.* 18. *Rule* 3. wherefore the true Side A is 3.

The true Side E is less than the Quotient 8— by *Monition* 2.

Example 5.

1qq — 1246000q = 089726256

That is Lqq — CqLq = Fqq.

0	8972	6256	(354
— 12	4600		—Cq
+ 81			Aqq
— 112	1400		—CqAq
— 31	1400		*Subtrahend.*
R 32	0372	6256	
10	8		4Ac
	54		6Aq
	12		4A
+ 11	352		
7	4760	0	—Cq2A
	1246	00	—Cq
— 7	6006	00	
+ 3	7514	00	*Divisor.*
54	0		4AcE
13	50		6AqEq
1	500		4AEc
	625		Eqq
+ 69	0625		
37	3800	0	—Cq2AE
3	1150	00	—CqEq
— 40	4950	00	
+ 28	5675	00	*Subtrahend.*
R 3	4697	6256	
1	7150	0	4Ac
	73	50	6Aq
		140	4A
+ 1	7223	640	
	8722	000	—Cq2A
	12	4600	—Cq
—	8734	4600	
+	8489	1800	*Divisor.*
6	8600	0	4AcE
	1176	00	6AqEq
	8	960	4AEc
		256	Eqq
+ 6	9784	9856	
3	4888	000	—Cq2AE
	199	3600	—CqEq
— 3	5087	3600	
+ 3	4697	6256	*Subtrahend.*

In this Example,
√q12,4 is 3+, by *Sect.* 18. *Rule* 3. Wherefore the true Side A is 3.

The true Side E, is less than the Quotient 9— by *Monition* 2.

Example 6.

1qq — 340c = 621066096

That is Lqq — BLc = Fqq.

	6	2106	6096	(354
—	3	40		—B
+	81			Aqq
—	91	80		BAc
—	10	80		*Subtrahend.*
R	17	0106	6096	
	10	8		4Ac
		54		6Aq
		12		4A
+	11	352		
	9	180		—B3Aq
		3060		—B3A
		34	0	—B
—	9	4894	0	
+	1	8626	0	*Divisor.*
	54	0		4AcE
	13	50		6AqEq
	1	500		4AEc
		625		Eqq
+	69	0625		
	45	900		—B3AqE
	7	6500		—B3AEq
		4250	0	—BEc
—	53	9750	0	
+	15	0875	0	*Subtrahend.*
R	1	9231	6096	
	1	7150	0	4Ac
		73	50	6Aq
			140	4A
+	1	7223	640	
	1	2495	00	—B3Aq
		35	700	—B3A
			340	—B
—	1	2530	7340	
+		4692	9060	*Divisor.*
	6	8600	0	4AcE
		1176	00	6AqEq
		8	960	4AEc
			256	Eqq
+	6	9784	9856	
	4	9980	00	—B3AqE
		571	200	—B3AEq
		2	1760	—BEc
—	5	0553	3760	
+	1	9231	6096	*Subtrahend.*

In this Example, The Lateral Coefficient 3, 4, squared-squaredly, multiplied and encreased 6, 2, is made 140, QQ: 3 +: by *Sect.* 18. *Rule* 4. Wherefore the true Side A is 3. The true Side E is less than the Quotient 9—, by *Monition* 2.

Example 7.

1qq — 771080001 = 085530576

That is Lqq — DcL = Fqq

	0	8553	0576	(426
—	77	1080	00	—Dc
+	256			Aqq
—	308	4320	00	—DcA
—	52	4320	00	*Subtrahend.*
R	53	2873	0576	
	25	6		4Ac
		96		6Aq
		16		4A
+	26	576		
—	7	7108	000	—Dc
+	18	8652	000	*Divisor.*
	51	2		4AcE
	3	84		6AqEq
		128		4AEc
		16		Eqq
+	55	1696		
—	15	4216	000	—DcE
+	39	7480	000	*Subtrahend.*
R	13	5393	0576	
	2	9635	2	4Ac
		105	84	6Aq
			168	4A
+	2	9741	208	
—		7710	8000	—Dc
+	2	2030	4080	*Divisor.*
	17	7811	2	4AcE
		3810	24	6AqEq
		36	288	4AEc
			1296	Eqq
+	18	1657	8576	
—	4	6264	8000	—DcE
+	13	5393	0576	*Subtrahend.*

In this Example.
√c77 is 4, by *Sect.* 18. *Rule* 3. Wherefore the true Side A is 4.

Example

Example 8.

$320\,0l - 1c = 46577$

That is CqL — Lc = Dc

An Ambiguous *Equation.*

46	577	(47 The greater Root.
32	00	Cq
— 64		—Ac
+128	00	CqA
+ 64	00	*Subtrahend.*
R- 17	423	
4	8	—3Aq
	12	—3A
— 4	92	
+ 3	200	Cq
— 1	720	*Divisor.*
33	6	—3AqE
5	88	—3AEq
	343	— Ec
— 39	823	
+ 22	400	CqE
— 17	423	*Subtrahend.*

In this Example,

√q32, is 5,65 in 32 is made 180,8. lacking 46,5. there remaineth 144.C: 5: by *Sect.* 18. *Rule* 4. But 144 exceedeth 46,5. Wherefore the true Side A is less than 5 by *Monition* 1.

The true Side E is less than the Quotient 10, by *Monition* 2.

Example 9.

$320\,0l - 1c = 46577$

That is CqL — Lc = Dc

The same Ambiguous *Equation.*

46	577	(15,7 The lesser Root.	
32	00	Cq	
— 1		—Ac	
+ 32	00	CqA	
+ 31	00	*Subtrahend.*	
R 15	577		
	3	—3Aq	
	3	—3A	
—	33		
+ 3	200	Cq	
+ 2	870	*Divisor.*	
1	5	—3AqE	
	75	—3AEq	
	125	— Ec	
— 2	375		
+ 16	000	CqE	
+ 13	625	*Subtrahend.*	
R 1	952	000	
	67	5	—3Aq
		45	—3A
—	67	95	
+	320	0	Cq
+	252	05	*Divisor.*
	472	5	—3AqE
	22	05	—3AEq
		343	— Ec
—	494	893	
+ 2	240	0	CqE
+ 1	745	107	*Subtrahend.*
R	206	893	000, &c.

In this Example,

The Solution is most easy by *Division*, according to *Sect.* 18. *Rule* 3.

Example

Example 10.

$5\dot{3}q - 1c = 1\dot{3}2\dot{5}4$

That is BLq — Lc = Dc

An Ambiguous *Equation.*

13	254	(47 The greater Root.
5	3	B
—64		—Ac
+84	8	BAq
+20	8	*Subtrahend.*
R—7	546	
4	8	—3Aq
	12	—3A
—4	92	
4	24	B2A
	53	B
+4	293	
—	627	*Divisor.*
33	6	—3AqE
5	88	—3AEq
	343	— Ec
—39	823	
29	68	B2AE
2	597	BEq
+32	277	
— 7	546	*Subtrahend.*

In this Example,

C: 5 : is 125, lacking 13, there remaineth 112, C: 5 —: by *Section* 18. *Rule* 4. But 112 exceedeth 13. Wherefore the true Side A is less than 5 by *Monition* 1.

The true Side E is less than the Quotient 12, by *Monition* 2.

Example 11.

$5\dot{3}q - 1c = 1\dot{3}2\dot{5}4$

That is BLq — Lc = Dc

The same Ambiguous *Equation.*

13	254	(20,05, &c. The lesser Root.		
5	3	B		
— 8		—Ac		
+21	2	BAq		
+13	2	*Subtrahend.*		
R	54	000	000	
	12	000	0	—3Aq
		6	00	—3A
—	12	006	00	
	21	200		B2A
		5	3	B
+	21	205	3	
	9	199	30	*Divisor.*
	60	000	0	—3AqE
		150	00	—3AEq
			125	— Ec
—	60	150	125	
	106	000		B2AE
		132	5	BEq
+	106	132	5	
	45	982	375	*Subtrahend.*
R	8	017	625	000, &c.

In this Example,

The Solution is most easy by *Division*, according to *Section* 18. *Rule* 3.

Example

Example 12.

600341 — 1c = 1023768

That is CqL — Lc = Dc.
An Ambiguous *Equation.*

1	023	768	(236. The greater Root.
6	003	4	Cq
— 8			—Ac
+12	006	8	CqA
+ 4	006	8	*Subtrahend.*
R— 2	983	032	
1	2		—3Aq
	6		—3A
— 1	26		
+	600	34	Cq
—	659	66	*Divisor.*
3	6		—3AqE
	54		—3AEq
	27		— Ec
— 4	167		
+ 1	801	02	CqE
— 2	365	98	*Subtrahend.*
R—	617	052	
	158	7	—3Aq
		69	—3A
—	159	39	
+	60	034	Cq
—	99	356	*Divisor.*
	952	2	—3AqE
	24	84	—3AEq
		216	— Ec
—	977	256	
+	360	204	CqE
—	617	052	*Subtrahend.*

In this Example.

√q 6 is 2+, in 6 is made 12 lacking 1, there remaineth 11, C : 2,5 by *Sect.* 18. *Rule* 4. But 11 exceedeth 1; wherefore the true Side A a little less than 2+ by *Monition* 1.

The true Side E is less than the Quotient 5− by *Monition* 2.

Example 13.

600341 — 1c = 1023768

That is CqL — Lc = Dc.
The same Ambiguous *Equation.*

	1	023	768	(17,13, *&c.* The lesser Root.		
		600	34	Cq		
		—1		—Ac		
+		600	34	CqA		
+		599	34	*Subtrahend.*		
R		424	428			
			3	—3Aq		
			3	—3A		
—			33			
+		60	034	Cq		
—		59	704	*Divisor.*		
		2	1	—3AqE		
		1	47	—3AEq		
			343	— Ec		
—		3	913			
+		420	238	CqE		
+		416	325	*Subtrahend.*		
R		8	103	000		
			86	7	—3Aq	
				51	—3A	
—			87	21		
+		6	003	4	Cq	
+		5	916	19	*Divisor.*	
			86	7	—3AqE	
				51	—3AEq	
				1	— Ec	
—			87	211		
+		6	003	4	CqE	
+		5	916	189	*Subtrahend.*	
R		2	186	811	000	
—			591	562	57	*Divisor.*
—		1	774	656	903	*Subtrahend.*
R			412	154	097	000, &c.

In this Example.

The Solution is most easy by *Division*, according to *Section* 18. *Rule* 3.

Example 14.

1qq—72c+238600l=8725815,7056.

That is Lqq—BLc+DcL=Fqq.

872	5815,	7056	(47,6
— 7	2		—B
+ 238	600		Dc
256			Aqq
954	400		DcA
+1210	400		
— 460	8		—BAc
+ 749	600		*Subtrahend.*
R 122	9815,	7056	
25	6		4Ac
	96		6Aq
	16		4A
23	8600		Dc
+ 50	4360		
34	56		—B3Aq
	864		—B3A
	72		—B
— 35	4312		
+ 15	0048		*Divisor.*
179	2		4AcE
47	04		6AqEq
5	488		4AEc
	2401		Eqq
167	0200		DcE
+ 398	9881		
241	92		—B3AqE
42	336		—B3AEq
2	4696		—BEc
— 286	7256		
+ 112	2625		*Subtrahend.*
R 10	7190,	7056	
1	7698	808	*Divisor.*
10	7190,	7056	*Subtrahend.*

In this Example,

QQ:7,2: is—2687; and √c238,6 is 6,2, whose QQ is +1480. Then —2687 +1480=—1207. This added to 872, giveth 2079, QQ:6+: by *Sect.*18.*Rule* 4. And because—2687 to be added, is greater than +1480 to be substracted, the true Side A shall be less than 6, by *Monition* 1. The true Side E is less than the Quotient 9, by *Monition* 2.

Example 15.

3l. — 1c = 1,258640782100.

That is CqL—Lc=Dc.

1,	258	640	782	100	(0,4499,&c.
3	3				Cq
—	64				—Ac
+1	2				CqA
+1	136				*Subtrahend.*
R	122	640	782	100	
	4	8			—3Aq
		12			—3A
—	4	92			
+	3				Cq
	25	08			*Divisor.*
—	19	2			—3AqE
	1	92			—3AEq
		64			— Ec
—	21	184			
+	12				CqE
+	98	816			*Subtrahend.*
R	23	824	782	100	
		580	8		—3Aq
		1	32		—3A
—		582	12		
+	3				Cq
	2	417	88		*Divisor.*
—	5	227	2		—3AqE
		106	92		—3AEq
			729		—Ec
—	5	334	849		
+	27				CqE
+	21	665	151		*Subtrahend.*
R	2	159	631	100	
		60	480	3	—3Aq
			13	47	—3A
—		60	493	77	
+		3			Cq
		239	506	23	*Divisor.*
		544	322	7	—3AqE
		1	091	07	—3AEq
				729	—Ec
—		545	414	499	
+	2	7			CqE
+	2	154	585	501	*Subtrahend.*
R		5	045	599	000, &c.

In this Example,

Because the lesser Root of the Ambiguous *Equation* is sought, the Coefficients, altho reduced, hinder not. The *Analysis* shall be made by *Division*, according to *Sect.*18.*Rule* 1.

Ex-

Example 16.

1qc − 5c + 5l = 1,147152872702092.

That is Lqc − CqLc + FqqL = Gqc.

1	14715	28727	02092	(0,2437 The Subtense of 14 Degrees.
+	5			Fqq
−	5			−Cq
	32			Aqc
1	0			FqqA
+1	00032			
−	40			−CqAc
	96032			*Subtrahend.*
R	18683	28727	02092	
	8	0		5Aqq
		80		10Ac
		40		10Aq
		10		5A
	5			Fqq
+	5008	8410		
	60			−Cq3Aq
	30			−Cq3A
		5		−Cq
−	630	5		
+	4378	3410		*Divisor.*
	32	0		5AqqE
	12	80		10AcEq
	2	560		10AqEc
		2560		5AEqq
		1024		Eqc
	20			FqqE
+	20047	62624		
	240			−Cq3AqE
	480			−Cq3AEq
	32	0		−CqEc
−	2912	0		
+	17135	62624		*Subtrahend.*
R	1547	66103	02092	&c.

The farther *Divisors* and Sums to be substracted, may in like sort be gotten for the other Figures of the Root after 24.

In this Example,

The Author's Note being the same with that on the last, needs not be repeated here; nor yet his Rules for the *Genesis* and *Analysis* of the *Six Binomials*, Chap. 16. of his *Clavis*, as being more proper for *Trigonometry* than *Arithmetick*. Wherefore having now waded thus far into the Deeps of that Curious, but Mysterious Mathematician, as to untie the Knots of *Affected Equations*, it is high time to desist: for whatsoever may seem to be omitted, the Ingenious may supply, by diligent Observation, and often Practice.

The mention of Side for Root in the *Sections*, and consequently L or l, for *Latus* the Side or Root, instead of A the Supposititious Root in the *Examples*, being frequent in *Species*, needs no Memorandum; So as a Question or two being added, wherein *Side used for Root.*

wherein *Affected Equations* will arise; as well all this general Survey of *Equations*, as the whole Review of *Arithmetick*, may be shut up together.

Q. Of a Number, what it is.

1. There is a Number, whose Square abated by 16, and the first Number augmented by 8; and the Total of one multiplied by the Remainder of the other, will produce 2560: what is that Number?

Resolution.

Answ. Twelve; For the Square of 12 being 144, lessened by 16, leaves 128; this multiplied by 12 and 8, that is 20, produceth 2560.

The Work by Cossicks.

Suppose 1ℨ: Then the Square is 1ʒ, abating 16, the Remain is 1ʒ—16. And the Number increased by 8, is 1ℨ + 8. And 1ʒ — 16 × 1ℨ + 8, makes 1φ+8ʒ—16ℨ—128=2560. And by Reduction, 1φ+8ʒ—16ℨ=2688.

The Work by Species.

— B 16. C 8. D 2560. Suppose the Number A, then the Square is Aq: And from thence abating 16, leaves Aq—B; and to the Number adding 8, makes the Total A + C. Then multiplying Aq — B into A + C, there is produced Ac + AqC — BA — BC, which are equal to D.

And by Reduction, Ac + AqC — BA = D + BC.

Or set after the other Mode, Lc + BLq — CqL = Dc.

The Resolution.

1c + 08q — 0161 ==	2	688	(12 Root.
		8	B
—		16	—Cq
	1		Ac
		8	BAq
	1	8	
—		16	—CqA
+	1	64	*Subtrahend.*
R	1	048	
		3	3Aq
		3	3A
		16	B2A
		8	B
+		498	
—		16	—Cq
+		482	*Divisor.*
		6	3AqE
		12	3AEq
		8	Ec
		32	B2AE
		32	BEq
+	1	080	
—		32	—CqE
	1	048	*Subtrahend.*

Q. Of a Fat of Wine mixed with Water, what drawn out at a time.

2. Suppose out of a Fat of Wine of 360 Gallons, be drawn out a certain Number of Gallons; and as many of Water as were drawn out be put into the Fat; and the like be done the second and third Times, and at last there be found to remain in the Fat of Wine (besides the Water mixed therewith) 208⅓ Gallons: how much Wine was drawn out at each time?

Answ.

Anſw. Sixty Gallons: As may be tried by *Alligation*, and *Tripled Proportions*, in the Second Part of this 4*th* Book; and by the *Mean Proportionals* in the third Part, where a like Queſtion to this is reſolved. Reſolution.

To the Reſolution, 2 Proportionals gotten (that is 1 leſs than the Draughts) between the whole Quantity 360, and the Remainder given $208\frac{1}{3}$, and let B be 360 and D $208\frac{1}{3}$; the Proportionals in *Species* ſtand thus:

$$D . \sqrt{c}BDq . \sqrt{c}BqD . B.$$

Then ſuppoſing the Draught ſought to be A, it ſhall be that

$$A = B - \sqrt{c}BqD.$$

And by exalting (as was ſhewed in *Reduction*) the plain *Species* to an equal Power with the other in this *Equation*, B—A ſhall be cubed.

And ſo $Bc - 3BqA + 3BAq - Ac = BqD.$

Tranſlated $Ac - 3BAq + 3BqA = Bc - BqD.$

Reduced into Numbers $1c - 1080q + 388800l = 19656000$, and reſolved.

19	656	000	(60
—	108	0	—3B
+ 3	888	00	3Bq
	216		AC
23	328	00	3BqA
+23	544	00	
— 3	888	0	—3BAq
19	656	00	*Subtrahend.*
R		000	

Several of the Queſtions in this Chapter reſolved by *Equations*, fall under ſome or other of the *Rules of Proportions disjunct* or *continued*, before handled, whereby the Truth of the Operations here may be tried. But if not, the *Reſolution of the Equations*, both by *Coſſicks* and *Species*, where both are uſed, evidence the Truth of both Concluſions by their Agreement. And where *Affected Equations* are reſolved by this latter Way of Mr. *Oughtred* only, the Reſolution agreeing in all things with the Tenor of the Queſtion, is as in all other Works, a Proof ſufficient of the Truth thereof. *Proof of the Reſolution of Equations.*

Partis quartæ & Libri quarti

FINIS.

AN APPENDIX
OF THE
Properties of ſome Numbers.

Abſtract Numbers of 3 ſorts.

ALL Abſtract whole Numbers being expreſſed by 9, ſignifying Figures and the Cipher, (as in the beginning of this Treatiſe was noted) the Numbers made up thereby were divided into three Sorts, *Digits*, *Articles*, and *Mixt Numbers*, of all which ſome peculiar Properties may be obſerved; but leſt it prove tedious, Content ſhall be taken with the firſt 12.

Digits. *Whence ſo called.*

A *Digit* is always wrote with one Figure; of which there being 9, with the Cipher 0, makes up the firſt Article 10, the Number of the Fingers in Latin known by the Name of *Digiti*, from whence the 9 Figures came to be called *Digits*.

Articles.

An *Article* hath always a Cipher in the firſt Place.

Mixt.

And a *Mixt Number* hath a Digit there.

Properties of the Unit.

One, Though ſome differ about its being a Number, yet all agree it is the entire Foundation, and the Root and Meaſure of every Number, every Number meaſuring another ſo many times as there are Units therein: For every Number, whom beſides the Unit, no other Number meaſureth, is meaſurable by no other than the Unit. And as 1 is the Foundation of Number, ſo abides he firm and unalterable, he cannot break, nor will be broken by others, but remains whole. He neither multiplieth nor divideth, nor will be multiplied nor divided by himſelf; if he be ſquared, cubed, &c. he is ſtill no more than 1.

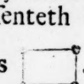
Of 2.

Two, The only even prime Number (all other even Numbers being compound) meaſureth every even Number; and is the only Integer that multiplied by or added to himſelf, makes the Total equal to the Product; for 2 and 2 is 4, and no more is twice 2. He is ſometime called the *firſt Lineary Number*, because a Line is bounded with two Points thus •——•.

Of 3.

Three, Is the firſt Simple odd Number made by *Addition* of Units, and not by *Multiplication*; and the firſt that multiplies an even Number to make the Product even, and an odd Number odd: As twice 3 is 6, and 3 times 5 is 15. It compoundeth and meaſureth every Number, whoſe ſeveral Notes taken by themſelves, and added to themſelves, are numbred from the ſame Ternary; as 39 . 54. &c. It is ſometime called the *Muſical Number*, becauſe the third Concord is the Chief in Muſick: Sometimes it is called a *Syſtatical*, or *Subſtantial Number*, becauſe all Sublunary Bodies conſiſt of the three principal Subſtances, *Sal*, *Sulphur*, and *Mercury*. Alſo it is the firſt that diſpoſed in Units, hath Beginning, Middle, and End; and in *Geometry* may repreſent the three Angles of a Triangle, thus; ▽ and ſo came to be called the *firſt Figural Number*, becauſe a Triangle is the firſt Figure in *Geometry*.

Of 4.

Four, Is the firſt even Compound Number, begotten by the Multiplication of 2 by 2, and the firſt proper Square Number, and repreſenteth the ſame, if the Units thereof be placed in oppoſition one to the other, as . It meaſureth compounded, and numbereth every Number, whoſe Figure comprehended under the two firſt Sides, it can number: As becauſe it can meaſure 16, it ſhall meaſure 69816. It is called ſometime a *Worldly*, or *Mundane Number*, becauſe the Sublunary World conſiſts of 4 Elements.

Of 5.

Five meaſureth and numbereth every Number, in whoſe firſt Place is 5 or 0; It is the ſecond Simple odd Number, and the firſt Circular Number; becauſe as a Circle turns to the Point whence it begun, ſo 5 multiplied by it ſelf, ends in 5: Wherefore whatever Equilateral figural Number hath 5 for his Root, in the firſt Place of his Quantity, will 5 be ſtill retained. Five is alſo called the *firſt Central Number*,

Number, becauſe 5 placed in the Center of a Circle, all the reſt of the Digits may be ſo diſpoſed about the Circle, that every two Oppoſites ſhall make 10, the firſt Article, and taking in the Central Number, make every way 15, thus:

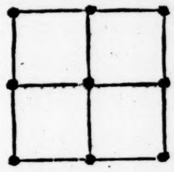
Six, Compoundeth and meaſureth every even Number, which 3 can meaſure. Of 6.
It is the ſecond even Compound and Circular Number, and the firſt perfect Number, whoſe even Parts are equal to himſelf, as $1 + 2 + 3 = 6$. Alſo Six is called the firſt *Pyramidal Number*; for the Units therein may be ſo placed, as to repreſent a Pyramis, thus:

.
. .
. . .

Seven, The old *Magi* called a *Virgin Number*, ſuppoſing the Force thereof great, as a Virgin in her full ſtrength: But this Force is diſcerned in things concrete, and dependeth not on the Quantity of 7; and ſo every 7*th* Year bringing ſome change in Nature, is Climacterical. It is ſometime called the *Sacred and Quiet*, or *Sabbatory Number*, becauſe in Sacred Writ, the Seventh Day and Year were appointed to be reſted in. Of 7.

Eight, Is the firſt proper Cubick Number, every Cube having 8 Corners. It compoundeth and numbereth every Number, whoſe Figure comprehended under the three firſt Figures it can number. It is a chief Note in Muſick, and taketh turns with 6 in the Termination of perfect Numbers, for they alternately end in 6 and 8. as 6. 28. 96. 8128. 130816. 2096128. 33550336. 536854528. &c. Of 8.

Nine, Is the ſecond Square Number, the firſt compound odd Number, and the laſt Digit. What Number ſoever it is applied to, rejecting the Nines of that whole Number taken in Groſs, or the Nines of the ſeveral Parts taken Simple, will leave the Remains alike. As 27 makes 3 times 9: Wherefore whether the Nines of 27 be rejected, or of 2 and 7, the Remain will be all one (to wit 0.) The Units in 9 regularly diſpoſed, repreſent 4 the Square of 2. And by an orderly placing in every Point and Corner one of the Digits, the whole 9 will not only be taken up, but counted up any way as they ſtand, will make 15. Of 9.



8	3	4
1	5	9
6	7	2

Scaliger called 9, the *chief and moſt perfect Number*; and *Exerc.* 365. *S.* 1. ſaith, It can be increaſed of none but the *Unit* to be made 10; and containeth in it ſelf all Species and Proportions of Quantity, as well Primary as Conſequent and Reſultant: For in it are *Length*, *Breadth*, *Depth*, *Perfect*, *Imperfect*, *Diviſible*, *Indiviſible*, *Triangle*, *Cube*, *Oblong*, *Plurilateral*, *Equality*, *Inequality*, *Abſolute*, *Comparate*, *Simple*, *Manifold*; and in *Specie* Double, *Seſquialter*, *Triple*, *Seſquitertia*, *Quadruple*, *Superpartiens*, &c.

Ten, Is the firſt Article, and by adjoining Ciphers to the right Hand, or increaſing the Unit to the left Hand, other Articles will be produced, as 10. 100. 1000, *&c.* or 20, 30, 40, *&c.* It meaſureth and compoundeth every Number, in whoſe Right-hand Place is a Cipher. It comprehendeth all the Digits. The firſt round Number, becauſe now they begin with the Digits again as in a Circle; and hence by the *Pythagoreans* called *Circular*, and counted a perfect Number. Of 10.

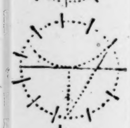
Eleven, multiplied by any Digit, beginneth and endeth alike; as 11 by 2, is 22; by 3, is 33, *&c.* Of 11.

Twelve, Is the Perimeter of a Triangle, whoſe *Area* is 6. For if 3, 4, 5, which make 12, be the Sides of a Triangle, half the Product of 3×4, which is 12, ſhall be the *Area*, as in Figural Numbers before was demonſtrated. Of 12.

In

Sir Balt. Gerbier's *Notes of the Circle.*

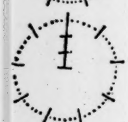
In imitation of these 12 Notes, because a Circle may be divided into such Portions, as the *Polygones* of several regular Works may be noted thereby. The Divisions of a Circumference in concordancy to 12, taken out of Sir *Baltasar Gerbier*, with a little Alteration, shall serve for a final Conclusion of all this Work.



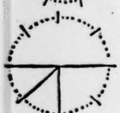
1. The Circle being a round Figure, representeth a Cipher or 0. The Center of which doth denote the *Unit*, or 1. From whence infinite Deductions are into Multitude; as from the Center infinite Lines may be drawn to the Circumference: And by cutting the whole Circle into Parts, shall be infinite Sections beneath the Whole; as from 1 broken into Pieces, ariseth infinite Fractions. And thus the Diameter terminating, the Circumference represents one half the Semicircle.

2. The Diameter drawn through the Center to the Circumference, divideth the Circle into two equal Parts.

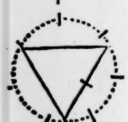
3. A Perpendicular falling from the Circumference on the Diameter in the Center, parteth the Circumference into 3 Parts, of which the lesser 2 are equal to the Third.



4. Two Diameters crossing each other at Right Angles in the Center, divide the Circle into four equal Parts, called *Quadrants*.

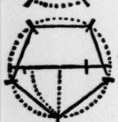


5. From the half of the Semidiameter, to the half of an Arch drawn from the Circumference to the touch of the other Semidiameter; the distance of the Touch to the first half, gives a Fifth of the Circumference, or very near it.



6. The Semidiameter applied to the Circumference, is little less than the sixth Part thereof.

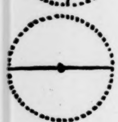
7. Half one of the Sides of the greatest Equilateral Triangle inscribed in a Circle, shall equal the seventh Part of the Circumference pretty exactly.



8. The Quadrant equally bissected, shall exactly part the Circumference into eight Parts.

9. Two thirds of the Semidiameter, parteth the Perimeter into 9 equal Divisions.

10. From the half of the Semidiameter, to the half of an Arch drawn from the Circumference to the touch of the other Semidiameter, the Distance of the Touch to the Center will give nigh the Tenth.



11. The Length of a Line from the Point where two Circles cut each other to the Semidiameter, may be taken from the eleventh Part of the Perimeter.

12. The third Part of the Quadrant is the twelfth Part of the Circle, and of the Semicircle the Sixth.

Totius Operis Finis.

Soli Deo Gloria.

AN Alphabetical Table,

Directing to some special Points noted in the *precedent Treatise*.

The TABLE.

Charges,

Gomed,

Intro-

Mixias,

N.

O.

Oitavos,

P.

Q.

Rope-

T.

Tripar-

The TABLE.

FINIS.

To the READER.

I Know no Reasons that can be assigned for suppressing the Errata : *unless to impose on a Credulous Buyer, that the Impression needs no Correction ; or that the Author (if found in an Error) may take Sanctuary under the Mistakes of the Press. But neither of these Motives, have any Prevalence with me ; as being fully assured, that the Success would prove as contemptible as the Design : When by the Discovery of some* Errata ; *the Reader would (as the natural Consequence of such an unjust Concealment) be tempted, to judge it unsafe to rely in any thing, either upon the Printer or the Author.*

I do therefore give this Advertisement ; That though the distance of my habitation from London, *permitted me not to correct the Sheets as they were printed off. Yet I have (before the Publication) carefully examined the whole Impression by the Original Manuscript left by my Father at his death. And have exactly noted all the deviations of the Press, that can possibly mislead the most unexperienced* Tyro. *So that I may perhaps incurr the censure of being unnecessarily scrupulous ; in that, together with the Errors which are more material ; I have also inserted those that the meanest* Genius *will scarce think worth the trouble of noting with his Pen ; as being only literal.*

London March *26th.*
1696.

S. Jeake.

ERRATA sic corrigas.

PAge 1. *line ult. read p. 66. p.* 2. *l.* 24. *r.* Hypsometria, *l.* 35. *r.* Hypogeiodia, *p.* 3. *l.* 23. *r.* ס. Sixty, *l.* 24. *r.* טו. Fifteen, *l.* 55. *r.* XVIII *for* XXII, *p.* 4. *l.* 14. *r.* Affections, *l.* 39. *r.* Lesse, *p.* 5. *l.* 35. *dele* the latter [into], *p.* 7. *l.* 1. *r.* Solid, & Marg. *l.* 1. *r.* 3. Solid, *p.* 8. *l.* 31. *r.* mixed either, *p.* 10. *l.* 17. *Lat. r.* Grana, *l.* 51. *r.* Æs, *p.* 11. *l.* 16. *r.* Imum, *p.* 12. *l. penult. r.* ☞ next Greater, *p.* 13. *l.* 1. *r.* .. *l.* 54. *r.* Second Book, *p.* 14. *l.* 53. *r.* Quintillion, *p.* 16. *l.* 16. *r.* —Prime Part of Composition, which is Addition, *p.* 19. *l.* 8. *r.* Cyphers, *l.* 17. *r.* leaves, *p.* 21. *l.* 2. *r.* 40, *l.* 14. *r.* preceding, *p.* 22. *l.* 15. *r.* If of two, *p.* 23. *l. penult. r.* The, *p.* 25. *l.* 28. *r.* substracted, *p.* 27. *l.* 29. *r.* Rectangles, *p.* 29. *l.* 14. Marg. *for* Geometry, *r.* Grounded on, *p.* 30. *l.* 46. *r.* Multiplee, *p.* 31. *l.* 54. *r.* figures, *p.* 32. *l.* 14. *r.* comprehended, *p.* 36. set the figures over the Dividends a little more to the Right hand, *p.* 44. *l.* 17. *r.* Fractions of Fractions, *p.* 46. *l.* 42. *for* $\frac{16}{12}$. *r.* $\frac{13}{16}$, and transpose the Comma that is there set after [16 and 12] setting it after [Denominator], *l. antepenult.* for $\frac{16}{13}$. *r.* $\frac{16}{12}$, *p.* 48. *l.* 31 In the latter Dividend, *r.* $\frac{15}{4}$ *p.* 49. *l.* 1. *for*, in, *r.* is, *and for*, is, *r.* in, *p.* 50. *l.* 25. Marg. *r.* pag. 59, *p.* 51. *l.* 12. *r.* Case, *l.* 15. *r.* $\frac{2}{3}$ & $\frac{3}{4}$ & $\frac{3}{2}$, And *r.* $\frac{8 \,\&\, 9 \,\&\, 18}{12}$. *l.* 19. at C. *for* $\frac{3}{5}$ *r.* $\frac{3}{4}$, *p.* 53. *l.* 21. *r.* Subtrahends, *p.* 59. *l. ult.* Marg. *r.* p. 50, *p.* 62. *l.* 8. *r.* considered, *l.* 16. *r.* Vide pag. 106. to 152, *p.* 63. *l.* 31. *r.* Verstegan, *p.* 66. *l.* 2. *r.* Tail, *l.* 22. *r.* if sold, *p.* 68. *l.* 40. *r.* Washers, *l. penult. r.* the Billet, *p.* 70. *l.* 13. *r.* Bushels, *p.* 73. *l.* 33. *r.* pag. 95, *p.* 74. *l.* 14. *r.* Wastel, *l.* 19. *r.* White-Bread, *l.* 35. *r.* such Horse loaves, *l.* 37. *r.* assized, *p.* 75. *l.* 43. in the last Column, Tit. Penny Houshold, *for* 2—02—$10\frac{1}{3}$ *r.* 2—02—$10\frac{2}{3}$, & *l.* 47. in the same Col. *for* 2—80—12, *r.* 2—00—12, *p.* 76. *l.* 3. *r.* upon it in one quarter, *l.* 34. Tit. Groats, Col. 6. *for* $1\frac{1}{2}$, *r.* $7\frac{1}{2}$, *l. penult. r.* or 13 *s.* 4 *d. p.* 77. *l.* 34. *r.* 1633, *l.* 40. Marg. *r.* old Coins, *p.* 78. *l.* 7. *for* —$03\frac{2}{3}\frac{3}{1}$, *r*, —$03\frac{2}{3}\frac{7}{1}$, *p.* 82. *l.* 24. *r.* Tsagad, *l.* 46. and 3 places more in the same page, *for* Estbang *r.* Etsbang, *l.* 52. *r.* Rabbins, *p.* 83. *l.* 21. *r.* Fingers, *l.* 43. *r.* Siculus, *p.* 84. *l.* 26. & also in the Marg. *r.* Lethec, *l.* 38. *r.* contain, *p.* 85. *l.* 2. *r.* Tables, *l.* 12. *r.* Pagnam, *p.* 86. *l.* 13. *for* Mæneh, *r.* Maneh. *l.* 21. *r.* Kodshah, or hakkedoshah, *p.* 88. *l.* 8. after the Table, *for* p. 84. *r.* p. 83, *p.* 89. *l.* 41. *r.* and Ponderal, *p.* 91. *l.* 1. *r.* Georgick Chœnix, *l.* 15. *r.* Chœnices, *l.* 16. *r.* Sextaries, *l.* 42. *r.* Georgick Kotyle, *l.* 52. *r.* Scapula, *l.* 57. *r.* Oxybaph, *p.* 92. *l.* 13. *r.* Salam : *l.* 16. *r.* Ponticus, *l.* 23. *r.* Laconica, *l.* 50. *r.* Sabitha, *l.* 53. *r.* Bœotick, *p.* 93. *l.* 1. & 3. *r.* Laconica, *l.* 19. *r.* Acetabule, *p.* 94. *l.* 23. *for* Table *r.* Tables, *l.* 41. *r.* Rosolus, *p.* 95 *l.* 14. *r.* i. Sitar, *l.* 28. *r.* Vitruvius, *p.* 97. *l.* 23. *r.* $4\frac{1}{2}$, *l.* 32. *r.* $1\frac{2}{3}$ ℥, *p.* 99. *l.* 4. *r.* Schoene, *l. antepenult. r.* he calls *Ulna Communis*, *p.* 100. *l.* 7. *r.* Unciæ, *l.* 13. *r.* An Inch, *l.* 29. *for* Pecks, *r.* Perches, *l. penult.* Marg. *r.* the, *p.* 101. *l.* 35. viz. *l.* 14. below the Tables, *dele* —Word were needless for, *l.* 37. *r.* diminutive, *l.* 39. *r.* word were needless for the same, *p.* 102. *l.* 30. Col. Libra's, *for* 152. *r.* 125, *p.* 103. *l. ult. dele*, it seems, *p.* 104. *l.* 29. *r.* Byzantium, *l.* 40. *r.* Ancient, *l.* 46. *r.* Valentinian's, *p.* 105. *l.* 2. *for*, the *r.* he, *l.* 40. & in the Marg. *r.* Ancient, *l.* 48. *r.* Siliqua, *p.* 106. *l.* 10. Put a Period after [Accompts], *p.* 107. *l.* 7. *r.* Nigritarum, *l.* 30. *r.* Cheshire, *p.* 108. Col. 1. *l.* 5. *r.* Guienne, *l.* 16. *r.* Turenne, *l.* 20. *r.* Borgoingne, Col. 2. *l.* 23. *r.* Embden, *l.* 57. *r.* Palatinates, *p.* 109. Col. 1. *l.* 2. *r.* Hale, *l.* 29. *r.* Switzerland, *l.* 57. *r.* Eubœa, Col. 2. *l.* 1. *r.* Campagnia, *l.* 8. *r.* Bolognia, *p.* 110. Col. 2. *l.* 27. *r.* Lootz, *l.* 32. *r.* Namur, *l.* 35. *r.* Swoll, *l.* 36. *r.* Utrecht, *p.* 111. Col. 1. *l.* 21. *r.* Baionne, *p.* 112. *l.* 4. *r.* $1\frac{1}{2}$, *l.* 26. *r.* comes to, *l.* 32. *r.* $6\frac{1}{4}$, and *for* who *r.* whom, *l.* 34. *r.* Land measures, *l.* 35. *r.* Dalica, *p.* 113. *l.* 11. dele the latter [for], *l.* 15. *r.* de Terre, *p.* 114. *l.* 32. *r.* Faneca, *p.* 115. *l.* 26. *r.* Pichy, *l.* 34. *r.* Covados, *l.* 45. *r.* Canvas, *p.* 116. *l.* 7. *r.* 10. Palmes, *l.* 57. *r.* Cannes, *p.* 118. *l.* 57. *r.* Puncheons, *p.* 119. *l.* 8. *for* Canado, *r.* Candado, *l.* 58. *r.* $7\frac{1}{7}$, *p.* 120. after *l.* 40. Insert, The Fat of Dantsick contains 180 Dantsick Stoops, or 81 Antwerp, *p.* 123. *l.* 1. *r.* 24 Muys, *p.* 124. *l. ult. for* $3\frac{1}{2}$, *r.* $3\frac{1}{3}$, *p.* 127. *l.* 22. *for* 1 ℔. *r.* 10 ℔. *l.* 56. *for* 48. *r.* 480, *l.* 59. *r.* 480 Aracoles at the rate of 80 Aracoles for, *p.* 129. *l.* 42. *for* Camporas, *r.* Campo, as, *p.* 130. *l.* 57. *r.* Spalato, *l. ult. r.* as Fez, *p.* 131. *l.* 1. *r.* Mina's, *l.* 28. *r.* Verona, *p.* 132. *l.* 4. r. Ounces, *l. penult.* dele 144, *p.* 133. *l. ult. r.* Breslaw, *p.* 134. *l.* 21. *r.* $102\frac{1}{2}$, Marks, the Kings Weight, *p.* 135. *l.* 1. *r.* 1. 14. Arragon, *l.* 31. *r.* Lyons, *l.* 37. *r.* Lyon, *p.* 137. *l.* 43. Marg. *r.* wrote, *p.* 138. *l.* 34. *r.* Sebastian, *p.* 139. *l.* 26. *r.* or Florin, *p.* 140. *l.* 13. *r.* Giuli, *p.* 142. Col. ult. *l.* 23. *for* 0 . 10 . 0, *r.* 1 . 10 . 0, *p.* 143. *l.* 11. *r.* Pancratius, *p.* 144. *l.* 12. *r.* Holland, *l.* 41. Col. *penult. for* 0 . 6 . 9, *r.* 0 . 6 . 6, *l.* 43. Col. 4. *for* 2 . $6\frac{10}{19}$, *r.* 2 . $2\frac{10}{19}$, *l.* 44. dele, others, *l.* 52. Col. 3. *for* $40\frac{1}{2}$, *r.* $49\frac{1}{2}$, *l.* 56. *r.* Royal, *l. penult.* Col. *ult. r.* 1 . $6\frac{3}{4}$, *l. ult.* Col. 4. & 5, *for* 2 . 15. & 2 . 16, *r.* 2 . 15. & 2 . $15\frac{3}{4}$, *p.* 145. *l.* 17. Col. *antepen. for* $\frac{9}{19}$, *r.* $\frac{9}{91}$, *l.* 21. *r.* as Bamberg, *p.* 146. *l.* 32. Col. 1. *r.* 60 Creutzers, *l.* 45. Col. *penult. for* 4 . 2, *r.* 4 . 9, *l.* 47. Col. 3. *for* $14\frac{1}{2}$, *r.* $14\frac{1}{4}$, *l.* 51. Col. 4. *for* 11 . $11\frac{1}{13}$, *r.* 18 . $11\frac{1}{13}$, *p.* 147. *l.* 23. Col. 4. *for* $10\frac{19}{41}$, *r.* $13\frac{19}{41}$, *l.* 27. Col. 4. *for* 9 . 6 . $13\frac{1}{11}$, *r.* 0 . 0 . $13\frac{1}{11}$, *l.* 43. *r.* Finferlin, *l.* 45. *r.* Florins, *p.* 148. *l.* 8. Col. 2. *for* 9 . 0, *r.* 6 . 0, *p.* 149. *l.* 6. Col. 1. *r.* Liure, *l.* 10. *r.* Genoa, *l.* 26. Col. 5. *for* $0\frac{1}{9}$— . *r.* $0\frac{1}{9}$. *l.* 36. *r.* Quartid'escue, *l.* 44. Col. 5. *for* 0 . $5\frac{1}{4}$, *r.* 1 . $5\frac{1}{4}$, *p.* 150. *l.* 8. Col. *ult.* for $2\frac{1}{4}$, *r.* $2\frac{1}{2}$, *l.* 9. Col. 4. *for* $\frac{14}{13}$, *r.* $\frac{14}{43}$, *l.* 35. Col. *ult. for* $2\frac{1}{4}$, *r.* $1\frac{1}{4}$, *p.* 151. *l.* 28. *r.* Montferrat, *l.* 40. *r.* de minimis non curat, *p.* 152. *l.* 6. *r.* part or parts. From *p.* 152. to 172. the Running Title to be corrected by the Titles of the several Chapters, *p.* 153. in the Example B, *for* 6. *r.* 60, *p.* 154. *l.* 10. *r.* Shillings, *p.* 156. *l.* 3. *r.* Denomination, *l.* 6. *r.* then multiply, *p.* 158. *l.* 32. *for* 334 : 12 : *r.* 434 : 12, *p.* 159. *l.* 33. *for* 6906, *r.* 6960, *p.* 160. *l. ult. r.* 51 l. 13 s. 5 d. as at H, *p.* 161. *l.* 26. under Averdupois, *for* gr. *r.* qrs, *l.* 39. *for* 236. *r.* 364, *l. antepen. for* when, *r.* with, *p.* 162. *l.* 28. *for* least *r.* last, *p.* 163. *l.* 31. for 6 s. *r.* 16 s. *p.* 164. *l.* 4. under Ton, *for* 29. *r.* 21, *l.* 21. dele, and Fractions, *l.* 22. *r.* of Integers and Fractions, *p.* 165. *l.* 27. At the end, add : And of the other, Book 1. Part 2. Chap. 5. *p.* 167. *l.* 23. *r.* both of one Nature, *l.* 32. Col. *ult. r.* 5. by $\frac{1}{2}$ or 5 half pounds—2 : 10 : 00, *l.* 33. Col. *ult. for* $8\frac{1}{0}$, *r.* $\frac{1}{80}$, *p.* 169. *l.* 35. *for* $\frac{1}{4}$ *r.* $\frac{1}{4}$ l. *p.* 170. Example E, draw a line between 12. and 626, *l.* 35. *r.* Denominations, *p.* 172. *l.* 1. *r.* Page 39, *p.* 173. *l.* 9. Marg. *r.* contract, *p.* 174. *l.* 44. *r.* Isosceles, *l.* 45. *r.*

Scalenum,

Scalenum, *p.* 175. *l.* 31. *r.* Diagonal, *l. ult. r.* $\frac{2}{3}$(3. *p.* 172. misprinted for 176, *l.* 3. *r.* Lines, *l.* 17. *r.* imply, *l.* 24. *r.* Cylinders, *l.* 26. *r.* $\frac{1}{16}$p, *r.* (the Flame, *l.* 32. (& elsewhere in *p.* 188, 189, &c.) *r.* Parallelipipedon, *l.* 47. *r.* a Cube) is, *l.* 51. *r.* Biquadrate, *l.* 55. *r.* Square, *p.* 177. *l.* 19. *dele* the Period, *l.* 26. *r.* as is plain, *p.* 179. *l.* 26. *r.* Ludolph, *p.* 182. *l.* 31. *for* 1, *r.* 12, *p.* 188. mark the first Figure with K, *p.* 194. *l.* 33. *r.* this Quotient, *p.* 198. *l.* 43. over the first Quotient *for* z *r.* 2, *p.* 200. *l.* 36. *r.* 796594176, *p.* 203. in the Third Example, under Zenz. *for* 1299 *r.* 1296, *p.* 205. *l. ult.* save 3, *for* yet *r.* get, *p.* 208. *l.* 23, 26, 29. *for* Coma *r.* Comma, *p.* 212. *l.* 39. set 982 over 1,00 in the Dividend, *p.* 217. *l.* 27. *r.* Compounds —0, 125 + 0,125, *p.* 227. *l.* 7. in the middlemost Examp. *for* 832 &c. *r.* 823 &c. *l. penult. dele* and. *p.* 229. *l.* 37. *r.* Sacro, *l.* 38. *r.* Peragration, *p.* 232. *l.* 41. *r.* Geodætical Reduction, *p.* 233. *l.* 37. *r.* ingenious, *l.* 39. *for* 245″. *r.* 945″. *p.* 241. *l.* 20. Marg. *r.* Luna, *p.* 242. *l. ult.* save 3, *dele* the Comma, and set it 3 words after. In the Sexagenary Table, Col. 39. at Top, & 29. at Side, in the Common Angle, *for* $^{19}{}_{51}$ *r.* $^{18}{}_{51}$, Col. 36. at Top, and 36. at Side, in Com. Ang. *for* $^{2}{}_{36}$ *r.* $^{21}{}_{36}$, Col. 52. at Top, and 39. at Side, in Com. Ang. *for* $^{32}{}_{48}$ *r.* $^{33}{}_{48}$, *p.* 247. *l.* 19. *r.* 06′, 0°, 20′. *p.* 251. *l.* 20. *r.* Comma, *p.* 252. Log. of 46. *for* 583 &c. *r.* 783 &c. *p.* 253. Log. of 227. *for* 485 &c. *r.* 585 &c. *p.* 254. Log. of 502. *r.* 2,70070, Log. of 404. *for* 3, *r.* 2, Log. of 469. *for* 1, *r.* 2, For Num. 573. Col. 2. *r.* 473, *p.* 255. Log. of 749. *r.* 2,87448, *p.* 265. *l.* 38. in the Quotient of the Log. of 15,0′. *dele* the Sign —, *l. ult.* in the Quotient of the Log. of 6,0. *dele* the Sign —, *p.* 267. *l.* 30. Tit. Cube, *r.* 00129, *p.* 269. *l.* 1. *r.* Ocular, *p.* 272. Beside the Characters there, set z. *for* Square, and ss. *for* Sursolide, *p.* 274. *l.* 5. *r.* Numbers, *p.* 276. *l.* 34. Examp. F, *for* 14. φ. *r.* 13. φ. *p.* 279. *l.* 1. *r.* 2.3. *p.* 285. in Example I. *for* 2 Zz. *r.* 6 Zz. *p.* 287. *l.* 6. *for* 8 N. *r.* 4 N. *p.* 304. *l.* 3. *r.* Homogeneity or Heterogeneity, *p.* 308. *l.* 24. *for* 28 + 316. *r.* 28 + √√ 316. *p.* 310. Col. 1. *l.* 8. *for* √√ 52. *r.* √√ 72. *l.* 34. Col. 2. insert before the Comma [in the other], *p.* 311. *l.* 15. *r.* *p.* 311. *p.* 315. Col. 1. insert [Remain] in the last Example, *p.* 320. *l.* 15. *for* —15028. *r.* —√√ 15028. *p.* 330. *l.* 9. *dele* the Comma's, and set a Comma after [Square] and *r.* and the Product, *p.* 334. *l.* 13. *r.* is in working, *l.* 30. *r.* Quesititious, *p.* 337. *l.* 36. *for* 4 — 2. *r.* 4 = 2. *p.* 344. *l.* 27. *for* F D. *r.* E D. *p.* 350. *l. ult.* save 5, *for* B 27. *r.* 9 B 27. *p.* 351. *l. ult.* right under $\frac{3}{4}$ A. *for* 1. *r.* 2. *p.* 356. *l.* 22. *for* without *r.* from, §. 2, 3, 4, &c. *l.* 36, &c. Names *i. e.* Parts, *p.* 357. *l.* 1. *for* and *r.* which, *l.* 5. *dele* mean, *l.* 38. *for* each of, *r.* it in, *l.* 42. upper Order, *i. e.* higher Rank, *l.* 44, 59. unequal *i. e.* odd, *l.* 45. equal *i. e.* even, *l.* 46. *r.* is made, *l.* 47. *dele* is, *p.* 358. *l.* 13. *r.* Concinnity, *p.* 359. *l.* 34. & 38. *dele* the Comma after [Quotient], *p.* 360. *l.* 3. wants a tittle under the last Figure of the Cube dividend, *p.* 361. *l.* 25. *r.* Examples, *p.* 363. *l.* 47. *r.* produces, *p.* 371. *l.* 1. *r.* 169 — 144. *p.* 375. *l.* 31. *for* 4Zq4. *r.* 4Zqq. *p.* 410. *l.* 26. *r.* Geodæticals, *p.* 411. *l.* 21. *for* $\frac{1}{7}\cdot\frac{1}{9}$ *r.* $\frac{1}{9}\cdot\frac{1}{7}$, *l.* 23. *for*, to the Third, *r.* to a Third, *p.* 412. *l.* 25. *r.* $\frac{2}{3}\cdot\frac{2}{1}$ *p.* 416. *l.* 32. *for* Compounds, *r.* Compound, *p.* 419. *l. antepen. r.* compared to, *p.* 425. *l.* 29. *r.* Backward, *p.* 428. *l.* 10. *for* $\frac{5}{6}$. *r.* $\frac{5}{8}$. *p.* 432. Exam. 2. *l. ult.* of Case 2. *r.* 85 : 3 : 4. *p.* 435. *l.* 6. *for* 35 l. *r.* 35 lb. *p.* 446. *l.* 24. *for* $\frac{207}{48}$. *r.* $\frac{207}{84}$. *p.* 447. *l.* 21. *for* $2\frac{1}{3}$. *r.* $2\frac{2}{3}$. *p.* 449. *l.* 33, 34. *r.* Supposititious, *p.* 453. *l.* 28. in the multipl. by 2, *for* 8. *r.* 84. *p.* 455. *l.* 36. in the Quot. *r.* 20 Crowns, *l.* 38. Divid. *for* 024. *r.* 240. *p.* 459. *l.* 18. *r.* abbreviate, *p.* 469. *l.* 2. *for* $\frac{1}{1}$. *r.* $\frac{1}{17}$. *l.* 10. *dele* the line between 1680. and 840. *l.* 21. *for* $\frac{1}{2}$. *r.* $\frac{1}{2}$. *l.* 27. *r.* orderly by, *p.* 472. *l.* 37. *for* three Sexes, *r.* several Sexes, *p.* 477. Col. *ult. l.* 6. *for* 33. *r.* 13. *l.* 7. *for* 23. *r.* 33. *p.* 478. *l.* 8. *for* 38. *r.* 14. *p.* 482. *l.* 44. *r.* added, *p.* 486. *l.* 11. *r.* 6566(40. *l.* 16. *dele* the line between 29184. and 3648. *p.* 503. *l.* 40. *for* $2\frac{1}{1}$. *r.* $2\frac{1}{2}$. *p.* 525. *l.* 3. & Marg. *r.* Isoperimeters, *l.* 8. *r.* Isoperimetral, *l.* 45. *r.* Spheroide, *p.* 531. *l.* 20. *for* of *r.* or, *p.* 534. *l.* 32. *r.* := T. *p.* 541. *l.* 30. *r.* allotted, *p.* 551. *l.* 2. *r.* he hath, *p.* 553. *l.* 31. *r.* Numbers, *p.* 560. *l.* 21. place a Semicolon after [Integers], *l.* 28. place the Comma before [is], *p.* 566. *l.* 23. *r.* in the Third, *p.* 567. *l.* 40. *r.* till 3. *l.* 42. *dele* and, *p.* 568. *l.* 37. place a Comma after [sought] and *dele* the latter [self], *l.* 43. *r.* Quotients, *p.* 572. *l.* 16. *for* ⊏ *r.* ⊏ *p.* 573. *l.* 16. place I. over $\frac{3}{4}$. R. over $1\frac{1}{2}$ &c. *for* P. 577, 578. *r.* 578, 579. *p.* 578. *for* 579. *l.* 22. *r.* Pence in a, *p.* 585. *l.* 43. *r.* thereto, *p.* 591. *l.* 34. *r.* take, *p.* 595. *l.* 8. *dele* the negative Sign —. *p.* 613, &c. *for* Equations, *r.* Æquations, *p.* 613. *l.* 23. *r.* Andreas, *p.* 622. *l.* 11. *for* + 3. *r.* + 23. *p.* 630. *l.* 8. *r.* Ports, *p.* 631. *l.* 44. *r.* C to be, *p.* 637. *l.* 11. *r.* that is, *l.* 48. *for* Sign, *r.* Sign —, *p.* 643. *l.* 19. *for* Number *r.* Magnitude given. *for* the Rectangle *r.* a Rectangle, *l.* 20. *for* of *r.* under, *l.* 23. 25. manifest, shewn, *i. e.* expounded, *l.* 25. *for* denied *r.* negatively, *or* being negative. *for* affirmed *r.* affirmatively, *p.* 644. *l.* 10. *r.* 5 F. *l.* 25. *r.* 2. Let any Æquation be propounded : As this, &c. *l.* 31. *r.* Of the Root L. to be found out of these, there are 2. parts, *l.* 33. *r.* equal, *l.* 39. *r.* Coëfficient, *l.* 49. *for* squared *r.* the Square, *l.* 50. *dele* of, *l.* 51. *dele* the first [of], *p.* 645. *l.* 2. *dele* then, *l.* 21. *r.* in the propounded Æquation, *l.* 33. *dele* Figure of the, *p.* 646. *l.* 36. *for* also *r.* although, *p.* 647. *l.* 19. *for* his own *r.* that same, *l.* 32. *for* may be *r.* be much, *l.* 46. *r.* used mixtly, *l. penult.* and *p.* 648. *l.* 15, 16. *for* may *r.* are to, *l.* 26. Marg. *for* of *r.* by, *l.* 30. *dele* sought, *l.* 36. *r.* 4879. *p.* 649. *l.* 2. *dele* sought. After *l.* 23. Add, Therefore the true Root falls between 34 and 36. *l.* 31. *for* to be *r.* be, *l.* 39. *for* they lessen *r.* lessen it, *p.* 650. *l.* 21. *r.* Sign of R. *l.* 29. *for* Or *r.* And, *p.* 652. Ex. 3. *l.* 2. place a tittle over 7 q. *l.* 35. *dele* it, *p.* 654. *l. ult.* save 3, place the Comma after [multiplied], *p.* 658. *l. ult.* save 3, Col. 2. *r.* and the Coeff : *l. antepen. dele* the Comma, and set it in the Period's place, *p.* 663. *l.* 13. *r.* Sabbatary, *p.* 664. The Diagrams are set the wrong side upwards, *l.* 11. *dele* the Comma, and insert it *l.* 12. after [half], *l.* 38. *for* from *r.* for.

THE Pointings of the Remains in Mr. Oughtred's *16.* Examples of Affected Æquations *(in the Lines marked* R*) are omitted here. If any be desirous to point them, they may have recourse to his* Clavis, Edit. 3. Oxon. 1652. *which I esteem the best.*

The Prefaces and Contents at the Beginning, and Table at the End of the Book; as also the preceding accompt of the Errata, have all been examined and corrected by me at the Press: And I do not know of any mistake in the Printing them.

S. J.